weight attaching to old data decays linearly and, after an interruption of demand such that $k\alpha > 1$, this weight actually becomes negative. This would be expected to initiate an exception routine.

Interestingly, there is a corresponding, though less stringent, limitation upon Johnston's more highly developed formula and he specifically cautions against adaptive factors which may, in some circumstances, be greater than unity.

My own formula clearly reverts to standard exponential smoothing so long as there are no 'gaps' in demand. Further, the values of the adaptive factor for low k seem consistent with those of Johnston. The formula also gives a rational response to fractional values of k (i.e. data collected during only part of a normal review period) although this aspect was not considered in the original application.

The writer for one would be interested to know whether such a simplified smoothing formula is suitable for Third World or small business situations—perhaps imposing some restrictions upon the maximum value of k—as a first step into genuinely adaptive forecasting, where demand has been too irregular for normal exponential smoothing to be perceived as working correctly.

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Reference

1. F. R. Johnston (1993) Exponentially Weighted Moving Average (EWMA) with irregular updating periods. *J. Opl Res. Soc.* **44**, 711–716.

SIMPLER EXPONENTIALLY WEIGHTED MOVING AVERAGES WITH IRREGULAR UPDATING PERIODS

Johnston[1] gives a lengthy derivation for a smoothing factor A for use in a moving average forecast when k periods of data have been combined together before including them in an EWMA recurrence relation, as follows:

$$m_{t+k-1} = (1 - A)m_{t-1} + AY_t/k$$

where

$$A = \frac{k(6\alpha + 3\alpha^2(k-1))}{6 + 6\alpha(k-1) + \alpha^2(2k^2 - 3k + 1)}. \tag{1}$$

Another approach is to assume that the combined value Y_t is evenly spread over the k periods. Updating each month individually and substituting in the one period ahead recurrence relation then gives

$$m_{t+k-1} = (1-\alpha)^k m_{t-1} + \alpha Y_t/k \sum_{i=0}^{k-1} (1-\alpha)^i,$$

resulting in

$$A = 1 - (1-\alpha)^k. \tag{2}$$

Formula (2) and its derivation are simpler than (1). It gives values of A which correspond closely to (1) for low values of α and k. For higher values of α and k, larger discrepancies occur as values of A from (1) tend towards or even exceed 1.0, while (2) remains obstinately on the side of common sense.

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Reference

1. F. R. Johnston (1993) Exponentially Weighted Moving Average (EWMA) with irregular updating periods. *J. Opl Res. Soc.* **44**, 711–716.