example shows that the blue force wins the combat earlier than in the case of Example 1, and that more tanks survive. This indicates that MacKay's model disperses firepower to attack subordinate targets and wastes firepower on invalid targets.

Conclusions

This viewpoint raises three comments gained from experience on MacKay's model and improves it. Two examples are used to illustrate the problems of MacKay's model and to demonstrate that the improved equations are able to make the attacker or defender assign firepower independently and effectively.

Response to 'Some comments on MacKay's Lanchester mixed forces model'

Journal of the Operational Research Society (2012) **63**, 1633.
doi:10.1057/jors.2012.10

The first point I would make in response is that what Liu *et al* characterize as 'MacKay's model' (their equations (1) and mine (12)) was only the first, provisional model in my paper, superseded by a better one in Section 4. Here I shall call this first model, the only model considered in the Viewpoint article, [M1]; the later model, in my Section 4, will be termed [M2]. I should emphasize that in my paper I did not particularly advocate either model as fully realistic; rather my intention, as with any prototypical Lanchester model, was to lay bare the mathematical argument by which the conditions embodied in the instantaneous loss-rates lead to different combat outcomes.

In all Lanchester models, a fundamental preliminary is to decide what is the unit of concentration. In [M1], in which fire is distributed at random in proportion to unit numbers, the unit of concentration is therefore that of *visibility*, of the ease with which the unit is acquired as a target. This might, for example, be a visual or radar cross-section. As Liu *et al* quite rightly point out, this neglects entirely the commander's ability to choose his/her fire distribution.

Doctrine and orders concerning fire distribution are clearly an important tactical means by which a commander can influence the outcome of a battle, and the purpose of [M2] was to explore them. The crucial feature of [M2] is that it allows the commander complete freedom to choose the fire distribution *before* the battle, but none at all to change it *during* the battle. This is an oversimplification, but, I would argue, is sufficiently common in real war to be a crucial model to understand. For example, consider the

References

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British Grand Fleet at Jutland, perhaps the nearest mankind has come to a perfectly Lanchestrian aimed-fire battle. Grand Fleet Battle Orders needed to be absolutely clear in advance about the required distribution of fire—for commanders' ability to nuance them in the fog of war and the smoke of battle was always going to be minimal.

In [M2], then, the initial freely-chosen fire distribution is thereafter influenced solely by the ease of finding targets of the required type. By exhibiting the quadratic conserved quantity of the model, analogous to that used in Lanchester's classic analysis, I found that the optimal solution, among *all* initial choices of fire distribution, is a hierarchical one: to target successive unit types in turn, ranked by the product of the mutual lethalties (ie, your ability to destroy your opponent multiplied by his to destroy you).¹

Liu *et al* do not make it clear how the fixed fire distribution in their model (2) is to be chosen, when or how it is to be changed, or what distribution is optimal in that model. Ironically, the strategy implied in their Example 2 appears to be exactly that of [M2] described above. The overarching point here is that, in the absence of battlefield feedback about the matrix of lethalties, such a strategy is optimal.

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¹Something similar is observed for the stochastic duel in Y. Friedman, Optimal Strategy for the One-Against-Many Battle, *Operations Research* 25 no. 5 (1977) 884–888. One might think of this as a precursor to a more sophisticated Gittins-index approach to targeting in which observation does feed back into changes of fire distribution: K. Glazebrook *et al.*, Index Policies for Shooting Problems, *Operations Research* 55 no. 4 (2007) 769–781.