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A stochastic dynamic programming model for valuing exclusivity from encroachment in franchising

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Valuing territorial exclusivity in franchising is difficult because of the uncertainty associated with variables such as future franchise sales and brand strength. We present a stochastic dynamic programming model to value the exclusivity option from the perspective of both the franchisor and the franchisee. When there is positive value to the franchisor of including the exclusivity option in the contract, and to the franchisee of purchasing this option, the likelihood of franchisor-franchisee encroachment-related conflict is reduced. We also discuss structural results and explain our results using a numerical example.

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Introduction

Business format franchise systems are an important part of the retailing sector in many developed countries. In these systems, the franchisor grants its franchisees the right to engage in a business developed by it using its brand name and trademark. Franchisees pay the franchisor a one-time fee when they join the system and pay ongoing royalties that are a percentage of their gross sales.

The franchise royalty payment structure misaligns the goals of the franchisor and franchisees. Because franchisors receive royalties on franchisees' gross sales, they seek to maximize system-wide gross sales whereas franchisees seek to maximize the net profits of their outlets. These goals are sometimes incompatible. A franchisor can authorize a new outlet in a location to increase system-wide sales and its royalty revenues, but this may negatively affect the sales and profits of existing franchisees in surrounding locations. This problem, termed territorial encroachment, is especially prevalent in mature franchise systems, where franchisors have few new domestic markets to exploit, thereby tempting them to seek growth by adding units in markets they already serve (Schneider *et al*, 1998).

Territorial encroachment creates conflict between the franchisor and franchisees. In fact, territorial encroachment related conflict is rampant in franchise systems (Hall, 2004). Retailing scholars have for long examined different approaches to manage the territorial encroachment problem in franchise systems. Recently Nair *et al* (2009) presented a novel approach to manage the encroachment problem. They propose that when a franchisee obtains the franchise rights for a particular location, the franchisor should offer the franchisee an option to buy territorial protection for an adjoining location that is presently marginal but could be a potential future location. They show that under certain circumstances a franchisor's foregone profit from franchising a new location is less than the franchisee's valuation of protection from future encroachment resulting from having a franchisee in the new territory, and therefore such an exclusivity option will be a win-win for both parties.

Nair *et al* (2009) model their approach using a simple two-period model. Their model enables them to validate their central thesis, but it is too simplistic to capture the many different paths in which the future could unfold. For modelling such problems, stochastic dynamic programming is ideally suited. In this paper we refine Nair *et al*'s (2009) deterministic model by developing a stochastic dynamic programming model (Bellman, 1957; Bertsekas, 1976) for valuing exclusivity from encroachment in franchising.

The remainder of the paper is organized as follows. We present an approach for valuing the exclusivity clause for a potential future location using a stochastic dynamic programming model. Next, we use a numerical example to illustrate our approach. We then present structural results and describe the sensitivity of the value of the option to

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various franchise system parameters. We conclude by discussing the limitations of our approach and the implications our approach has for reducing encroachment-related conflict.

Model

We focus on two locations A and B, separated by some distance from each other. All notations used in the model are listed in the appendix. Location A is the primary location being considered by the franchisor and franchisee to locate an outlet, and location B is a potential future site. The current and future population dynamics of these locations and other factors, such as franchise brand strength, will determine the initial decision of locating in A and not in B. We seek to determine the value to the franchisor and franchisee of locating in A only and the value of locating in A now but locating in B during any future period in the time horizon of the model. Since there may be incremental sales from locating at B in the future, the franchisor's value for locating in B at a future period will likely be positive, since royalties are based on franchisee revenue. Let the difference in value to the franchisor of locating in A now, and B in the future, and to locate in A alone be O_r. The franchisee on the other hand may lose sales to the location in B and will see a reduction in profits, if the location at B is given to another franchisee. Let this reduction in profits to the franchisee be O_e.

If at the time of entering into the contract of locating at A, the franchisor offers the franchisee an option of exclusive rights to location B, the cost of this option would be the value of foresaking the right of the franchisor to locating at B at any future period, that is O_r. On the other hand, the value to the franchisee of exclusivity to locations A and B is O_e. The franchisee will purchase this option as long as $O_e > O_r$. If O_r is non-negative, the franchisor also gains from selling this option. This upfront contract has a win-win outcome, which is likely to reduce potential discord in the franchisor–franchisee relationship.

We next present the dynamics at play and notation for our model.

The population dynamic

Suppose the population in locations A and B in period t are n_t^A and n_t^B, respectively. The population growth at A and B can be modelled using growth rates observed for the areas in a deterministic or stochastic manner. The stochastic modelling of population growth can be accomplished by using a distribution of population for each future period independent of the past, or as a Markov transition matrix dependent on the population in the previous period. We adopt the latter approach, without loss of generality of the methodology to the former approach. We assume that the population in A can be modelled as being in one of s^A discrete states, and the population in B as being in one of s^B discrete states. Suppose p_{ij}^A is the probablity that the population in location A is in state j in period $t+1$ given that it was in state i in period t, where $1 \leqslant i, j \leqslant s^A$ and $\sum_j p_{ij}^A = 1$ for every i. Also, there is a function ϕ^A that converts the state to actual head count of population in state i in period t, $n_t^A = \phi^A(i)$. Similarly p_{ij}^B represents the transition probabilities and ϕ^B the function that converts state to actual population for location B.

