# Papers

# Customer value modelling: Synthesis and extension proposals

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**Abstract** Customer lifetime value is a key concept in relationship marketing. This paper develops, synthesises and organises formulae for computing this lifetime value in a progressive manner and continues a research direction started some years ago. Calculations are organised using a double taxonomy: customer relationship behaviour models (retention and migration models) and algebraic and matrix computing methods. The formulae suggested allow for the explicit representation of recency-censored customer migration processes, the flexible integration of desynchronised financial flows and extend customer value optimisation procedures from the retention to the migration model.

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## INTRODUCTION

The development of 'customer oriented' interactive database marketing brings to the fore, in most contexts, the need for customer value and customer portfolio assessment methods. These methods have either been fine-tuned in direct marketing and catalogue sales or developed in sales force effort allocation models. There are many ways in which they could be improved but they require systematisation and unification first.

From a customer attraction and retention perspective, a profitable customer is a customer whose income, generated during the commercial relationship, exceeds with an acceptable amount costs supported to attract, satisfy and keep him. This amount is called customer lifetime value (LTV).

Customer value calculations help enterprises solve various fundamental problems like budgeting customer acquisition expenses, selection of recruiting media (the LTV is generally different according to the media used), or of types of offer or distributing efforts between prospecting and preserving customers. Well-conducted LTV analysis can also help build a competitive strategic advantage.[1]

The modelling of customer value depends on the context and on the customer's relationship behaviour (whether the situation is contractual or not). A brief analysis shows that

 

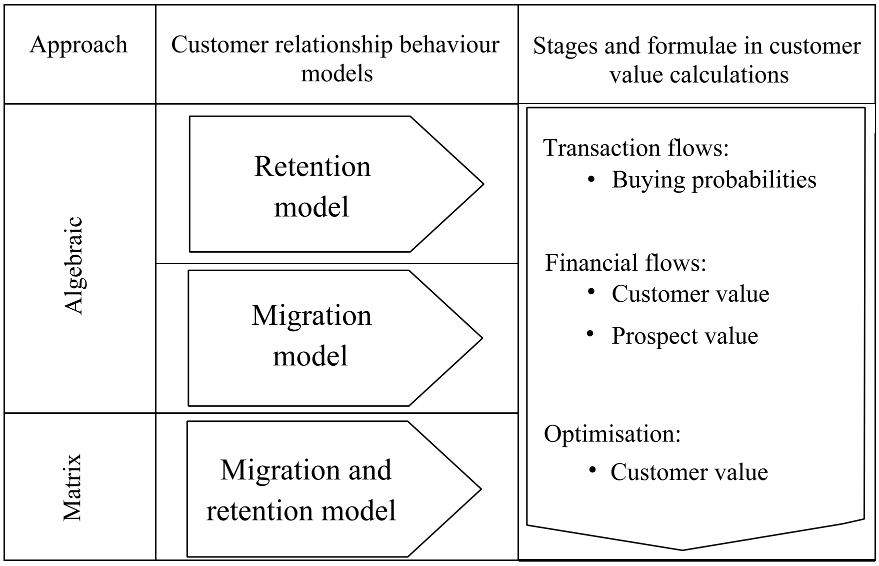


**Figure 1** Modelling customer value, a research framework

modelling efforts are based on the distinction between customer retention and migration.

This study of existing models shows also that, in a first set of contributions, the mathematical developments are algebraic and focus on retention models while, in another set, they are based on matrix approaches and applied mainly to the migration model.

The need to desynchronise financial flows (expenses and gains) in the customer relationship, is also put forward by these contributions.

Based upon results emerging from a broad examination of these models, this paper guides towards systematisation and unification. It finds elements that are common and identifies those that are not. It adds components that seem absent from the other approaches. In this way, a set of formulae that complete existing solutions is developed and adapted.

These, existing or new, formulae are grouped in a progressive customer value calculation framework. It is presented schematically in Figure 1 and in detail in Appendix 1.

The algebraic approach and the matrix approach are treated apart. Formulations progressively integrate transactional flows expressing probabilities to pass orders, financial flows composed of gains and expenses and customer and prospect value optimisation procedures based upon long-term calculations.

The use of certain representation artefacts (migration trees and diagrams, transition probability matrixes) facilitates the development of some recursive algebraic formulae for LTV calculations.

Several additional aspects concerning dynamics of customer migration, like right and left censored migration processes or mechanics of purging the customer list, are treated explicitly.

## EXISTING MODELS AND NATURE OF MODELLING APPROACHES

### Retention, migration and mathematical approaches of customer value

Dwyer[2] presents examples of LTV calculations drawn from the two major contexts that were distinguished by

Jackson[3] in the industrial buying environment: buyers of type 'always a share' and buyers of type 'lost for good'.

On this occasion Dwyer[4] reveals the more general character of this typology, extends it to the consumer market, and shows that the 'always a share' behaviour, that he associates with the 'migration model', is generally representative for catalogue buying; while the 'lost for good' behaviour, that he associates with the 'retention model', is representative for financial services, press subscriptions, etc.

Berger and Nasr[5] use this distinction between customer retention and migration and present mathematical LTV calculation formulae for four situations implying retention models and for one context requiring a migration model.

Blattberg and Deighton[6] build a model that computes 'customer equity' and finds the optimal balance between customer acquisition (investments to convert prospects into customers) and customer retention (investments to convince active customers to continue to buy) expenses. This model uses an infinite time horizon (long term) and LTV formulas are therefore simple, easy to use and avoid all summing operations that limited time horizon (life time) models suppose. The concept of customer equity is further developed in Blattberg, Getz and Thomas.[7]

Building upon these works of Berger and Nasr[8] and of Blattberg and Deighton,[9] Pfeifer and Carraway[10] apply Markov chains to customer relationship modelling and LTV calculations. They insist on the flexibility of models based on Markov chains and show that all situations modelled by the previous authors can be solved with Markov chain models. While the previous contributions are limited to recency-based customer value models, these authors illustrate the link between Markov chains and the RFM (recency, frequency and monetary) modelling framework that dominates industry practice in direct marketing and catalogue sales. Some developments that are more specific to these sectors can also be found in Birtran and Mondschein.[11,12]

A comparative analysis of these contributions brings several points to the forefront, which underline the need of a systematisation and unification effort:

— Berger and Nasr insist on the desynchronisation of financial flows (expenses and gains) during a customer relationship. They adopt an algebraic approach that they apply essentially to the retention model and neglect the migration model to a certain extent
— Blattberg and Deighton's developments are also algebraic. They are limited to the retention model, insist on customer value optimisation in a long-term perspective and focus on prospects
— the matrix solutions of Pfeifer and Carraway are applied primarily to the migration model. They adapt some long-term actual value calculations from Blattberg and Deighton to the matrix approach, but the optimisation procedures adopted by them are different (specific to Markov decisions processes).

## The nature and mechanics of the customer's individual response

The two types of temporal behaviour of customers in a relationship with a company (that have been named differently according to authors: 'lost for good'/'always has share' by Jackson, 'retention model'/'migration model' by Dwyer or 'contractual'/'non-contractual' by Reinartz and Kumar),[13] display different migration patterns.

The *retention model* considers that a

person or a company remains a customer as long as they generate transactions. This means that if at some given moment customers do not renew their contracts or do not generate any transaction they can be considered as 'lost for good' or as 'ex-customers'. It also means that if an ex-customer buys again they are considered as a new customer and one deals with an acquisition rather than a customer retention issue. The evaluation of customer potential in a relation of this kind will only take into account the customer's probability to remain active from one period to another or what Blattberg and *et al*. call the customer's survival probability. The customer lifetime corresponds to the number of successive periods during which the customer is and remains active.

The *migration model* considers that customers can reappear (turn up again) after some periods during which they did not make transactions and traces their probability of 'reactivating'. The evaluation of the customer's potential in such a relationship relies on the joint probabilities of remaining and becoming again an active customer after a fixed number of periods or, in other words, on the 'survival' and 'reactivation' probabilities. The lifetime of the customer corresponds to the number of successive periods during which the customer, according to the company's estimations, either remains active or reactivates. For a customer to preserve this status after several periods of inactivity their probability of reactivating, as estimated by the company, must be superior to the prospects' response probability. Alternatively, the customer is ticked off the customer database and they become an ex-customer or a 'purge' if one adopts a certain terminology used by the direct marketing profession.

### Distinction between transaction and financial flows

In order to encourage a progressive assessment of customers' potential and to facilitate the resolution of problems with increasing complexity it is convenient to distinguish transactional and financial flows that were generated by customers during the number of periods considered. The first will help estimate the number of transactions incurred by a customer during the relationship with the company and the others will facilitate profitability calculations for the customer relationship.

