
Empirical investigation of the VaR of hedge funds using daily data

Florent Pochon*, and Jérôme Teïletche

*IXIS Corporate Investment Bank, 47 Quai d'Austerlitz, 75648, Paris Cedex 13, France.
Tel: +33 1 58 55 14 69; E-mail: fpochon@ixis-cib.com

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Florent Pochon is a quantitative economist at IXIS Corporate Investment Bank, Paris, France.

Jérôme Teïletche is Professor of Finance at the Université Montesquieu Bordeaux IV, Bordeaux, France. At the time of writing this study, he was a quantitative economist at IXIS Corporate Investment Bank, Paris, France.

Practical applications

On top of operational risk, market risk is substantial for hedge fund investors. As hedge fund returns exhibit highly non-Gaussian features, the issue of choosing a good model to compute and forecast VaR for hedge funds is crucial. This has become all the more important as hedge funds now deliver weekly or daily NAV estimates as compared to monthly or quarterly ones', a few years ago. This paper uses a high-frequency dataset for a CTA index. We show that the efficiency of one-day-ahead VaR primarily depends on the type of quantiles used for computing VaR forecasts, not on the variance forecasting model. More precisely, among popular models, the use of Cornish-Fisher expansion appears to be the most performing method to compute adequate quantiles.

Abstract

We compare the performance of several Value-at-Risk (VaR) models when applied to a high-frequency hedge fund index. Our analysis is carried out on the Barclay/Calyon CTA daily index available since early 2000. We use 1-day-ahead VaR forecasts for various thresholds (10, 5 and 1 per cent) and apply univariate and multivariate VaR backtesting procedures. Our results show that the efficiency of VaR forecasts primarily depends on the type of quantiles used for computing VaR forecasts. The choice of the model used to forecast volatility (simple smoothing average, EWMA, symmetric or asymmetric GARCH models) proves much less important in that specific case. Our results also show that the most flexible form is the

Cornish-Fisher expansion for 10 and 5 per cent thresholds, whereas Student quantiles are the best to forecast efficiently 1 per cent VaRs.

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INTRODUCTION

While alternative management is acknowledged for its surprisingly high and recurrent returns, it has also been associated with less favourable episodes, such as the dramatic bankruptcy of star fund LTCM. Alternative management investors

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have to contend with numerous risk factors like liquidity risk, operational risk and credit risk. For these risks are perhaps even more substantial in alternative management than in conventional management, because of its unique legal status since, they are often offshore companies subject to less restrictive regulations requiring high entry tickets and restrictive exit conditions, and because of their utilisation of sophisticated instruments with a major leverage effect.

Market risk is also found in the strategies of hedge funds. This is all the more the case for directional strategies that lead to unhedged positions based on an expectation of how the market as a whole is going to move. We will focus this study on market risk and primarily on the canonical measure of market risk, Value-at-Risk (VaR) that is defined as the maximum loss that one can expect to incur according to a certain degree of confidence over a given horizon.

Despite all the afore-mentioned interest in hedge funds, their VaR for the time being has led to few analyses, notably in comparison with other asset classes. Lhabitant¹ proposes a multifactorial approach *à la* Sharpe,^{2,3} where the sensitivity of each hedge fund is estimated according to indices of the various strategies. In other words, the VaR of individual hedge funds is explained according to that of strategies, and this curbs the model's predictive capacities. Pochon and Teiletche^{4,5} condition the returns of the various strategies of hedge funds on the returns of standard assets, in this case the S&P and the Baa/Aaa spread, enabling nonlinear relationships to be used via a mixture of Gaussian distributions. Faced with the non-normality of the returns of hedge funds, Signer and Favre⁶ propose drawing on the Cornish–Fisher expansion to calculate VaR as Zangari⁷ had proposed for standard assets. Bacmann and

Gawron,⁸ Bali and Gockan⁹ confirm the superiority of this approach, as well as the one based on the Extreme Value Theory over the more conventional approaches located within the Gaussian framework. All these previous approaches are primarily backward looking, that is, VaR is estimated from the distribution of *past* variations in prices (albeit varying over time). In practice, risk management in large institutions is often based on forward-looking approaches, that is, they seek to predict VaR. To do so, GARCH temporal series models or constrained versions of such models (EWMA model of RiskMetrics) are frequently used. Monteiro¹⁰ compares various GARCH models and various specifications of conditional distribution. He shows that an estimation of simple (unconditional) variance combined with a Cornish–Fisher approach offers the best results.

A common limitation to all these studies is the fact that they are based solely on monthly frequency data. This is linked to the fact that hedge funds for a long time simply published monthly track records. For various reasons, notably the larger presence of institutional investors, the development of fund platforms, the increasing professionalism and improvement in risk management within funds, etc, increasingly publish track records with a higher frequency, whether daily or weekly, notably when they invest in liquid vehicles. The main contribution of this research is that it is, to the best of our knowledge, the first to draw on this type of data to analyse the VaR of hedge funds. More precisely, we use the Barclay/Calyon index covering the performances of a set of CTA funds. This leads to a more sophisticated understanding of the market risk hedge funds are exposed to and enhanced adequacy of academic research on the other assets that is just about systematically based on daily data. Looking

forward, this enables us to implement more complete back-testing than in previous studies.