The brand strength dynamic

With increased opening of outlets, advertising and offering superior customer value, the franchise brand gets strengthened. This results in additional sales potential that can increase the profitability of all franchisees. Many of these issues are exogeneous to the decision problem we are considering. Much like with population dynamics, we could therefore model brand strength in future periods based on past performance in a deterministic manner using growth rates (there is the possibility of brand strength declining as well) or stochastically using independent distributions of brand strength in any period, or as a transition matrix with the brand strength dependent on the brand strength in the previous period. In the marketing literature, the Markov transition matrix has been used extensively for brand switching models (eg, Vilcassim and Jain, 1991; Lilien *et al*, 1992; Roy *et al*, 1996). As with population dynamics, suppose brand strength in a particular period t, b_t, can be modelled as being in one of B discrete states and q_{ij} is the probablity that the brand strength is in state j in period $t+1$ given that it was in state i in period t, $1 \leqslant i, j \leqslant B$ and $\sum_j q_{ij} = 1$ for every i.

The sales dynamic

Typically, potential sales at a location are a function of the population, brand strength, advertising, promotions and the like. Customers who have a choice between two alternate locations of the same brand will prefer one location over the other based on travel distance, traffic density, etc. In our model, we focus on population, brand strength and purchase preference between the two locations A and B, if both are open. Suppose the expected sales in dollars in a time period per 1000 population in either location depends on the brand strength and is denoted by $s(i)$, where i is the brand strength in that period.

Suppose if only location A is open, the purchase preference (the proportion of the population in a particular location who will purchase at that location) of potential customers living in and around location i to purchase at an outlet in location A is $u(i)$ where $i \in \{A, B\}$. If both locations A and B are open, suppose the purchase preference of potential customers living in and around location i to purchase at an outlet in location j is $v(i, j)$ where $i, j \in \{A, B\}$.

Other parameters in the model

Let the initial upfront fee that the franchisor charges the franchisee be I, the development costs borne by the franchisor for each location be D, the per period fixed cost for running an outlet for the franchisee be F, and the variable cost per 1000 customers be V. Let the royalty fee charged by the franchisor be ρ% of the total sales and let the one period discount factor be β.

Stochastic dynamic programming model

In any period t, the state of the system can be characterized as (m^A, m^B, b, l), where m^A and m^B are the states of population in location A and B, respectively, b is the brand strength, and l is the number of locations open, where $l = 1$ when only location A is open, and $l = 2$ when both locations A and B are open. The one-period revenue to the *franchisor* when only A is open, $R_t^r(m^A, m^B, b, 1)$, can be computed to be

$$R_t^r(m^A, m^B, b, 1) = s(b)\rho \left[\sum_{z \in \{A,B\}} u(z)\phi^z(m^z) \right] \Big/ 1000 \qquad (1)$$

When outlets are open at both locations A and B, the one-period revenue to the *franchisor*, $R_t^r(m^A, m^B, b, 2)$, can be computed to be

$$R_t^r(m^A, m^B, b, 2) = s(b)\rho \left[\sum_{y \in \{A,B\}} \sum_{z \in \{A,B\}} v(y, z)\phi^y(m^y) \right] \Big/ 1000 \qquad (2)$$

We assume that the outlet in location A is opened at the intial period, $t = 0$, at which time the initial fee is collected and the development costs are incurred by the franchisor. Let the model horizon be T. Suppose $f_t^r(m^A, m^B, b, l)$ is the discounted present value of net profits to the *franchisor* from being in state (m^A, m^B, b, l) in period t and taking optimal actions in each period until the time horizon of the problem, T. Then this value can be computed using the following recursive equation:

$$f_t^r(m^A, m^B, b, l) = \max \begin{cases} Open\_at\_B: & I - D + R_t^r(m^A, m^B, b, 1) + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} f^r_{t+1}(i, j, k, 2) & t > 0, l = 1 \\ DoNothing: & R_t^r(m^A, m^B, b, l) + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} f^r_{t+1}(i, j, k, l) & t > 0 \\ Open\_at\_A: & I - D + R_t^r(m^A, m^B, b, 1) + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} f^r_{t+1}(i, j, k, 1) & t = 0, l = 1 \end{cases} \qquad (3)$$

There are three actions possible in each state, as shown in the three rows of (3). In the initial period, the action is to open an outlet at A. There are no other actions allowed in the initial period. In subsequent periods, the actions are either to do nothing or to open an outlet at location B. In the two actions pertaining to opening outlets, the franchisor collects an initial fee, invests in development costs and collects royalty revenue. The last term in each row is the value of being in any future period having taken the optimal decision in the current period. These values are discounted to the current period using the discount factor β. The populations in and around the two locations and the brand strength probabilistically unfold into the future, as reflected in the three transition probability terms. If the location in B is opened, the state in the next period will have two locations open, $l = 2$, as shown in the first row.

