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## Original Article

# Implied transaction costs by Leland option pricing model: A new approach and empirical evidence

Received (in revised form): 22nd August 2012

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**ABSTRACT** Estimation of transaction costs in a stock market is an important issue for stock trading, asset pricing, stock market regulation and so on, and it is often done by combining the bid-ask spread estimate with commissions and other fees provided by market participants, which can be subjective. This study aims to offer an innovative alternative method to estimate the transaction costs in stock trading via the implied transaction costs by using the Leland option pricing model. The effectiveness of this new approach is tested by using the S&P/ASX 200 index call options data. On the basis of the actual transaction costs estimates on the Australian Securities Exchange (ASX) documented by previous studies and Roll's model, the empirical results reveal that this new approach can provide a reliable transaction costs estimate on stock trading on the ASX. Furthermore, the accuracy of the implied transaction costs across option moneyness and maturity and the variation of the implied transaction costs during the recent global financial crisis period are investigated.

*Journal of Derivatives & Hedge Funds* (2012) 18, 333–360. doi:10.1057/jdhf.2012.12

**Keywords:** implied transaction costs; Leland option pricing model; index options; empirical

## INTRODUCTION

Bid-ask spreads, commissions or brokerage fees, execution costs, and other costs related to securities trading can be collectively referred to as transaction costs. Transaction costs are an important factor in trading options, stocks or any other asset. They affect the equilibrium asset prices and hence the mean asset returns.

Consequently, estimating transaction costs is an important topic for empirical analyses. Smith and Whaley (1994) pointed out the importance of understanding and accurately measuring transaction costs for the purpose of business and regulatory decisions regarding market operations.

Given the importance of transaction costs in securities trading, this study aims to offer a new approach to estimating the transaction costs used by the market for the selling and buying of an asset. This study differs from others in the literature and considers estimating transaction costs from an option pricing model together with empirical evidence.

The Black-Scholes-Merton (BSM) model for option pricing has made a great impact on securities trading in the real world. However, the BSM model suffers from the unrealistic assumption of no transaction costs. There are many studies that have explored ways to deal with option payoff replication and hedging errors owing to transaction costs.

Leland (1985) developed a hedging strategy in which the price of a call option should be given by the BSM model with an adjusted volatility. The adjusted volatility depends on the proportional transaction costs rate, the volatility of the underlying asset and the rebalancing interval of the replicating portfolio. He claimed that the hedging error can be arbitrarily small, if the length of the rebalancing frequency tends to

zero and also if one uses the BSM delta-hedging with the adjusted volatility. Despite the critiques of Leland's strategy, a number of studies have empirically shown that the modified Leland strategy exceeded the accuracy of the BSM model, such as Primbs and Yamada (2006) and Zakamouline (2008). Further, Leland (2007) provided two adjustments to the original Leland (1985) formula by incorporating the initial costs of trading with the assumptions that the initial portfolios are all cash and all stock positions. The adjustments were made because the original Leland (1985) model did not explicitly consider initial costs of trading. In addition, Leland's model does not depend on the investor's risk preferences and has a closed-form solution.

There are also other studies dealing with hedging errors but using a preference-dependent hedging strategy, which is called the utility-maximisation approach to option pricing with transaction costs. Although the models of the utility-based approach pioneered by Hodges and Neuberger (1989) are successful in terms of the optimality of the hedging strategies and good empirical performances, they have a few disadvantages that restrict their broad application in practice. Among the disadvantages are the fact that they are slow to compute as they usually result in three- or four-dimensional free boundary problems (Whalley and Wilmott, 1999); they appear to be difficult to handle and impractical as they are time consuming to compute (Atkinson and Alexandropoulos, 2006); the investor's risk must be specified, the market must be continuously monitored and there are computational problems in deriving the parameters of the bounded area (Gregoriou *et al*, 2007); and they lack closed-form solutions and the calculations of the optimal hedging

are time consuming (Zakamouline, 2006, 2008).

Taking into account the complexity and the few disadvantages of utility-dependent option pricing models, this study considers estimating the transaction costs per trade implied by an option price observed in the market using the Leland (1985) model. This study is further motivated by the development of the Leland option pricing model, which is a modified strategy of the widely used BSM model, and also the unresolved questions of whether Leland's method can be used effectively to price and hedge options with realistic transaction costs and rebalancing intervals.

In practice, traders incur several types of transaction costs every time they trade. These include commissions, bid-ask spread and other costs that are related to the price impact of trades. To the best of our knowledge, all studies in the literature estimate the transaction costs by combining the bid-ask spread estimate with commissions and other fees provided by market participants, which can be subjective. In contrast, the implied transaction costs proposed in this study provide a total transaction costs estimate in one go. This is based on the realised volatility and the market prices, which are all objective.

This study uses the S&P/ASX 200 index options data to demonstrate the effectiveness of the proposed approach. The S&P/ASX 200 index options are chosen for this study because they are the most popular and liquid index options on the Australian Securities Exchange (ASX). Furthermore, index options offer a wider choice of exercise prices compared to options on individual stocks, which typically trades with a limited number of exercise prices. The implied transaction costs are estimated from the Leland

model with respect to option moneyness and time to maturity, respectively. We perform statistical tests on whether the estimated implied transaction costs differ between option moneyness and time to maturity groupings. Then the option moneyness and time to maturity groupings that best estimate the transaction costs can be identified.

The accuracy of the implied transaction costs is assessed based on a few benchmarks. First, we consider the bid-ask spread estimate by using the best-known Roll (1984) model, which should be a lower bound for our estimate as the bid-ask spread is only one part of the implied transaction costs obtained here. We also compare the bid-ask spread estimate obtained from Roll (1984) with that of the actual stock market bid-ask spread estimate reported in Cummings and Frino (2011). Second, we compare the implied transaction costs estimate with the actual round-trip transaction costs estimates for large stocks on the ASX reported in previous studies, including Aitken and Frino (1996) and Comerton-Forde *et al* (2005). These actual transaction costs are also recently documented in Chen *et al* (2010). We contacted a few brokerage firms in order to obtain indicative round-trip costs, but we did not receive any response. Therefore, third, we consider comparing the implied transaction costs estimate with a study by Fong *et al* (2010) on the brokerage services in Australia. Fong *et al* (2010) conducted the study from 1995 to 2007. The nature of these studies is discussed in a later section of this article.

Compared to the previous studies, a few features of the implied transaction costs approach stand out. It is more objective as it does not need to estimate the commission and many other fees based on the information provided by market participants. It is relatively straightforward as it is

based on the historical volatility and other market-observable variables. The empirical results also reveal that the new approach is reliable.

The US subprime mortgage crisis in mid-2007 led to a global financial crisis that lasted until about the end of 2008. Volatility of the underlying asset and transaction costs rose significantly during this crisis. Therefore, this study also investigates the implied transaction costs during this crisis period. Thus, we investigate the implied transaction costs using the S&P/ASX 200 index call options for the period from 2 April 2001 to

31 December 2010, which covers the recent global financial crisis. We divide the sample into three groups: pre-crisis, during crisis and post-crisis. We hypothesise that the implied transaction costs are higher during the crisis period. This is because there are high levels of uncertainty about market future movement and enormous transaction costs associated with the trading of the underlying asset. We find that during the crisis, the implied transaction costs increase more than double the rate before the crisis. However, the implied transaction costs decrease around 40 per cent after the crisis, but the costs are still higher than those before the crisis.

The remainder of this article is organised as follows. The next section briefly reviews the transaction costs structure in the capital markets. The description of the Leland model, the methodology to estimate the transaction costs and the description of the data are provided in the following three sections, respectively. The empirical results and conclusions are then presented.

## TRANSACTION COSTS IN CAPITAL MARKETS

Generally speaking, transaction costs in capital markets consist of the following three main

components: commissions or brokerage fees, bid-ask spreads and market impact costs.

In addition, there are other components of transaction costs in capital markets, such as opportunity costs and desk-timing costs (see Freyre-Sanders *et al*, 2004; Kissell *et al*, 2004; Kissell, 2006).