The status of a customer is given by an initial transaction. In order to assess transactional flows it is appropriate to count the transactions generated during several periods or business cycles by a number of initial transactions $t_0$ or customers. Let us note $t_j$ the number of transactions generated by $t_0$ customers or initial transactions after $j$ periods. The customer's response probability after $j$ periods is given by the number of transactions generated after $j$ periods divided by the number of initial transactions or $t_j$ when $t_0 = 1$. The transaction potential of a customer will then be given by the expected number of transactions generated during the lifetime of a customer. This corresponds at each period to the sum of probabilities to generate a transaction:

$$T_j = \sum_{k=0}^{j} t_k.$$

## THE RETENTION MODEL

The 'lost for good' type of behaviour implies that the customer remains active until the moment they leave the company. Simplified, this means that at each business cycle those who remain customers are going to generate a transaction.

**Table 1:** The magazine subscriptions case (growing retention rate)

| Periods from acquisition | Retention rate | Survival rate (II 2) | Expected number of active customers (n*3) | Profit per customer and period (m − c) ($) | Discounted profit per customer and period (5/(1 + d)^1) ($) | Total discounted profit per period (4*6) ($) |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | 1 | 1 | 1000.000 | 12 | 12 | 12000 |
| 1 | 0.71 | 0.708 | 707.701 | 12 | 10 | 7077 |
| 2 | 0.79 | 0.561 | 560.602 | 12 | 8 | 4485 |
| 3 | 0.83 | 0.463 | 463.325 | 12 | 7 | 3243 |
| 4 | 0.84 | 0.389 | 389.395 | 12 | 6 | 2336 |
| Lifetime | Transactions | 3.121 | 3121.000 | | Value | 29141 |

## General case (retention probability variable with time)

The management of subscribers to a consumer magazine represents this situation well and raises a generic problem of customer retention and attrition. The customer potential and the customer value come from individuals with whom the relation persists. In an example drawn from the consumer magazine subscription business, Dwyer presents a situation in which the probability to remain a customer by renewing the subscription increases slightly with time and becomes stable after some periods.

### Numeric example

Dwyer gives a numeric example that is repeated by many of the other authors mentioned before. This example is summarised and adapted in order to reveal essential mechanisms of the retention model and to allow for evaluation of similar situations in other industries by decision calculus. It deals with a company in the magazine subscription business which evaluates the transactional potential and the value of its customers for an estimated lifetime of five periods. The margin by retained customer is \$40 of which \$20 are income from subscriptions and \$20 are advertising revenues. The marketing costs by retained customer are \$28 and the discount rate is 20 per cent.

### Formalisation of calculations

The survival probability at each period $j$ is

$$\prod_{k=0}^{j} p_k$$

the product of probabilities to remain a customer during all those periods. It indicates the expected number of transactions per customer on each period ($t_j$).

The cumulated expected number of transactions per customer during the analysed periods is the sum of survival probabilities:

$$T_j = \sum_{k=0}^{j}\left[\prod_{l=0}^{k} p_l\right].$$

In order to facilitate customer value calculation the notion of discounted transactions[14] is introduced. The expected and discounted number of transactions during a customer lifetime is given by the discounted sum of the customer's survival probabilities:

$$T_j^a = \sum_{k=0}^{j}\left[\prod_{l=0}^{k} \boldsymbol{p_l}/(1+\boldsymbol{a})^k\right]$$

where $a$ is the discount rate and $j$ is the lifetime of a customer expressed number

**Table 2:** The magazine subscriptions case (constant retention rate)

| Periods from acquisition | Retention rate | Survival rate (2..) | Expected number of active customers (n*3) | Profit per customer and period (m – c) ($) | Discounted profit per customer and period (5/(1 + d)^1) ($) | Total discounted profit per period (4*6) ($) |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | 1 | 1 | 1000 | 12 | 12 | 12000 |
| 1 | 0.8 | 0.8 | 800 | 12 | 10 | 8000 |
| 2 | 0.8 | 0.64 | 640 | 12 | 8 | 5120 |
| 3 | 0.8 | 0.512 | 512 | 12 | 7 | 3584 |
| 4 | 0.8 | 0.41 | 409.6 | 12 | 6 | 2458 |
| Lifetime | | Transactions | 3362 | | Value | 31162 |
| Long-term | | | 5000 | | | 36000 |

of periods. In the example, the expected number of transactions during the lifetime of a customer is 3.12 (see Table 1) and its discounted version is 2.43. The expected value of the customers ($V_j$) is the profit of a transaction multiplied by the discounted expected number of transactions:

$$V_j = (m - c)\sum_{k=0}^{j}\left[\prod_{l=0}^{k} p_l/(1+a)^k\right].$$

## The case where retention probabilities remain constant

In mature market situations one can suppose that for a constant marketing effort purchase probabilities remain constant from one period to another. Under these circumstances formulae are simplified and make it easier to estimate long-term evolutions. As a result survival probabilities can be expressed as powers of the constant retention probability and no more as a product of variable probabilities.

### Numeric example

Dwyer's example adapted to a situation with constant retention probability leads to the following calculations (see Table 2).

### Formalisation of calculations

The survival probability after $j$ periods is $\boldsymbol{p}^j$.

The sum of survival probabilities gives the transaction potential of a customer

$$\sum_{k=0}^{j} \boldsymbol{p}^k$$

which is here

$$\sum_{k=0}^{j} 0.8^k = 3.36.$$

The discounted transaction potential is then

$$\sum_{k=0}^{j} [\boldsymbol{p}/(1+\boldsymbol{a})]^k$$

that is

$$\sum_{k=0}^{4} [0.8/(+0.2)]^k = 2.6.$$

The net present value of a customer results from the profit of a transaction multiplied by the discounted transaction potential:

$$(\boldsymbol{m} - \boldsymbol{c})\sum_{k=0}^{j} [\boldsymbol{p}/(1+\boldsymbol{a})]^k$$

that is (\$40 − \$28) 2.6 = \$31.2

### 'Long-term' customer value

When purchase probabilities remain constant from one period to the other, it is convenient to compute the long-term present value of a customer. The long term indicates a hypothetical situation where the customer's lifetime would be endless. Computing the long-term transaction potential and customer value is not of great interest in itself. It helps optimise marketing efforts: avoiding spending too much in customer retention programmes with no budget constraints or finding the optimum retention budget when budget constraints exist.

It can be shown that the long-term transaction potential

$$\lim_{n \to \infty} \sum_{j=0}^{n} \boldsymbol{p}^j \text{ is } 1/(1-p).$$

In the long term, the discounted transaction potential will then be

$$\lim_{n \to \infty} \sum_{j=0}^{n} [\boldsymbol{p}/(1+\boldsymbol{a})]^j = (1+a)/(1+a-p).$$

For example, if the (constant) retention probability is 0.8 then the long-term transaction potential of a customer is $1/(1-0.8) = 5$. This means that if the lifetime of a customer lasts forever that customer would generate five transactions. The discounted transaction potential is $(1+0.2)/(1+0.2-0.8) = 3$ and the customer's present value will be $(m-c)(1+a)/(1+a-p)$ or $(\$40-\$28)*3 = \$36$. If one remembers that the discounted transaction potential of a customer was 2.6 transactions and the customer's present value was \$31.2, it becomes clear that these long-term measures must be seen as limits towards which tend the cumulated transactions and the customer value.

These limits offer a fast computing instrument that avoids fastidious summations and produces some indicative values of the transaction potential and of the customer.

### Optimisation of the long-term present customer value

Up to this point the calculations have used given retention rates. It is yet justified to consider that the company can influence retention rates through marketing actions oriented towards its customers. This supposes to express the customer retention probability as a function of the marketing effort (budget) directed towards the customers during each period. The retention probability expressed as a function of the retention budget increases with a growth rate that decreases progressively towards the end when the retention probability reaches its ceiling (see Figure 2). A simple exponential function like

$$p_r = \text{ceiling}*(1-\exp(b*R))$$

can be estimated by decision calculus using subjective estimations from managers.

The optimum retention budget for a customer $R$ can be found by maximising the present long-term customer value, that is

$$\max(m-R/p_r)\,(1+a)/(1+a-p_r)$$

where $p_r = f(R)$.