The boundary condition, at $t = T$, for the above recursion for all possible states is

$$f_T^r(m^A, m^B, b, l) = 0 \qquad (4)$$

Let $\tilde{l}_t \equiv \delta_t^r(m^A, m^B, b, l_t)$ be the optimal action taken by the *franchisor* in each state.

The profits of the *franchisee* who invests in location A can be determined by following the paths that the optimal franchisor decisions take. Suppose $f_t^e(m^A, m^B, b, l)$ is the profit made by the franchisee in each state (m^A, m^B, b, l) in period t. If $R_t^e(m^A, m^B, b, l)$ is the one-period revenue earned by the franchisee, it can be computed by

$$\eta(m^A, m^B, l) = \begin{cases} \sum_{z \in \{A,B\}} u(z)\phi^z(m^z) & l = 1 \\ \sum_{y \in \{A,B\}} \sum_{z \in \{A\}} v(y, z)\phi^y(m^y) & l = 2 \end{cases} \qquad (5)$$

$$R_t^e(m^A, m^B, b, l) = -F + \eta(m^A, m^B, l) \times [-V + s(b)(1 - \rho)]/1000 \qquad (6)$$

For notational convenience we introduce

$$\kappa_t^e(m^A, m^B, b, l) \equiv R_t^e(m^A, m^B, b, l) + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} f^e_{t+1}(i, j, k, l) \qquad (7)$$

Then, $f_t^e(m^A, m^B, b, \tilde{l})$ can be calculated using the following equation.

$$f_t^e(m^A, m^B, b, \tilde{l}_t) = \begin{cases} \kappa_t^e(m^A, m^B, b, \tilde{l}_{t+1}) & t > 0 \\ -I + \kappa_t^e(m^A, m^B, b, \tilde{l}_{t+1}) & t = 0 \end{cases} \quad (8)$$

The boundary condition would be $f_T^e(m^A, m^B, b, \tilde{l}_t) = 0$ for all states. From Equation (5), it should be clear that $\eta(i,j,1) - \eta(i,j,2)$, usually a negative number, is the loss in total customers if only one location is opened.

Suppose the recursion in (3) is solved without the Open_at_B action available to give $\hat{f}_t^r(m^A, m^B, b, l)$ and the corresponding solution to (8) is $\hat{f}_t^e(m^A, m^B, b, \tilde{l}_t)$. Then the value of the option to the franchisor of giving the franchisee exclusivity to areas A and B can be computed as:

$$O^r = f_0^r(m^A, m^B, b, 1) - \hat{f}_0^r(m^A, m^B, b, 1) \quad (9)$$

And the value to the franchisee of obtaining the option of exclusivity at both locations A and B can be computed as:

$$O^e = \hat{f}_0^e(m^A, m^B, b, 1) - f_0^e(m^A, m^B, b, 1) \quad (10)$$

As long as $O^e > O^r$ the franchisee has an incentive to purchase this option, and as long as $O^r > 0$, the franchisor should be willing to sell this option, since the franchisor receives up front the stream of revenue that would accrue from a future opening of an outlet at location B.

A numerical example

We next present a numerical example to illustrate our approach. Suppose the various dynamics present are as detailed below.

The population dynamic

We use a Markov transition matrix to model the population growth at locations A and B. We assume that the population at A can be modelled as being in one of 10 discrete states, and the population in B as being be in one of 10 discrete states. The transition matrices for A and B are shown below.

Population growth transition matrix at location A

A	1	2	3	4	5	6	7	8	9	10
1	0.7	0.3	0	0	0	0	0	0	0	0
2	0.2	0.6	0.2	0	0	0	0	0	0	0
3	0	0.2	0.6	0.2	0	0	0	0	0	0
4	0	0	0.2	0.6	0.2	0	0	0	0	0
5	0	0	0	0.2	0.6	0.2	0	0	0	0
6	0	0	0	0	0.2	0.6	0.2	0	0	0
7	0	0	0	0	0	0.2	0.6	0.2	0	0
8	0	0	0	0	0	0	0.2	0.6	0.2	0
9	0	0	0	0	0	0	0	0.2	0.6	0.2
10	0	0	0	0	0	0	0	0	0	1

