Original Article

The procyclicality of hedge fund alpha and beta

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ABSTRACT   Traditional financial institutions as banks follow procyclical risk strategies, that is, they increase their leverage in economic expansions and reduce it in contractions, which leads to a procyclical behaviour for their betas and other risk and financial performance measures. We study the cyclical aspects of hedge fund strategies, an issue quite overlooked in the literature. We rely on two procedures: conditional modelling and Kalman filtering of hedge funds' alpha and beta. We find that hedge fund betas are usually procyclical. Our results also show that the alpha is often high at the beginning of a market upside cycle but, as the demand pressure increases, it progressively shrinks, which suggests that the alpha puzzle ought to be analysed in a dynamic setting.

INTRODUCTION

Procyclical risk analysis is now one of the main concerns for researchers working in the field of financial institutions, especially in banking research and macroprudential analysis (Amato and Furfine, 2004; Heid, 2007; Rajan, 2005, 2009; Shin, 2009; Adrian and Shin, 2010; Jacques, 2010; Gennaioli et al, 2011). However, these analyses are often cast in a static setting. Moreover, the hedge fund industry, despite its growing share in the world financial system,[1] is quite neglected in these analyses of risk procyclicality or merged with shadow banking. For instance, the principal factor at the root of the

recent financial crisis in the United States emanated from the subprime mortgage sector in which hedge funds are much involved. Indeed, according to Adrian and Shin (2010), the share of hedge funds in the origination of US subprime mortgages by the leveraged financial sector was then as high as 32 per cent, which suggests that hedge funds may generate important financial shocks having repercussions throughout the whole economy.

Hedge funds have thus become one of the pillars of the financial system, especially in the United States. The linkages between the banking sector and the hedge fund industry are so tight that a financial crisis originating from hedge funds may endanger financial stability, especially via the repos market (for example, the LTCM[2] episode). Moreover, pension funds' savings are increasingly invested in hedge funds, the return on conventional savings vehicles being on a downward trend.[3] Hence, the social impact of the bankruptcy of a major hedge fund should not be overlooked because it may impair seriously the pensioners' well-being, as was the case during the last subprime crisis. Moreover, a failure of an important financial institution may entail externalities for the whole financial system (Vives, 2010). In this respect, the risk taking of a major hedge fund may create aggregate shocks that could give raise to contagion effects (Vives, 2010; Gennaioli et al, 2011). Macroprudential analysis must give rise to policies that endogenize these systemic risks.[4]

To shed more light on the procyclical risk in the hedge fund industry, we test two hypotheses using data on the Greenwich[5] US hedge funds. First, we document an unknown stylized fact in the hedge fund industry. Indeed, we observe a high volatility at business cycle frequencies in the returns spectrum of many hedge fund strategies.

We thus check this apparent procyclicality of risk management in the hedge fund industry by using two kinds of approaches: the conditional coefficient model and a state-space model based on the Kalman filter. We use these setups to study the cyclical behaviour of the hedge funds' alphas and betas categorized by strategies. Our methodology allows us studying the cyclical components of hedge fund performance for the whole industry and then by strategy. Financial procyclicality is related to the amplification of aggregate risk by financial institutions at the business cycle frequencies (Altman and Saunders, 2001; Berger and Udell, 2004; Jacques, 2010). Since hedge fund are known to be very leveraged, there is a suspicion that this procyclical effect exists as the beta is positively related to financial leverage, a ratio which usually increases in economic expansions and decreases in contractions.

The second hypothesis we test concerns the prevalence of the alpha puzzle in a dynamic setting, especially at business cycle frequencies, a problem also neglected in the literature (Racicot and Théoret, 2007b, 2009). Indeed, the alpha puzzle is mostly studied in a static setting, which may bias the estimated alpha. We rather cast the alpha analysis in a dynamic setting to study more closely the alpha puzzle. We test if the alpha has also a tendency to be procyclical. In this respect, we conjecture that the alpha increases at the beginning of an economic recovery but, as demand pressures increase, it decreases progressively thereafter, the increase in the financial institutions' leverage contributing to the rise of demand pressures, the rise in asset prices and the fall in returns (Danthine and Donaldson, 2002; Geanakoplos, 2010). Unfavourable macroeconomic shocks related to a developing recession or crisis should also depress the alpha.

We thus suspect that the resulting overestimated alphas trend downward with the maturation of an economic recovery. The puzzle related to the bloated alpha must thus be analysed in a dynamic setting.[6] Moreover, this decrease in the alpha at the end of an economic recovery may be used as a leading or forward-looking performance indicator in the hedge fund industry.

The organization of this article is as follows. In the next section, we present the dynamic models of hedge fund returns. The subsequent section provides the stylized facts on hedge fund returns and our empirical results. The last section concludes.

CYCLICAL FRAMEWORK FOR ANALYSING HEDGE FUND RETURNS

The Kalman filtering approach

A Kalman filter model is usually made up of a signal or observation equation and of state or transition equations for the unobserved variables or coefficients.[7] To explain the expected hedge fund excess returns, we choose, as signal equation, the three-factor Fama and French (1992, 1993 and 1997) model:[8]

$$\left(R_p - R_f\right)_t = \alpha_t + \beta_{1t}\left(R_m - R_f\right)_t + \beta_2 SMB_t$$
$$+ \beta_3 HML_t + \beta_4\, gprod_t + \varepsilon_t \qquad (1)$$

where $R_p - R_f$ is the excess return of a portfolio, R_f being the risk-free return; $R_m - R_f$, the market risk premium; SMB, a portfolio that mimics the 'small firm anomaly', which is long in the returns of selected small firms and short in the returns of selected big firms; HML, a portfolio that mimics the 'value stock anomaly', which is long in returns of stocks of selected firms having a high book value/market value ratio (value stocks) and short in selected stocks having a low book

value/market value ratio (growth stocks); $gprod$, the annual industrial production growth rate exponentially smoothed with the Holt–Winter method. α_t is the time-varying alpha, β_{1t}, the time-varying beta, and ε_t, the innovation.

To explain the excess return of a portfolio, Fama and French added to the CAPM market risk premium two other risk factors: the SMB and HML ones. We omit in this equation the momentum factor proposed by Jegadeesh and Titman (1993) and Carhart (1997) because the influence of this factor is weak for the majority of hedge fund strategies, a state space model estimated with the Kalman filter having to be parsimonious.[9]

A positive sign for SMB in equation (1) suggests that a portfolio manager, here a hedge fund, prefers stocks of small firms over stocks of bigger ones, which is usually the case for hedge funds. Moreover, a positive sign for the variable HML would be symptomatic of a preference for stocks with a high book-to-market value ratio over stocks with a low book-to-market value ratio, which is also a frequent preference in the hedge fund industry.[10]

The transition equations give the representation of the time-varying alpha (α_t) and beta (β_{1t}).[11] We first assume that the state variables follow a pure random walk and then we extend this recursive process with financial variables which convey valuable information to the hedge fund manager.

A pure recursive model

We start our modelization with a purely recursive representation of the time-varying coefficients of equation (1), which is the simplest way for making coefficients variable. This is the model generally used in the Kalman filter literature.

The respective equations of the alpha and beta are therefore:

$$\alpha_t = \alpha_{t-1} + \xi_t \qquad (2)$$

$$\beta_{1t} = \beta_{1,t-1} + \nu_t \qquad (3)$$

with ξ_t and ν_t being respectively the innovations terms of equations (2) and (3). If $\xi_t \sim iid(0, \sigma_\xi^2)$ and $\nu_t \sim iid(0, \sigma_\nu^2)$, which amount to pure white noise variables, these equations would represent pure random walk processes.