In the example with a \$23 retention budget $R$ by customer the company achieves a 0.8 retention rate, this corresponds to \$28 by retained customer and gives a long-term present value of the customer of $(40-28)*(1+0.2)/(1+0.2-0.8) = \$36$. If the response to the retention budget is expressed by the function $p_r = 0.81(1-\exp(-0.16*R))$, one can see that this result is not an optimum one. In such a situation, where the retention rate is limited upwards to 0.81 and marks an accelerated growth (coefficient of elasticity 0.16) with an effort of only \$7 per customer a retention rate of 0.785 is obtained. This

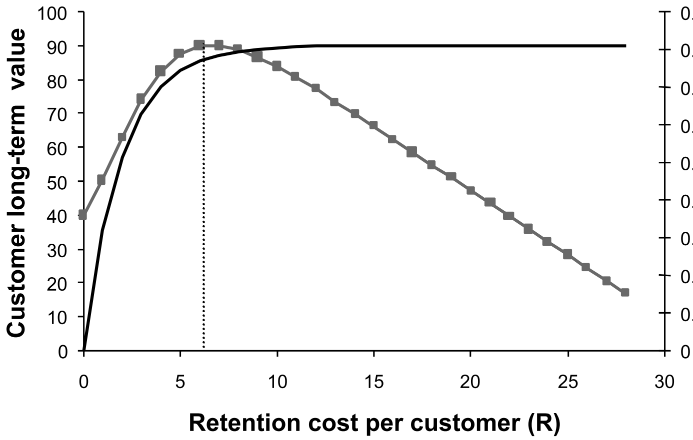


**Figure 2** Maximisation of the long-term customer value (without the acquisition stage)

results in \$8.9 per retained customer. The long-term value becomes then $(40 - 8.9)*(1 + 0.2)/(1 + 0.2 - 0.785) =$ \$90 which is optimum.

### Finding the balance between customer acquisition and retention

In order to have the complete picture of customer relationship profitability, one cannot ignore the customer acquisition stage that transforms prospects into customers. Blattberg and Deighton[15] call the measure of customer profitability 'customer equity'. By putting the problem in a prospect's[16] perspective they suggest a model that makes it possible to optimise customer acquisition and retention.

$$CE_j = p_a\left[m - c_a + (m - c_r)\sum_{k=1}^{j}[p_r/(1 + a)]^k\right] \text{and} \lim_{n \to \infty} CE_j = p_a[m - c_a + (m - c_r)*p_r/(1 + a - p_r)]$$

Following the reasoning previously applied to retention costs, customer acquisition costs $c_a$ can be expressed as expenses by prospect divided by the acquisition probability $p_a$, which itself is a function of these expenses.

Insuring customer relationships profitability reduces to successive optimisation of the customer acquisition and retention value.

$$\text{Max}[p_a m - A + p_a(mp_r - R)/(1 + a - p_r)]$$

where $p_a = f(A)$ $p$ and $p_r = f(R)$. The optimisation procedure, that has been adapted from Blattberg and Deighton, finds first prospect expenses that maximise the acquisition value $(p_a m - A)$ of a customer and then uses the resulting acquisition rate $(p_a')$ in order to compute the retention value $(p_a'(m\ p_r - R)/(1 + a - p_r))$. The maximum customer value is the sum of the optimum acquisition and retention values calculated in this way. Due to its simplicity and conciseness this model is well adapted to decision calculus.

In order to set the Blattberg and Deighton problem in the dual perspective of customer acquisition and retention, it is first necessary to express the acquisition probability as a function of acquisition expenses by prospect, for

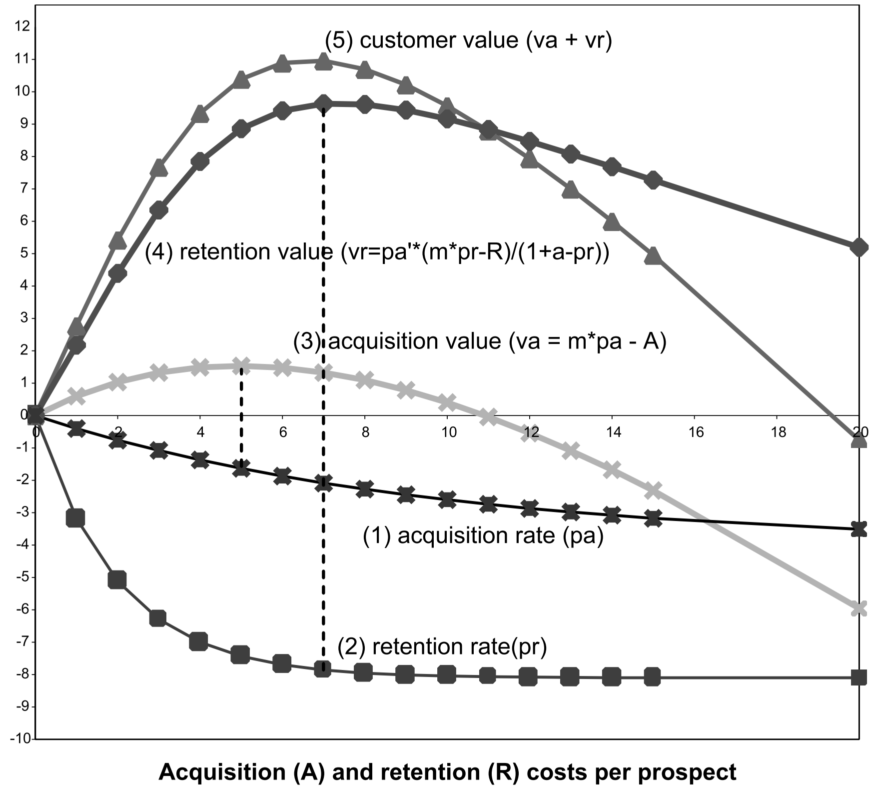


**Figure 3** Maximisation of the customer long-term value (with the acquisition stage)

example $p_a = 0.4(1 - \exp(-0.1*\mathrm{A}))$. The ceiling acquisition rate (probability) and its elasticity should be lower than their corresponding parameters in the retention rate function. The optimised values can be found graphically (see Figure 3).

With a \$5 budget per prospect a 0.16 response probability (acquisition rate) is reached. It gives the best acquisition value for a prospect 0.16*40 − 5 = \$1.53 and can be read on the negative scale in Figure 2. This acquisition rate enters the calculation of the retention value which becomes maximum when retention expenses by customer are \$7 and result in a retention rate of 0.78. In this way, the maximum retention value that can be achieved per prospect is \$9.62. The total value of a prospect is the sum of their acquisition and retention values \$1.53 + \$9.62 = \$11.16.* The customer value can be easily derived here by dividing the prospect value by the acquisition rate (probability) \$11.16/0.16 = \$68.32. The result is coherent with the \$90 customer value obtained when acquisition costs were ignored.

### Desynchronisation of the financial flows in the retention model

In their survey of retention models Berger and Nasr[17] pay special attention to the distinction between the moments when input (gains) and output (expenses) flows intervene. In the previous examples it was considered that gains (m) and promotional expenses (c) intervened at the same moment during a business cycle where from the formula

$$V_j = (m - c) \sum_{k=0}^{j} [p/(1 + a)]^k.$$

**Table 3:** Cases that illustrate commercial cycle particularities and the desynchronisation between the financial input (gains) and output (expenses) flows in different industries

| Case | Insurance (1) | Health club (2a) | Car leasing(2b) |
|---|---|---|---|
| Gains ($m$) | 260 | 125 | 7000 |
| Number of gain cycles per year ($nm$) | 1 | 2 | 0.33 |
| Expenses ($c$) | 50 | 25 | 95 |
| Number of expense cycles per year ($nc$) | 1 | 2 | 1 |
| Retention probability ($p$) | 0.75 | 0.8 | 0.3 |
| Number of years ($j$) | 10 | 4 | 12 |
| Discount rate ($a$) | 0.2 | 0.2 | 0.2 |
| Customer value ($V$) | 569 | 355 | 8273 |

A more attentive analysis of the alternation of gains and expenses of a company during the life of a customer, indicates the following timing. If acquisition costs that occur in the previous cycle are ignored, the customer brings gains at the beginning of the cycle and the company makes its promotional expenses at the middle of the cycle in order to assure repeat purchase. Cycle lengths can be equal, lower or superior to one year. There can be different cycles for gains and for expenses.

The following formula generalises and regroups several formulas suggested by Berger and Nasr,[18] in order to deal with various situations:

$$V_j = m \sum_{k=0}^{j^*nm} [(p)^k/(1+a)^{k/cm}] - c \sum_{k=1}^{j^*nm} [(p)^{(k-1)^*nm/nc}/(1+a)^{(k-0,5)/nc}]$$

where *cm* and *cc* are the number of cycles per year for gains and for expenses.

Berger and Nasr[19] give detailed calculations for the three cases summarised here and for two other cases, presenting situations where profits per retained customer are not constant but variable with time (Case 3) and situations where financial flows are not discrete but continuous (Case 4).