Population growth transition matrix at location B

B	1	2	3	4	5	6	7	8	9	10
1	0.3	0.7	0	0	0	0	0	0	0	0
2	0.2	0.2	0.6	0	0	0	0	0	0	0
3	0	0.2	0.2	0.6	0	0	0	0	0	0
4	0	0	0.2	0.2	0.6	0	0	0	0	0
5	0	0	0	0.2	0.2	0.6	0	0	0	0
6	0	0	0	0	0.2	0.2	0.6	0	0	0
7	0	0	0	0	0	0.2	0.2	0.6	0	0
8	0	0	0	0	0	0	0.2	0.2	0.6	0
9	0	0	0	0	0	0	0	0	0.2	0.8
10	0	0	0	0	0	0	0	0	0	1

Also, the function used to convert the state to actual head count of population is a linear function, $\phi^A(i) = 5000 + 1000(i)$. For example, at state 3, the population will be $5000 + 1000 \times 3 = 8000$. Similarly, the function that converts state to actual population for location B is $\phi^B(i) = 2500 + 500(i)$.

The brand strength dynamic

Suppose brand strength is at three levels, high, medium and low, denoted by states H, M and L, respectively. Suppose the transition matrix for brand strength is

	H (%)	*M (%)*	*L (%)*
H	70	20	10
M	10	80	10
L	0	20	80

The sales dynamic

Suppose the expected dollar sales per 1000 population in any time period is based on brand strength, $s(i)$ is

	H	*M*	*L*
Sales per 1000 pop ($)	15,000	25,000	50,000

If only location A is open, the purchase preference $u(i)$ to purchase at an outlet in location A is

	Shopping in
Living in	Area A
Area A	80%
Area B	50%

If both locations A and B are open, the purchase preference matrix $v(i, j)$ is

	Shopping in	
Living in	Area A	Area B
Area A	80%	20%
Area B	20%	80%

Other parameters in the model

Assume the initial upfront fee that the franchisor charges the franchisee, I, is \$35 000, the development costs borne by the franchisor for each location, D, is \$50 000, the per period fixed cost for running an outlet for the franchisee, F, is \$25 000 and the variable cost, V, per 1000 customers is \$12 000. Let the royalty fee charged by the franchisor, ρ, be 8% of the total sales and let the one period discount factor, β, be 90%.

Results

On running the stochastic dynamic programming model in (3) for a 10-period horizon with and without the Open_at_B action, and from (9), we obtain the following results

$$f_0^r(5,5,2,1) = 141,524$$
$$\hat{f}_0^r(5,5,2,1) = 119,227$$
$$O^r = 22,297$$

On running the recursion in (8) for a 10-period horizon with and without the Open_at_B action, and from (10), we obtain the following results:

$$f_0^e(5,5,2,1) = 360,898$$
$$f_0^e(5,5,2,1) = 405,387$$
$$O^e = 44,489$$

Since $O^e > O^r$, the franchisee will purchase the exclusivity option offered by the franchisor for \$22 297 for a 10-year period, because she values exclusivity in that location at \$44 489. This signifies a win-win outcome for both the franchisor and the franchisee, which is likely to result in the avoidance of conflict.

Structural results

In this section, we examine some structural results for our model and the sensitivity of the value of exclusivity to various model parameters. We start by presenting two lemmas.

Lemma 1 For all i, j, k and $t<T, f_t^r(i,j,k,2)-f_t^r(i,j,k,1)$ is

(a) non-decreasing in population of A and B, and is independent of $\eta(i,j,1)$;
(b) non-decreasing in sales at brand strength b, $s(b)$, and brand strength b;
(c) non-decreasing in royalty fee, ρ;
(d) a step function in variable and fixed costs of operating a franchise, V and F, respectively.

Proof From (3), we have

$$\begin{aligned} f_t^r(i,j,k,2) - f_t^r(i,j,k,1) &= [R_t^r(i,j,k,2) - R_t^r(i,j,k,1)] \\ &+ \min\Big\{ D - I,\ \beta \sum_x \sum_y \sum_z p_{ix}^A p_{jy}^B q_{kz} \\ &\times [f_{t+1}^r(x,y,z,2) - f_{t+1}^r(x,y,z,1)]\Big\} \end{aligned} \quad (11)$$

It can be verified that $[R_t^r(i,j,k,2)-R_t^r(i,j,k,1)]$ is non-decreasing in the population of A and B and is independent of the values of $u(i)$ and $v(i,j)$. We can also verify from (4) that all the results (a)–(d) are true at T. Suppose the result was true at $t+1$. Then by induction we can conclude that (a) is true at t.To prove (b) and (c), note from (1) and (2) that

$$\begin{aligned} R_t^r(m^A, m^B, b, 2) &- R_t^r(m^A, m^B, b, 1) \\ &= s(b)\rho \Bigg[\sum_{y\in\{A,B\}} \sum_{z\in\{A,B\}} v(y,z)\phi^y(m^y) \\ &- \sum_{z\in\{A,B\}} u(z)\phi^z(m^z) \Bigg] \Big/ 1000 \end{aligned} \quad (12)$$

And since $s(b)$ is increasing in b, substituting in (11) gives the result by induction.