We show that efficient 1-day-ahead VaR forecasts are achievable. In our case, the results primarily relies on the type of quantiles used for VaR computations, with the Cornish–Fisher quantiles being the most flexible ones for 10 and 5 per cent VaRs, and the Student’s the most suitable for 1 per cent VaR. Interestingly, the choice of the models used to forecast volatility proves much less important. After presenting our methodology in the next section, we give detailed results in the penultimate section and draw a conclusion in the final section.

METHODOLOGY

General principle

Generally speaking, the VaR of an investment represents the maximum loss that can be considered, given a certain degree of confidence at a given horizon. More formally, if we denote by $1-\alpha$ the degree of confidence and r_t the return linked to the investment as recorded at date t , VaR is defined by the following relationship:

$$\Pr[r_t < VaR_{t|t-1}(\alpha)] = \alpha$$

Note that VaR is defined here in an *ex ante* manner, that is, the VaR expected in period t conditional on the information available in period $t-1$. For we are trying here to test which models enable an efficient VaR to be constructed with respect to a track record $t = 1, \dots, T$. More precisely, we introduce the indicative function associated with violations of VaR:

$$I_t(\alpha) = \begin{cases} 1 & \text{if } r_t < VaR_{t|t-1}(\alpha) \\ 0 & \text{otherwise} \end{cases}$$

It is accepted that the problem of the validity of the VaR model is then summarised by the verification of two hypotheses (Christoffersen,¹¹ Campbell¹²):

Unconditional coverage

The probability that an *ex post* loss exceeds VaR forecasts must be exactly equal to α , that is, $\Pr[I_t(\alpha) = 1] = E[I_t(\alpha)] = \alpha$.

Independence

Violations of VaR at various periods must be independent over time. In particular, $I(\alpha)$ has to be independent from past returns and past values of VaR.

Berkowitz *et al.*¹³ show that these two hypotheses can be regrouped in a single hypothesis, so-called conditional coverage. It is expressed in statistical terms as the fact that demeaned violations of the VaR threshold represent a martingale difference:

$$E[I_t(\alpha) - \alpha | \Omega_{t-1}] = 0$$

where the information set Ω_{t-1} includes past values of violations of VaR, $I_{t-1}(\alpha)$, $I_{t-2}(\alpha)$, ..., but also any other variables known in period $t-1$, such as the levels of VaR or past returns.

Several tests of the conditional coverage hypothesis have been proposed in the literature. Christoffersen¹¹ starts off from the transition matrix of violations of VaR,

$$\Pi = \begin{pmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{pmatrix}$$

where $\pi_{ij} = \Pr[I_t(\alpha) = j | I_{t-1}(\alpha) = i]$. He shows that under the hypothesis of nil conditional efficiency of VaR, the two linear restrictions $\pi_{01} = \pi_{11} = \alpha$ must be verified, and this can be

tested via a likelihood ratio:

$$LR_{CC} = -2 \log[L(\Pi_2; I_1(\alpha), \dots, I_T(\alpha)) - L(\Pi_1; I_1(\alpha), \dots, I_T(\alpha))] \sim \chi^2$$

with

$$L(\Pi_1; I_1(\alpha), \dots, I_T(\alpha)) = (1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}$$

for n_{ij} the number of cases where $I_t(\alpha) = j$ for $I_{t-1}(\alpha) = i$ and $L(\Pi_2; I_1(\alpha), \dots, I_T(\alpha)) = (1 - \pi_2)^{n_{00}+n_{10}} \pi_2^{n_{01}+n_{11}}$ with $\pi_2 = (n_{01} + n_{11}) / (n_{00} + n_{10} + n_{01} + n_{11})$. The Christoffersen test is thus as a joint test of a correct unconditional coverage noted LR_{UC} , where under the null $I_t \sim i.i.d \text{ Bernouilli}(\alpha)$, and a test of independence denoted LR_{IND} , where under the null $\pi_{01} = \pi_{11}$: $LR_{CC} = LR_{UC} + LR_{IND}$ where LR_{UC} and $LR_{IND} \sim \chi^2$. The Christoffersen test LR_{CC} presents however the drawback of dealing only with first-order dependencies — and is therefore not very general — as well as a second drawback of not allowing dependency with other variables such as returns.

The Engle and Manganelli test,¹⁴ the so-called CaViar test, solves this problem simply by proposing a regression of demeaned violations, so-called *Hits* with $Hit_t(\alpha) = I_t(\alpha) - \alpha$, on their past values and any other variable:

$$Hit_t(\alpha) = \lambda + \sum_{k=1}^K \beta_k Hit_{t-k}(\alpha) + \sum_{k=1}^L \gamma_k g[Hit_{t-k}(\alpha), Hit_{t-k-1}(\alpha), \dots, r_{t-k}, \dots] + \varepsilon_t$$

$g[\cdot]$ stands for any function. The conditional coverage hypothesis is then simply tested via Student or Fisher tests of the nullity of regression coefficients including the constant. We use the Wald statistics in association with the test of

conditional efficiency hypothesis, we note EM_{CC} :

$$EM_{CC} = \frac{\hat{\psi}' Z' Z \hat{\psi}}{\alpha(1 - \alpha)} \sim \chi^2(K + L + 1),$$

with $\psi = (\lambda, \beta_1, \dots, \beta_k, \gamma_1, \dots, \gamma_l)'$ and Z the matrix of the corresponding explanatory variables. In our application, we set $K=5$ and $g[Hit_{t-k}(\alpha), Hit_{t-k-1}(\alpha), \dots, r_{t-k}, \dots] = VaR_{t|t-1}$ (ie the ex ante VaR).