The filtering of the time-varying coefficients in the framework of a pure recursive process is easy to understand. For instance, in equation (3), the coefficient estimated at time $(t-1)$ serves as a seed value (a guess) for the estimation of the coefficient at time t. However, the filtered coefficient at time t is computed optimally with the Kalman filter following the flow of new information which piles up from one period to the next (Ljungqvist and Sargent, 2004). Therefore the estimated coefficient $\hat{\beta}_{1t}$ may be quite different from the corresponding coefficient estimated one period earlier, that is $\hat{\beta}_{1,t-1}$, even in a pure random walk setting.

Recursive model combined with conditioning financial market variables

Economic and financial information may be added in equations (2) and (3) by relying on more elaborate recursive processes. These variables allow monitoring the reaction of the time-varying coefficients to the conditioning market information. In this model, we express the time-varying alpha and beta as follows:

$$\alpha_t = \alpha_{t-1} + \phi_1 r_{t-1} + \phi_2 \left(R_m - R_f \right)_{t-1} + \xi_t \qquad (4)$$

$$\beta_{1t} = \beta_{1,t-1} + \phi_3 r_{t-1} + \phi_4 \left(R_m - R_f \right)_{t-1} + \phi_5 VIX_{t-1} + \nu_t \qquad (5)$$

with r, the level of short-term interest rate; $(R_m - R_f)$, the market risk premium, and VIX, the implicit volatility of the S&P500 index. The conditioning variables are lagged on period, our aim being to track the reaction of the time-varying coefficients to the conditioning market information. The retained financial variables, which are lagged one period, are thus known at time t.

Note that equation (4), like equation (5), may be transformed in first-differences. For instance, equation (4) can be written as:

$$\left(\alpha_t - \alpha_{t-1} \right) = \phi_1 r_{t-1} + \phi_2 \left(R_m - R_f \right)_{t-1} + \xi_t. \qquad (6)$$

The revision of the alpha from one period to the next is thus the function of three elements: the interest rate observed one period earlier, the market risk premium also observed with a one period lag and an innovation. The coefficients φ_1, φ_2 and the innovation result from the searching process of the filter.

Let us specify the expected sign of the variables incorporated in equations (4) and (5). An increase in the interest rate might be perceived as good news or bad news by hedge funds. If this increase is seen as a forthcoming deterioration of the stock market upward trend or as an indicator of inflation, that is bad news. But for hedge funds that follow call-like option strategies, which bet on the stock market volatility, that might be good news. The signs of φ_1 and φ_3 are thus indeterminate in equations (4) and (5). As shown in the empirical section, the sign is quite related to the specific hedge fund strategies.

In equation (4), an increase in the market risk premium at time $(t-1)$ tends to signal market strengthening. Indeed, if the market risk premium follows a martingale process,

we can write:

$$E\left[\left(R_m - R_f\right)_t | \Omega_t\right] = \left(R_m - R_f\right)_{t-1} \qquad (7)$$

with Ω_t the information set available at time t. According to equation (7), an increase in the market risk premium a time $(t{-}1)$ is viewed as a strengthening of the stock market. This may induce hedge funds to position themselves for an increase in their alpha, this positioning being dependent on the portfolio manager skills. In this case, the sign of φ_2 is positive. But if the alpha is not manageable, this coefficient should be close to 0. However this should not be the case for the time-varying beta which is viewed as a control or decision variable. As a signal of market strengthening, an increase in the market risk premium should encourage hedge funds to take more risk and therefore to increase their beta. The sign of φ_4 would thus be positive in equation (5).

In addition to the standard variables usually incorporated in models with time-varying coefficients, we also include the implicit volatility of the S&P500, that is, the *VIX*, to explain the time-varying beta. Indeed, an increase in market volatility[12] should usually induce hedge funds to bear less risk, and therefore to decrease their beta. However, an increase in market volatility might be welcomed by some hedge fund strategies which are very involved in option-based strategies. Another specification not used in this article to introduce volatility in the equation of the time-varying beta but often found in the literature on market timing relies on the square of the market risk premium as an indicator of stock market volatility, the second moment of the market risk premium:[13]

$$\beta_{1t} = \beta_{1,t-1} + \phi_3 r_{t-1} + \phi_4 \left(R_m - R_f\right)_{t-1} + \phi_6 \left(R_m - R_f\right)_{t-1}^2 + \nu_t \qquad (8)$$

Nevertheless, note that, according to Treynor and Mazuy (1966), the squared market risk premium might serve to detect good or bad market timing,[14] a good market timing being associated to a positive sign for this variable. The sign of the squared market risk premium is thus theoretically indeterminate in equation (8).

The conditional approach

There is another way to compute time-varying coefficients. Following the conditional approach (Ferson and Schadt, 1996; Christopherson *et al*, 1998; Ferson and Qian, 2004), time-varying alpha and beta are obtained by substituting equations (4) and (5) in equation (1), replacing α_{t-1} and $\beta_{1,t-1}$ by α_0 and β_0. Equation (9) obtains:

$$
\begin{aligned}
R_{pt} - R_{ft} = {} & \alpha_0 + \varphi_1 r_{t-1} + \varphi_2 \left(R_m - R_f\right)_{t-1} \\
& + \beta_0 \left(R_m - R_f\right)_t + \varphi_3 r_{t-1}\left(R_m - R_f\right)_t \\
& + \cdots + \varphi_4 \left(R_m - R_f\right)_{t-1}\left(R_m - R_f\right)_t \\
& + \varphi_5 VIX_{t-1}\left(R_m - R_f\right)_t + \beta_2 SMB_t \\
& + \beta_3 HML_t + \beta_4 gprod_t + \varepsilon_t \qquad (9)
\end{aligned}
$$

To estimate equation (9), we resort to a parsimonious approach, the OLS estimation. The conditional alpha and beta are easily identified using equations (4) and (5). Equation (9) may be viewed as a very good approximation of the Kalman filter when using the same explanatory variables.

Note that there is another current method to make the regression coefficients time-variable: the rolling or recursive regression. This method was used by McGuire *et al* (2005) to study the dynamics of the hedge funds risk exposures. While this parsimonious method is comparable with the Kalman filter, it is not an optimal procedure like the filter since there is no optimal revision of the covariance matrix period after period.

EMPIRICAL RESULTS

The data

This study is based on a sample of the indices of US Greenwich Alternative Investment hedge fund strategies, a leader in hedge fund databases and the first institute to collect data on the broad universe of hedge funds. Note that we compared hedge fund databases in previous studies (Racicot and Théoret, 2007a, b) and the empirical results are very close, especially with respect to the Hedge Fund Research Institute (HFR) database.[15] Statistical information on this sample is reported in Table 1. Our observation period for the monthly returns of these hedge fund indices runs from January 1995 to March

2010, for a total of 183 observations for each index (strategy). The risk factors that appear in the Fama and French equation – which are the market risk premium and the two mimicking portfolios: *SMB* and *HML* – are drawn from French's website.[16] The interest rate used to test the models is the US three months Treasury bill rate and the selected market portfolio index, the S&P500. The period we analyse was plagued by four major financial crises: (i) the Asian financial crisis (1997); (ii) the Russian/LTCM crisis (1998); (iii) the bursting of the high–tech market bubble (2000); (iv) the 2007–2009 subprime market crisis, related to high risk mortgages. Our period of analysis is therefore rich in major stock market corrections. Despite

Table 1: Descriptive statistics of the Greenwich indices returns, 1995–2010[a]