It can be also shown that beyond a particular value of V and F, the location B will never be opened. Until that point, the expression is independent of F, proving (d) by induction. □

Lemma 2 For all i, j, k and $t<T$, $f_t^e(i,j,k,1)-f_t^e(i,j,k,2)$

(a) varies by population of A depending on $\eta(i,j,1)-\eta(i,j,2)\ldots$ if $\eta(i,j,1)-\eta(i,j,2)$ is non-decreasing (constant, non-increasing) in population of A, then $f_t^e(i,j,k,1)-f_t^e(i,j,k,2)$ is non-decreasing (constant, non-increasing) in population of A;
(b) is non-decreasing in population of B;
(c) is non-decreasing in sales at brand strength b, $s(b)$ and brand strength b;
(d) is non-increasing in royalty fee, ρ, and variable costs V;
(e) is a step function in fixed costs of operating a franchise, F.

Proof From (8), we have

$$\begin{aligned} f_t^e(i,j,k,1) - f_t^e(i,j,k,2) &= [R_t^e(i,j,k,1) - R_t^e(i,j,k,2)] \\ &+ \beta \sum_x \sum_y \sum_z p_{ix}^A p_{jy}^B q_{kz} \\ &\times [f_{t+1}^e(x,y,z,1) - f_{t+1}^e(x,y,z,2)] \end{aligned} \quad (13)$$

We can verify from the boundary condition that all the results (a)–(e) are true at T. From (5) we know that $[R^e_t(i,j,k,1) - R^e_t(i,j,k,2)]$ is a function of $\eta(i,j,1) - \eta(i,j,2)$ is non-decreasing (constant, non-increasing) in population of A, then $[R^e_t(i,j,k,1) - R^e_t(i,j,k,2)]$ is non-decreasing (constant, non-increasing) in population of A. Suppose (a) was true at $t+1$. Then by induction we can conclude that (a) is true at t. It can be shown that $\eta(i,j,1) - \eta(i,j,2)$ is always non-decreasing in the population of B. Using the above logic, (b) is proved. To prove (c) and (d), note from (6) that

$$\begin{aligned} R^e_t(m^A, m^B, b, 1) - R^e_t(m^A, m^B, b, 2) \\ = [\eta(m^A, m^B, 1) - \eta(m^A, m^B, 2)] \\ \times [-V + s(b)(1-\rho)]/1000 \end{aligned} \quad (14)$$

And since $s(b)$ is increasing in b, substituting in (13) gives the result (c) to be true at t. Similarly it is evident from (14) that (d) is true at t. The results then follow by induction. It can be also shown that beyond a particular value of F, the location B will never be opened. Until that point, the expression is independent of F, proving (e) by induction. □

We next use these two lemmas to prove the two main results of the paper.

Theorem 1 For all i, j, k and $t < T$, the value of the option of exclusivity to the franchisor, O^r, is

(a) non-decreasing in population of A and B, and is independent of $\eta(i,j,1)$;
(b) non-decreasing in sales at brand strength b, $s(b)$ and brand strength b;
(c) non-decreasing in royalty fee, ρ;
(d) a step function in variable and fixed costs of operating a franchise, V and F, respectively;
(e) non-decreasing in initial fee, I.

Proof From (9) and (3), we know that

$$O^r = \max \begin{cases} I - D + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} [f^r_{t+1}(i,j,k,2) \\ \quad - f^r_{t+1}(i,j,k,1)] \\ 0 \end{cases} \quad (15)$$

Notice from (11) and (15) that the initial fee has a net effect of $(1-\beta)I$ on the value of the option. The results then directly follow this observation and from Lemma 1 using induction. □

Theorem 2 For all i, j, k and $t < T$, the value of the option of exclusivity to the franchisee, O^e,

(a) varies by population of A depending on $\eta(i,j,1) - \eta(i,j,2)$. If $\eta(i,j,1) - \eta(i,j,2)$ is non-decreasing (constant, non-increasing) in population of A, then $f^e_t(i,j,k,1) - f^e_t(i,j,k,2)$ is non-decreasing (constant, non-increasing) in population of A;
(b) is non-decreasing in population of B;
(c) is non-decreasing in sales at brand strength b, $s(b)$, and brand strength b;
(d) is non-increasing in royalty fee, ρ, and variable costs V;
(e) is a step function in fixed costs of operating a franchise, F;
(f) is independent of initial fee, I.

Proof From (10), (8) and (7), we have

$$O^e = \beta \begin{cases} 0 & \tilde{l}_t = 1 \\ R^e_t(m^A, m^B, b, 1) - R^e_t(m^A, m^B, b, 2) & \\ \quad + \beta \sum_i \sum_j \sum_k p^A_{m^A i} p^B_{m^B j} q_{bk} [f^e_{t+1}(i,j,k,1) f^e_{t+1}(i,j,k,2)] & \tilde{l}_t = 2 \\ \quad [f^e_{t+1}(i,j,k,1) f^e_{t+1}(i,j,k,2)] & \end{cases} \quad (16)$$

The results then directly follow from Lemma 2 using induction and the observation that the above expression is independent of the initial fee, I. □

Next we illustrate these results using the same data as the numerical example, but varying one variable at a time.