Lastly, Berkowitz *et al.*,¹³ propose carrying out simple tests of the absence of autocorrelation in the time series of Hit_t 's via a standard Ljung–Box test for which the statistic is given by $LB(K) = T(T + 2) \sum_{k=1}^K \rho_k^2 / (T - k) \sim \chi^2(K)$ and they recommend having $K = 5$. Hurlin and Tokpavi¹⁵ propose widening the test to a multivariate case, that is the various levels of coverage α , via the statistic of the Li and McLeod test¹⁶

$$Q_m(K) = T \sum_{k=1}^K (vec R_k) (R_0^{-1} \otimes R_0^{-1}) \times (vec R_k) \sim \chi^2(Km^2)$$

where m stands for the different levels of coverage. The authors recommend drawing on $m = 3$ for $\alpha = \{1 \text{ per cent}, 5 \text{ per cent}, 10 \text{ per cent}\}$. We denote this test HT in the Tables below.

Construction of volatility forecast: GARCH models and distributional hypotheses

In compliance with academic and professional practice, all the VAR forecasting models we use in this study are based on the following formula:

$$VaR_{t|t-1}(\alpha) = z_{t|t-1}(\alpha) \times h_t$$

The VaR forecast in $t-1$ at horizon t is thus calculated as the product of the volatility forecast in $t-1$ at horizon t , denoted h_t , with a scalar that

depends on the level of confidence, that is the α -quantile denoted $z_{t|t-1}(\alpha)$.

With respect to volatility, we have chosen three types of models:

- (i) Estimators based on simple sample standard deviation estimated on the latest n returns:

$$h_t = n^{-1} \sum_{\tau=1}^n r_{t-\tau}^2.$$

In practice, we use three alternative windows: 22 days ($n = 22$, denoted SMA22), 66 days ($n = 66$, denoted SMA66) and 250 days ($n = 250$, SMA250).

- (ii) Estimators based on GARCH models estimated over the last 250 trading days. In practice, we have drawn on five types of models that are presented in detail in the Table 1 (more details about the estimated models are available from the authors on request). These models stand out because of their specification, with notably a distinction between asymmetric models and symmetric models.
- (iii) Estimators of an EWMA moving average type: $h_t^2 = \lambda h_{t-1}^2 + (1 - \lambda)r_{t-1}^2$. In compliance with recommendations made by RiskMetrics, we pick $\lambda = 0.94$. Note

that the EWMA model has common characteristics with conventional estimators, since the latter is a specific case of EWMA where all the observations are equally weighted ($\lambda = 1/n$) and with the GARCH estimators, since the EWMA model is obtained as a GARCH with nil long-term variance ($\omega = 0$) and integrated ($\alpha + \beta = 1$).

Note that in the calculation of VaR and the estimation of volatilities; whether in historical models or GARCH models, we omit the impact of the short-term average of returns. However, it is well known that the addition of the average, which is unstable over time and nearly nil on average in the long term, leads to adding noise, and this lowers the efficiency of volatility estimators (Canina and Figlewski¹⁷, Jorion¹⁸).

For the quantile $z_{t|t-1}(\alpha)$, we have chosen three measures:

- Quantiles resulting from the normal standard distribution.
- Quantiles stemming from a Student distribution. Student distribution, which includes normal distribution as a specific case for an infinite number of degrees of freedom,

Table 1: Univariate GARCH models

<i>Model</i>	<i>(A)Symmetric response depending on returns sign</i>	<i>Specification</i>
GARCH	Symmetric	$h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1}$
AV-GARCH – Absolute value	Symmetric	$h_t^{1/2} = \omega + \alpha r_{t-1} + \beta h_{t-1}^{1/2}$
NARCH – Nonlinear GARCH	Symmetric	$h_t^{\lambda/2} = \omega + \alpha r_{t-1} ^\lambda + \beta h_{t-1}^{\lambda/2}$
EGARCH - Exponential GARCH	Asymmetric if $\gamma \neq 0$	$\log(h_t) = \omega + \alpha \frac{ r_{t-1} }{h_t} + \gamma \frac{r_{t-1}}{h_t} + \beta \log(h_{t-1})$
GJR-GARCH	Asymmetric if $\gamma \neq 0$	$h_t = \omega + \alpha r_{t-1}^2 + \gamma I_{[r_{t-1} < 0]} r_{t-1}^2 + \beta h_{t-1}$

makes it possible to take into account the non-normality of the series. In particular, the distribution associated with Student distribution can be compatible with the phenomenon of excess kurtosis, as witnessed in daily returns of CTAs (cf below), once the number of degrees of freedom is reduced. It partly explains the non-normality of the series, but distribution remains symmetric (asymmetric versions of Student distribution do exist, however; cf Hansen¹⁹). When we estimate the parameters of Student distribution throughout the period for the various GARCHs models, we find an average degree of freedom equal to 8.43 (ranging from 7.37 to 9.10). This is why we draw on a Student distribution with eight degrees of freedom for the calculation of the quantile.