Commissions or brokerage fees form a part of the total transaction cost in stock trading and they vary across brokers. The majority of brokers will tier their fees on the size of the trade or the trade value. For example, for a low-trade value transaction, an investor could pay a higher commission rate compared to that of a large-trade value transaction. In fact, Fong *et al* (2010) reported that the brokerage is not just dependent on trade size, but also on the ordering route method and other considerations. Furthermore, Johnsen (1994) argued that the execution costs of trading are often bundled with 'soft dollars' that pay for research, which may or may not be related to a specific trade.

Developing this theme further, in addition to the commission paid to the broker, an investor also incurs the bid-ask spread, which is the difference between the bid price and the ask price of the stock. These costs occur for investors who wish to buy at the best ask price or sell at the best bid price, incurring a liquidity cost. Liquidity refers to the ease with which a stock can be bought or sold without disturbing the price (Zakamouline, 2008).

Market impact costs, on the other hand, represent the movement in the price of the stock caused by a particular trade or order. The costs occur for two reasons: first, the liquidity demands of the investor, and second, the information content of the order (Kissell, 2006). Market impact costs occur when investors trade too aggressively or when they buy and sell large

positions. These large transaction orders impact on the stock price. This usually occurs in stocks that are not very liquid and are considered less significant with liquid stocks. Market impact costs are further deemed to be the sum of two components: temporary and permanent price effects (Zakamouline, 2008).

Given the structure of transaction costs in the capital markets, it is not easy to find a representation of realistic transaction costs. Therefore, as stated in Zakamouline (2008), to simplify the treatment of transaction costs, most studies assume that transaction costs are proportional to the value of trading. This assumption is valid only for large investors who trade in liquid stocks. In this situation, the bid-ask spread is the main component of transaction costs and the market impact costs are considered negligible.

Therefore, this study assumes that the estimated transaction costs are proportional to the value of trading, as Leland (1985) proposed in his model. This assumption is valid for large investors who trade in liquid stocks. The estimated transaction costs obtained are considered to cover more than the commission charged by the broker. The estimated transaction costs will include the bid-ask spread and other related trading costs, which in practice are sometimes difficult to identify and estimate.

A few studies have analysed the transaction costs associated with stock trading on the ASX. Aitken and Frino (1996) examined the magnitude and determinants of execution costs associated with institutional trades on the ASX. Their results suggested that the factors explaining the magnitude of the execution costs are brokerage commissions, trade size, stock liquidity (proxied by the bid-ask spread), broker identification, the proportion of the trade

executed using market orders and the duration of the trade. Comerton-Forde *et al* (2005) also examined the magnitude and determinants of execution costs on the ASX using institutional trade data. Their results revealed that trade complexity, stock liquidity, bid-ask spread and brokerage commission are the significant factors influencing the execution costs.

## LELAND OPTION PRICING MODEL

The strategy of Leland (1985) is identical to the BSM strategy, except that it incorporates transaction costs through an adjustment to the volatility of the underlying asset. It is assumed that transaction costs are proportional, and that the portfolio is rebalanced at discrete time intervals that are  $\delta t$  apart. The adjusted volatility is as follows

$$\sigma^* = \sigma \left( 1 + \frac{k\sqrt{\frac{2}{\pi}}}{\sigma\sqrt{\delta t}} \right)^{1/2} \quad (1)$$

where  $\delta t$  is the rebalancing interval,  $k$  is the round-trip proportional transaction costs rate (measured as a fraction of the value of transactions) and  $\sigma$  is the volatility of the underlying asset.

When there are transaction costs, the price of buying an asset is higher than the actual price of the asset and the proceeds from selling the assets are lower than the price of the asset. Essentially, when transaction costs are included in this situation, the effective prices are more volatile than without transaction costs. This was what Leland did by increasing the volatility as above. Therefore, Leland (1985) addressed the transaction costs shortcoming of the BSM strategy. Both the BSM and Leland models assume that the risk-free interest rate and the

underlying asset volatility are constant over the life of the option.

One of the assumptions in Black and Scholes (1972) is that the stock pays no dividend. However, dividends on some stocks may be substantial, and can have a significant effect on the valuation of options whose stocks make such payments during the life of the options. Therefore, a dividend adjustment must be allowed for in the option pricing formula.

Merton (1973) generalised the Black and Scholes (1972) model by relaxing the assumption of no dividend. Merton (1973) allowed for a constant continuous dividend yield on the stock and stock index. Thus, in this study we refer to the models of Black and Scholes (1972) and Merton (1973) as the BSM model. Replacing  $S_0$  by  $S_0e^{-qT}$  in the Leland (1985) model, the prices of the call,  $c$ , and the price put,  $p$ , on an index providing a dividend yield at rate  $q$  are as follows:

$$c = S_0e^{-qT}N(d_1^*) - Ke^{-rT}N(d_2^*) \quad (2)$$

$$p = Ke^{-rT}N(-d_2^*) - S_0e^{-qT}N(-d_1^*). \quad (3)$$

The  $d_1^*$  and  $d_2^*$  are obtained as in (4) and (5), respectively, by replacing  $\sigma$  with the adjusted volatility  $\sigma^*$ , and after adjusting for dividend yield at rate  $q$ . The function  $N(\cdot)$  is the cumulative probability distribution function for a standardised normal distribution.

$$d_1^* = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^{*2}}{2}\right)T}{\sigma^*\sqrt{T}} \quad (4)$$

$$d_2^* = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q - \frac{\sigma^{*2}}{2}\right)T}{\sigma^*\sqrt{T}} = d_1^* - \sigma^*\sqrt{T} \quad (5)$$

## Variables

We now consider the variables required by the Leland option pricing model.

### *Time to maturity*

According to Hull (2009), the life of an option should be measured in trading days rather than calendar days. The normal assumption in equity markets is that there are 252 trading days per year. Li and Yang (2009) used 252 trading days in their study on the Australian index options market. Thus, we employ 252 days as the number of trading days in a year for stocks.

In this study, the time to maturity,  $T$ , is measured by the number of trading days between the day of trade and the day immediately before expiry day divided by the number of trading days per year.

### *Realised volatility*

Since the recognition that the BSM option price depends upon only one unobservable parameter, that is, the volatility of stock returns, considerable attention has been paid by researchers to measuring volatility. Return volatility is widely used in portfolio construction, options pricing and trading, volatility-related derivatives' trading, and risk management. Many different methodologies and theories have been developed in the past decades.

As in Hull (2009), the realised volatility approach is used to determine the return standard deviation (volatility). This measure is only a proxy of true but unknown realised volatility. The same realised volatility formula was used by Li and Yang (2009).

Let  $n$  be the number of trading days before the expiration of an option. Daily return of the index is calculated as:

$$R_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad (6)$$

for  $i = 2, 3, \dots, n$ . Let  $S_i$  be the index level and  $R_i$  be the log-return on the  $i$ th day during the remaining life of the option.

Therefore, the annualised realised volatility can be expressed as:

$$\sigma_{r,t} = \sqrt{\frac{252}{n-2} \sum_{i=2}^n (R_{i,t} - \bar{R}_t)^2} \quad (7)$$

where  $\bar{R}_t$  denotes the mean of daily index log-returns during the period  $t$ .

There are a few other alternative approaches to estimating volatility for the input to a model. Using low-frequency daily return data, Parkinson (1980), Rogers *et al* (1994), Yang and Zhang (2000) and other researchers developed the methodologies for realised volatility estimates. However, these approaches do not necessarily lead to optimal estimates.

Among other different measures of volatility are the generalised autoregressive conditional heteroscedasticity (GARCH) models and stochastic volatility models. The original GARCH model was developed by Bollerslev (1986). Others such as Nelson (1991) and Bollerslev and Mikkelsen (1996) contributed significantly to the GARCH models. Heston and Nandi (2000), Yung and Zhang (2003) and Barone-Adesi *et al* (2008) are among the few empirical studies that adopted GARCH models in the option pricing model. There is a large amount of literature on GARCH models that aims to measure volatility, but the models cannot be reviewed as a variant of the BSM model, given that it is a firmly discrete-time theory.

The stochastic volatility model is where the volatility is allowed to be random. This is because of the fact that the BSM model treats the volatility as a constant. Hull and White (1987), Heston (1993), Danielsson (1994), Kim *et al* (1998) and many others have studied the stochastic volatility models. Among the few empirical studies that adopted stochastic models are Bakshi *et al* (1997), Sarwar and Krehbiel (2000), Kim and Kim (2004) and Sharp *et al* (2010).