## THE MIGRATION MODEL

The 'always a share' type of behaviour supposes that the customer does not have to be active during all the analysed period and that the customer can reactivate with a probability that increases with recency of the last transaction.[20] In order to get a better understanding and use of retention/reactivation mechanisms and of stochastic processes that characterise this kind of behaviour, several artefacts like decision trees, migration trees, diagrams and matrixes of transition probabilities are used. They facilitate the development or adaptation of a series of algebraic and matrix formulae.

### Algebraic approach

With time, customers have purchase patterns that alternate active and inactive periods. A prolonged inactivity can lead to elimination from the customer list. The progressive shaping of purchase profiles is seen in Figure 4.

By regrouping customer purchase patterns at each stage according to recency criteria, by first separating the active customers from those that are not active and by segmenting afterwards the inactive ones according to the recency of their last purchase a customer migration tree is obtained. It is a key representation

| Period 1 decision | Probability of buying decision | Period 2 decision | Probability of buying decision | Period 3 decision | Probability of buying decision | Period 4 decision | Probability of buying decision | Buying profile | Final probability of path | Number of purchases | Profile present value |
|---|---|---|---|---|---|---|---|---|---|---|---|
| buy | 0.3 | buy | 0.3 | buy | 0.3 | buy | 0.3 | 1111 | 0.008 | 4 | 11.953 |
| | | | | | | no buy | 0.7 | 1110 | 0.019 | 3 | 11.776 |
| | | | | no buy | 0.7 | buy | 0.182 | 1101 | 0.011 | 3 | 11.303 |
| | | | | | | no buy | 0.818 | 1100 | 0.052 | 2 | 11.005 |
| | | no buy | 0.7 | buy | 0.182 | buy | 0.3 | 1011 | 0.011 | 3 | 9.412 |
| | | | | | | no buy | 0.7 | 1010 | 0.027 | 2 | 9.161 |
| | | | | no buy | 0.818 | buy | 0.11 | 1001 | 0.019 | 2 | 8.348 |
| | | | | | | no buy | 0.89 | 1000 | 0.153 | 1 | 7.724 |
| no buy | 0.7 | buy | 0.182 | buy | 0.3 | buy | 0.3 | 0111 | 0.011 | 3 | 1.846 |
| | | | | | | no buy | 0.7 | 0110 | 0.027 | 2 | 1.595 |
| | | | | no buy | 0.7 | buy | 0.182 | 0101 | 0.016 | 2 | 0.926 |
| | | | | | | no buy | 0.818 | 0100 | 0.073 | 1 | 0.504 |
| | | no buy | 0.818 | buy | 0.11 | buy | 0.3 | 0011 | 0.019 | 2 | –2.278 |
| | | | | | | no buy | 0.7 | 0010 | 0.044 | 1 | –2.693 |
| | | | | no buy | 0.89 | buy | 0.067 | 0001 | 0.034 | 1 | –4.511 |
| | | | | | | no buy | 0.993 | 0000 | 0.475 | 0 | –6.02 |

**Figure 4** Stochastic decision tree and purchase patterns

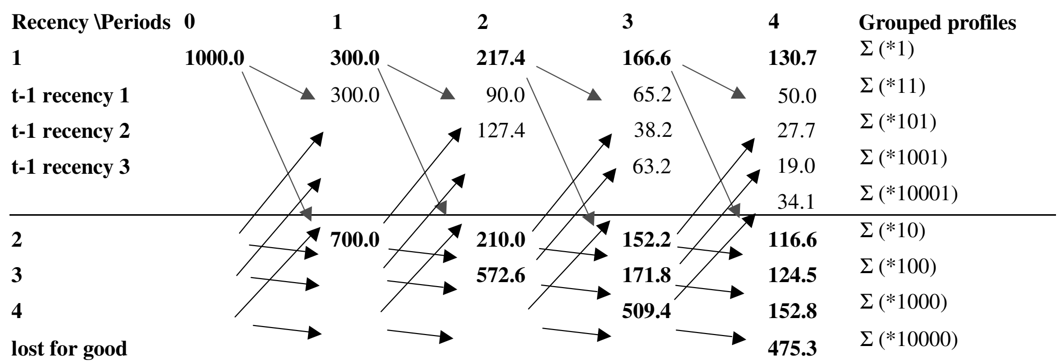


**Figure 5** Customer migration tree

form for understanding transition mechanisms in this model.

The migration tree shows particularly well (see Figure 5) the recurrent character of the transition process to which the customer is submitted. This facilitates the development of powerful and flexible algebraic formulae for computing the transactional and financial flows associated to the migration model.

The expected number of transactions generated by a customer after $j$ periods is the sum of reactivation probabilities of transactions initiated by the same customer in the previous periods. It is a recurrent expression that has as a starting point the first transaction, the one that turns a prospect into a customer:

$$t_j = \sum_{k=1}^{\min(j,R)} \left[ t_{j-k} p_k \prod_{l=1}^{k-1} (1 - p_l) \right]$$

where $t_0 = 1$.

This formulation[21] is more general and more concise than the formulas given by Berger and Nasr[22] and Dwyer.[23] It integrates a recency limit, R, that enables right censoring of the migration process and makes this expression and the other ones that have been derived therefrom comparable to the matrix formulations suggested by Pfeifer and Caraway.[24]

Censorship according to recency is largely used by the industry. When a segment or a customer crosses this recency limit, the customer's response probability becomes lower or equal to the prospect's. Consequently, there are no more reasons to consider that individual as a customer and by convention the customer's reactivation probability ($P_R$) is fixed to zero.

Table 4 illustrates these aspects. Calculations are inspired from Dwyer's example, which has been slightly modified in order to facilitate estimating the reduction of response probability with recency by decision calculus.[25]

A company is forecasting the behaviour of 1,000 customers, knowing their purchase probability according to the recency of the last purchase. The marketing margin $m$ is \$40 by transaction and promotion expenses are \$4 by customer. For simplicity these values are kept constant.

The Table shows that with a migration model the retention probability is generally low and that companies rely less on their survival probability of a customer than on the customer's reactivation probability in order to boost sales and to increase the customer value.

The calculation of the purchase probability in column 5 uses the above-mentioned formula in order to illustrate this recurrent retention and reactivation process: $t_1 = [1*0.3] = 0.300$. $t_2 = [t_1*0.3 + t_0*(1 - 0.3)*0.182] = 0.217$ etc. This means that an initial transaction $t_0 = 1$ generates 0.300 transactions after one period and 0.217 transactions after two periods. In the second period the 0.217 transactions come from the two evoked sources: $t_1*p_1 = 0.9$ expresses the retention or the survival and $t_0*(1 - p_1)*p_2 = 1*(1 - 0.3)*0.182 = 0.127$ expresses reactivation of customers that were inactive during the first period. In the fifth period the whole process rests essentially on the reactivation process, as the survival probability diminishes (0.002); at the same time intervenes the censorship of customers who passed the recency limit ($R$) because they have not reactivated for five periods (their reactivation probability becomes too small).

The notation $t_j$ expresses a customer migration process that starts with a number of manifest customers, that is customers that have just ordered or in other words customers for which the recency of the last order is one. The starting number of customers or the number of generating transactions can be noted $t_{r,j}$. In a customer database large numbers of latent customers or temporarily inactive customers are kept whose recency of the last order is greater than one. The transaction potential of these customers needs to be explicitly considered in certain situation and justifies extending lifetime value calculations to customer recency greater than one.

$$t_{r,j} = \sum_{k=1}^{\min(j,R-r+1)} \left[ t_{j-k} p_{r+k-1} \prod_{l=1}^{r+k-2} (1 - p_l) \right]$$
$$\text{where } t_{1,j} = t_j,\ t_{r,0} = 0$$

In this formula transactional flows with recency greater than one ($t_{r,j}$, $r > 1$) are expressed as a function of recency one transactional flows. Notations $t_j$ and $t_{1,j}$ are equivalent. It is a generalisation of the previous formula that accepts left censored migration processes.