- So-called Cornish–Fisher quantiles, which are based on an expansion around the normal distribution in order to take into account the asymmetry and the fat tails of the distribution. Formally, the quantile associated with the Cornish–Fisher expansion is given by:

$$z_{CF}(\alpha) = z_{normal}(\alpha) + \frac{1}{6}(z_{normal}(\alpha)^2 - 1)SK + \frac{1}{24}(z_{normal}(\alpha)^3 - 3z_{normal}(\alpha))(KU - 3) - \frac{1}{36}(2z_{normal}(\alpha)^3 - 5z_{normal}(\alpha))SK^2$$

with $z_{normal}(\alpha)$ the quantile of the normal distribution, SK the skewness coefficient and KU the kurtosis coefficient. In the above estimates, we use the values of these two parameters found over the whole of the period.

Finally, the quantiles used for the various thresholds and the various methods are

Table 2: Quantiles

<i>VaR thresholds</i>	<i>Gaussian</i>	<i>Student (8 degrees of freedom)</i>	<i>Cornish-Fisher Expansion</i>
1%	−2.33	−2.90	−2.99
5%	−1.64	−1.86	−1.67
10%	−1.28	−1.40	−1.16

summarised in the Table 2 (values rounded off to simplify their presentation).

RESULTS

Data

Our analysis is based on the Calyon Financial Barclay index, which is an equiweighted index, covering the returns of a set of large CTA funds that are open to new investments. In July 2006, the index was made up by 14 funds, while 17 belonged to it when the index was launched in January 2000. The list is reviewed at the beginning of each year. The index is not investable. The index and associated log-returns are represented in Figure 1. The sample ranges from January 2000 to June 2006 offering 1,693 daily returns. Table 3 shows descriptive statistics of these returns.

With a standard deviation of 0.58 per cent per day, the volatility of the index is close to 9 per cent per year. This is a high figure for an overall index of hedge funds, but typical for a CTA index. Even though the average return and the median return are very similar, distribution is significantly asymmetric in the direction of negative returns. It also presents kurtosis significantly above 3 so that the overall

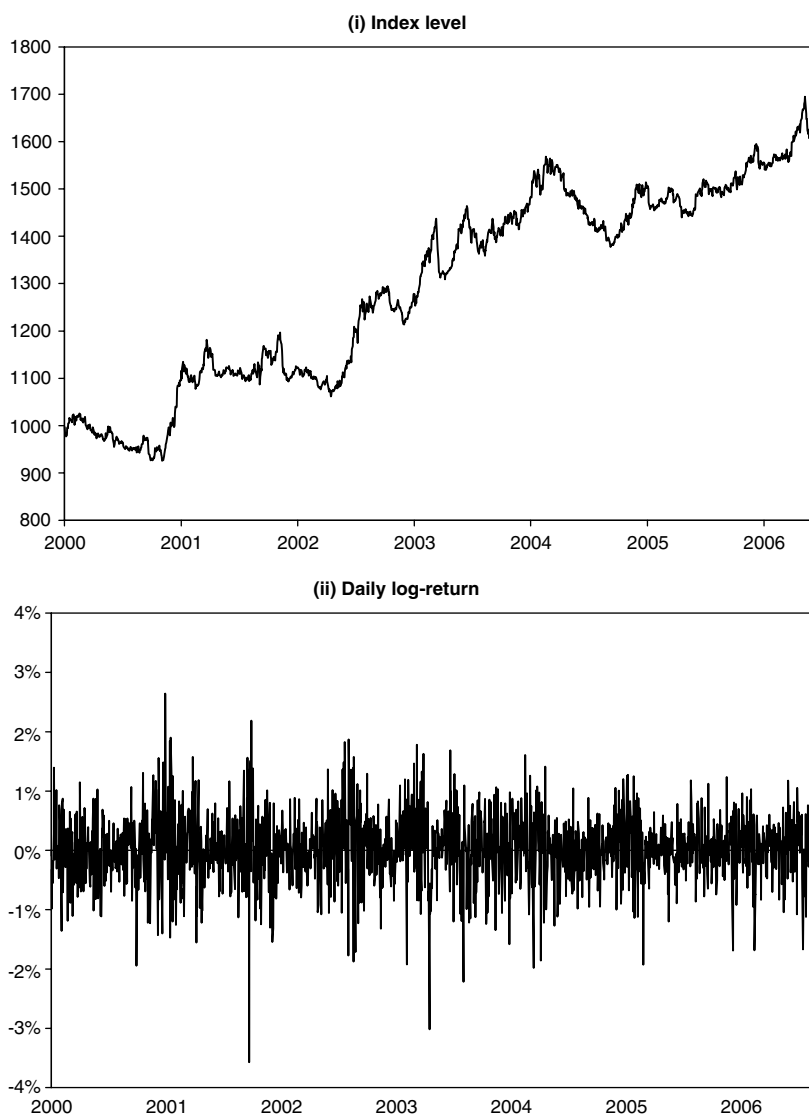


Figure 1: Calyon Financial Barclay CTA Index.
 Source: <http://www.calyonfinancial.com/tools/barclay.html>

distribution is significantly different from a normal distribution. Such non-normality of unconditional distribution of returns might be linked to the presence of ARCH effects but could also combine with non-normality of conditional distribution. This is why we consider various distributional hypotheses,

including some that are typical of fat tail or asymmetric distribution, and in particular, a Student distribution and a Cornish–Fisher expansion (see above). For their part, Ljung–Box statistics show that the series of returns does not display any dependence on average but high dependence on variance. This finding,

combined with an ARCH-LM test, provides a justification for using GARCH models — without an ARMA-like component.