It is concluded that with all the different methodologies to measure volatility, the challenge remains to find the 'best' approach for estimating volatility to be used in an option pricing model. In addition, Brailsford and Faff (1996) undertook a comprehensive study to test a number of different models,<sup>1</sup> which include GARCH models, and assessed their predictive performance against different measures of prediction error.<sup>2</sup> No one model was found to be consistently the best, but the GARCH model did perform relatively well.

### **Dividends**

According to the Reserve Bank Bulletin (2003)<sup>3</sup> dividend yields on the ASX 200 have averaged 3.6 per cent since 1997 up to 2003, and from the Reserve Bank Australia statistics<sup>4</sup>, the dividend yields S&P/ASX 200 index reported for 2004–2010 are in the range of 3.38–7.44 per cent. In this study, the dividend yields are converted to continuous compounding dividend yields.

### **Risk-free interest rate**

For the risk-free interest rate, the Australian 90-day Bank Accepted Bill (BAB) rate is used as a proxy. Daily BAB yields data are obtained from the Reserve Bank of Australia. Australia 90-day

BAB is probably close to the rate faced by option traders and the maturity matches the S&P/ASX 200 option's maturity well (Li and Yang, 2009). The BAB yields are in the range of 3.00–8.12 per cent. The risk-free interest rates are then converted to continuous compounding risk-free interest rates.

### Rebalancing interval

This study aims at estimating the implied transaction costs for the buying and selling of an asset. This means that the costs for the buying and selling of an asset should be the same regardless of the rebalancing interval.

## METHODOLOGY

This study offers a new way to estimate the transaction costs per trade by matching the market-observed prices of options with the model prices of the corresponding options. The implied transaction cost is in fact the proportional transaction costs rate for the buying and selling of the asset in rebalancing a portfolio replicating an option. This section describes the methodology used in estimating the round-trip transaction costs,  $k$ , and the benchmarks used.

### Estimation of transaction costs per trade using Leland option pricing model

The estimation of the proportional round-trip transaction costs rate,  $k$ , of stock trading will be the same regardless of the rebalancing interval, whether the rebalancing is done on a quarterly or daily basis. Therefore, this study will only consider daily rebalancing.

The implied adjusted volatility,  $\sigma^*$ , from the Leland model is estimated. The implied adjusted volatility is calculated using a Visual Basic for

Application function in which an iterative algorithm using a procedure called the bisection search method is adopted from Kwok (1998). Using Leland model, the theoretical model call price is computed and equalised with the market-observed option price of the S&P/ASX 200 index as follows:

$$C_{market} = C_{model(S,K,T,r,\sigma^*(\sigma,k,t),q)} \quad (8)$$

where  $C_{market}$  is the market-observed call price and  $C_{model}$  is the computed call price, which is based on a set of variables: security price ( $S$ ), strike price ( $K$ ), option time to maturity ( $T$ ), risk-free interest rate ( $r$ ), adjusted volatility ( $\sigma^*$ ) and dividend yield ( $q$ ).

It should be pointed out that when we calculate  $k$  based on the above approach, one condition must be fulfilled, that is,  $k < (1/2)\sigma\sqrt{2\pi\delta t}$ . If this condition is violated, then the partial differential equation governing the option pricing becomes mathematically ill-posed (for details, refer to Wilmott *et al* (1995) and Kwok (1998)).

### The benchmarks

The benchmarks are (1) the bid-ask spread estimated from Roll's model; (2) the actual stock market bid-ask spread estimate reported in Cummings and Frino (2011); (3) the actual round-trip transaction costs estimates for large stocks on the ASX reported in Aitken and Frino (1996) and Comerton-Forde *et al* (2005); and (4) the Australian brokerage commission charges documented by Fong *et al* (2010).

### Roll's model

One of the methods that has been developed to measure the bid-ask spread is the use of serial covariance of asset price change, such as



Roll (1984) and its extensions: Glosten (1987), Choi *et al* (1988), Stoll (1989), George *et al* (1991), Chu *et al* (1996), Chen and Blenman (2003) and Holden (2009).

The Roll (1984) model estimates the effective spread implied in a sequence of trades. The effective spread was calculated from the observed serial correlation of transaction prices. The two major assumptions of Roll's model are that the asset is traded in an informationally efficient market and the observed price changes are stationary. Under these assumptions, Roll showed that the trading costs induce negative serial dependence in successive observed market price changes. Further, he assumed that the underlying true value of the security lies at the centre of the spread. The possible paths of observed transaction price changes are assumed to be restricted, whereby the transaction prices can only bounce either at the ask price or at the bid price. Roll's bid-ask spread estimator is given by

$$s = 2\sqrt{-Cov(\Delta P_t, \Delta P_{t-1})}. \quad (9)$$

where  $s$  is the spread, and  $\Delta P_t$  and  $\Delta P_{t-1}$  are defined as the observed stock price changes at times  $t$  and  $t-1$ , respectively. Roll's model assumed that the next observed price is equally likely to go up by  $s$  or down by  $s$ , or remain the same. The negative covariance term was used because successive price changes are assumed to be negatively correlated to each other.

Similarly, in order to obtain a relative spread, the covariance of successive return is

$$Cov(R_t, R_{t-1}) = -\frac{s^2}{4} - \frac{s^4}{16} \quad (10)$$

However, the last term ( $s^4/16$ ) is very small and is ignorable.

Using the serial covariance estimator, the bid-ask spreads of the data will have both positive and negative serial covariances. The disadvantage of using Roll's measure as well as its extensions is that if a bid-ask spread of the data has positive serial covariances, a problem of imaginary root exists. Therefore, the spread is undefined.

The extensions of Roll's model differ from Roll's model only in scale because, with appropriate parameter substitutions, the models do in fact reduce to Roll's model and therefore seem to be perfectly correlated (Anand and Karagozoglou, 2006). Hasbrouck (2004, 2009) improved Roll's estimator by using the Gibbs sampler and Bayesian estimation, but these measures require an iterative process and are computationally intensive.

Thus, this study employs the Roll (1984) model as representative of the serial covariance-based estimator to measure the bid-ask spread. Although Harris (1990) argued that Roll's model cannot provide estimates for more than half of the firms listed on the New York Stock Exchange (NYSE)/American Stock Exchange (AMEX), it is in fact regarded as one of the most appealing and easy to use spread measurement models. It is able to measure directly from a time series of market prices.

With respect to the bid-ask spread of the data that have positive serial covariances that lead to the spread being undefined, Goyenko *et al* (2009), Hasbrouck (2009) and Holden (2009) solved this problem by substituting a default numerical value of zero. Therefore, we use a modified version of Roll's bid-ask spread as follows:

$$s = \begin{cases} 2\sqrt{-Cov(R_t, R_{t-1})} & \text{if } Cov(R_t, R_{t-1}) < 0 \\ 0 & \text{if } Cov(R_t, R_{t-1}) \geq 0 \end{cases} \quad (11)$$

### ***Bid-ask spread estimate by Cummings and Frino (2011)***

Cummings and Frino (2011) examined the mispricing of Australian stock index futures. In one element of their study, they have measured the percentage bid-ask spread in the stock market. They measured the percentage bid-ask spread on each of the constituent stocks in the index. The study reported that the mean bid-ask spread of the stock market is 0.17 per cent. Cummings and Frino (2011) obtained the quote data for the index constituents using the daily list of Bloomberg from the period of 1 January 2002–15 December 2005.

It should be noted that the time period of their study coincides with ours and that their estimate of the bid-ask spread of stocks can be used for comparison to our estimate of the index spread using Roll's model. Therefore, we consider 0.17 per cent as the benchmark for the bid-ask spread for stock trading.

### ***The actual round-trip transaction costs estimates by Aitken and Frino (1996) and Comerton-Forde et al (2005)***

Aitken and Frino (1996) analysed the magnitude and determinants of execution costs associated with institutional trades on the ASX. In terms of the transaction costs estimate, they used data that extend the period from 1 April 1991 to 30 June 1993. Their sample includes the 70 top stocks by market capitalisation. There were 6996 institutional purchases and 8032 sales analysed in this study. They reported that the magnitude of execution costs was small and that the costs were 0.27 per cent as the value of round-trip transaction costs.