A customer with recency two, for example, will have a zero purchase probability at the beginning (column 5 in Table 4), in period two probability to reactivate is 0.18, in period three the purchase probability of 0.145, will be composed of the survival (remain active)

**Table 4:** The catalogue sales case

| Periods from acquisition | Response probability and recency | Survival probability (2..) | Reactivation probability (5-3) | Purchase probability | Number of remaining customers (*n*-7) | Number of purged customers | Expected number of active customers (5**n*) | Profit per customer and period (*m**5-c(1-7) ($) | Discounted profit per customer and period (9/1 + d)^1) ($) | Total discounted profit per period (6*10) ($) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 10 | 1 | 1 | 0 | 1 | 1000 | 0 | 1000 | 36 | 36 | 36000 |
| 1 | 0.3 | 0.3 | 0 | 0.3 | 1000 | 0 | 300 | 8 | 6.667 | 6667 |
| 2 | 0.182 | 0.09 | 0.127 | 0.217 | 1000 | 0 | 217 | 4.69 | 3.26 | 3260 |
| 3 | 0.11 | 0.027 | 0.14 | 0.167 | 1000 | 0 | 167 | 2.66 | 1.542 | 1542 |
| 4 | 0.067 | 0.008 | 0.123 | 0.131 | 524.67 | 475.33 | 131 | 3.13 | 1.51 | 792 |
| Lifetime | 1.659 | 1.425 | 0.39 | 1.815 | 4525 | 457.33 | 1815 | 54.5 | 48.98 | 48979 |

 

probability of $0.055 = 0.18*0.3$ and the probability to reactivate with recency three $0.09 = 0.92*0.11$. After five periods (0 to 4) the purchase probability or the expected number of generated transactions $t_{2,4}$ becomes 0.062 which is naturally less than $t_{1,4} = 0.131$ for a customer whose migration process started with the active state (recency 1). The total number of generated transactions $T_{2,4}$ during the five periods becomes 0.5 as compared to $T_{1,4} = 1.815$. Thus, a customer of recency two can still bring a transaction out of two during the five following periods, while an active customer can bring two transactions.

This algebraic formulation of transactional flows in a migration model is equivalent with the matrix formulas suggested by Pfeifer and Carraway and makes a direct link to the matrix approach presented in the next section.

## Financial flows and customer value calculations

From the number of transactions incurred during each period following the launching of a relationship marketing programme, it is possible to calculate the customer's profitability for the estimated duration of the relationship that is the lifetime value. The separation of input (gains) and output (promotion expenses) financial flows in the migration model is needed not only in order to mark the desynchronisation between the two flows (central topic in Berger and Nasr's work) but also in order to reflect the different transactional basis that applies to the two flows.

By definition, the migration model considers as a customer every individual having ordered already and not having yet exceeded the pre-established recency limit. This implies that all customers defined in this way are eligible for promotional activities conducted by the company during several periods even if they were not active in the period that precedes these actions. The marketing margin comes only from the active customers while promotional costs are incurred by default with all eligible customers and not only with the active ones as seems to be suggested by the formula developed by Berger and Nasr.[26] By eliminating the desynchronisation aspect in the timing of gains and expenses, their formula can be reduced to

$$V_{1,j} = \sum_{k=0}^{j} \{(m - c)t_{1,k}/(1 + a)^k\}.$$

It reflects a very particular case where promotion costs apply only to the active customers, which is a reduction, compared to the calculations presented by Dwyer, which the authors have followed.

For reasons of clarity the discount rate is set to zero. The formula becomes then

$$(m - c)\sum_{k=0}^{j} t_{1,k}$$

or again $(m - c)T_{r,k}$. The customer value that results ($40 − $4)*1.815 = $65.34, is largely greater than the value in the general case that is presented in Table 4. As might be observed, the Berger and Nasr formulation is limited to a very specific situation. Rare are the industries that can count on the spontaneous reactivation of customers without having to make the marketing efforts to stimulate this process.

Companies spend money in order to maintain the retention and reactivation process described by the migration tree. The simplified formula becomes then

$$\sum_{k=0}^{j} (mt_k - c) \text{ or } mT_j - c_j.$$

By applying it to the data given in Table 4, the customer value after five periods becomes $40*1.815 − $4*5 = $52.4.

Censorship is needed for economic reasons because, if one continues to engage in promotional actions towards all members of a cohort, costs by active customer increase up to a point where the margin, even if it remains constant, is no longer enough to cover these costs (see column 9).

With censorship included, the customer value formula is slightly modified to become

$$\sum_{k=0}^{j}(mt_k - c(1-q_k))] \text{ or } mT_k - cR + \sum_{k+R}^{j}[c(1-q_k)],$$

because when the period $j$ arrives to the recency limit $R = 4$ customers that did not reactivate are purged (excluded). The customer value becomes \$40*1.815 − 4\$*4 − \$4*(1 − 0.475) = \$54.5.

The customer value formula suggested here allows for residual value calculations for customers that are inactive at the assessment time (left bound censorship), takes into account the promotional costs that are maintained by default during the lifetime of a customer and integrates customer elimination.

$$V_{r,j} = \sum_{k=0}^{j}\{[mt_{r,k} - c(1-Q_k)]/(1+a)^k\}, \quad \text{where } Q_k = \sum_{l=0}^{k} q_k$$

and

$$q_k = \begin{cases} t_{1,k-R}\prod_{l=1,k\geq R}^{R}(1-p_l),\ r=1 \\ \text{or, else} \\ \prod_{l=1,k=R-r+1}^{R}(1-p_l) + t_{r,k-R}\prod_{l=1,k\geq R+1}^{R}(1-p_l), r>1 \end{cases}$$

where: $V_{r,j}$ = value of a customer with recency r; $t_{r,j}$ = expected transactions generated by a customer of recency $r$ in period $j$; $m$ = net margin on a transaction; $c$ = mailing cost; $p_l$ = response probability of a segment of recency $l$; $R$ = recency limit beyond which a customer is ticked off the database; $a$ = discount rate; $q_k$ = cumulated probability for a customer to be eliminated (purged) after $k$ periods.

This calculation cumulates the present gains produced by transactions that were generated during the projection period and systematically deducts mailing costs. It supposes that mailings are sent to customers only if they are maintained in the database, which is a standard marketing policy in a migration model.

The formula also integrates the modelling of the elimination process for customers who exceed the recency limit. Elimination intervenes when there is a succession of non-response (no-purchase) periods equal to the recency limit.

For an active customer (recency 1) the probability to be eliminated intervenes after $R$ periods and corresponds at each period $k$ to the probability to generate a transaction after $k - R + 1$ periods multiplied with the probability of the non-response succession. In the example in Table 4, an active customer can be eliminated starting with period 4 (the recency limit), with a probability of 0.475, that is the product of the probability to generate a transaction $t_{1,4-4} = 1$ and the probability of the no-answer succession $(1 - 0.3)(1 - 0.18)(1 - 0.11)(1 - 0.06) = 0.475$.

For an inactive customer ($r > 1$) the probability to be eliminated intervenes in two stages. The first stage intervenes after $R - r + 1$ periods. For a customer of recency 3 the first elimination intervenes after two periods $(4 - 3 + 1)$. The second stage begins after $R + 1$ periods (here $4 + 1 = 5$). The first stage eliminates customers who do not reactivate any more with a probability that corresponds to the non-response succession between the recency of the customer and the recency limit. A customer of recency 3 has a probability to be eliminated at the first stage of $(1 - 0.11)(1 - 0.06) = 0.83$. The second stage begins with the reactivation of customers from a cohort with given recency and continues up to the elimination of all customers from this cohort.

The probability to be eliminated at each period $k$ is the product of the probability to generate a transaction after $k - R + 1$ periods that multiplies the probability of the non-response succession. A customer of recency 3 will have in period 5 a probability to be eliminated of $0.0524 = 0.11*0.475$. It is the product of the probability to generate a transaction $t_{3,5-4} = 0.11$ and of the probability of the non-response succession 0.475.

### Matrix approach

The matrix approach suggested by Pfeifer and Carraway[27] suits both retention and migration models. While for the retention models, the algebraic formulations are simple and straightforward enough, for the migration model the matrix approach represents a flexible and elegant alternative to the algebraic formulae.

The transition probabilities matrix P in Table 5a summarises the migration process. It expresses transition probabilities, between two periods, from a given recency state either towards recency one (buying customer) or towards a recency state incremented by one (customer who does not buy). According to Markov Chain theory the same matrix $P$ to the power 4 (Table 5b) indicates transition probabilities after four periods. If after one period transition possibilities from a given recency state are limited to two states: recency one and the given recency incremented by one, after two periods possible transitions include three states and after four periods all the five states become accessible. The sum of the $P$ matrixes to powers incremented from 0 to 4 (Table 5c) contains the cumulated probabilities during five periods. It should be noticed that the starting period is considered equal to zero and that matrix $P$ to the power zero corresponds to the identity matrix. It can be shown that in the long term the sum of the probability matrixes $P$ at incremented powers tends towards the inverse matrix of the difference between the identity matrix and $P$ (Table 5d).

In order to obtain a strict equivalence with the algebraic formulae presented before and summarised in Table 6, the matrixes in Table 5 are multiplied with the vector noted $\boldsymbol{1}_1$, that has the following shape [1, 0. 0…]. In this way the expected number of transactions vector is extracted. It consists of first column elements of the transition probabilities matrix and of its transformations.