Out-of-sample VaR comparisons

Results are reported in Tables 4–6, with univariate tests in panel As and the HT multivariate test in panel Bs. All the tests were computed on 1-day-ahead VaR forecasts. For every VaR threshold (10, 5 and 1 per cent), we compare the ability of various models to forecast volatility and VaR (historical smoothing averages, EWMA and GARCH models). Given the GARCH models are estimated on 250-day rolling windows, we are left with 1,442 out-of-sample forecasts.

We report the asymptotic p -values of the test statistics. Berkowitz *et al.*¹³ show that for small sample sizes and for 1 per cent VaR, the asymptotic critical values can be significantly misleading and thus use Dufour²⁰ Monte Carlo technique to compute correct p -values. However, complementary analyses (not reported) showed us that the sample bias was not significant, given our sample size (1442 observations), and that Monte-Carlo p -values were very similar to asymptotical ones. We

consider the null hypothesis can be rejected when p -values are smaller than 0.10.

Table 4 reports the test results when VaRs are computed using the Gaussian quantiles (−1.28 for 10 per cent, −1.64 for 5 per cent VaR, −2.33 for 1 per cent VaR), and GARCH models are normal GARCH. Results are not clearcut and depend on the VaR threshold. For a 10 per cent threshold, the worst VaR models are performed by the 250-day simple historical smoothing average (SMA250) and the GJR-GARCH models. Both (i) overestimate significantly the VaR and fail the unconditional coverage test (the ex post violation rate (VR) is significantly below 10 per cent) and (ii) fail most univariate tests (LR_{CC} , EM_{CC} , $LB(5)$ and even $LB(10)$ tests for the former). On the other hand, the best performers are the EWMA and the NARCH, which successfully pass every test. Other models have mixed results depending on the test statistics: the AVGARCH and the GARCH in particular exhibit p -values below the significance level for LR_{CC} .

For the 5 per cent threshold, results are similar although less clearcut. The EWMA and the NARCH models remain the best models along with the GARCH model and more surprisingly,

Table 3: Descriptive statistics for returns

Mean	Median	Maximum	Minimum	Std. Dev.	Skewness (p -value)	Kurtosis (p -value)	Jarque-Bera (p -value)
0.0277%	0.0270%	2.642%	−3.572%	0.5789%	−0.25179 (0.000)	5.15889 (0.000)	346.6715 (0.000)
$Q(r_i;5)$ (p -value)	$Q(r_i;10)$ (p -value)	$Q(r_i^2;5)$ (p -value)	$Q(r_i^2;10)$ (p -value)	ARCH- LM(5)			
9.168 (0.133)	10.978 (0.359)	109.74 (0.000)	150.34 (0.000)	75.95 (0.000)			

Table 4: Normal quantiles and normal GARCH

	<i>VR (%)</i>	<i>VaR (%)</i>	<i>LR_{UC}</i>	<i>LR_{IND}</i>	<i>LR_{CC}</i>	<i>EM_{CC}</i>	<i>LB(5)</i>	<i>LB(10)</i>
<i>(a) Univariate tests (1-day VaR)</i>								
<i>VaR 10%</i>								
SMA22	9.57	0.71	0.590	0.231	0.423	0.048	0.603	0.792
SMA66	9.15	0.73	0.282	0.036	0.062	0.193	0.200	0.407
SM250	7.63	0.75	0.002	0.022	0.001	0.010	0.015	0.010
EWMA	8.88	0.72	0.150	0.350	0.230	0.296	0.807	0.847
GARCH	8.88	0.73	0.150	0.039	0.042	0.171	0.227	0.170
EGARCH	8.60	0.74	0.071	0.249	0.101	0.181	0.186	0.232
GJR-GARCH	7.98	0.75	0.008	0.047	0.004	0.042	0.093	0.150
AVGARCH	8.74	0.73	0.105	0.106	0.073	0.433	0.415	0.463
NARCH	8.95	0.73	0.178	0.150	0.143	0.413	0.663	0.369
<i>VaR 5%</i>								
SMA22	5.83	0.91	0.159	0.310	0.221	0.115	0.537	0.483
SMA66	5.13	0.94	0.814	0.733	0.918	0.044	0.299	0.297
SM250	5.13	0.96	0.814	0.042	0.123	0.388	0.206	0.168
EWMA	5.55	0.92	0.345	0.214	0.296	0.121	0.418	0.417
GARCH	5.27	0.94	0.636	0.299	0.522	0.461	0.523	0.555
EGARCH	4.72	0.94	0.621	0.004	0.015	0.039	0.011	0.048
GJR-GARCH	4.72	0.96	0.621	0.004	0.015	0.071	0.026	0.160
AVGARCH	4.99	0.94	0.995	0.010	0.035	0.160	0.076	0.300
NARCH	5.20	0.94	0.723	0.516	0.760	0.308	0.512	0.571
<i>VaR 1%</i>								
SMA22	2.01	1.29	0.001	0.586	0.003	0.000	0.782	0.896
SMA66	1.66	1.32	0.020	0.357	0.045	0.005	0.838	0.910
SM250	1.46	1.36	0.102	0.420	0.190	0.303	0.716	0.832
EWMA	1.73	1.31	0.011	0.338	0.025	0.011	0.812	0.901
GARCH	1.25	1.33	0.360	0.216	0.307	0.479	0.597	0.901
EGARCH	1.53	1.34	0.062	0.398	0.123	0.142	0.883	0.921
GJR-GARCH	1.39	1.35	0.162	0.442	0.280	0.411	0.686	0.920
AVGARCH	1.32	1.33	0.247	0.465	0.391	0.723	0.935	0.989
NARCH	1.53	1.33	0.062	0.336	0.111	0.062	0.738	0.921