Comerton-Forde *et al* (2005) examined the institutional trading costs on the ASX and the impact of broker ability on the cost of institutional trading. The data used were provided by an active institutional investor that consists of 42 229 institutional trades (18 773 purchases and 23 526 sales) made by 41 different actively managed portfolios from 15 May 2001 to 15 May 2002 on the ASX. The results of their study revealed that the transaction costs for large stocks on the ASX are around 0.50 per cent.

We note that our study period does not coincide with that of Aitken and Frino (1996) but does coincide with that of Comerton-Forde *et al* (2005). However, the transaction costs estimate can be taken as a reference. Therefore, we consider transaction costs for large stocks on the ASX between 0.27 and 0.50 per cent as another benchmark to the implied transaction costs estimate obtained in our study.

### ***The brokerage commission charges in Australia documented by Fong et al (2010)***

Fong *et al* (2010) studied the brokerage service and individual investor trade performance in Australia. In one element of their study, they studied the commissions charged by brokers. Using the Australian Stock Exchange data over a 13-year period from 1 January 1995 to 31 December 2007, they identified the types of brokers and also distinguished the classes of investors. They categorised brokers into (1) institutional brokers, (2) retail discount brokers and (3) full service retail brokers. They also distinguished the trades made by (1) individual investors at discount brokerage firms, (2) individual investors at full service brokerage firms and (3) institutional investors.

On the basis of their research of institutional investors, websites and telephone surveys, Fong *et al* (2010) found that the commission rates in Australia range between 0.1 and 0.5 per cent for institutional brokers, 0.11 and 0.66 per cent for retail discount brokers, and 1 and 2 per cent for full service retail brokers.

Our study assumes that the estimated transaction costs are proportional to the value of trading, as Leland (1985) proposed in his model. This assumption is valid for large investors who trade in liquid stocks. Thus, referring to the findings by Fong *et al* (2010), we consider taking the commission charged by the broker to institutional investors trading in large stocks to be between 0.1 and 0.5 per cent. When doubled, the commission charges are 0.2 and 1 per cent. Thus, the minimum brokerage fee charged by brokers for large stocks trading on the ASX is 0.2 per cent. On the basis of this, we expect that the implied transaction costs rate for stock trading is greater than 0.2 per cent.

## DATA

This section describes the data used in this study. The first subsection describes the S&P/ASX 200 index option data. The second and third subsections describe the data sampling procedure, as well as the sample statistics.

### Data description

S&P/ASX 200 index options are chosen for this study because they are highly popular and liquid. S&P/ASX 200 index option prices are European in style and cash-settled with quarterly expirations. They are available over a wide variety of exercise prices and several maturities. The quarterly expiry cycles are March, June, September and December. The expiration day is

the third Thursday of the expiry month or the following business day if an expiry Thursday happens to be a public holiday, unless otherwise specified by ASX. Trading of expiry contracts ceases at 12:00 noon on the expiry date. Trading continues after the settlement price has been determined. The options are quoted in index point and each index point is valued at AUD \$10.<sup>5</sup>

For S&P/ASX 200 index options, the period before 2 April 2001 was a period of excessive movements owing to the changes in the underlying asset of S&P/ASX 200 index options. On 15 November 1985, ASX first listed options on the All Ordinaries Index (XAO), which was the main benchmark for its listed stocks. The first trading of options was on 8 November 1999, and since then the trading of index options has grown tremendously. On 3 April 2000, the underlying index for ASX index options was changed from All Ordinaries Index to the S&P/ASX 200 index. During the period from 3 April 2000 to 31 March 2001, a continuation of the former All Ordinaries Index was calculated and disseminated by the ASX to allow for the maturity of futures contracts based on the superseded index. During this period the ASX re-listed index options on the All Ordinaries Index where they had been delisted twice largely owing to thin trading. From 31 March 2001, the S&P/ASX 200 index was formally used as the underlying asset of index options on the ASX.<sup>6</sup>

Owing to the excessive movements and changes occurring in the underlying asset of S&P/ASX 200 index options before 2 April 2001, we begin our sample period from 2 April 2001. Our sample data cover the period from 2 April 2001 to 31 December 2010. This sample period covers the recent global financial crisis

that began on 1 July 2007 and ended at the end of 2008.<sup>7</sup> For our analysis, we divide our sample into three periods. We consider the pre-crisis period as the starting date of our sample, 2 April 2001 until 30 June 2007, while the post-crisis period is from 1 January 2009 until the end of the sample period.

In this study, the closing price of the S&P/ASX 200 index on each day is used as the underlying price, while the closing price of the option is taken as the actual option price. In this study, daily index options data that consist of trading date, expiration date, closing price, strike price and trading volume for each trading option are collected from the Securities Industry Research Centre of Asia-Pacific. We refer to a few Australian empirical studies that used daily data in their analysis, such as Do (2002), Do and Faff (2004), Li and Yang (2009) and Sharp *et al* (2010), as well as to other studies conducted in markets other than Australia, such as Sarwar and Krehbiel (2000) and Li and Pearson (2007).

The next subsection discusses the possible problem of non-synchronous data arising from using daily closing prices for both the options and the S&P/ASX 200 index, as well as the steps taken in order to reduce such problems. We acknowledge that using the potentially non-synchronous data may yield noisier results and weaken the conclusions of the analysis. On the other hand, the noise caused by non-synchronous data has not been shown to be systematic, and studies that eliminate the problem still show the presence of the pricing biases (see Rubinstein, 1985; Bakshi *et al*, 1997; Lam *et al*, 2002; Lehar *et al*, 2002; Kim and Kim, 2004).

### **Data sampling procedure**

Using closing prices for both the options and the S&P/ASX 200 index may result in

non-synchronous data. To reduce the non-synchronous data problem, we conduct the following sampling procedure and also employ some filter rules to remove any offending daily option prices.

First, in this study, the daily closing price of the option is taken as the actual option price. When this study was done, high-frequency data or transactions data were not obtainable. Owing to the unavailability of data, this study uses daily closing option prices. The daily closing option price represents the price of the last trade of an option contract during the trading session. The last option trade does not often correspond to the closing time of the market, and could occur anytime during the trading hours. This leads to potentially non-synchronous data because option prices and the closing index level may be non-synchronous as the closing times for the two markets differ. We will explain the procedure to reduce the non-synchronous problems later in the article.

This study does have bid-ask quotes data, but not every quote becomes a trade. We noted that there are studies that consider the midpoint of bid-ask quotes in order to reduce non-synchronous problems, such as Heston and Nandi (2000), Yung and Zhang (2003), Li and Pearson (2007) and Barone-Adesi *et al* (2008). Midpoints are based on bid and ask quotes, which are more frequently refreshed than trade prices. However, as mentioned, not every quote becomes a trade. Brown and Pinder (2005) pointed out that the representation of an option's value with the midpoint of the bid-ask spread results in an overestimation of the option's value. Thus, based on this, in our study, we use daily closing option price as the actual option price.

Second, option prices and the closing index levels may be non-synchronous because the

closing times for the two markets differ. The option contracts can be traded during normal trading hours between 06:00 hours and 17:00 hours and night trading hours between 17:30 hours and 20:00 hours.<sup>8</sup> The underlying stock market closes at 16:05 hours and this creates a problem of non-synchronous closing prices for the options and equities markets. We acknowledge that using time-stamped intraday prices on both the S&P/ASX 200 index and its options would perhaps be preferable. However, at the time of the study, high-frequency data were not available. Thus, in order to reduce the problem of synchronicity between option and index prices, any significant differences between the option prices of the S&P/ASX 200 index at the closing time of the equity market and at the closing time of the options market are investigated. As we do not have access to high-frequency data during the study period, we monitored for 1 week for any significant differences in the S&P/ASX index option prices at 17:00 hours and 20:00 hours. We found that there are no significant differences between the option prices of the S&P/ASX 200 index at 17:00 hours and 20:00 hours, and the prices remain the same most of the time. Furthermore, it appears that the options are not actively traded when the equity market closes. Thus, this shows that the problem of non-synchronicity between option and index prices may not be significant.