The vector $\mathbf{t}_1$ expresses the expected number of transactions that have been generated after one period by customers of different recency while the vector $\mathbf{t}_4$ expresses transactions generated after 4 periods. In the given example a customer of recency two can generate 0.182 transactions after one period and 0.117 after four periods. The transaction potential after four periods is contained in the vector $\mathbf{T}_4$ and the long-term transaction potential is contained in the vector $\mathbf{T}_n$. In the given example a customer of recency two can generate 0.507 transactions during the four following periods and 0.674 transactions in the long term. The sum of transition probabilities during several periods can

**Table 5:** Transition probabilities matrix and its transformations

**a) After one period**

| | r1 | r2 | r3 | r4 | r5 |
|---|---|---|---|---|---|
| r1 | 0.3 | 0.7 | 0 | 0 | 0 |
| r2 | 0.182 | 0 | 0.818 | 0 | 0 |
| r3 | 0.11 | 0 | 0 | 0.89 | 0 |
| r4 | 0.067 | 0 | 0 | 0 | 0.933 |
| r5 | 0 | 0 | 0 | 0 | 1 |

$\boldsymbol{P}$

**b) After 4 periods**

| | r1 | r2 | r3 | r4 | r5 |
|---|---|---|---|---|---|
| r1 | 0.131 | 0.117 | 0.124 | 0.153 | 0.475 |
| r2 | 0.065 | 0.081 | 0.083 | 0.093 | 0.679 |
| r3 | 0.031 | 0.029 | 0.053 | 0.056 | 0.83 |
| r4 | 0.011 | 0.01 | 0.011 | 0.034 | 0.933 |
| r5 | 0 | 0 | 0 | 0 | 1 |

$\mathbf{P}^4$

**c) Cumulated: periods 0 to 4**

| | r1 | r2 | r3 | r4 | r5 |
|---|---|---|---|---|---|
| r1 | 1.815 | 1.179 | 0.869 | 0.662 | 0.475 |
| r2 | 0.507 | 1.31 | 1.005 | 0.82 | 1.358 |
| r3 | 0.276 | 0.171 | 1.116 | 0.946 | 2.49 |
| r4 | 0.113 | 0.071 | 0.05 | 1.034 | 3.732 |
| r5 | 0 | 0 | 0 | 0 | 5 |

$\sum_{k=0}^{4} \boldsymbol{P}^k$

**d) Cumulated: long-term**

| | r1 | r2 | r3 | r4 |
|---|---|---|---|---|
| r1 | 2.103 | 1.472 | 1.204 | 1.072 |
| r2 | 0.675 | 1.472 | 1.204 | 1.072 |
| r3 | 0.357 | 0.250 | 1.204 | 1.072 |
| r4 | 0.141 | 0.099 | 0.081 | 1.072 |

$(\mathbf{I} - \mathbf{P})^{-1}$

**Table 6:** The vector of the expected number of transactions in the transition probabilities matrix and its derivatives

**a) After one period**

| | $\boldsymbol{t}_{1,1}$ | $\boldsymbol{t}_{2,1}$ | $\boldsymbol{t}_{3,1}$ | $\boldsymbol{t}_{4,1}$ |
|---|---|---|---|---|
| r1 | 0.300 | 0.182 | 0.110 | 0.067 |

$\boldsymbol{t}_1 = \boldsymbol{P}\mathbf{1}_1$

**b) After 4 periods**

| | $\boldsymbol{t}_{1,4}$ | $\boldsymbol{t}_{2,4}$ | $\boldsymbol{t}_{3,4}$ | $\boldsymbol{t}_{4,4}$ |
|---|---|---|---|---|
| r1 | 0.131 | 0.117 | 0.124 | 0.153 |

$\boldsymbol{t}_4 = \boldsymbol{P}^4\mathbf{1}_1$

**c) Cumulated: periods 0 to 4**

| | $\boldsymbol{T}_{1,4}$ | $\boldsymbol{T}_{2,4}$ | $\boldsymbol{T}_{3,4}$ | $\boldsymbol{T}_{4,4}$ |
|---|---|---|---|---|
| r1 | 1.815 | 0.507 | 0.276 | 0.113 |

$\boldsymbol{T}_4 = \sum_{k=0}^{4} \boldsymbol{P}^k\mathbf{1}_1$

**d) Cumulated: long-term**

| | $\boldsymbol{T}_{1,n}$ | $\boldsymbol{T}_{2,n}$ | $\boldsymbol{T}_{3,n}$ | $\boldsymbol{T}_{4,n}$ |
|---|---|---|---|---|
| r1 | 2.103 | 0.674 | 0.357 | 0.141 |

$\lim_{n \to \infty} \boldsymbol{T}_n = (\boldsymbol{I} - \boldsymbol{P})^{-1}\mathbf{1}_1$

be interpreted also as the expected number of periods spent in a state before transiting to the following state, the last state being the absorbing state. The line total of the long-term sum of probabilities matrix is the expected absorption time for the state represented by this line. In the example a customer in recency state one has an expected absorption time in the 'ex-customer' state of 5.85 periods.

The value for the anticipated lifetime of a customer can be easily calculated by multiplying the discounted sum of probability matrixes with the rewards vector $R$. By considering gains and promotional expenses constant, the vector $R1$ looks like this: $[m - c, - c, - c, \ldots]$, meaning that customers of recency one bring gains in addition to promotional costs because they buy, while for customers of superior recency there are only costs.

$$\mathbf{V}_j = \sum_{k=0}^{j} [\mathbf{P}^k/(1+a)^k]\mathbf{R}$$

$$\downarrow$$

| $\mathbf{V}_{1,4}$ | $\mathbf{V}_{2,4}$ | $\mathbf{V}_{3,4}$ | $\mathbf{V}_{4,4}$ |
|---|---|---|---|
| 48.974 | 2.524 | −0.714 | −1.350 |

By applying the customer value calculation formula to Dwyer's example using a five periods time horizon ($k = 0$ to 4) that seems to be this industry's estimated customer lifetime, a customer of recency one has a value of $48.97, a customer of recency two has a value of $2.52 and customers with a recency superior to two have negative values. The long-term calculations confirm these results.

$$\lim_{j \to \infty} \mathbf{v}_j = [\mathbf{I} - \mathbf{P}/(1+a)]^{-1}\mathbf{R}$$

$$\downarrow$$

| $\mathbf{V}_{1,4}$ | $\mathbf{V}_{2,4}$ | $\mathbf{V}_{3,4}$ | $\mathbf{V}_{4,4}$ |
|---|---|---|---|
| 50.765 | 3.556 | −0.211 | −1.166 |

## Optimisation of the customer value

By applying the logic used with retention models it is possible to represent the purchase probability with recency as a function of the marketing effort. To reduce complexity, let us suppose that by controlling the number of mailings sent to customers it is possible to modify the ceiling purchase probability leaving the other parameters unaltered.

In the example the function of purchase probability with recency was 0.3 exp(−0.5*recency − 1), where 0.3 is the ceiling (pp) the function reaches when the recency is equal to one and the mailing costs per customer are $4. If one spends more for mailings this ceiling can grow to a limit of 0.5 and if one spends less it can fall to 0. The ceiling purchase probability expressed as a customer cost function is pp = 0.3 (1 − exp( − 0.5*C)). When the budget is $4 the ceiling is 0.3 as in Dwyer's example, when the budget is $1 the ceiling becomes 0.1 and when the budget increases to $10 the ceiling approaches the limit of 0.5. In these circumstances, finding the optimum cost per customer consists of maximising the long-term present value of a recency one customer:

$$\max V_1 = [\mathbf{I} - \mathbf{P}/(1+a)]^{-1}\mathbf{R}\ \mathbf{1}_1$$

where

$$R' = [(m - C/\text{pp}), -C/\text{pp}, -C/\text{pp}, -C/\text{pp}, 0]$$

and

$$p_{11} = \text{pp} = f(C),$$

and

$$P = f(\text{pp}, \text{recency}).$$

The marketing expenses $c$ are $C$/pp; pp is a function of expenses per customer $C$ and the purchase probability depends on the ceiling pp and the recency.

By applying this optimisation procedure to the given example it is optimum to spend $3 per customer and not $4. The graphical solution is presented in Figure 6. Details of the calculations are given in Appendix 2.

It is easy to extend this reasoning to the acquisition of new customers and to find an optimum balance between customer acquisition and retention budgets. In this way the Blattberg and Deighton[28] optimisation procedure is adapted to customer migration situations.