	Mean	Median	Max	Min	SD	Skew	Kurtosis	CAPM-beta
Distressed securities	0.68	1.04	4.79	−7.44	2.06	−1.47	6.86	0.22
Equity market neutral	0.87	0.80	8.10	−2.53	1.41	1.33	8.95	0.08
Futures	0.67	0.21	7.71	−6.80	3.10	0.18	2.72	−0.08
Macro index	0.55	0.66	4.00	−2.95	1.45	0.29	3.06	0.27
Market neutral group	0.93	0.92	7.20	−6.06	1.48	−0.61	8.99	0.20
Short–sellers	0.18	−0.10	11.41	−6.88	3.61	0.56	3.46	−1.01
Value index	0.61	1.11	5.68	−9.65	2.54	−1.21	5.94	0.56
Arbitrage index	0.87	0.90	4.10	−8.58	1.38	−2.40	17.99	0.16
Convertible arbitrage index	0.31	0.60	6.55	−19.31	2.98	−3.78	26.43	0.40
Growth index	1.04	1.19	20.10	−12.99	4.53	0.43	5.50	0.76
Long–short	1.09	1.31	13.20	−9.24	3.02	0.04	5.20	0.52
Mean of indices	0.71	0.78	8.44	−8.40	2.51	−0.60	8.65	0.19
Weighted composite	0.56	0.90	4.75	−5.96	1.86	−1.01	5.18	0.37
S&P500	0.46	1.29	11.06	−18.47	4.62	−1.13	5.99	1.00

[a]The statistics reported in this table are computed on the monthly returns of the Greenwich indices over the period running from January 1995 to March 2010. The weighted composite index is computed over the whole set of the Greenwich indices (strategies). The CAPM beta is estimated using the simple market model, that is: $R_{it} - R_{ft} = \alpha + \beta(R_{mt} - R_{ft}) + \varepsilon_{it}$, where R_i is the return of the index i, R_m is the S&P500 return, R_f is the riskless rate and ε_i is the innovation.

these market collapses, Table 1 reveals that the Greenwich hedge funds performed quite well during this period. The mean monthly return of these indices was 0.71 per cent over this period, for an annual rate of 8.5 per cent. This rate is higher than the annual mean return of the S&P500 over the same period, which amounted to 5.5 per cent. The low performers over this period were the short-sellers, the convertible arbitrage and macro strategies, whereas the high performers were the long-short, growth and market-neutral group strategies. Moreover, the standard deviation of returns differs greatly from one index to the next. The indices returns standard deviations are generally below the S&P500 one.

Several researchers argue that the strategies followed by hedge funds are similar to option-based strategies (Fung and Hsieh, 1997, 2004; Weisman, 2002; Agarwal and Naik, 2000, 2004). And effectively, Table 1 reveals that some hedge fund strategies are actually similar to hedged option ones, like the covered call and protective put ones. These option-based strategies have a beta which is quite low, of the order of 0.6 for at-the money options, and may yet offer all the same quite high returns which approximate those shown in Table 1.[17] The equity market-neutral and the arbitrage strategies have a very low beta compared with other funds but their return exceeds the overall mean return.

Furthermore, plain vanilla puts, to which the short-seller strategy is linked, have a negative expected return. That might explain the low mean return of the short-seller index over the period of analysis. At 0.18 per cent, it is well below the mean return of the whole set of strategies. Incidentally, the CAPM beta of the short-seller index, equal to −1.01, is negative and quite high in absolute value over the sample

period. According to the CAPM, the excess return of a portfolio having a negative beta should be low and even negative: this is the case of the short-seller index.

Furthermore, according to Table 1, the composite index of hedge funds has less kurtosis than the market index given by the S&P500. However, this characteristic is not shared by all hedge fund strategies, the convertible arbitrage index having a kurtosis as high as 26.43. A high kurtosis means that rare or extreme events are more frequent than for the normal distribution, which suggests that payoffs are very nonlinear. Once more, we may relate these statistics to those associated to the cash-flows of option-based strategies. Their payoffs have a relatively low standard deviation but a high degree of kurtosis compared with the returns of the stock market index, which is priced in their returns.

Stylized facts provided by spectra of macroeconomic variables

Looking at time series spectra is a good way to establish their periodicity. In this respect, Figure 1 provides the spectra of selected US macroeconomic variables, returns factors and returns of hedge fund strategies. The spectra of the two macroeconomic variables used in this study, the industrial production growth and the unemployment rate, have their usual plot. Industrial production growth displays high volatility at high frequencies while the unemployment rate, being measured in level since it is a stationary time series, has the most of its volatility concentrated at very low frequencies. These plots are consistent with the seminal business cycle analysis performed by Stock and Watson (1990).[18]
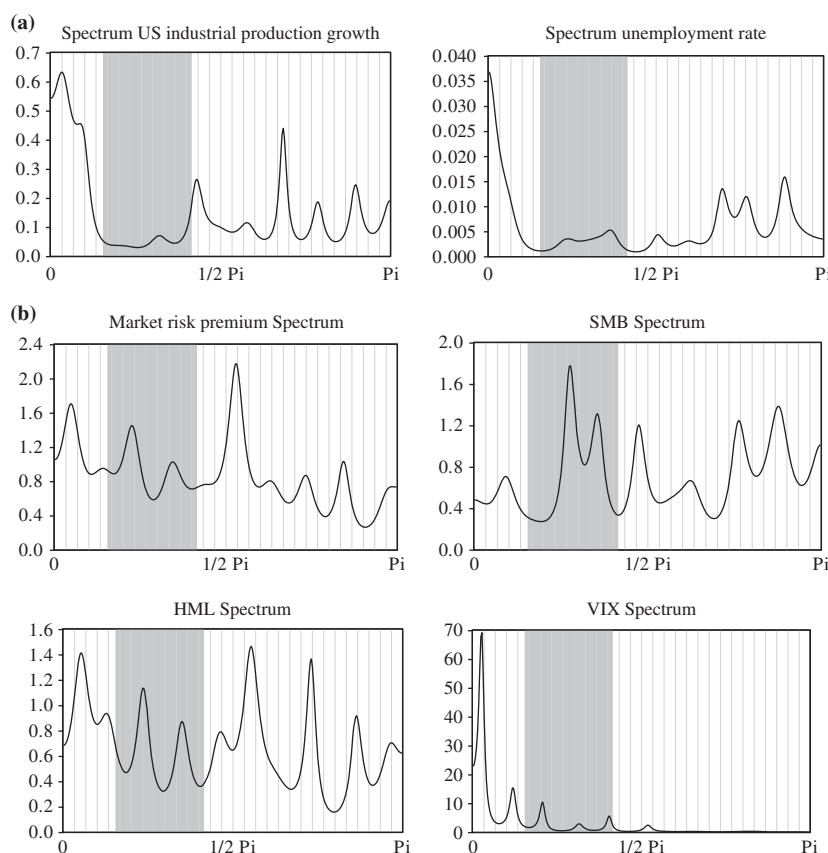
(a)

(b)

Figure 1: (a) Macroeconomic variables spectra; (b) Return factors spectra.

Figure 1 also presents the spectra of the factors entering in the Fama and French equation (equation (1)). Similarly to the analysis of Stock and Watson (1990), the spectrum of the most important variable explaining hedge fund returns, that is the market risk premium, suggests that this variable is procyclical. A simple moving average of the market risk premium also indicates that it is procyclical (Figure 2). It thus has a predictable component.[19] However, according to its spectrum, the *SMB* factor is much more cyclical than the market index. This may be explained by the composition of this index which is weighted by small and big firms. The *HML* variable has a shorter cycle than the *SMB* one. The differences

between the spectra of *SMB* and *HML* factors are thus interesting from the standpoint of econometric analysis. Moreover, the spectrum of the *VIX* shows dampened fluctuations, which asymptotically vanish.[20]

The results

Kalman filter results

In this section, we consider in detail the estimation of equation (1) using the Kalman filter. The specification of the time-varying alpha and beta is given in equations (4) and (5).[21]

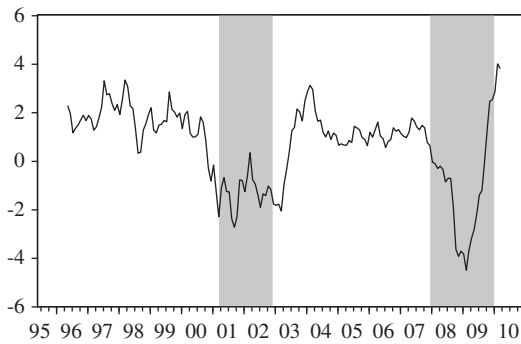The Kalman filter estimation of this system of equations, which is reported in Table 2, reveals

Figure 2: US market risk premium moving average.
Note: US recessions are shaded.

that this system performs quite well. The overall performance of the model is the highest for the weighted composite index with the highest likelihood ratio (*L*). The *L* statistic is also quite high[22] for three of the indices: market-neutral, long–short and value index. The model performs moderately well for the distressed securities and growth strategies. The model is less satisfying for the futures index.