Third, all observations that do not satisfy the minimum value arbitrage constraints are removed (Bakshi *et al*, 1997; Sharp *et al*, 2010):

$$c(\tau) \geq \max[0, S_0 - KB(\tau)] \quad (12)$$

where  $c(\tau)$  is the price of a call maturing in  $\tau$  periods (years),  $p(\tau)$  is the price of a put maturing in  $\tau$  periods (years),  $K$  is the exercise price of the option,  $S_0$  is the initial index level,

$r$  is the risk-free rate of return and  $B(\tau)$  is the current price of a \$1 zero coupon bond with the same maturity as the option.

It should be noted that the removal of the observation violating equation (12) alleviates the problem of non-synchronicity between option and index prices.

Fourth, all observations that have less than 6 days to maturity are removed in line with Bakshi *et al* (1997) because these very short-term options may introduce bias; their prices are noisy. Furthermore, the implied volatilities of options with short time to maturity behave erratically (Sarwar and Krehbiel, 2000).

Lastly, low exercise price options (LEPOs) are also removed from the sample. Certain S&P/ASX index options series have an exercise price of zero. These are all calls, and ASX termed them LEPOs.<sup>9</sup> LEPOs require no payment on exercise and are always in the money. They behave like forward contracts.

Given these facts and the data filtering process above, it should be noted that the problem owing to the non-synchronicity of trading data is alleviated to a large extent. Thus, the sample data should be reasonable for the purpose of this study.

## Sample statistics of the S&P/ASX 200 index options data

The ASX index options market can sometimes be illiquid. Options that are deep out of the money and deep in the money may induce liquidity-related biases. We divide the option data into several categories across moneyness and time to maturity.

In essence, an option's moneyness is intended to reflect its probability of being in the money at maturity. Typically, in previous studies,

moneyness is measured as  $S/K$ , where  $S$  is the index level and  $K$  is the exercise price. The greater (lower) the level of moneyness, the more likely a call (put) will be exercised at maturity. Referring to Bollen and Whaley (2004), the usual expression  $S/K$  fails to account for the fact that the probability that the option will be in the money at expiration depends heavily on the volatility rate of the underlying asset and the time remaining to expiration of the option. This makes comparisons of the implied volatility function across the index problematic. Thus, to account for these effects, Bollen and Whaley (2004) and Brown and Pinder (2005) measured moneyness using the option's delta.

Referring to Hull (2009), the delta of a call option on an asset that provides a dividend yield at rate,  $q$ , is

$$\Delta_c = e^{-qT} N(d_1) \quad (13)$$

with the usual notation of  $d_1$  in the BSM formula.

According to Bollen and Whaley (2004), deltas range from zero to one, and can be loosely interpreted as the risk-neutral probability that the option will be in the money at expiration. Deltas are computed for each option using the parameter assumptions described earlier. On the basis of the deltas, the options are categorised in moneyness groups. Options with deltas greater than 0.98 or less than 0.02 are excluded from the analysis (Bollen and Whalley, 2004; Brown and Pinder, 2005). Thus, the problem owing to the non-synchronicity of trading data is further alleviated.

Table 1 lists the upper and lower bound of the moneyness categories while Table 2 presents the summary statistics of the calls sample.

The average option moneyness, option time to maturity, daily volume, open interest and

**Table 1: Moneyness category definitions for call options**

<i>Moneyness category</i>	<i>Delta range</i>
Deep-out-of-the-money (Deep-OTM)	$0.02 < \Delta_c \leq 0.125$
Out-of-the-money (OTM)	$0.125 < \Delta_c \leq 0.375$
At-the-money (ATM)	$0.375 < \Delta_c \leq 0.625$
In-the-money (ITM)	$0.625 < \Delta_c \leq 0.875$
Deep-in-the-money (Deep-ITM)	$0.875 < \Delta_c \leq 0.98$

*Note:* Listed are the moneyness category and the corresponding delta ranges in our sample. Options with deltas greater than 0.98 or less than 0.02 are excluded from the analysis (Bollen and Whalley, 2004; Brown and Pinder, 2005).

number of series traded per day are also reported in Table 2. It should be noted that the average market price for call options increases with time to maturity.

The average maturity for the calls sample is 59 days. The options are classified into three maturity categories: (i) short term (<30 days), (ii) medium term (30–90 days) and (iii) long term ( $\geq 90$  days).

There are altogether 42012 call options in the sample, with deep-OTM and OTM call options accounting for 49.41 per cent of the total sample. This implies that OTM call options are actively traded. This is in fact similar to those observed in Barone-Adesi *et al* (2008). Barone-Adesi *et al* (2008) documented that OTM options on the S&P 500 index are more actively traded than ITM options, and they used OTM options as their sample data. Further, in our sample data, long-term and in-the-money options appear least frequently.

**Table 2: Sample properties of S&P/ASX 200 index call options**

Moneyness	Time to maturity in days ( $T$ )			
	$T < 30$	$30 \leq T < 90$	$T \geq 90$	Total
Delta ( $\Delta_c$ )				
Deep-OTM ( $0.02 < \Delta_c \leq 0.125$ )	12.31 (2542)	18.86 (2313)	41.08 (1150)	20.34 (6005)
OTM ( $0.125 < \Delta_c \leq 0.375$ )	40.11 (4857)	53.69 (6748)	86.40 (3151)	56.21 (14756)
ATM ( $0.375 < \Delta_c \leq 0.625$ )	99.37 (4062)	135.24 (7276)	204.89 (3094)	140.07 (14432)
ITM ( $0.625 < \Delta_c \leq 0.875$ )	167.33 (2132)	232.55 (2429)	394.86 (672)	226.82 (5233)
Deep-ITM ( $0.875 < \Delta_c \leq 0.98$ )	271.38 (804)	465.57 (599)	872.21 (183)	414.05 (1586)
Total	83.68 (14397)	115.35 (19365)	167.07 (8250)	114.65 (42012)

  

Sample average				
Delta ( $\Delta_c$ )	Maturity (days)	Volume	Open interest	Series traded per day
0.39	59.12	115.36	1115.52	11.80

Note: The average prices (in index points) and the number of observations in parentheses that fall into each category are provided. Some sample averages are also reported. The sample period is from 2 April 2001 to 31 December 2010. Deep-OTM, OTM, ATM, ITM and deep-ITM stand for deep-out-of-the-money, out-of-the-money, at-the-money, in-the-money and deep-in-the-money, respectively.

## EMPIRICAL RESULTS

The discussion of the results is divided into two subsections. The first subsection is the estimation of bid-ask spread in stock trading using Roll's model. As defined earlier, there are other components of transaction costs, which include the bid-ask spread. Thus, it is expected that the transaction costs estimates obtained in this study across the pre-crisis, during crisis and post-crisis periods should be greater than the bid-ask spread estimate using Roll's model. Rather than only using estimates from Roll's model, we also use other transaction costs estimates documented from other studies by Aitken and Frino (1996), Comerton-Forde *et al* (2005), Cummings and Frino (2011) and Fong *et al* (2010).

The second subsection is the discussion on the transaction costs rates,  $k$ , implied by the Leland

model across different periods: pre-crisis, during crisis and post-crisis. We determine the option moneyness and time to maturity groupings that best estimate the transaction costs across the different periods.

### Roll's bid-ask spread estimate

We use Roll's model to estimate the bid-ask spread in stock trading. Ideally, the best way to find the relative bid-ask spread for stock trading is in fact by analysing the 200 stocks underlying the index individually or by estimating the spread using a number of assets as a sample. This will be very cumbersome. However, it is known that the S&P/ASX 200 index is recognised by professional investors as the leading benchmark in Australia for broad market movements in the

**Table 3:** Bid-ask spread estimate using Roll's model

<i>Sample period</i>	<i>Total no. of observations</i>	<i>No. of positive covariances</i>	<i>No. of negative covariances</i>	<i>Average relative spread (s)</i>
April 2001–15 December 2005	1149	552	597	0.1729%

*Note.* This table reports the spread estimate from Roll (1984) given by  $s = 2\sqrt{-Cov(R_t, R_{t-1})}$  where  $s$  is the spread. Roll's spread is calculated yearly using daily returns from 2 April 2001 to 15 December 2005. The sample period used is to coincide with Cummings and Frino (2010) who calculated the bid-ask spread for the stock market. Following Goyenko *et al* (2009), Hasbrouck (2009) and Holden (2009), if the covariance is positive, the spread is set to zero.

stock market. Therefore, in this study we consider taking the average index spread as representative of the average bid-ask spread of stock trading.