Varying population in A and B

Figure 1 shows the value of the option of exclusivity when the population of location A increases over the years. It confirms the result in Theorem 2 that the value to the franchisee with population increase in A depends on how $\eta(i,j,1) - \eta(i,j,2)$ changes with population in A. Following Theorem 1, we see that the value to the franchisor is always non-decreasing. As the preference for purchasing at location A, $v(1,1)$, increases, the value of $\eta(i,j,1) - \eta(i,j,2)$ decreases with increase in population at A, making the franchisee less likely to value the option of exclusivity.

Figure 2 shows the effect of increase in the population of B. As shown in Theorems 1 and 2, the value to both the franchisor and the franchisee is non-decreasing with population in B.

Varying brand strength and sales rate

Figure 3 shows the effect of increasing brand strength and sales rate. Notice that as sales rate per 1000 population or brand strength increases, the value of the option of exclusivity is non-decreasing for both the franchisee and the franchisor, as is predicted in Theorems 1 and 2. The value, however, increases faster for the franchisee. This indicates that if franchisee and franchisor estimate that the brand strength and sales potential are likely to increase, they will be more willing to purchase the option of exclusivity.

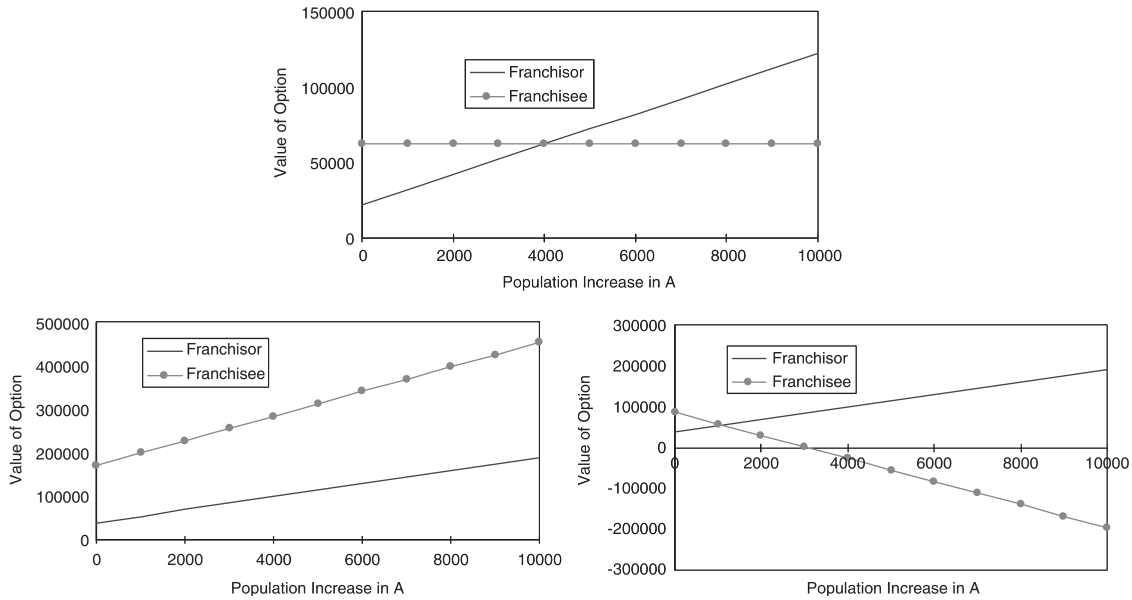

Figure 1 Varying population in A, for (clockwise from top) $\eta(i, j, 1)-\eta(i, j, 2)$ constant, non-increasing and non-decreasing.

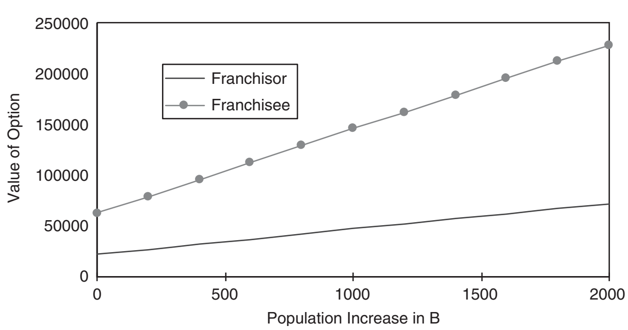

Figure 2 Varying population in B.

Varying fixed and variable costs

Consistent with Theorems 1 and 2, Figure 4 shows that the value of the exclusivity option to both the franchisee and franchisor does not change with increasing fixed costs until a point beyond which the value of the option is zero for both, since at those costs location B will never be opened. However, as shown in Theorem 2, increasing the variable costs will reduce the value of the option to the franchisee since the cost of doing business increases significantly. This variable cost increase, however, has no effect on the value of the option to the franchisor. As with the fixed cost, beyond a certain point increasing variable cost is too great for the franchisee to remain profitable, then, of course, the value of the option is zero.