Table 4: Continued

<i>(b) Multivariate test</i>		
	<i>HT</i>	<i>P-val</i>
SMA22	40.33	0.670
SMA36	41.42	0.625
SMA250	67.56	0.016
EWMA	45.60	0.447
GARCH	37.90	0.764
EGARCH	67.45	0.017
GJR-GARCH	44.77	0.482
AVGARCH	48.38	0.338
NARCH	31.51	0.936

Notes: *VR* stands for the percentage of time returns prove inferior to the ex ante VaR forecast (ex post violation rate). *VaR* stands for the average value of the 1-day-ahead VaR forecasts. *LR_{UC}*, *LR_{IND}*, *LR_{CC}* stand, respectively, for *p*-values of the unconditional coverage test statistic, the independence test statistic and Christoffersen's conditional coverage test statistic; *EM_{CC}* stands for the Engle and Magnanelli's test *p*-values (5 lags). *LB(k)* stands for the Ljung-Box's *p*-value with *k* lags. GARCH models are estimated using a 250-day rolling window.

Notes: *HT* stands for the multivariate portmanteau's *p*-value (Li and McLeod¹⁶) as used by Hurlin and Tokpavi.¹⁵ We use *k*=5 lags and use three coverage rates $\alpha=1\%,5\%,10\%$. GARCH models are estimated using a 250-day rolling window.

SMA22; the GJR-GARCH, the AV-GARCH are still among the least performing models, along with the EGARCH.

At the 1 per cent level, the ex post VR is larger than the initial coverage rate (1 per cent) for every model and reflects the leptokurtic feature of the Calyon index. The *LR_{UC}* test, however, indicates that this difference is only significant for the SMA22, SMA66, EWMA and NARCH models. SMA22, SMA66 and EWMA also fail the *LR_{CC}* and *EM_{CC}* tests. SMA250, GARCH, EGARCH, AVGARCH and GJR-GARCH face no rejections.

All in all, no clear distinction emerges between the models, neither between GARCH and simple moving average estimators, nor between symmetric or asymmetric GARCH

models — when relying on the univariate statistics. Panel 4b reports the HT multivariate statistic computed for three thresholds (1, 5 and 10 per cent) and with five lags. The larger *p*-values are obtained by the NARCH and the GARCH models indicating that they are the best VaR performers throughout the thresholds. SMA250 and EGARCH both fail the test.

Turning to the VaRs computed using the Cornish-Fisher quantiles (GARCH models are still normal GARCHs), results in Table 5 turn a bit more unanimous. Quite surprisingly, with the use of these quantiles, almost every forecast method considered pass the univariate tests, including simple historical moving averages, no matter the threshold (10, 5 or 1 per cent). The only failure exceptions are the SMA250 for 10

Table 5: Cornish–Fisher expansion quantiles with normal GARCH

	VR (%)	VaR (%)	LR_{UC}	LR_{IND}	LR_{CC}	EM_{CC}	$LB(5)$	$LB(10)$
<i>(a) Univariate tests (1-day VaR)</i>								
<i>VaR 10%</i>								
SMA22	11.37	0.64	0.087	0.326	0.142	0.015	0.256	0.178
SMA66	10.61	0.66	0.439	0.175	0.295	0.158	0.205	0.299
SM250	9.36	0.68	0.420	0.056	0.116	0.021	0.003	0.003
EWMA	11.03	0.65	0.197	0.306	0.258	0.536	0.722	0.472
GARCH	10.47	0.66	0.547	0.220	0.393	0.659	0.713	0.238
EGARCH	10.47	0.66	0.547	0.141	0.282	0.510	0.356	0.435
GJR-GARCH	10.12	0.67	0.868	0.130	0.313	0.626	0.422	0.609
AVGARCH	10.47	0.66	0.547	0.049	0.120	0.502	0.368	0.469
NARCH	10.61	0.66	0.439	0.175	0.295	0.582	0.673	0.257
<i>VaR 5%</i>								
SMA22	5.55	0.92	0.345	0.214	0.296	0.089	0.358	0.502
SMA66	5.06	0.95	0.909	0.718	0.931	0.064	0.349	0.394
SM250	4.99	0.98	0.995	0.085	0.227	0.470	0.277	0.197
EWMA	5.41	0.94	0.478	0.354	0.506	0.163	0.365	0.414
GARCH	4.99	0.96	0.995	0.205	0.448	0.360	0.400	0.543
EGARCH	4.65	0.96	0.537	0.013	0.037	0.102	0.032	0.106
GJR-GARCH	4.65	0.97	0.537	0.003	0.011	0.052	0.019	0.136
AVGARCH	4.85	0.95	0.803	0.022	0.070	0.262	0.132	0.434
NARCH	4.99	0.96	0.995	0.421	0.724	0.218	0.461	0.523
<i>VaR 1%</i>								
SMA22	0.90	1.65	0.704	0.614	0.819	0.981	0.988	1.000
SMA66	0.76	1.70	0.346	0.667	0.585	0.970	0.994	1.000
SM250	0.62	1.75	0.124	0.723	0.287	0.932	0.998	1.000
EWMA	0.76	1.68	0.346	0.667	0.585	0.990	0.994	1.000
GARCH	0.69	1.71	0.217	0.695	0.432	0.978	0.996	1.000
EGARCH	0.62	1.72	0.124	0.723	0.287	0.942	0.998	1.000
GJR-GARCH	0.55	1.74	0.064	0.751	0.171	0.853	0.999	1.000
AVGARCH	0.69	1.71	0.217	0.695	0.432	0.978	0.996	1.000
NARCH	0.69	1.71	0.217	0.695	0.432	0.978	0.996	1.000

