
Original Article

Commodity models and investment under uncertainty: The optimal contract determination

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ABSTRACT Investment projects involving commodities typically require a large amount of capital, last many years and include clauses that resemble call and put options. Therefore, the price dynamics behavior assumed for the commodity is essential to consider in valuing such an investment project. In this article, optimal contract determination is analyzed assuming several models proposed in the literature: the basic AR(1) model and the two- and three-factor models developed by Schwartz and Smith and Cortazar and Schwartz, respectively. These models are applied to contracts involving the WTI and Brent oilfields and natural gas in the Henry Hub fields. The results indicate that the model's assumptions, especially those that have to do with volatility, play crucial roles in negotiating this kind of contract, as the differences obtained between models can in some cases represent hundreds of millions of dollars.

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INTRODUCTION

When a company is planning to develop a crude oil or natural gas field, the investment is significant and production usually lasts many years; however, the main investment has to be made initially in order for there to be any return (see for example Jahn *et al*¹ or Smit,² among

others). Consequently, the company needs a sell contract that contains clauses including a minimum price to guarantee that the seller will recover the value of the investment. It is also common that these sorts of contracts include a maximum price to hedge against unexpected and steeper price increases for the buyer. These

clauses are put and call options in each liquidation period, and as a result the stochastic behavior of commodity prices plays a crucial role in their valuation. As volatility is a decisive parameter in option valuation, the model volatility assumptions are critical.

As Schwartz³ states, commodity prices take into account mean reversion. Therefore, the simplest model to use includes mean reversion; it is called the AR(1) model. In recent years, however, several authors have proposed more sophisticated models in which the commodity price is assumed to be the sum of several factors. The ones we review are the two-factor model by Schwartz and Smith⁴ and the three-factor model by Cortazar and Schwartz.⁵ This article analyzes the relative performances of these models.

This comparative study of models is of critical importance for investment under conditions of uncertainty because we apply these models to fictitious contracts involving the WTI and Brent oilfields and natural gas in the Henry Hub fields. The results indicate that the model's assumptions with regard to the stochastic behavior of the commodity price, and particularly the volatility assumptions, play crucial roles in negotiating this kind of contract, as the differences obtained between models can represent hundreds of millions of dollars in some cases.

This article is organized as follows. The comparative study between models is contained in the next section. The subsequent section deals with investment under uncertainty, and the main conclusions can be found in the last section.

COMMODITY MODELS

The AR(1) model

If we assume that the commodity log-spot price (p_t) follows an AR(1) model, it is easy to see that

p_{t+j} , with a high 'j' is a random variable with variance $\sigma^2 \Delta t / (1 - \rho^2)$. Consequently, this model assumes that the volatility is fenced, that is, that it does not grow with time.

This model will be estimated using three commodity price series: those for WTI and Brent crude oil and Henry Hub natural gas. Cortazar and Naranjo⁶ found that there was a mean reversion in commodity prices until 1999, whereas afterwards, commodity prices have exhibited a random walk behavior. Therefore, we are going to assume that there is a structural break; as a result, we are going to use only data for the period before that date. Table 1 includes the model parameter estimates for the three commodities. As can be understood from Figure 1, this model is much too simple, as the empirical volatility of futures returns does not reach zero when time to maturity extends to

Table 1: AR(1) parameter estimates

	<i>Brent crude oil</i>	<i>WTI crude oil</i>	<i>Henry Hub natural gas</i>
ρ	0.98 (0.008)	0.98 (0.007)	0.96 (0.01)
C	2.93 (1.35)	2.9 (1.18)	0.69 (0.23)
σ	28%	30%	45%
R^2	96%	96%	92%
Number obs.	601	783	509

The table shows the parameter estimates of the AR(1) model. Weekly observations of one-month futures contracts are used in all cases: NYMEX WTI crude oil from 1/1/1985 to 12/27/1999, NYMEX Henry Hub natural gas from 4/2/1990 to 12/27/1999 and ICE Brent crude oil from 6/27/1988 to 12/27/1999. Standard deviations in parentheses.