We assess the reliability of this approach by comparing it with the bid-ask spread estimate reported by Cummings and Frino (2011). As stated earlier, Cummings and Frino (2011) reported that the mean bid-ask spread in the stock market is 0.17 per cent. They used a sample period ranging from 1 January 2002 to 15 December 2005. Thus, we also consider estimating the bid-ask spread of stock trading using the S&P/ASX 200 index from our sample period from 2 April 2001 to 15 December 2005.

The spread for each day is calculated based on the yearly (a fixed length of 252 days) return series by applying equations (10) and (11). The result of Roll's average spread is presented in Table 3. There are 1149 observations during the sample period from 2 April 2001 to 15 December 2005. Out of these 1149 observations, 552 (or 48.04 per cent) are of positive serial covariance and 597 (or 51.96 per cent) are of negative serial covariance. The average spread is approximately equal to 0.17 per

cent, which is the same as the bid-ask spread reported by Cummings and Frino (2011). Thus, this implies that our approach to estimating the bid-ask spread for stock trading using the S&P/ASX 200 index data is reliable.

Consequently, we extend our estimate of the bid-ask spread using Roll's model across the pre-crisis, during crisis and post-crisis periods. We hypothesise that the bid-ask spread for stock trading would be higher during the crisis period. Table 4 presents the results.

The results show that the bid-ask spread during the crisis period with the average spread of 0.31 per cent is higher than that before the crisis with the average spread of 0.21 per cent. In both the pre- and during crisis periods, the rate of negative serial covariance is higher than that of the positive serial covariance. However, the average spread during the post-crisis period is much lower with a value of 0.08 per cent. This low value of spread is a result of the fact that the rate of positive covariance is higher than that of the negative covariance.

As explained earlier, Roll's spread is set to zero when there is a positive serial covariance. Setting the spread to zero may underestimate the spread



**Table 4:** Bid-ask spread estimate using Roll's model across three different periods: pre-, during and post-crisis

Sample period	Total no. of observations	No. of positive covariances	No. of negative covariances	Average relative spread ( <i>s</i> ) (%)
Pre-crisis	1531	659	872	0.2110
During crisis	380	147	233	0.3148
Post-crisis	504	356	148	0.0788

*Note.* This table reports the spread estimate from Roll (1984) given by  $s = 2\sqrt{-Cov(R_t, R_{t-1})}$  where  $s$  is the spread across pre-, during and post-crises periods. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010. Following Goyenko *et al* (2009), Hasbrouck (2009) and Holden (2009), if the covariance is positive, the spread is set to zero.

when there is a large number of positive covariances. This is in fact one of the likely disadvantages of using Roll's model as a measure for a bid-ask spread and consequently as a measure for transaction costs. To remedy this problem, we propose a new approach to estimating transaction costs as outlined in this article.

### Implied transaction costs rate estimates, $k$

The empirical results of the average round-trip transaction costs rate,  $k$ , estimated from the Leland model in each pre-crisis, during crisis and post-crisis period, are discussed in this subsection. The implied transaction costs rates,  $k$ , are first investigated across different moneyness groupings. Then the implied transaction costs rates,  $k$ , are investigated across different time to maturity groupings. We perform a statistical test to investigate whether there is any significant difference in the value of  $k$  between any two of the moneyness and time to maturity groupings.

Table 5 displays the implied transaction costs rates,  $k$ , estimated from the Leland model for call options across different moneyness groupings in each of the pre-crisis, during crisis and post-crisis periods. The total number of observations being studied under these three periods is 31 675, which is less than the total number of observations displayed in Table 1. This is because only these observations do not violate the condition that  $k$  must be less than  $(1/2)\sigma\sqrt{2\pi\delta t}$ .

We conduct a pairwise statistics test on whether there are any statistically significant differences between the  $k$  values estimated from any two of the moneyness groupings. Table 6 displays the results.

We summarise the results as follows:

- (1) We hypothesise that the transaction costs are higher during the crisis period. Using our approach in estimating the transaction costs, the results in Table 5 support our hypothesis that during the crisis period, the transaction costs estimates are higher than those of the

**Table 5:** Summary of estimated implied transaction costs rate,  $k$ , for the whole call options across different moneyness groupings

<i>Moneyness</i>		<i>Pre-crisis</i>	<i>During crisis</i>	<i>Post-crisis</i>
Deep-OTM	Mean (%)	0.29	0.91	0.61
	SD (%)	0.18	0.67	0.36
OTM	Mean (%)	0.32	1.08	0.63
	SD (%)	0.22	0.82	0.40
ATM	Mean (%)	0.36	1.14	0.66
	SD (%)	0.25	0.78	0.40
ITM	Mean (%)	0.34	0.84	0.66
	SD (%)	0.23	0.62	0.43
Deep-ITM	Mean (%)	0.36	0.88	0.78
	SD (%)	0.25	0.58	0.46
Total $n$		14 880	6424	10 371

*Note:* The moneyness is defined in terms of option delta as described in Table 1. Let  $n$  be the number of observations. Only these observations do not violate the condition that  $k$  must be less than  $(1/2)\sigma\sqrt{2\pi\delta t}$ . Mean is the reported value of the average implied transaction costs rate in that particular moneyness groupings. SD is the standard deviation. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

- pre-crisis and the post-crisis periods across all moneyness groupings.
  - (2) In the pre-crisis period, the  $k$  values implied from using the deep-ITM call options are not significantly different from those of the ATM and ITM call options.
  - (3) During the crisis period, (i) the  $k$  values implied from using the deep-OTM call options are not significantly different from those of the ITM and deep-ITM call options; and (ii) the  $k$  values implied from using the ITM call options are not significantly different from those of the deep-ITM call options.
  - (4) In the post-crisis period, only the  $k$  values implied from using the ATM call options are not significantly different from those of the ITM call options.
- If the Leland models are perfectly theoretically correct, then the transaction costs rate should be the same across the different moneyness groupings regardless of the three periods. From Table 6, we clearly see that the implied

**Table 6:** Summary of pairwise test statistics of average implied transaction costs rate ( $k$ ) between moneyness categories across different periods: pre, during and post-crisis

<i>Moneyness</i>	<i>Deep-OTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>Deep-ITM</i>
<i>Panel A: Pre-crisis</i>					
Deep-OTM	—	<b>0.0001</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
OTM	—	—	<b>0.0000</b>	<b>0.0008</b>	<b>0.0067</b>
ATM	—	—	—	<b>0.0025</b>	0.9185
ITM	—	—	—	—	0.2352
Deep-ITM	—	—	—	—	—
<i>Panel B: During crisis</i>					
Deep-OTM	—	<b>0.0000</b>	<b>0.0000</b>	0.0770	0.7117
OTM	—	—	<b>0.0054</b>	<b>0.0000</b>	<b>0.0089</b>
ATM	—	—	—	<b>0.0000</b>	<b>0.0008</b>
ITM	—	—	—	—	0.6276
Deep-ITM	—	—	—	—	—
<i>Panel C: Post-crisis</i>					
Deep-OTM	—	<b>0.0306</b>	<b>0.0000</b>	<b>0.0009</b>	<b>0.0000</b>
OTM	—	—	<b>0.0040</b>	<b>0.0377</b>	<b>0.0000</b>
ATM	—	—	—	0.7995	<b>0.0005</b>
ITM	—	—	—	—	<b>0.0014</b>
Deep-ITM	—	—	—	—	—

*Note.* This table reports the test-statistics ( $P$ -values) of  $k$  values between two different moneyness categories. Each bold number implies that the  $P$ -value is less than 0.05 and that the two mean  $k$  values are different. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

transaction costs rates,  $k$ , are mostly significantly different across the deep-OTM, OTM, ATM, ITM and deep-ITM call options in each of the three different periods. The different findings of implied transaction costs rates across the moneyness groupings can be related to the measurement of the realised volatility undertaken in this study. The volatility of the underlying asset may be underestimated,

which would lead to systematic measurement errors in the estimated implied transaction costs rate across the different moneyness groupings.