Varying initial and royalty fees

Figure 5 shows that changes in the initial fee do not affect the value of the option to the franchisee, confirming what we observed in Theorem 2. On the other hand, the value to the franchisor of conceding exclusivity increases, as we proved in Theorem 1, since the fee the franchisor collects from opening B increases. Figure 5 also confirms the results on the impact of royalty fee as we showed in Theorem 2. As the royalty fee increases, the value of the option of exclusivity to the franchisee decreases, since less of the revenue can be retained by the franchisee. The opposite is true for the franchisor as shown in Theorem 1.

Conclusion and managerial implications

Summary

In this paper, we refine the approach of Nair *et al* (2009) for business format franchisors to increase system wide revenues without antagonizing existing franchisees by including uncertainty and a number of additional parameters in the model. Nair *et al* (2009) propose that the franchisor provide a new franchisee an option to buy territorial exclusivity for an adjoining location that is initially marginal, but may develop into a franchise location in the future. We develop a stochastic dynamic programming model to value this exclusivity from the viewpoint of both the franchisor and franchisee. The stochastic dynamic programming model enables us to capture the different possibilities of how sales at a particular franchise location might unfold over time, something that cannot be done using traditional discounted cash flow and net present value analyses. We examine some structural results and explain the results using a numerical example and then run a sensitivity analysis to show how the value of exclusivity changes when different parameters of the model vary. Our

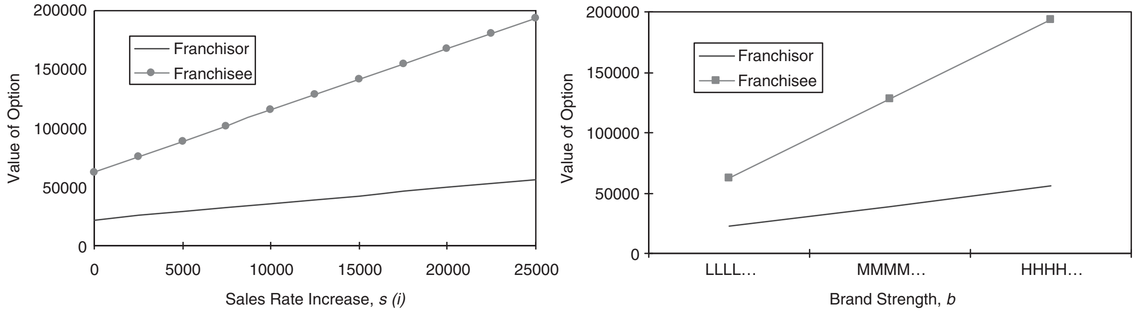

Figure 3 Varying sales rate and brand strength.

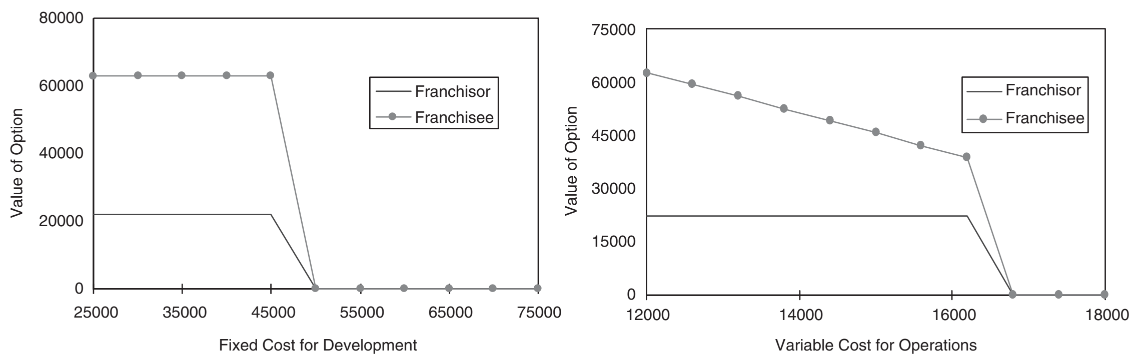

Figure 4 Varying fixed and variable costs for franchises.