In the estimation of the measurement equation given by equation (1), the market risk premium is the most important risk factor impacting on hedge fund returns, followed by the *SMB* one. In Table 2, the final state beta provided by the *sv2* variable coefficient has a high of 1.10 for the growth strategy and a low of 0.16 for the equity market-neutral strategy. Most of the strategies betas are below the market portfolio one, being 1 by definition, hedge funds usually reducing their market exposure with hedging operations. Let us notice that the betas reported in Table 2 are those corresponding to the final state of our estimation period. To analyse the dynamics of the alphas and betas corresponding to the Greenwich indices, we must build the state series of these coefficients.

Before considering this subject, note in Table 2 that the *SMB* factor is significant at the 99 per cent confidence level for all strategies analysed except for the futures one for which it is not significant. Since hedge funds are in search of abnormal returns, they prefer to hold stocks of small firms over those of bigger ones.[23] The *HML* factor is for its part significant at the 90 per cent confidence level for only three indices: the growth index, the long–short index and the weighted composite one. The impact of this factor is generally lower than *SMB* in most of the empirical studies using the Fama and French model to estimate hedge fund returns.

One of our main contributions in this study is to analyse the procyclicality of risk-taking in the hedge fund industry. In that respect, we have introduced the smoothed growth of industrial production in the Fama and French model. This variable is significant at the 95 per cent confidence level for the weighted composite index, which stands for the performance of the representative hedge fund, and for the value index. Consistent with economic theory, the sign of this variable is negative. Indeed, the growth of industrial production may be considered as an inverse proxy for the degree of risk aversion, a countercyclical variable (Cochrane, 2005). In expansions, risk aversion decreases following the increase in the industrial production growth, and thus expected returns decrease *ceteris paribus*; in contractions, the inverse relationship holds.

Let us now consider the estimated states series of the filtered alpha and beta. We begin our discussion with the beta state series which is *a priori* more manageable than the alpha state variable. In Table 2, the final state beta (*sv2*), associated to the market risk premium is very significant for every index as indicated by the *p-values* of this coefficient. It is equal to 0.48 for

Table 2: Kalman filter estimation[a]

	Time-varying alpha			Time-varying beta				SMB	HML	gprod	L
	sv1	*r(−1)*	*(Rm−Rf)(−1)*	*sv2*	*r(−1)*	*(Rm−Rf)(−1)*	*VIX*				
Distressed securities	0.3097	−0.0097	−0.0068	0.3593	−0.0014	0.0000	0.0043	0.1689	0.0403	−0.0121	−344.64
	0.0034	*0.5989*	*0.3735*	*0.0000*	*0.9172*	*0.7714*	*0.0016*	*0.0000*	*0.2641*	*0.7643*	
Market neutral	−0.1904	−0.0493	−0.0165	0.1596	−0.0145	0.0001	0.0027	0.1299	−0.0022	−0.0183	−295.16
	0.0183	*0.0003*	*0.0056*	*0.0000*	*0.2183*	*0.3054*	*0.0114*	*0.0000*	*0.9384*	*0.5847*	
Long-short	−0.0571	−0.0525	0.0225	0.6152	−0.0150	0.0001	0.0041	0.2618	−0.0669	−0.0563	−296.72
	0.4823	*0.0001*	*0.0001*	*0.0000*	*0.0300*	*0.0484*	*0.0001*	*0.0000*	*0.0380*	*0.0588*	
Value index	−0.0367	−0.0560	0.0193	0.3985	0.0072	0.0025	−0.0002	0.2815	0.0364	−0.0861	−300.67
	0.6592	*0.0005*	*0.0021*	*0.0000*	*0.5283*	*0.0321*	*0.2502*	*0.0000*	*0.2049*	*0.0171*	
Growth index	−0.4483	−0.0700	0.0314	1.1032	−0.0591	0.0055	0.0007	0.3416	−0.2984	−0.0472	−372.19
	0.0003	*0.0006*	*0.0004*	*0.0000*	*0.0007*	*0.0014*	*0.0014*	*0.0000*	*0.0000*	*0.3196*	
Futures index	0.1807	0.0145	−0.0134	1.0342	−0.1540	0.0108	0.0019	0.0041	0.0596	0.1132	−503.46
	0.4628	*0.6376*	*0.3856*	*0.0000*	*0.0000*	*0.0001*	*0.0000*	*0.9602*	*0.4725*	*0.1447*	
Weighted composite	0.0292	−0.0229	0.0125	0.4782	−0.0216	0.0040	0.0002	0.1995	−0.0424	−0.0449	−260.06
	0.6623	*0.0002*	*0.0084*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0937*	*0.0408*	

[a]The model we use is explained in the second section. The equations are estimated resorting to the Kalman filter. *sv1* and *sv2* are the final state values of the alpha and beta. For each strategy, the first line of numbers provides the estimated coefficients of the variables and the second line gives the corresponding *p*-values (in italics). The L statistic is the log likelihood associated to an estimation. The *sv1* coefficient is the final state of the alpha and the *sv2* coefficient is the final state of the beta.

the weighted composite index, the beta of hedge funds being usually quite moderate relatively to traditional managed stock portfolios like the mutual funds ones. The indices having the highest beta are the growth index and the futures one. These betas are greater than one. The indices having the lowest beta are the market neutral, as expected for such a strategy, and the distressed securities index.

Regarding the conditioning variables, we note in Table 2 that the interest rate (r_f) has a negative impact on hedge funds betas, which support our hypothesis since an increase in interest rate signals a market deterioration, which lead hedge funds to take less risk. In other respects, according to the market variable ($R_m - R_f$), hedge funds take more risk when the market returns, as measured by the S&P500 index, increases, but this effect is quite low and not significant for the distressed securities and market-neutral strategies. Finally, financial market volatility, as measured by *VIX*, impacts positively and significantly on the market returns of all strategies except the value index one, for which the exposure to volatility is negative and insignificant. In light of our previous comments about the sign of this variable, hedge funds seem conditioned by the payoffs related to forward market volatility, the value of an option being dominated by its volatility.

Turning to the alpha, we note in Table 2 that the estimated final state alpha given by *sv1* is generally low and even negative for most of the strategies. Indeed, our sample ends at the beginning of 2010, which corresponds to the start of an economic recovery in the United States following the subprime crisis, which explain the low level of *sv1* for the majority of the strategies. This result does not support the alpha puzzle. Note however that the final state alpha of the

distressed securities strategy is quite high, at 0.31, which is related, as shown later, to the specificity of this strategy. Moreover, consistent with the results obtained for the beta, the alpha responds generally negatively (and significantly) to an increase in the interest rate and positively (and significantly) to a strengthening of the stock market conditions. Like the beta, the hedge fund alpha thus seems procyclical.

