Viewpoints

DANTZIG–WOLFE DECOMPOSITION ALGORITHM

MR. GONCALVES's attempt to derive a criterion[1] for recognizing when a basic feasible solution to the master program in Dantzig–Wolfe's decomposition method corresponds to a non-basic feasible solution of the original problem is both inaccurate and unhelpful.

The original problem is

$$\text{Max}\, x_0 = \mathbf{C}_1\mathbf{X} + \mathbf{C}_2\mathbf{Y}$$

subject to

$$\mathbf{A}_1\mathbf{X} + \mathbf{A}_2\mathbf{Y} = \mathbf{b} \ldots m \text{ equations,}$$

$$\mathbf{AX} \leqslant \mathbf{p} \ldots n \text{ inequalities,} \quad \mathbf{X} \geqslant 0, \quad \mathbf{Y} \geqslant 0.$$

$\mathbf{X}$ and $\mathbf{Y}$ are column vectors of dimension r and s respectively.

If we take a column vector $\mathbf{Z}$ of dimension n and transform

$$\mathbf{AX} \leqslant \mathbf{p} \quad \text{into} \quad \mathbf{AX} + \mathbf{IZ} = \mathbf{p}$$

then a basic feasible solution is obtained if and only if

$$V = V_x + V_y + V_z \geqslant r + s - m,$$

where $V_x =$ the number of vanishing x_i,

$V_y =$ the number of vanishing y_i,

and $V_z =$ the number of vanishing z_i

in the solution.

Mr. Goncalves omits V_z and writes $V = V_x + V_y \geqslant r + s - m$ as necessary and sufficient condition.

However, his equation for V_x is really the equation for $V_x + V_z$, so that the two errors compensate for each other, producing the correct result.

But what is the point of it all? To use his criterion we have to go from a basic feasible solution of the master program, to the corresponding solution of the original problem. Having done that, why is it better to use a formula requiring the dimension of the face that contains $\mathbf{X}$ as in the convex polyhedron $\mathbf{AX} \leqslant \mathbf{p}$, $\mathbf{X} \geqslant 0$, the degree of degeneracy of that face and of the basic feasible solution to the master program, and the number of not-vanishing t_i variables in the master program than to count simply $V_x + V_y + V_z$?

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REFERENCE

[1] A. S. GONCALVES (1968) Basic feasible solutions and the Dantzig–Wolfe decomposition algorithm. *Opl Res. Q.* **19**, 465.