should not combine both into a continuous function. Finally, to make sure the existences of both T_1 and T_3, we need to assume that

$$2S + DM_1^2[c(1-r)I_c - pI_d] > 0$$

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The William Patterson University of New Jersey J-T Teng

Complexity results for single-machine scheduling with positional learning effects

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In the paper Lin (2007), it is claimed that a single machine scheduling problem of minimizing the number of late jobs with a positional learning effect is strongly NP-hard. To prove it, the author provided a reduction from 3-PARTITION to the decision version of this problem. However, we will show that the proof is incorrect, since the reduction is not a pseudopolynomial one. Nevertheless, we will provide a correct proof.

Throughout the paper, we will keep the notation and terminology used by Lin (2007). There is given a set of jobs $N=\{J_1, J_2, \ldots, J_n\}$ to be processed on a single machine. Each job J_i is associated with a due date d_i. The processing time of any job depends on the position at which it is arranged in a particular schedule. If job $J_i \in N$ is scheduled in the jth position $(1 \leqslant j \leqslant n)$, then its processing time is p_{ij}. Owing to the learning effect, the processing time of a job is non-increasing with respect to the positions, that is, $p_{i1} \geqslant p_{i2} \geqslant \cdots \geqslant p_{in}$ for any $J_i \in N$. The completion time of job J_i is denoted by C_i. The job is late if $C_i > d_i$. The problem is to determine a schedule that has the minimum number of late jobs. The problem according to the three-field notation scheme is denoted by $1|LE|\sum U_j$.

Let us recall the significant fragments of the strong NP-hardness proof. First, the definition of 3-PARTITION will be given.

3-PARTITION (Garey and Jonson, 1979): Given non-negative integer B and a set of $3m$ non-negative integers $A = \{x_1, x_2, \ldots, x_{3m}\}$ with $B/4 < x_i < B/2$ for each x_i and $\sum_{i=1}^{3m} x_i = mB$, is there a partition $A_1, A_2, \ldots, A_m$ of set A such that for each subset A_k, $\sum_{x_i \in A_k} = B$?

Based on an instance of 3-PARTITION, the author created an instance of $4m$ jobs ($3m$ ordinary and m enforcer). For each element $x_i \in A$, the author created job J_i, $1 \leqslant i \leqslant 3m$, such that:

$$p_{ij} = (m - \lceil j/4 \rceil + 1)\omega + x_i v^{\lceil j/4 \rceil - 1}, \quad 1 \leqslant j \leqslant 4m$$

$$D = 3\omega \sum_{k=1}^{m} (m-k+1) + 2B \sum_{k=1}^{m} v^{k-1} - Bv^{m-1}$$

where $v = 2mB$, $\omega = v^m$ and D is a due date that is the same for all ordinary jobs. Since the definition of the ordinary jobs is enough to show that the given proof is incorrect, then we will omit the definition of enforcer jobs.

Recall now a condition from a definition of a pseudopolynomial reduction. Let π_1 and π_2 are two decision problems. Let D_{π_1} and D_{π_2} denote their sets of all possible instances, $\mathrm{Max}(I)$ denotes the maximum value for an instance I and $N(I)$ is the size of I. Let $f: D_{\pi_2} - \rightarrow D_{\pi_1}$ denote the reduction from π_2 to π_1. One of the requirements for f to be pseudopolynomial is such that there must exist a polynomial Q of two variables that holds:

$$\forall I \in D_{\pi_2}: \mathrm{Max}(f(I)) \leqslant Q(\mathrm{Max}(I), N(I)) \quad (1)$$

It means that the values of any instance I of the problem π_2 cannot increase in an exponential manner if π_2 is reduced to π_1.

Let π_2 denote 3-PARTITION and π_1 is the considered scheduling problem. It is obvious that for the given reduction and $I \in D_{\pi 2}$, we have $\mathrm{Max}(I) = B$, $N(I) = 3m$, $\mathrm{Max}(f(I)) > \omega = (2mB)^m$ and $N(f(I)) = 4m$ (ie $3m$ ordinary and m enforcer jobs). Thus, there does not exist such Q for which (1) holds, thereby the reduction cannot be pseudopolynomial. Therefore, the proof of the strong NP-hardness is incorrect. Observe that in this way it is proved that the scheduling problem is at least NP-hard, but it is already established by Theorem 1 (Lin, 2007).

Nevertheless, the strong NP-hardness of $1|LE|\sum U_j$, follows straightforward from the strong NP-hardness of the problem $1|LE|L_{\max}$ proved by Cheng and Wang (2000).

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Wrocław University of Technology Adam Janiak
Radosław Rudek