The plots of the betas indicate that they are far from being constant, as suggested by the conventional CAPM, and exhibit a procyclical behaviour. In Figure 3, we compare the profile of the state beta of the weighted composite index with the US unemployment rate, a well-known countercyclical time series. As there is a training period for the Kalman filter, there are usually spikes at the start of the plots of the state betas or alphas. We note that the state beta is negatively correlated to the unemployment rate. More precisely, the beta has an inverse cycle compared with unemployment: it is thus procyclical. However, remind that the unemployment rate is a lagged indicator of economic activity while the beta, being the result of an investment decision process, is forward-looking. In that respect, the

Figure 3: US unemployment rate and Greenwich hedge fund composite index. *Note*: US recessions are shaded.

behaviour of the beta during the 2007–2009 subprime crisis is very instructive. During the first half of the crisis, the unemployment rate jumps significantly while the beta drops substantially, which suggest that the representative hedge fund reduced greatly its exposure to market risk. But being a lagged indicator, the unemployment rate continued to increase in the last phase of the crisis, while the beta resumed its increase, in expectation of the upcoming economic expansion. In this sense, the beta is forward-looking.

As shown in Figure 4, the state beta of the weighted composite index decreased during the 1997 Asian crisis before resuming its increase in 1998. Thereafter, following the first US recession of the millennium, the beta decreased from the beginning of 2000 till the end of 2002, which paved the way to a market recovery. It almost doubled from 2003 to the middle of 2005. It decreased progressively thereafter in expectation of an economic slowdown and in reaction to the corporate accounting scandals. This beta dynamics is comparable with the one obtained by McGuire *et al* (2005) during the period 1997–2004 with respect to hedge fund

risk exposure, which is a leveraging during the upward trend of the stock markets or the economic expansions followed by a deleveraging during the crises.

The state alpha related to the weighted composite index has a profile similar to the beta but is more volatile (Figure 4). The alpha decreased after the Asian crisis, the decrease gaining momentum during the technological bubble. During this episode, the estimated alpha dropped from a high of 1 per cent monthly to a low close to 0 per cent, which suggests that the alpha puzzle must be studied in a dynamic setting. Our procyclical approach seems thus more relevant to study the alpha process. As in the case of beta, the profile of the alpha is particularly interesting during the 2007–2009 subprime crisis. According to Figure 4, it decreases to a low of −0.5 per cent in the middle of the crisis before recovering thereafter, a profile similar to the beta. In summary, alpha and beta co-move positively, a result in line with the common factors that drive these two performance measures.

Conditional models

In this section, we compare in this section the cycle of state and conditional beta and alpha. Figure 5 provides this comparison for the Greenwich weighted composite index. We note that the conditional beta (equation (5)) has a procyclical profile that is very similar to the one of the state beta, which supports the Kalman filter estimation. Both betas are very synchronized during an economic slowdown, as during the technological bubble burst and the subprime crisis. But when the economy recovers, we note that the state beta is forward-looking with respect to the conditional one. For example, during the
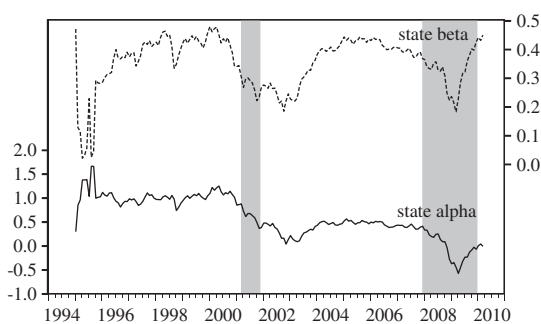

Figure 4: State series for the alpha and beta of the Greenwich weighted composite index. *Note*: US recessions are shaded.
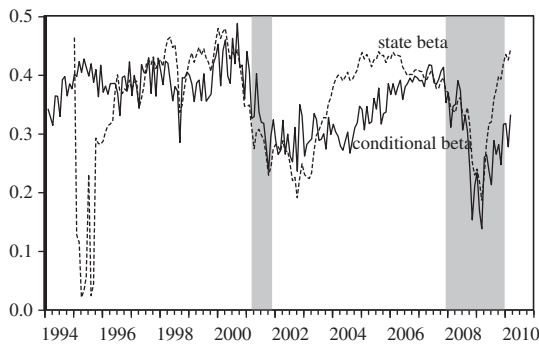
Figure 5: State series and conditional model for the beta of the Greenwich weighted composite index.
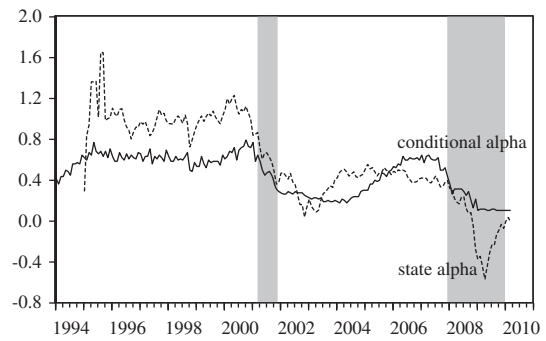*Note*: US recessions are shaded.

Figure 6: State and conditional series for the alpha of the Greenwich weighted composite index.
Note: US recessions are shaded.

2003–2007 economic recovery, the increase in the conditional beta lagged the increase in the state beta by many months. The same phenomenon is observed during the last period of the subprime crisis, whereas the state beta jumps while the conditional beta increases only modestly. Note also that the state beta began its decrease well before the subprime crisis, which is less the case for the conditional beta. The Kalman filter thus seems better appropriate to model the forward-looking properties of time-varying betas.

Figure 6, which plots the Kalman filtered and conditional alphas for the Greenwich weighted composite index, sheds more light on the alpha puzzle in a dynamic setting. It seems that the alpha puzzle is much less problematic in our procyclical framework for analysing hedge fund returns. Contrary to the previous empirical analyses of hedge fund returns, the alpha may become nil or even negative in periods of economic contraction. This pattern is more pronounced for the Kalman filtered series than the one obtained for the conditional model. It is thus interesting to note that the alpha is positive in economic expansions, even moving on an
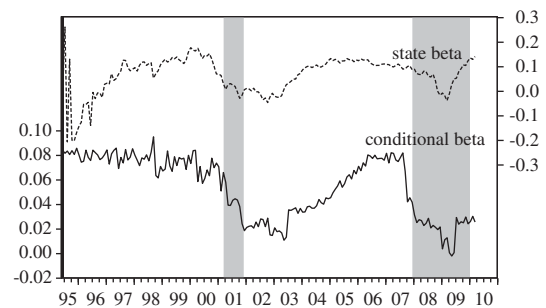

Figure 7: State and conditional beta of the equity market-neutral strategy.
Note: US recessions are shaded.

upward trend in the first phase of economic expansions. Note that in the 1990s, there actually seems to be an alpha puzzle but it would be related to the long period of economic expansion which prevailed then. Note also in Figure 7 that the cyclical behaviour of the alpha is asymmetric, having a tendency to decrease more in contractions than it increases in expansions. Even if our sample is not long enough to be conclusive, the alpha seems mean-reverting towards a level which declines slowly.

The cyclical properties of the weighted index return are shared by the returns of the specific strategies analysed. We note in Figures 7 and 8
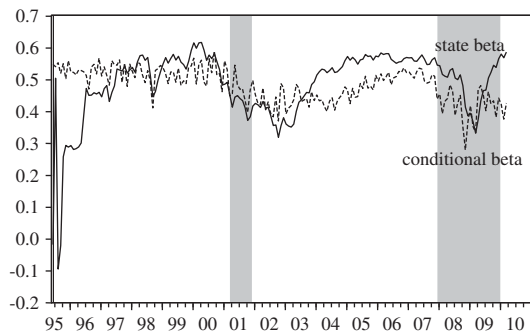
Figure 8: State and conditional beta of the long-short strategy.
Note: US recessions are shaded.