Table 5: Continued

(b) *Multivariate test*

	<i>HT</i>	<i>P-val</i>
SMA22	47.66	0.365
SMA36	45.37	0.457
SMA250	64.15	0.032
EWMA	47.02	0.390
GARCH	44.90	0.476
EGARCH	55.05	0.145
GJR-GARCH	54.67	0.153
AVGARCH	47.03	0.389
NARCH	39.97	0.684

Notes: *VR* stands for the percentage of time returns prove inferior to the ex ante VaR forecast (ex post violation rate). *VaR* stands for the average value of the 1-day-ahead VaR forecasts. *LR_{UC}*, *LR_{IND}*, *LR_{CC}* stand, respectively, for *p*-values of the unconditional coverage test statistic, the independence test statistic and Christoffersen's conditional coverage test statistic; *EM_{CC}* stands for the Engle and Magnanelli's test *p*-values (5 lags). *LB(k)* stands for the Ljung-Box's *p*-value with *k* lags. GARCH models are estimated using a 250-day rolling window.

Notes: *HT* stands for the multivariate portmanteau's *p*-value (Li and McLeod¹⁶) as used by Hurlin and Tokpavi.¹⁵ We use *k*=5 lags and use three coverage rates $\alpha=1\%,5\%,10\%$. GARCH models are estimated using a 250-day rolling window.

per cent VaR estimates, the EGARCH and GJR-GARCH models for the 5 per cent VaR estimates, which all exhibit significant violation clustering (cf panel 3a). Note that contrary to the pure Gaussian case in Table 2, the ex post VR are much closer to the ex coverage rate for the 10 and 5 per cent thresholds with only one rejection for *LR_{UC}*. These results are largely confirmed by the multivariate HT statistic reported in panel 3b with only one rejection for the SMA250; the NARCH model has the highest *p*-value.

Table 6 reports the results for the VaR estimates using the Student quantiles with eight degrees of freedom. GARCH volatility forecasts are also taken from *t*-GARCH estimates. Panel 4a shows that for large VaR thresholds (10 or 5

per cent), no variance forecasting model is optimal. Interestingly, every model fails *LR_{CC}* and *EM_{CC}* tests at these thresholds. For the former test, most of failures stem from the failure of the unconditional coverage tests: using Student quantiles induce a substantial overestimation of the VaR of the CTA index for 10 and 5 per cent thresholds, whatever the models used to make volatility forecasts (the results are unchanged using seven degrees of freedom instead of eight). With a more stringent threshold of 1 per cent, the results get much better since every model then pass the univariate tests successfully. Interestingly, at this threshold level, *p*-values are almost systematically larger than those obtained in Tables 4 and 5 by the Gaussian and Cornish-Fisher methods, showing