This logic seeks the best trade-off between long-term efforts and gains. For the migration model, other logics with shorter temporal horizons can also be envisaged. Pfeifer and Carraway[29] present optimisation procedures extracted from the rich literature on Markov decision

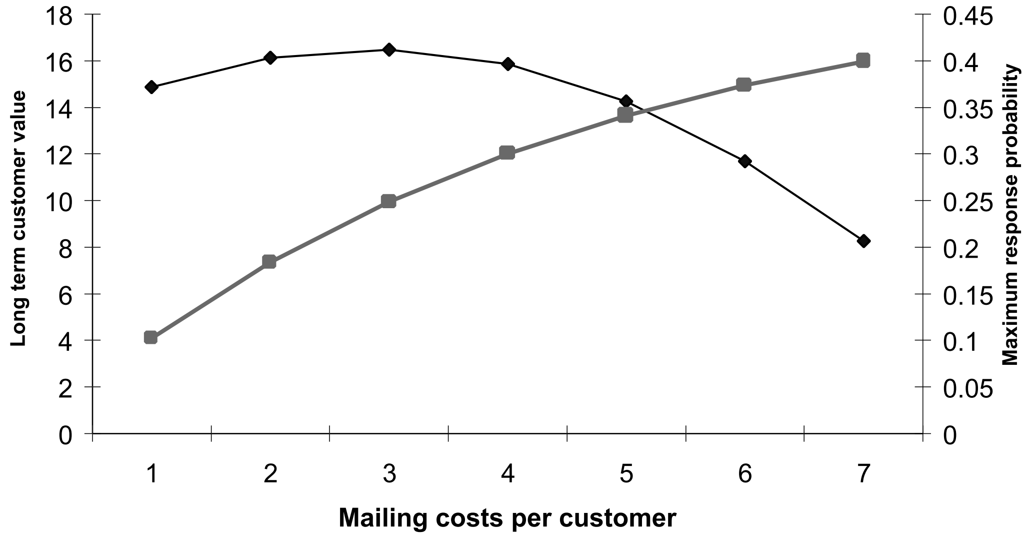


**Figure 6:** Maximisation of the long-term customer value for the migration model

processes. Birtran and Mondschein[30] formalise the direct marketing problem (that integrates a particular migration model) as a dynamic programming problem and develop optimisation heuristics adapted to this operations research problem. These approaches use RFM stratification and vary coverage and intensity of mailing campaigns in order to maximise the value of customers.

It can be shown that without budgetary constraints the optimal policy is to send mailings to all customer segments with positive lifetime value. As for monotonously decreasing response probabilities the customer value is also decreasing, when margins and mailing costs are constant, a breakeven response rate can be calculated for which the lifetime value is equal to zero. Using this property and the breakeven response rate, it is possible to calculate a profitability index from the response rate of each layer/segment.

## CONCLUSIONS

The formulae for computing the economic value of a customer or the customer lifetime value that have been developed in this paper are organised in systematic and progressive way according to a double taxonomy: the one of customer relationship behaviour models and the one of calculation methods. The retention/migration model dichotomy is based on strong theoretical foundations in consumer behaviour that have been underlined by the previous studies.

Besides the evoked behavioural characteristics, the comparative analysis of the two categories of customer relationship models reveals their economic substrata. In a migration model the retention probability is usually relatively low and companies rely less on the customer's survival probability than on the customer's reactivation probability in order to increase sales and customer value.

The two calculation methods, algebraic and matrix based, are stepwise and progressively applied to both relationship behaviour models. On that occasion several stages are distinguished

— a physical, quantitative level of transaction flows that are stochastically treated

— a monetary value level of financial flows, the temporal effects of which are studied
— a decisions level, the one of optimisation calculations.

In this way calculations become easier to compare and their formalisation in order to solve problems of increasing complexity becomes easier.

The algebraic formulae developed for the migration model represent an alternative to the matrix formulations. They make it possible to approach in an explicit manner the desynchronisation of financial flows and the censorship of migration processes on recency. Calculations to estimate eliminations from the customer list formalise right censorship in a migration process; computing the probabilities to generate transactions for inactive customers formalises left censorship. These aspects are only implicitly treated in the matrix approach of Pfeifer and Carraway[31] and are absent from the algebraic approach suggested by Berger and Nasr.[32]

The optimisation procedure developed for retention models as well as the Blattberg and Deighton procedure that optimally balances marketing efforts between customer acquisition and retention have also been adapted to the migration model using the properties of long-term transition matrixes.

As shown by Mülhern[33] customer LTV models cannot be applied to all situations. They require customers to have some persistent relations with enterprises and that financial flows (gains and expenses) can be forecast with a certain precision at an individual level. When these conditions are not satisfied, the historic analysis of profitability can replace LTV calculations. Yet, under the combined and interdependent impulses from information technology progress and from the adoption of a customer-centred marketing, situations for which models of LTV are applicable become dominant. This incites pursuing the customer evaluation and dynamic management models' systematisation and unification efforts.

## Note

* Please note that where formulae contain currency values, these have been rounded to two decimal places.

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14 As suggested by Pfeifer and Carraway (2000) *op. cit.* in their matrix solutions.
15 Blattberg and Deighton (1996) *op. cit.*
16 The Blattberg and Deighton model applies only to situations where acquisition of a new customer

remains profitable, which is a rather special situation. In many industries, like direct marketing and mail order, acquiring new customers induces a cost that must be covered by income from existing customers.

17 Berger and Nasr (1998) *op. cit*.

18 *Ibid*.

19 *Ibid*.

20 As for recency a reverse notation is conventionally used, that is, the last business cycle has recency one, the last-but-one cycle has recency two and so on. We can say that there is a negative correlation between the recency and the customer response rate.

21 For a better understanding of the way algebraic formulae concerning the migration model have been derived, visual support and live examples can be found at http://claree.univ-lille1.fr/cocoon/slides_ltv_uk/slides.xml

22 Berger and Nasr (1998) *op. cit*., equation 10, p. 25.

23 Dwyer (1997) *op. cit*., p. 12.

24 Pfeifer and Carraway (2000) *op. cit*.

25 The relation between the buying probability and the recency of the last transaction is formalised as the following exponential function $p = \min + (\max - \min)*\exp(-b*\text{recency})$, if recency $>= 1$. Recency zero means active customer thus buying probability $= 1$.

26 Formula (11) of the customer value Berger and Nasr (1998) *op. cit*. p. 26. (see equation below.) $GC$ = Gross marketing contribution per customer, $M$ = promotional costs per customer, $Ci$ = no. customers in period $i$, $d$ = discount rate, $CLV$ = Customer Lifetime Value.

27 Pfeifer and Carraway (2000) *op. cit*.

28 Blattberg and Deighton (1996) *op. cit*.

29 Pfeifer and Carraway (2000) *op. cit*.

30 Birtran and Mondschein (1996) *op. cit*.

31 Pfeifer and Carraway (2000) *op. cit*.

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$$CLV = \left\{GC^*\left[C_0 + \sum_{i=1}^{n}\left[\sum_{j=1}^{i}\left[C_{i-j}^*P_{i-j}^*\prod_{k=1}^{j}(1-P_{t-j+k})\right]\Big/(1+d)^i\right]\right]\right\}$$
$$-\left\{M^*\left[[C_0/(1+d)^{0,5}] + \sum_{i=1}^{n}\left[\sum_{j=1}^{j}\left[C_{i-j}^*P_{t-j}^*\prod_{k=1}^{j}(1-P_{t-j+k})\right]\Big/(1+d)^{j+0,5}\right]\right]\right\}/C_0$$

(see Ref. 26 above).

 