Next, we investigate the estimated transaction costs rate,  $k$ , implied by the Leland model across time to maturity groupings in each pre-crisis, during crisis and post-crisis period. Table 7 reports the results.

**Table 7:** Summary of estimated implied transaction costs rate,  $k$ , for the whole call options across different time to maturity groupings

<i>Maturity</i>		<i>Pre-crisis</i>	<i>During crisis</i>	<i>Post-crisis</i>
≤ 29 days	Mean (%)	0.37	1.06	0.58
	SD (%)	0.29	0.79	0.39
30–49 days	Mean (%)	0.33	1.04	0.65
	SD (%)	0.23	0.78	0.40
50–69 days	Mean (%)	0.34	1.27	0.72
	SD (%)	0.22	0.84	0.36
70–89 days	Mean (%)	0.33	0.99	0.75
	SD (%)	0.20	0.77	0.41
90–109 days	Mean (%)	0.34	0.76	0.61
	SD (%)	0.20	0.54	0.41
110 to 129 days	Mean (%)	0.29	1.20	0.73
	SD (%)	0.19	0.45	0.40
≥ 130 days	Mean (%)	0.29	1.05	0.66
	SD (%)	0.20	0.64	0.36
Total $n$		14 880	6424	10 371

*Note.* The Let  $n$  be the number of observations. Only these observations do not violate the condition that  $k$  must be less than  $(1/2)\sigma\sqrt{2\pi\delta t}$ . The time to maturity is defined as ≤ 29 days as short term, 30–89 days is medium term and ≥ 130 days is long term. Mean is the reported value of the average implied transaction costs rate in that particular time to maturity groupings. SD is the standard deviation. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

We also conduct pairwise statistics tests on whether there are any statistically significant differences between the  $k$  values estimated from any two of the time to maturity groupings. Table 8 displays the results.

We summarise the results as follows:

- (1) We hypothesise that the transaction costs are higher during the crisis period. Using our approach in estimating the transaction

**Table 8:** Summary of pairwise test statistics of average implied transaction costs rate ( $k$ ) between pre-, during and post-crisis periods for call options across all maturities

Maturity (days)	$\leq 29$ days	30–49 days	50 to 69 days	70 to 89 days	90 to 109 days	110 to 129 days	$\geq 130$ days
<i>Panel A: Pre-crisis</i>							
$\leq 29$ days	—	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>
30–49 days	—	—	0.3811	0.4335	0.2513	<b>0.0000</b>	<b>0.0000</b>
50–69 days	—	—	—	0.0977	0.6769	<b>0.0000</b>	<b>0.0000</b>
70–89 days	—	—	—	—	0.0713	<b>0.0000</b>	<b>0.0000</b>
90–109 days	—	—	—	—	—	<b>0.0000</b>	<b>0.0000</b>
110–129 days	—	—	—	—	—	—	0.8033
$\geq 130$ days	—	—	—	—	—	—	—
<i>Panel B: During crisis</i>							
$\leq 29$ days	—	0.4484	<b>0.0000</b>	0.0918	<b>0.0000</b>	<b>0.0003</b>	0.8730
30–49 days	—	—	<b>0.0000</b>	0.2392	<b>0.0000</b>	<b>0.0001</b>	0.7764
50–69 days	—	—	—	<b>0.0000</b>	<b>0.0000</b>	0.1773	<b>0.0000</b>
70–89 days	—	—	—	—	<b>0.0000</b>	<b>0.0001</b>	0.2417
90–109 days	—	—	—	—	—	<b>0.0000</b>	<b>0.0000</b>
110–129 days	—	—	—	—	—	—	<b>0.0046</b>
$\geq 130$ days	—	—	—	—	—	—	—
<i>Panel C: Post-crisis</i>							
$\leq 29$ days	—	<b>0.0000</b>	<b>0.0000</b>	<b>0.0000</b>	0.1378	<b>0.0000</b>	<b>0.0000</b>
30–49 days	—	—	<b>0.0000</b>	<b>0.0000</b>	0.0929	<b>0.0047</b>	0.4636
50–69 days	—	—	—	0.1950	<b>0.0000</b>	0.8097	<b>0.0002</b>
70–89 days	—	—	—	—	<b>0.0000</b>	0.5013	<b>0.0001</b>
90–109 days	—	—	—	—	—	<b>0.0006</b>	<b>0.0442</b>
110–129 days	—	—	—	—	—	—	<b>0.0191</b>
$\geq 130$ days	—	—	—	—	—	—	—

Note. This table reports the  $P$ -values of  $k$  between two different maturity categories. Each bold number implies that the  $P$ -value is less than 0.05 and that the two mean  $k$  values are different. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

costs, the results in Table 7 support our hypothesis that during the crisis period, the transaction costs estimates are higher

than those of the pre-crisis and post-crisis periods across all time to maturity groupings.

- (2) In the pre-crisis period, the  $k$  values implied from using (i) the call options with maturity between 30 and 109 days are not significantly different from each other; and (ii) the call options with maturity between 110 to 129 days are not significantly different from those of the options with maturity greater than 130 days.
- (3) During the crisis period, the  $k$  values implied from using (i) the call options with maturity less than 29 and up to 49 days are not significantly different from those of the options with maturity between 70 and 89 days and greater than 130 days; (ii) the call options with maturity between 50 and 69 days are not significantly different from those of the options with maturity between 110 and 129 days; and (iii) the call options with maturity between 70 and 80 days are not significantly different from those of the options with maturity greater than 130 days.
- (4) In the post-crisis, the  $k$  values implied from using (i) the call options with maturity less than 29 and up to 49 days are not significantly different from those of the options with maturity between 90 and 109 days; (ii) the call options with maturity between 50 and 69 days are not significantly different from those of the options with maturity between 70 and 89 days; (iii) the call options with maturity between 50 and 89 days are not significantly different from those of the options with maturity between 110 and 129 days; and (iv) the call options with maturity between 30 and 49 days are not significantly different from those of the options with maturity greater than 130 days.

From these results, across the three different periods, the implied  $k$  values are not significantly

different between the majorities of the option time to maturity groupings. The possible explanation behind this is that different maturity options will have different realised volatilities, but on the other hand, realised volatility is assumed to be the same for call options with the same time to maturity. This is the reason that the values of  $k$  are not very different between any two of the time to maturity groupings. This is in contrast to the values of  $k$  implied by options in different moneyness groupings.

Further, looking at the standard deviation values, the deviation from the average value of  $k$  is higher for short-term options compared to long-term options. This suggests that the implied adjusted volatility of short-term options behaves erratically (Sarwar and Krehbiel, 2000).

Table 9 reports the various transaction costs rate estimates implied from various option moneyness and time to maturity groupings for calls. The empty cells in Panels B and C means that there are no options that fall in this category.

Using the results from Tables 6 and 8 and referring to Table 9, we conclude that:

- (1) In the pre-crisis period, all option moneyness and time to maturity groupings are good to be used to estimate the implied transaction costs rate,  $k$ , except short-term deep-OTM and OTM call options.
- (2) During the crisis period, all option moneyness and time to maturity groupings are good to be used to estimate the implied transaction costs rate,  $k$ , except OTM and ATM call options with maturity between 90 and 109 days.
- (3) In the post-crisis period, all option moneyness and time to maturity groupings are good to be used to estimate the implied transaction costs rate,  $k$ .