Table 6: Student quantiles and t-GARCH

	<i>VR (%)</i>	<i>VaR (%)</i>	<i>LR_{UC}</i>	<i>LR_{IND}</i>	<i>LR_{CC}</i>	<i>EM_{CC}</i>	<i>LB(5)</i>	<i>LB(10)</i>
<i>(a) Univariate tests (1-day VaR)</i>								
<i>VaR 10%</i>								
SMA22	7.98	0.77	0.008	0.451	0.023	0.038	0.587	0.908
SMA66	7.63	0.80	0.002	0.097	0.002	0.009	0.066	0.098
SM250	6.80	0.82	0.000	0.040	0.000	0.003	0.117	0.205
EWMA	7.63	0.78	0.002	0.649	0.007	0.048	0.829	0.821
GARCH	7.07	0.80	0.000	0.070	0.000	0.009	0.254	0.264
EGARCH	7.00	0.80	0.000	0.061	0.000	0.002	0.080	0.126
GJR-GARCH	6.80	0.81	0.000	0.016	0.000	0.000	0.002	0.004
AVGARCH	7.56	0.80	0.001	0.285	0.003	0.021	0.136	0.221
NARCH	7.42	0.80	0.001	0.030	0.000	0.014	0.175	0.244
<i>VaR 5%</i>								
SMA22	4.30	1.03	0.214	0.060	0.079	0.026	0.104	0.303
SMA66	3.74	1.06	0.023	0.470	0.057	0.010	0.189	0.115
SM250	2.84	1.09	0.000	0.117	0.000	0.007	0.348	0.732
EWMA	3.68	1.04	0.016	0.437	0.040	0.098	0.664	0.411
GARCH	3.40	1.06	0.003	0.712	0.012	0.111	0.806	0.901
EGARCH	2.98	1.07	0.000	0.009	0.000	0.001	0.013	0.081
GJR-GARCH	3.12	1.08	0.000	0.059	0.000	0.021	0.200	0.622
AVGARCH	3.05	1.06	0.000	0.011	0.000	0.002	0.014	0.057
NARCH	3.47	1.06	0.005	0.345	0.012	0.068	0.476	0.707
<i>VaR 1%</i>								
SMA22	0.97	1.60	0.913	0.587	0.858	0.990	0.983	0.999
SMA66	0.83	1.65	0.511	0.640	0.722	0.950	0.992	1.000
SM250	0.69	1.69	0.217	0.695	0.432	0.975	0.996	1.000
EWMA	0.83	1.63	0.511	0.640	0.722	0.992	0.992	1.000
GARCH	0.76	1.66	0.346	0.667	0.585	0.988	0.994	1.000
EGARCH	0.76	1.66	0.346	0.667	0.585	0.984	0.994	1.000
GJR-GARCH	0.62	1.69	0.124	0.723	0.287	0.915	0.998	1.000
AVGARCH	0.69	1.65	0.217	0.695	0.432	0.978	0.996	1.000
NARCH	0.69	1.66	0.217	0.695	0.432	0.978	0.996	1.000

Table 6: Continued

(b) Multivariate test

	<i>HT</i>	<i>P-val</i>
SMA22	38.63	0.737
SMA36	49.55	0.297
SMA250	48.08	0.349
EWMA	37.22	0.789
GARCH	43.70	0.527
EGARCH	58.45	0.086
GJR-GARCH	71.10	0.008
AVGARCH	72.24	0.006
NARCH	42.24	0.590

Notes: *VR* stands for the percentage of time returns prove inferior to the ex ante VaR forecast (ex post violation rate). *VaR* stands for the average value of the 1-day-ahead VaR forecasts. *LR_{UC}*, *LR_{IND}*, *LR_{CC}* stand, respectively, for *p*-values of the unconditional coverage test statistic, the independence test statistic and Christoffersen's conditional coverage test statistic; *EM_{CC}* stands for the Engle and Magnanelli's test *p*-values (5 lags). *LB(k)* stands for the Ljung-Box's *p*-value with *k* lags. GARCH models are estimated using a 250-day rolling window.

Notes: *HT* stands for the multivariate portmanteau's *p*-value (Li and McLeod¹⁶) as used by Hurlin and Tokpavi.¹⁵ We use *k*=5 lags and use three coverage rates $\alpha=1\%,5\%,10\%$. GARCH models are estimated using a 250-day rolling window.

logically that Student quantiles are better to forecast extreme losses. The HT statistics in panel 4b indicate three rejections for the EGARCH, GJR-GARCH and N-GARCH models.

To conclude, our results first show that for the Barclay/Calyon CTA index, efficient VaR forecasts depends much more on the quantile used for computing VaR, than on the choice of the model used to forecast volatility (simple smoothing average, EWMA, symmetric or asymmetric GARCH models). Second, and in line with Monteiro,¹⁰ the Cornish-Fisher quantiles appear the easiest and the most flexible way to obtain efficient VaR forecasts: even combined with very simple volatility estimators, VaR forecasts manage to respect the unconditional coverage and violation

independence conditions at least for 10 and 5 per cent thresholds.

CONCLUSION

As NAV were formerly exclusively available on a monthly basis, there is a growing pressure for hedge funds to issue weekly or even daily NAV estimates, so as to improve transparency as well as risk management capacity. The main contribution of this paper is to conduct an empirical investigation of hedge fund VaR using daily returns of an hedge fund index, namely the Barclay CTA index, which starts as early as January 2000.

For various thresholds, we consider several methods to compute 1-day-ahead VaR forecasts

using different types of quantiles (Gaussian, Cornish–Fisher expansion, Student) and volatility forecasting models (historical averages, EWMA, GARCH and t-GARCH). We compare these estimates, using univariate and multivariate back-testing procedures aimed at checking their efficiency both in terms of unconditional coverage and independence of VaR violations.

Our results show that for the Barclay/Calyon CTA index, the efficiency of VaR forecasts primarily depends on the type of quantiles used for computing VaR. The choice of the model used to forecast volatility (simple smoothing average, EWMA and symmetric or asymmetric GARCH models) proves much less important in that specific case. Our results also show that the most flexible form is the Cornish–Fisher expansion, since it enables even very simple volatility models to pass successfully back-testing tests for 10, 5 or even 1 per cent VaR thresholds. Thus, our results seem in line with those of Monteiro¹⁰ on monthly data. On the other hand, Student VaRs are the best to compute 1 per cent VaR forecasts, although they are ill-adapted for larger thresholds (with large VaR overestimations). All in all, if on the one hand, our results need confirmation before being generalised, they are, on the other hand, somewhat reassuring for risk managers as they show that making good VaR estimations for hedge funds seems achievable.

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