Figure 9: State and conditional beta of the value index strategy.
Note: US recessions are shaded.
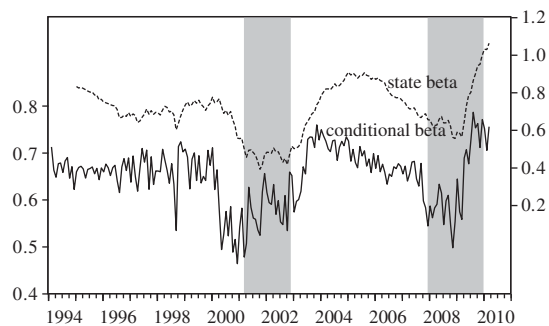
Figure 10: State and conditional beta of the growth index strategy.
Note: US recessions are shaded.

that the betas of the equity market–neutral and long-short strategies display procyclicality, a counterfactual result given the nature of these strategies.[24] In line with the weighted composite index, the betas of both strategies returned on an upward trend well before the end of the 2007–2009 subprime crisis, the swing being more pronounced with the Kalman filter. Moreover, the conditional beta seems more sensitive to market conditions than the Kalman filtered one, especially for the equity market–neutral strategy.

It is also interesting to study the procyclical behaviour of two strategies having higher betas than the representative hedge fund, which, like the market-neutral and the long–short strategies, have a relatively low market exposure. Figures 9 and 10 provide the cyclical behaviour of the value index and the growth index whose CAPM betas are respectively 0.56 and 0.76. We note that the procyclical behaviour of these two strategies is quite different. The value index has a procyclical profile which is similar to the weighted composite index and is quite sensitive to market conditions. Moreover, as the weighted composite, the value index is forward-looking, especially before and during the 2007–2009
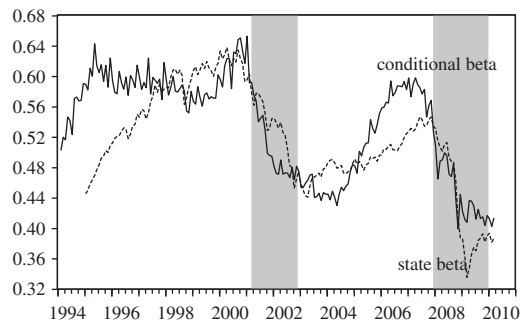
subprime crisis. However, contrary to the weighted composite index, note that the value index conditional beta is more forward-looking than the Kalman filtered beta, which suggests that the value index strategy is very sensitive to financial market conditions. Another interesting aspect related to the cyclical profile of the growth index is that during the expansion 2002–2007, its beta, both computed by the Kalman filter and the conditional model, began to decrease well before the end of the expansion after having peaked at a level near 1, which is not the case for the value index which follows economic conditions more closely.
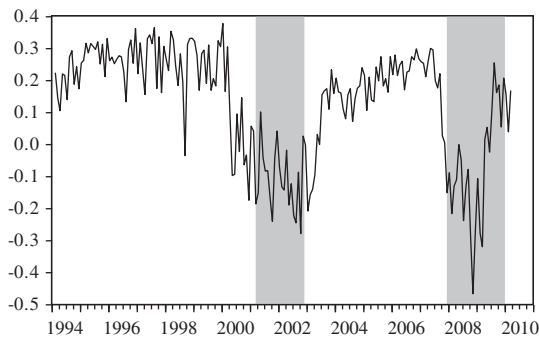
Figure 11: Conditional beta of the futures index strategy.
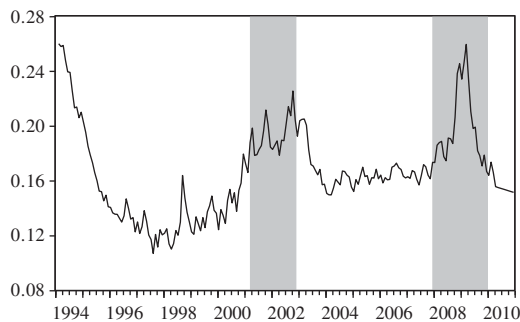*Note*: US recessions are shaded.

Figure 12: Conditional beta of the distressed securities strategy.
Note: US recessions are shaded.

Finally, we consider two very specialized strategies: the futures and the distressed securities ones (Figures 11 and 12). Owing to the instability of their returns, it was difficult to apply the Kalman filter to these strategies so we resorted only to the conditional model. The procyclical profile of the futures index beta is close to the one given by the weighted composite index. However, the fluctuations of the futures index beta are much wider, being in a range of [−0,5, 0.4] in Figure 11. This profile is also very different from the ones of the long-short and market-neutral strategies, which also explains

why the Kalman filter has difficulties to capture the time profile of the futures index. In other respects, even if it is quite less volatile than the futures index, one interesting dimension of the distressed index is that, contrary to the other indices, its beta increases during contractions. Actually, the distressed securities strategy focuses on contraction periods so distressed funds take more risk during economic slowdowns. As its beta tends also to decrease in an economic expansion, the distressed securities strategy seems to be countercyclical.

CONCLUSION

In this article, we revisit hedge fund return models relying on a procyclical setting based on two dynamic procedures: the Kalman filter and the conditional model approach. These procedures appear more rigorous than the standard OLS one usually used to compute the conditional alpha and beta. In the Kalman filter approach, the model coefficients are updated each period in an optimization framework taking into account all information accumulated until this period. We also consider the conditional model as an approximation to the Kalman filter.

To the best of our knowledge, our study is the first to consider in detail the cyclical profile of hedge fund returns. Indeed we resort to a procyclical approach to study hedge fund returns, introducing cyclical variables in our models, such as the growth of industrial production, unemployment rate, the rate of vacancy and the *VIX*, among others. We find that these variables capture quite well the cyclical behaviour of the hedge fund risk-return trade-off.

Our study shows that the state and conditional alphas are quite responsive to the business cycle, usually increasing during expansions and

decreasing during contractions. The Kalman filter and the conditional model both support this procyclical profile. We may thus conclude that the absolute hedge fund return, as measured by the Jensen alpha, displays a cyclical behaviour, a new aspect for this performance measure not found in the literature. Our experiments also show that the alpha tends to mean-revert from one business cycle to the next. In order to check for its robustness, this result needs further investigation.

The time-varying beta of hedge fund strategies also displays a pronounced procyclical profile for both the Kalman filter and the conditional model. The beta responds positively to the market risk premium and negatively to the interest rate level, a quite rational behaviour. Moreover, the beta is conditioned by a cycle which may be explained quite easily by the behaviour of the macroeconomic variables we use. Incidentally, the performance of models relying on conditioning information is usually much better to explain the expected excess returns of the hedge funds because the time-varying beta is significantly related to macroeconomic information. A promising research avenue is to consider the impact of procyclicality of hedge fund risk on the business cycle itself. Actually, do hedge funds exacerbate the amplitude of the business cycles? This is a very important question from the point of view of macroprudential policy that deserves further investigation.