**Table 9:** Implied round trip transaction costs,  $k$ , estimated from Leland option pricing model using call options across pre-, during and post-crisis periods

		<i>Moneyness</i>				
		<i>Deep-OTM (%)</i>	<i>OTM (%)</i>	<i>ATM (%)</i>	<i>ITM (%)</i>	<i>Deep-ITM (%)</i>
<i>Panel A: Pre-crisis</i>						
Time to maturity (days)	≤ 29	0.30	0.37	0.41	0.36	0.39
	[30–49]	0.28	0.30	0.37	0.34	0.29
	[50–69]	0.29	0.33	0.36	0.31	0.37
	[70–89]	0.27	0.31	0.34	0.36	0.37
	[90–109]	0.26	0.32	0.36	0.37	0.31
	[110–129]	0.25	0.27	0.30	0.33	0.25
	≥ 130	0.38	0.26	0.29	0.30	0.39
<i>Panel B: During crisis</i>						
Time to maturity (days)	≤ 29	0.86	1.07	1.19	0.93	1.01
	[30–49]	0.94	1.04	1.10	0.78	0.73
	[50–69]	1.27	1.47	1.24	0.73	0.69
	[70–89]	1.14	0.95	1.01	0.79	0.90
	[90–109]	0.59	0.71	0.87	0.57	—
	[110–129]	1.31	1.31	1.23	0.57	—
	≥ 130	1.08	0.96	1.12	1.06	0.69
<i>Panel C: Post-crisis</i>						
Time to maturity (days)	≤ 29	0.52	0.56	0.63	0.65	0.81
	[30–49]	0.61	0.67	0.64	0.67	0.72
	[50–69]	0.72	0.73	0.74	0.57	0.56
	[70–89]	0.77	0.70	0.79	0.77	0.89
	[90–109]	0.72	0.53	0.63	0.83	0.78
	[110–129]	0.76	0.77	0.63	0.51	1.27
	≥ 130	0.68	0.64	0.66	0.75	—

*Note.* The average implied round trip transaction costs,  $k$ , (in per cent) estimated from Leland option pricing model using call options reported across moneyness and maturity. The moneyness is defined in terms of option delta as described in Table 1. The time to maturity is categorised as: less than or equal to 29 days is short term, 30–89 days is medium term and greater than or equal to 90 days is long term. We consider the pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

**Table 10:** The average implied round trip transaction costs,  $k$ , estimated from Leland option pricing model using call options across pre-, during and post-crisis periods

Implied transaction costs	Pre-crisis (%)	During crisis (%)	Post-crisis (%)
Average $k$	0.33	1.08	0.63
SD	0.22	0.76	0.39

*Note.* The average implied round trip transaction costs,  $k$ , (in per cent) estimated from Leland option pricing model using call options reported across different period: pre-crisis, during crisis and post-crisis. The pre-crisis period is from 2 April 2001 to 30 June 2007, during crisis period is from 1 July 2007 to 31 December 2008 and post-crisis period is from 1 January 2009 to 31 December 2010.

Given these findings, we estimate the single value,  $k$ , which will be the implied transaction costs rate for the buying and selling of the asset in rebalancing a portfolio replicating an option. We tabulate the results in Table 10.

We find that during the crisis, the average implied transaction costs increase more than double the rate before the crisis from around 0.33 to around 1.08 per cent. The increase in the transaction costs estimate is a result of the high levels of uncertainty about market future movement and the enormous transaction costs associated with the trading of the underlying asset during the crisis period.

However, the implied transaction costs decrease by around 40 per cent after the crisis to a value of 0.63 per cent, but the costs are still higher than those before the crisis. This may be due to the fact that the financial market situation in the year 2009–2010 was gradually returning to normal.

The value of the implied transaction costs rates is assessed against the benchmarks. The round-trip transaction costs estimates of 0.33 per cent pre crisis, 1.08 per cent during the crisis and 0.63 per cent post crisis are considered good estimates for the following reasons.

First, they are well above Roll's bid-ask spread estimate of 0.17 per cent, and also above the actual stock market bid-ask spread of 0.17 per cent in Cummings and Frino (2011). Second, in the pre-crisis period, our estimate lies between the actual transaction costs estimate for large stocks on the ASX of 0.27 per cent in Aitken and Frino (1996) and 0.50 per cent in Comerton-Forde *et al* (2005). Third, our estimate is above the minimum brokerage fees of 0.20 per cent charged by brokers in Australia to institutional investors trading in large stocks on the ASX documented by Fong *et al* (2010).

## SUMMARY AND CONCLUSION

Estimation of transaction costs is an important topic in empirical analyses of market efficiency and microstructure. Petersen and Fialkowski (1994) discussed the importance of accurately measuring transaction costs to assess market efficiency, asset pricing models and theories of spread behaviour analyses. Obviously, transaction costs affect investment returns and volatility. Therefore, a reliable estimate of transaction costs would significantly enhance market efficiency and microstructure research.

This study has two objectives. The first objective is to offer a new way to estimate transaction costs observed in the market for the buying and selling of a stock. The transaction costs per trade are estimated using an option pricing model. To the best of our knowledge, no similar study has attempted to estimate the



transaction costs per trade via an option pricing model. The option pricing models used here is the Leland (1985) model. The transaction costs are implied by matching the market-observed option prices with the model option prices. Estimating transaction costs is done using the implied adjusted volatility, which is dependent on the volatility of the underlying asset. Here the volatility of the underlying asset is measured using the historical volatility of the underlying asset over the remaining life of the option. One key feature of the proposed approach is that it does not need to obtain information on commissions and other fees from market participants, which can be subjective and different.

The implied transaction costs approach is tested empirically based on the S&P/ASX 200 index call options data covering the period from 2 April 2001 to 31 December 2010. The implied transaction costs rate estimate is judged to be reasonable based on the bid-ask spread estimate based on Roll (1984), the actual stock market bid-ask spread estimated by Cummings and Frino (2011), the actual transaction costs for large stocks on the ASX documented by Aitken and Frino (1996), Comerton-Forde *et al* (2005) and Chen *et al* (2010), and the brokerage service fees charged by brokers in Australia documented by Fong *et al* (2010).

Our sample data cover the period of the global financial crisis from the middle of 2007 to end of 2008. Thus, the second objective of this study is to investigate the implied transaction costs during this crisis period. During the crisis, the implied transaction costs increase more than double the rate before the crisis. This confirms our hypothesis that the implied transaction costs are higher during the crisis than those before the crisis. The higher transaction costs during the crisis are a result of the high levels of uncertainty

about future market movements and the enormous transaction costs associated with the trading of the underlying asset. Further, volatility of the underlying asset can rise significantly during the crisis period. However, the implied transaction costs decrease by around 40 per cent after the crisis, but the costs are still higher than those before the crisis. This may be a result of the fact that the conditions in the financial market have improved over the course of 2009 and 2010.

In conclusion, the implied transaction costs approach presented in this article can offer a practical and viable way to estimate the transaction costs per trade. This new technique for estimating transaction costs is particularly valid for large traders and can be expected to be also valid for other stock markets.

## NOTES

1. In addition to GARCH models, there are random walk, historical mean, moving average, exponential smoothing, weighted moving average and simple regression.
2. The prediction errors used were the mean errors, mean absolute error, root mean squared error and mean absolute percentage error.
3. Reserve Bank bulletin 'Statement on monetary policy', November 2003 (Box B). Sample period 1997–2003, [www.rba.gov.au/publications/bulletin/2003/nov/pdf/bu-1103-1.pdf](http://www.rba.gov.au/publications/bulletin/2003/nov/pdf/bu-1103-1.pdf), accessed 24 September 2009.
4. These dividend yields are extracted from the Reserve Bank Australia: Alphabetical Index Statistics, 'S&P/ASX 200–Share Market-F7', [www.rba.gov.au/Statistics/Bulletin/F07.pdf](http://www.rba.gov.au/Statistics/Bulletin/F07.pdf), accessed 24 September 2009 and 13 March 2011.

5. Australian Securities Exchange (ASX) (2009). Index options, [www.asx.com.au/products/index-options.htm](http://www.asx.com.au/products/index-options.htm), accessed 1 December 2009.
6. The description of the changes in the underlying asset of S&P/ASX 200 index option is extracted from Li and Yang (2009).
7. The choice of the starting date of the subprime mortgage crisis is arbitrary but consistent with the market consensus that the crisis started in the summer of 2007. Moreover, we refer to a featured article in 1301.0 – Year Book Australia 2009–10 (2010), ‘Feature Article: The Global Financial Crisis and its impact on Australia’, contributed by the Reserve Bank of Australia that mentioned that the subprime mortgage crisis emerged around the middle of 2007 and intensified in 2008, and that the conditions in financial markets improved over the course of 2009. Source is from [www.abs.gov.au/AUSSTATS/abs@.nsf/Lookup/1301.0Chapter27092009%E2%80%9310](http://www.abs.gov.au/AUSSTATS/abs@.nsf/Lookup/1301.0Chapter27092009%E2%80%9310), accessed 21 May 2012.
8. All times are Sydney local times.
9. Australian Securities Exchange (ASX) (2009).

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