## Appendix 1: Formulae for customer lifetime value calculations

| Approaches | | Algebraic formulae | Matrix formulae |
|---|---|---|---|
| Steps\models | Retention models (1) | Migration models (2) | Valid for both models (3) |
| | | **Buying probabilities varying with time** | |
| 1 Transactions at period $j$ | $t_j = \prod_{k=0}^{j} p_k$ | | $\mathbf{t_j} = \prod_{k=0}^{j} \mathbf{P}_k, P_0 = I$ |
| 2 Value after $j$ periods | $V_j = (m-c)\sum_{k=0}^{j}\left[\prod_{l=0}^{k} p_l/(1+a)^k\right]$ | | $\mathbf{V_j} = \sum_{k=0}^{j}\left[\prod_{l=0}^{k} \mathbf{P}_l/(1+a)^k\right] R$ |
| | | **Buying probabilities constant with time** | |
| | | Transaction flows: Buying probability | |
| 3 Flows at period $j$ | $t_j = p^j$ | $t_j = \sum_{k=1}^{\min(j,R)}\left[t_{j-k} p_k \prod_{l=1}^{k-1}(1-p_l)\right], t_0 = 1$ | $\mathbf{t_j} = \mathbf{P}^j \mathbf{1}_1$ |
| | | $t_{r,j} = \sum_{k=1}^{\min(j,R-r+1)}\left[t_{j-k} p_{r+k-1} \prod_{l=1}^{r+k-2}(1-p_l)\right]$ ou $t_{1,j} = t_j, t_{r,0} = 0$ | |
| 4 Cumulated flows after $j$ periods | $T_j = \sum_{k=0}^{j} p^k$ | $T_{r,j} = \sum_{k=0}^{j} t_{r,k}$ | $\mathbf{T_j} = \sum_{k=0}^{j} \mathbf{P}^k \mathbf{1}_1$ |
| 5 Long-term cumulated flows | $T = \lim_{j \to \infty} T_j = 1/(1-p)$ | | $T = \lim_{j \to \infty} \mathbf{T}_j = (\mathbf{I} - \mathbf{P})^{-1}\mathbf{1}_1$ |
| | | Financial flows: Customer value | |
| 6 Value at period $j$ | $V_j = (m-c)\sum_{k=0}^{j}[t_k/(1+a)^k]$ | $V_{r,j} = \sum_{k=0}^{j}[(mt_{r,k} - c(1-Q_k))/(1+a)^k]$, ou $Q_k = \sum_{l=0}^{k} q_k$ | $\mathbf{V}_j = \sum_{k=0}^{j}[\mathbf{P}^k/(1+a)^k]\mathbf{R}$ |
| | | where $q_k = \begin{cases} t_{1,k-R} \prod_{l=1,k \geq R}^{R}(1-p_l), r = 1 \\ \prod_{l=r,k=R-r+1}^{R}(1-p_l) + t_{r,k-R}\prod_{l=1,k \geq R+1}^{R}(1-p_l), r > 1 \end{cases}$ | |
| 7 Long-term value | $V = \lim_{j \to \infty} V_j = (m-c)(1+a-p)$ | | $V = \lim_{j \to \infty} \mathbf{V}_j = [\mathbf{I} - \mathbf{P}/(1+a)]^{-1}\mathbf{R}$ |

## Appendix 1: continued

| Approaches | Algebraic formulae | Matrix formulae |
|---|---|---|
| | Financial flows: Prospect value (customer equity) | |
| 8 CE after $j$ periods | $CE_j = p_a\left[m - c_a + (m - c_r)\sum_{k=1}^{j}[p_r/(1+a)]^k\right]$ $CE_j = p_a\left[(m - c_a) - \sum_{k=0}^{j}[(mt_{r,k} - c_r(1 - Q_k))/(1+a)^k]\right]$ | (6,3) with P and R modified P includes the prospect state and $R' = [m - c_a, m - c_r, -cr, \ldots]$ |
| 9 Long term CE | $\lim_{j \to \infty} CE_j = p_a[m - c_a + (m - c_r)*p_r/(1 + a - p_r)]$ | (7,3) with $P$ and $R$ modified like in (8,3) |
| | Optimisation: Customer/prospect value | |
| 10 Optimal retention costs $R$ | $\max(m - R/p_r)(1 + a)/(1 + a - p_r)$ *where* $p_r = f(R)$ | max (7,3) where $P = f(R)$ and $R' = [m - R/p_{11}, -R/p_{11}, \ldots]$ |
| 11 Optimal retention ($R$) and acquisition ($A$) costs | $\max\ [p_a m - A + p_a(mp_r - R)/(1 + a - p_r)]$ *where* $p_a = f(A)$ *et* $p_r = f(R)$ | max (9,3) where $P = f(R)$ and $R' = [m - A/p_{11}, m - R/p_{21}, -R/p_{21} \ldots]$ |

Legend:
$a$ = discount rate, $A$ = Acquisition costs per prospect, $c$ = promotion (mailing) costs, $c_a$ = acquisition cost for a new customer, $c_r$ = retention cost for a retained customer, $CE$ = customer equity, $j$ = lifetime value of a customer (no. periods), $m$ = marketing margin, $q$ = customers eliminated (purged) from the list, $Q$ = cumulated purged customers, $p$ = buying probability, $p_a$ = prospect's acquisition probability, $p_r$ = customer retention probability, **P** = transition (probabilities) matrix, $r$ = recency, $R$ = retention cost per customer, $\boldsymbol{R}$ = gains vector, $t_j$ = expected transactions after $j$ periods, $T_j$ = cumulated expected transactions after $j$ periods, $T$ = long-term cumulated expected transactions, $V_j$ = customer lifetime value, $V$ = customer long-term value, $\mathbf{V}_j$ = customer lifetime value vector, **V** = customer long-term value vector.

 

## Appendix 2: Matrix calculations for customer long-term value optimisation

| Cost per customer | Transition probabilities | | | | | | Rewards | | Customer value | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 1€ | r1 | 0.102 | 0.898 | 0.000 | 0.000 | 0.000 | r1 | 30 | r1 | 14.872 |
| | r2 | 0.062 | 0.000 | 0.938 | 0.000 | 0.000 | r2 | −10 | r2 | −22.223 |
| | r3 | 0.038 | 0.000 | 0.000 | 0.962 | 0.000 | r3 | −10 | r3 | −16.915 |
| | r4 | 0.023 | 0.000 | 0.000 | 0.000 | 0.977 | r4 | −10 | r4 | −9.489 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r3 | r5 | | ***R*** | | r1 |
| 2€ | r1 | 0.184 | 0.816 | 0.000 | 0.000 | 0.000 | r1 | 29 | r1 | 16.127 |
| | r2 | 0.111 | 0.000 | 0.889 | 0.000 | 0.000 | r2 | −11 | r2 | −22.722 |
| | r3 | 0.068 | 0.000 | 0.000 | 0.932 | 0.000 | r3 | −11 | r3 | −18.008 |
| | r4 | 0.041 | 0.000 | 0.000 | 0.000 | 0.959 | r4 | −11 | r4 | −10.335 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 3€ | r1 | 0.248 | 0.752 | 0.000 | 0.000 | 0.000 | r1 | 28 | r1 | 16.479 |
| | r2 | 0.151 | 0.000 | 0.849 | 0.000 | 0.000 | r2 | −12 | r2 | −23.725 |
| | r3 | 0.091 | 0.000 | 0.000 | 0.909 | 0.000 | r3 | −12 | r3 | −19.385 |
| | r4 | 0.055 | 0.000 | 0.000 | 0.000 | 0.945 | r4 | −12 | r4 | −11.313 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 4€ | r1 | 0.300 | 0.700 | 0.000 | 0.000 | 0.000 | r1 | 27 | r1 | 15.853 |
| | r2 | 0.182 | 0.000 | 0.818 | 0.000 | 0.000 | r2 | −13 | r2 | −25.324 |
| | r3 | 0.110 | 0.000 | 0.000 | 0.890 | 0.000 | r3 | −13 | r3 | −21.110 |
| | r4 | 0.067 | 0.000 | 0.000 | 0.000 | 0.933 | r4 | −13 | r4 | −12.452 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 5€ | r1 | 0.341 | 0.659 | 0.000 | 0.000 | 0.000 | r1 | 25 | r1 | 14.242 |
| | r2 | 0.207 | 0.000 | 0.793 | 0.000 | 0.000 | r2 | −15 | r2 | −27.557 |
| | r3 | 0.125 | 0.000 | 0.000 | 0.875 | 0.000 | r3 | −15 | r3 | −23.211 |
| | r4 | 0.076 | 0.000 | 0.000 | 0.000 | 0.924 | r4 | −15 | r4 | −13.765 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 6€ | r1 | 0.373 | 0.627 | 0.000 | 0.000 | 0.000 | r1 | 24 | r1 | 11.687 |
| | r2 | 0.227 | 0.000 | 0.773 | 0.000 | 0.000 | r2 | −16 | r2 | −30.422 |
| | r3 | 0.137 | 0.000 | 0.000 | 0.863 | 0.000 | r3 | −16 | r3 | −25.694 |
| | r4 | 0.083 | 0.000 | 0.000 | 0.000 | 0.917 | r4 | −16 | r4 | −15.255 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |
| | | r1 | r2 | r3 | r4 | r5 | | ***R*** | | r1 |
| 7€ | r1 | 0.399 | 0.601 | 0.000 | 0.000 | 0.000 | r1 | 22 | r1 | 8.260 |
| | r2 | 0.242 | 0.000 | 0.758 | 0.000 | 0.000 | r2 | −18 | r2 | −33.885 |
| | r3 | 0.147 | 0.000 | 0.000 | 0.853 | 0.000 | r3 | −18 | r3 | −28.542 |
| | r4 | 0.089 | 0.000 | 0.000 | 0.000 | 0.911 | r4 | −18 | r4 | −16.915 |
| | r5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | r5 | 0 | r5 | 0.000 |