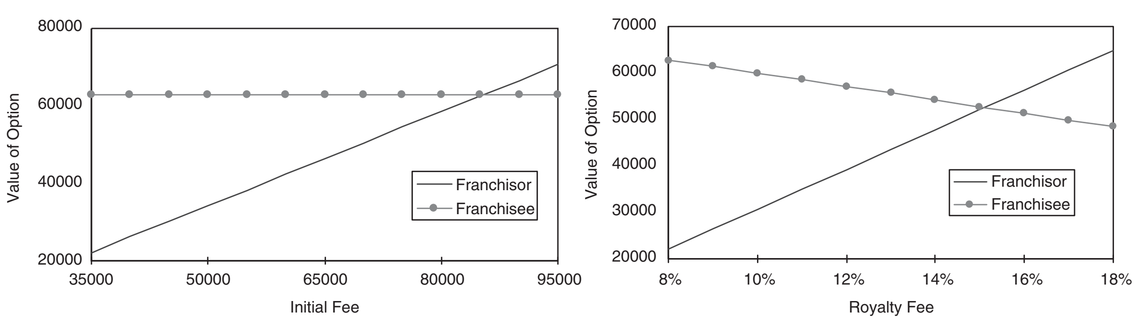

Figure 5 Varying initial and franchise fee.

results show that the proposed exclusivity option creates positive value for both the franchisor and the franchisee, and often the value of the option to the franchisee is greater than the value to the franchisor, thereby indicating that franchisees would find value in buying territorial protection options if these were offered by the franchisor.

Managerial implications

As discussed in the introduction section, territorial encroachment is a source of conflict between franchisors and franchisees. In this paper, we provide an approach that franchisors can use to mitigate encroachment-related conflict. We provide conditions under which this conflict may be mitigated. As can be seen from the figures in the paper, it is not always true that the franchisee's value of the exclusivity option is higher than the franchisor's value. In such cases, the exclusivity contract cannot be ratified by both parties. Adopting our model will require the franchisor to plan ahead and forecast population growth and other parameters such as brand strength, which they probably already do. Our model then allows them to put these inputs in a model in a meaningful way and obtain results that can help them grow their franchises while reducing channel conflict. The process of forecasting these parameters in and of itself will be beneficial to the franchisor, if they are not doing so already.

The exclusivity valuation is based on projections of how the future unfolds. The actual events may differ from the projections. To manage any fallout from forecasting errors, franchisors should write franchise contracts that allow for revision of initial terms contingent upon how the future unfolds.

The franchisor can offer exclusivity contracts only if the contract creates value for the franchisee. Our structural results show that population increase in the secondary location, greater brand strength and increasing sales rate make the option valuable to the franchisor, but make it even more valuable to the franchisee. On the other hand, increases in variable cost reduce the value for the franchisee without affecting the franchisor, and increase in royalty rates reduces the value for the franchisee but increases the value for the franchisor. Clearly population dynamics are not in the franchisor's control, but parameters such as brand strength are. This suggests that for growing the franchise with minimal channel conflict, a franchisor will have to pay attention to factors within its control that it otherwise might not have worried about, even if it will have to sacrifice some of its short-term self-interest.

Limitations

One drawback of offering territorial exclusivity is that it is difficult for the franchisor to retract the decision should the need arise. In our approach, the future values of the option to the franchisee could be computed in advance for various possible states of the system to enable the franchisor to purchase back the option of exclusivity at a future time, if such a need arises. Likewise, our analysis is based on offering the option to a new franchisee at the time of the contract. Should a franchisor want to offer the option to existing franchisees, it is possible to use our approach to value such an option.

Our option-based proposal is motivated by the growing research on real options, where the principles of financial option analysis are used to factor real world uncertainties into investment decisions. Although, our analysis cannot technically be labelled real options analysis, it does have many significant parallels of a real options analysis.

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Appendix

Notations

A	the primary location being considered
B	a potential future site
I	the initial upfront fee that the franchisor charges the franchisee
D	the development costs borne by the franchisor for each location
F	the per period fixed cost for running an outlet for the franchisee
V	the variable cost per 1000 customers
ρ	the royalty fee charged by the franchisor as a % of the total sales
β	the one period discount factor
O_r	the cost to franchsior of the exclusive rights option offered to the franchisee
O_e	the value to the franchisee of exclusivity
n_t^A (n_t^B)	the population in the location A (B) in period t
s^A (s^B)	the number of discrete states of the population in A (B)
p_{ij}^A (p_{ij}^B)	the transition probablity that the population in location A (B) is in state j in period $t+1$ given that it was in state i in period t, where $1 \leqslant i, j \leqslant s^{A(B)}$
ϕ^A (ϕ^B)	the function that converts the state to actual head count of population for location A (B) in state i in period t, $n_t^A = \phi^A(i)$
b_t	the brand strength in a particular period t,
B	the number of discrete states of brand strengh
q_{ij}	the transition probablity that the brand strength is in state j in period $t+1$ given that it was in state i in period t, $1 \leqslant i, j \leqslant B$
$s(i)$	the expected sales in dollars in a time period per 1000 population in either location depending on the brand strength i
$u(i)$	the purchase preference of potential customers living in and around location i to purchase at an outlet in location A when only location A is open, where $i \in \{A, B\}$.
$v(i, j)$	the purchase preference of potential customers living in and around location i to purchase at an outlet in location j when both locations A and B are open, where $i, j \in \{A, B\}$

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