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Comment on 'the strong NP-completeness of 3-PARTITION problem with B $\geqslant k^m$' by Zhongyi Jiang, Fangfang Chen, Chunqing Wu

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The note of Jiang and colleagues is concerned with the following problem.

3-*PARTITION*: Given $3m+1$ positive integer numbers $x_1, \ldots, x_{3m}$ and B such that $\sum_{j=1}^{3m} x_j = mB$ and $B/4 < x_j < B/2$ for $j=1, \ldots, 3m$, is there a partition of the set $A=\{1, \ldots, 3m\}$ into m disjoint subsets A_l such that $\sum_{j \in X_l} x_j = B$ for $l=1, \ldots, m$?

The main result of the authors is Theorem 1 stating that 3-PARTITION is NP-hard in the strong sense even if $B \geqslant k^m$, where $k \geqslant 2$ is a given integer. This result is incorrect. In the proof, Jiang and colleagues use a series of transformations of 3-PARTITION, each time obtaining a new problem by resetting $B := kB$ and $x_j := kx_j$, $j=1, \ldots, 3m$. In the first new problem, B and x_j are multiples of k; in the second new problem, they are multiples of k^2 and so on. Each new problem is a special case of 3-PARTITION. It remains NP-hard in the strong sense until B and x_j are multiples of k^c, where c is a given positive integer. However, passing from a problem with B and x_j being multiples of k^c to a problem with B and x_j being multiples of k^m cannot be done by using multiplier k, and their proof fails. This passage needs multiplier k^{m-c}, but then the reduction is not pseudopolynomial.

Furthermore, if the logic of the authors is correct, then $\mathscr{P} = \mathscr{NP}$. Indeed, substituting k and m by m and $3m$, respectively, does not change the logic of the proof of Theorem 1. Then the statement of this theorem would be '3-PARTITION is NP-hard in the strong sense even if $B \geqslant m^{3m}$'. However, 3-PARTITION can be solved in $O(m^{3m})$ time by full enumeration. This means that the strongly NP-hard problem is solvable in $O(B)$ time, which is pseudopolynomial, and $\mathscr{P} = \mathscr{NP}$.

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