ACKNOWLEDGEMENTS

NOTES

1 Hull (2006) reports that one trillion $ was invested in this industry in 2004. In 2008, this amount peaked to 1.93 trillion (Sadka, 2010).
2 LTCM is the acronym of Long Term Capital Management, a highly levered US hedge fund that sustained massive losses in 1998.
3 According to Devasabai (2010), the share of hedge fund investments in US pension funds, which is presently about 2.5 per cent, will increase to 15 per cent by the end of the decade.
4 For instance, regulators could constrain the financial institutions to pay for the externalities they generate. In that respect, when a big financial institution increases its leverage, it can give raise to negative externalities for the other institutions, which are not endogenized in its behaviour. Actually, these externalities could entail an important reduction in liquidities for the whole financial system, liquidity being a public good.
5 More specifically, the full name of this company is: Greenwich Alternative Investment.
6 Note also that the alpha puzzle was detected over the 1990s, while the Great Moderation was at its peak. The high observed alpha might be explained by these favourable business conditions.
7 The appendix provides a general description of the Kalman filter procedure. For an introduction to the Kalman filter and its uses in finance, see Rachev et al (2007), chap.11 for a hedge fund return modelling application. According to L'Habitant (2004), the Kalman filter is similar to a least squares

estimation except that the coefficients of the model are updated at every period following the arrival of new information. The applications of the Kalman filter in finance go back to the beginning of the 1980s. At this time, the filter was used to forecast the real rate of interest and the risk premia in forward and futures markets (Fama and Gibbons, 1982; Hsieh and Kulatilaka, 1982). Bassett *et al* (1991) used the Kalman filter to forecast forward prices of nontraded securities. There are also extensive applications in the fields of exchange rates and term structure of interest rates where the Kalman filter is used to forecast volatility and other key variables (Pennacchi, 1991). There are nevertheless few studies in the hedge fund area that analyse their style dynamics, especially the time-varying dimensions of alpha and beta (Swinkels and van der Sluis, 2001; Giamouridis and Vrontos, 2007).

8 Our experiments show that the Fung and Hsieh's (2004) factors have only a marginal explanatory power on hedge fund returns and do not affect the cyclical behaviour of the alphas and betas, the main object of our study. Since we resort to a Kalman filter framework in this article, we must use a parsimonious approach so we do not retain the Fung and Hsieh factors.

9 While the Fama and French device is still a popular choice to model hedge fund returns, other more parsimonious models have appeared recently in the literature, for instance the one proposed by Berkelaar *et al* (2009). These authors suggest that hedge fund strategies could be modelled using the following parsimonious model: $r_{h,t} = \alpha_{h,t} + \beta_{h,t}^{EQ} r_{h,t}^{EQ} + \beta_{h,t}^{FI} r_{h,t}^{FI} + \varepsilon_t$. This simple two-factor model uses the returns on equity

(r^{EQ}) and those on fixed income (r^{FI}) securities, as many studies have shown that a significant portion of hedge fund return strategies is well explained by these two factors. The parameter $\alpha_{h,t}$ represents the time-varying alpha of strategy h and the other components represent the systematic beta components. Thus, the performance at every time period of a particular strategy can be explained by the manager skills (alpha), the returns on equity (stocks) and the returns on fixed income securities (for example, bonds). Because of the time-varying nature of their model, they rely on the Kalman filter to estimate the model parameters. For a similar approach, see also Eling and Faust (2010).

10 The signs of the factor loadings are seldom discussed in the studies of the Fama and French model. This gap must be filled because the signification of these signs is partly a matter of interpretation.

11 For another approach to dynamize alphas and betas, see Giamouridis and Vrontos (2007), Mamaysky *et al* (2007, 2008).

12 Note that an increase in market volatility usually takes place during an economic contraction, when the volume of market transactions skyrockets. For more details, see Le Grand and Ragot (2010).

13 Market volatility is then another factor contributing to the revision of the time-varying alpha and beta from one period to the next.

14 In this situation, the squared market risk premium represents the co-asymmetry of a given portfolio return with the market return.

15 These results are available upon request.

16 The address of the French's website is: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

17 For a discussion of the beta and the mean return of option-based strategies, see Whaley (2006), chapter 10.

18 See also Racicot (2011) for a review of spectral analysis techniques applied to financial data.

19 Note that the spectrum of a random variable, being by nature unpredictable, is flat.

20 We also built the spectra on hedge fund returns. See Racicot and Théoret (2012).

21 Note that, for robustness check, we estimated random walks for the state variables given by Equations (2) and (3). For the sake of simplicity, we do not report the results here but these results were poor compared with the estimations of Equations (4) and (5) in which we add control variables to account for the cycles and the funds' forward looking strategies (Racicot and Théoret, 2007a, b).

22 To see the correspondence between the R2 and the log-likelihood statistic in Table 2, note, for instance, that the equation of the excess return of the equity hedge strategy has a R2 of 0.83 when estimated by the OLS method and a log-likelihood statistic of −173 when estimated by the Kalman filter.

23 Indeed, there is a lack of accounting information for small firms compared with the big ones. This 'opacity' might explain the small firm anomaly, which is an abnormal return on small firms stocks after accounting for systemic risk.

24 Indeed we might believe that these hedge funds should 'hedge procyclicality'.

References

Adrian, T. and Shin, H.S. (2010) The Changing Nature of Financial Intermediation and the Financial Crisis of 2007–2009, Working Paper, Federal Reserve Bank of New York.

Agarwal, V. and Naik, N.Y. (2000) Multi-period performance persistence analysis of hedge fund. *Journal of Financial and Quantitative Analysis* 35(3): 327–342.

Agarwal, V. and Naik, N.Y. (2004) Risk and portfolio decisions involving hedge funds. *Review of Financial Studies* 17(1): 63–98.

Altman, E. and Saunders, A. (2001) An analysis and critique of the BIS proposal on capital adequacy and ratings. *Journal of Banking and Finance* 25(1): 25–46.

Amato, J.D. and Furfine, C.H. (2004) Are credit ratings pro-cyclical? *Journal of Banking and Finance* 28(11): 2641–2677.

Bassett, G.W., France, W.G. and Pliska, S.R. (1991) Kalman filter estimation for valuing nontrading securities, with applications to MMI cash future spread on October 19 and 20, 1987. *Review of Quantitative Finance and Accounting* 1(2): 135–151.

Berger, A.N. and Udell, G.F. (2004) The institutional memory hypothesis and the procyclicality of bank lending behaviour. *Journal of Financial Intermediation* 13(4): 458–495.

Berkelaar, A., Kobor, A. and Kouwenberg, R. (2009) Asset allocation for hedge fund strategies: How to better manage tail risk. In: G.N. Gregoriou (ed.) *The VaR Modeling Handbook*. New York: McGraw-Hill.

Carhart, M.M. (1997) On persistence in mutual fund performance. *Journal of Finance* 52(1): 57–82.

Christopherson, J.A., Ferson, W.E. and Glassman, D.A. (1998) Conditioning manager alphas on economic information: another look at the persistence of performance. *Review of Financial Studies* 11(1): 111–142.

Cochrane, J.H. (2005) *Asset Pricing*. Revised edn. Princeton, NJ: Princeton University Press.

Danthine, J.P. and Donaldson, J.B. (2002) Labour relations and asset returns. *Review of Economic Studies* 69(1): 41–64.

Devasabai, K. (2010) Pension funds set to drive hedge fund growth. *Hedge Funds Review* 4 January.

Eling, M. and Faust, R. (2010) The performance of hedge funds and mutual funds in emerging markets. *Journal of Banking and Finance* 34(11): 1993–2009.

Fama, E.F. and French, K.R. (1992) The cross-section of expected stock returns. *Journal of Finance* 47(2): 427–465.

Fama, E.F. and French, K.R. (1993) Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33(1): 3–56.

Fama, E.F. and French, K.R. (1997) Industry costs of equity. *Journal of Financial Economics* 43(2): 153–193.

Fama, E.F. and Gibbons, M.R. (1982) Inflation, real returns and capital investment. *Journal of Monetary Economics* 9(3): 297–323.

Ferson, W.E. and Qian, M. (2004) Conditional evaluation performance: Revisited. The Research Foundation of CFA Institute (mimeo).

Ferson, W.E. and Schadt, R.W. (1996) Measuring fund strategy and performance in changing economic conditions. *Journal of Finance* 51(2): 425–461.

Fung, W. and Hsieh, D.A. (1997) Empirical characteristics of dynamic trading strategies: The case of hedge funds. *Review of Financial Studies* 10(2): 275–303.

Fung, W. and Hsieh, D.A. (2004) Hedge fund benchmarks: A risk based approach. *Financial Analyst Journal* 60(5): 65–80.

