Viewpoint

Comment on Makis and Jardine (1992): Optimal replacement policy for a general model with imperfect repair

In this viewpoint we obtain a new way of finding the optimal policy of a maintenance model with minimal repair and increasing repair cost which depends on the number of the repair in a cycle. Recall that a cycle is the time between two neighbouring replacements.

Boland and Proschan¹ proposed this model and considered a periodic replacement policy. However, this policy was not optimal. Later, Makis and Jardine² considered a general model with imperfect repair which included the previous model as a special case. In this article the optimal replacement policy is given. To achieve this goal, Makis and Jardine formulated the replacement problem in the framework of semi-Markov decision processes. Using well-known results from optimal stopping theory, we can find the same optimal policy. It is hoped that the result of the present analysis, which is based on more elementary arguments, will complement Makis and Jardine's paper.

We first fix the notation to be used:

- c_r = replacement cost;
- c_n = repair cost at the n th failure in a cycle;
- $m(x)$ = mean residual life;
- T_n = n th failure time in a replacement cycle.

The following will be assumed throughout:

1. $c_r > 0$, c_n is a non-decreasing sequence.
2. $m(0) < +\infty$.
3. The failure rate is increasing.

It should be noted that only policies replacing at failure need be considered. In order to find the optimal policy, we adopt the usual long-run average cost criterion. By classical arguments from renewal theory, the long-run average reward R of the system under a replacement policy T is given by

$$V_T = E(R_T)/E(T) \quad (1)$$

Let $V^* = \inf_{T \in \Gamma} R_T$, where Γ is the set of all policies, where replacement is made at failure. A replacement policy T^* is optimal if

$$V_{T^*} = \inf_{T \in \Gamma} R_T \quad (2)$$

Obviously, $E(R_T) \geq V^*E(T)$, for every $T \in \Gamma$, and T^* is optimal if

$$E(R_{T^*} - V^*T^*) = \inf_{T \in \Gamma} E(R_T - V^*T). \quad (3)$$

Using (3) and noting again that only replacement policies which stop at failure have to be taken into account, we can formulate the optimal policy problem for the model examined by Boland and Proschan as follows.

To find the optimal stopping rule for the stochastic sequence

$$Z_n = c_r + \sum_{i=1}^{n-1} c_i - V^*T_n \quad n \geq 1 \quad (4)$$

First, we shall show that we are in the monotone case (see eg Chow *et al*³), ie

$$\begin{aligned} Z_n &\leq E(Z_{n+1} | Z_1, \dots, Z_n) \Rightarrow Z_{n+1} \\ &\leq E(Z_{n+2} | Z_1, \dots, Z_{n+1}) \end{aligned} \quad (5)$$

It is clear that, $Z_{n+1} = Z_n + c_n - V^*(T_{n+1} - T_n)$. Then, because of the above assumptions, $Z_n \leq E(Z_{n+1} | Z_1, \dots, Z_n)$ is equivalent to $c_n \geq V^*m(T_n)$. Also, since $c_{n+1} \geq c_n$ and $m(T_n) \geq m(T_{n+1})$, we have $Z_{n+1} \leq E(Z_{n+2} | Z_1, \dots, Z_{n+1})$, as desired. Thus the stopping rule

$$N = \inf\{n \geq 1 | Z_n \leq E(Z_{n+1} | Z_1, \dots, Z_n)\} \quad (6)$$

is optimal. In other words, the optimal replacement policy is

$$N = \inf\{n \geq 1 | c_n \geq V^*m(T_n)\} \quad (7)$$

The above policy is equal to the policy found by Makis and Jardine. In contrast to the method proposed by Makis and Jardine the present result uses well-known properties of optimal stopping theory and is based on a more elementary argument.

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References

- 1 Boland PJ and Proschan F (1982). Periodic replacement with increasing minimal repair costs at failure. *Opns Res* **30**: 1183–1189.
- 2 Makis V and Jardine KS (1992). Optimal replacement policy for a general model with imperfect repair. *J Opl Res Soc* **43**: 111–120.
- 3 Chow YS, Robbins H and Siegmund D (1971). *Great Expectations: The Theory of Optimal Stopping*. Houghton-Mifflin: New York.