Geanakoplos, J. (2010) The Leverage Cycle. Working paper, Cowles Foundation for Research in Economics.

Gennaioli, N., Shleifer, A. and Vishny, R.W. (2011) A Model of Shadow Banking. Working Paper, NBER no. 17115.

Giamouridis, D. and Vrontos, I.D. (2007) Hedge fund portfolio construction: A comparison of static and dynamic approaches. *Journal of Banking and Finance* 31(1): 199–217.

Heid, F. (2007) The cyclical effects of Basle II requirements. *Journal of Banking and Finance* 31(12): 3885–3900.

Hsieh, D.A. and Kulatilaka, N. (1982) Rational expectations and risk premia in forward markets: Primary metals at the London metals exchange. *Journal of Finance* 37(5): 1199–1207.

Hull, J.C. (2006) *Options, Futures, and Other Derivatives*, 6th edn. New Jersey: Pearson.

Jacques, K. (2010) Procyclicality, bank lending, and the macroeconomic implications of a revised Basle accord. *The Financial Review* 45(4): 915–930.

Jegadeesh, N. and Titman, S. (1993) Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* 48(1): 65–91.

Le Grand, F. and Ragot, X. (2010) Prices and Volumes of Options: A Simple Theory of Risk-Sharing When Markets are Incomplete. Working paper, Banque de France.

L'Habitant, F. (2004) *Hedge Funds: Quantitative Insights*. New York: Wiley.

Ljungqvist, L. and Sargent, T.J. (2004) *Recursive Macroeconomic Theory*, 2nd edn. Massachusetts: MIT Press.

Mamaysky, H., Spiegel, M. and Zhang, H. (2007) Improved forecasting of mutual fund alpha and beta. *Review of Finance* 11(3): 1–42.

Mamaysky, H., Spiegel, M. and Zhang, H. (2008) Estimating the dynamics of mutual fund alpha and betas. *Review of Financial Studies* 21(1): 233–264.

McGuire, P., Remolona, E. and Tsatsaronis, K. (2005) Time-varying exposures and leverage in hedge funds. *BIS Quarterly Review* 59–72.

Pennacchi, G.G. (1991) Identifying the dynamics of real interest rates and inflation: Evidence using survey data. *Review of Financial Studies* 4(1): 53–86.

Rachev, S.T., Mittnik, S., Fabozzi, F.J., Focardi, S.M. and Jasic, T. (2007) *Financial Econometrics*. New York: Wiley.

Racicot, F.E. (2011) Low-frequency components and the weekend effect revisited: Evidence from spectral analysis. *Aestimatio: The IEB International Journal of Finance* 3: 2–19.

Racicot, F.E. and Théoret, R. (2007a) A study of dynamic market strategies of hedge funds using the Kalman filter. *Journal of Wealth Management* 10(3): 94–106.

Racicot, F.E and Théoret, R. (2007b) The beta puzzle revisited: A panel study of hedge fund returns. *Journal of Derivatives and Hedge Funds* 13(2): 125–146.

Racicot, F.E. and Théoret, R. (2009) Integrating volatility factors in the analysis of the hedge fund alpha puzzle. *Journal of Asset Management* 10(1): 37–62.

Racicot, F.E. and Théoret, R. (2012) Risk Procyclicality and Dynamic Hedge Fund Strategies: An Application of Kalman Filter to Time-Varying Alpha and Beta. Working Paper, Telfer School of Management, University of Ottawa.

Rajan, R.G. (2005) Has Financial Development Made the World Riskier? Working Paper, NBER.

Rajan, R.G. (2009) The credit crisis and cycle-proof regulation. Federal Reserve Bank of St-Louis, September-October: 397–402.

Sadka, R. (2010) Liquidity risk and the cross-section of hedge-fund returns. *Journal of Financial Economics* 98(1): 54–71.

Shin, H.S. (2009) Securitization and financial stability. *Economic Journal* 119(536): 309–332.

Stock, J.H. and Watson, M.W. (1990) Business Cycle Properties of Selected U.S. Economic Time Series, 1959–1988. NBER Working Paper No. 3376.

Swinkels, L.A.P. and van der Sluis, P.J. (2001) Return–Based Style Analysis with Time-Varying Exposure. Working Paper, Tilburg University.

Treynor, J. and Mazuy, K. (1966) Can mutual funds outguess the market? *Harvard Business Review* 44(4): 131–136.

Vives, X. (2010) Strategic Complementarity, Fragility and Regulation. Working Paper, IESE Business School.

Weisman, A. (2002) Informationless investing and hedge fund performance measurement bias. *Journal of Portfolio Management* 28(4): 80–91.

Whaley, R.E. (2006) *Derivatives: Markets, Valuation and Risk Management*. New York: Wiley.

APPENDIX

The Kalman filter procedure

Our main procedure relies on the Kalman filter. A brief description of this method follows. Assume an observable time series γ_t. This variable may be for instance a financial asset return. It depends on the variable h_t which is unobservable

or latent. This variable could be the stochastic volatility of y_t. Since we cannot observe h_t, we have to simulate it. The variance of h_t, denoted by ω_t, is also unobservable. The model can be represented as follows: $y_t = \theta_1 + \theta_2 h_t + \varepsilon_t$; $h_{t+1} = \theta_3 + \theta_4 h_t + \eta_t$, where θ_i are the parameters to estimate, ε_t stands for a Gaussian noise whose variance is v_{1t}, and η_t is a Gaussian noise with variance v_{2t}. The first equation is the measurement or observation equation whereas the second is the state or transition equation. Let us now consider the case of time-variable coefficients. At time $(t-1)$, estimations of h_{t-1} and its variance ω_{t-1} as well as coefficients θ_{it-1} are predetermined. At time 0, we must have a preliminary estimation of h_0 and ω_0. But because these values are unknown, the software EViews, used in this study, put a zero value to h_0 and a high value to ω_0 in order to account for the uncertainty related to the estimation of h_0. Let us step back to time $(t-1)$ of the simulation or of the filtering. The three steps of the procedure followed by the Kalman filter are then: forecasting, updating and parameter estimation. In the first step, we make the following two

forecasts: $h_t|t-1$, that is the forecast of h_t conditional to the information set at time $(t-1)$, and $\omega_t|t-1$, that is the forecast ω_t conditional to the information set at time $(t-1)$. These forecasts, which are unbiased conditional estimations, obtain: $h_{t|t-1} = \theta_{3,t-1} + \theta_{4,t-1} h_{t-1}$; $\omega_{t|t-1} = \theta_{4,t-1}^2 \omega_{t-1} + v_{2,t-1}$. The second step is the updating one. At time t, we have a new observation of y, that is, y_t. We can thus compute the prediction error $v_t = y_t - \theta_{1,t-1} - \theta_{2,t-1} h_{t|t-1}$. The variance of v_t, denoted by ψ_t, is given by: $\psi_t = \theta_{2,t-1}^2 \omega_{t|t-1} + v_{1,t-1}$. We use the parameters v_t and ψ_t to update h_t and its variance ω_t as follows: $h_t = h_{t|t-1} + \theta_{2,t-1} \times \omega_{t|t-1} \times v_t / \psi_t$; $\omega_t = \omega_{t|t-1} + \theta_{2,t-1}^2 \times \omega_{t|t-1}^2 / \psi_t$. The last two equations are conditionally unbiased and efficient estimators. The Kalman filter is thus optimal because it is the best estimator in the class of linear estimators. The third step deals with parameter estimation. To estimate the parameter θ_i, we use the maximum likelihood method. The likelihood function can be written as follows: $l = -1/2 \sum_t \log(\psi_t) - 1/2 \sum_t v_t^2 / \psi_t$. To complete the procedure, we go to time $(t+1)$ and repeat the three-step procedure up to period n.