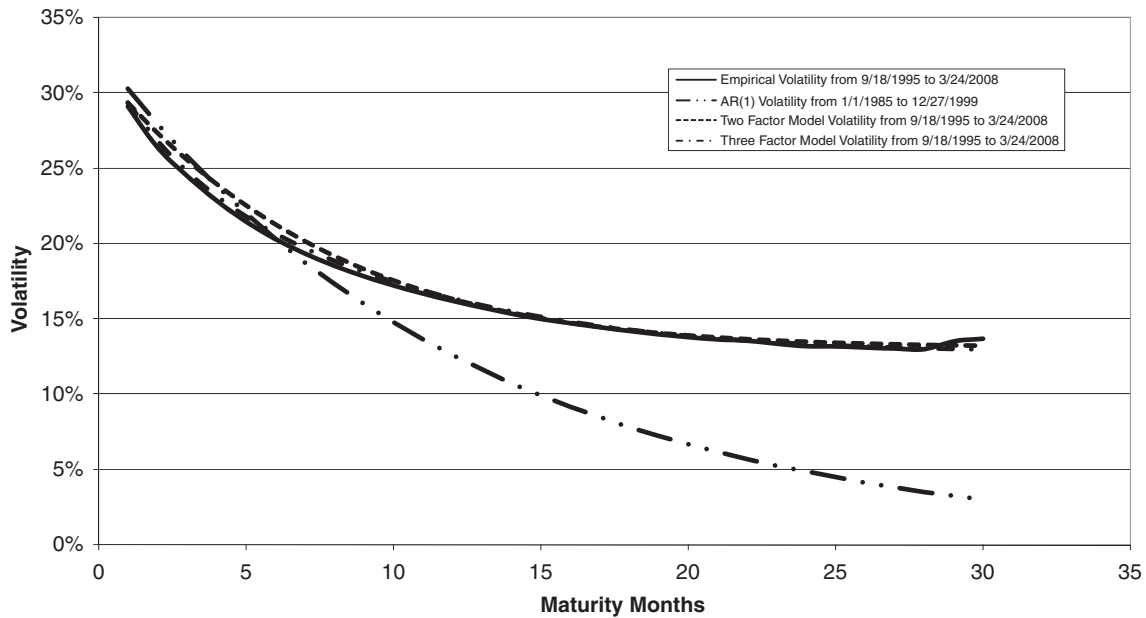


Figure 1: WTI volatility.

infinity as the model predicts (for the sake of brevity, we are only going to present the results for WTI crude oil; however, the results for the other commodities are available from the authors on request). Even more importantly, models considering a single source of uncertainty are not very realistic since they imply that futures prices at different maturities should be perfectly correlated, which defies existing evidence.

The two-factor model

In this model, it is assumed that the log-spot price is the sum of two components (or factors): one short-term factor (χ_t) that follows an Ornstein-Uhlenbeck process and one long-term factor (ε_t) that follows a standard Brownian motion (see Schwartz and Smith⁴). Under these model assumptions, the long-term volatility is $\sigma_\varepsilon\sqrt{t}$. Therefore, this model assumes that volatility grows with time; consequently, it is not fenced.

This implies that the volatility of futures returns does not go to zero when time to maturity goes to infinity, which is a desirable property.

As there is no market quotation for factors that make up the spot price of the three commodities presented above, the estimate has been performed using the Kalman filter methodology (see, for example, Harvey⁷). The data set employed in the estimation procedure consists of weekly observations of Henry Hub natural gas and WTI crude oil futures prices traded at NYMEX and Brent crude oil futures prices traded at ICE.

Table 2 includes the model parameter estimates for the three commodities. The results are consistent with those obtained by Schwartz and Smith.⁴ As can be understood from Figure 1, this model estimates the volatility of futures returns more accurately than did the previous one. The in-sample predictive power ability of the two-factor model can be analyzed

Table 2: Two-factor model parameter estimates

<i>Contract</i>	<i>Brent</i>	<i>Henry Hub</i>	<i>WTI</i>
μ_{ξ}	0.1641*** (0.0293)	0.1524*** (0.0286)	0.1376*** (0.0247)
k	0.8854*** (0.0071)	1.1879*** (0.0425)	1.0598*** (0.0083)
σ_{ξ}	0.1441*** (0.0033)	0.1553*** (0.0059)	0.1315*** (0.0027)
σ_{χ}	0.3038*** (0.0081)	0.6057*** (0.0240)	0.2905*** (0.0066)
$\rho_{\xi\chi}$	-0.1736*** (0.0342)	-0.1791*** (0.0585)	-0.0240*** (0.0310)
μ_{ξ}^*	-0.0269*** (0.0008)	-0.0601*** (0.0016)	-0.0219*** (0.0007)
λ_{χ}	0.0792 (0.0630)	-0.0225 (0.1327)	0.1120** (0.0547)
σ_{η}	0.0122*** (0.0001)	0.0735*** (0.0006)	0.0127*** (0.0001)
Log-likelihood	30 847	17 635	47 232
AIC	30 831	17 619	47 215
SIC	30 797	17 588	47 180
Number obs.	537	330	654

The table shows the parameter estimates of the two-factor model. Standard errors in parentheses. The estimated values are reported with *denoting significance at 10 per cent, **denoting significance at 5 per cent and ***denoting significance at 1 per cent. The data set consists of: NYMEX WTI crude oil futures contracts from 9/18/1995 to 3/24/2008, NYMEX Henry Hub natural gas futures contracts from 12/3/2001 to 3/24/2008, and ICE Brent crude oil futures contracts from 12/15/1997 to 3/24/2008. Weekly observations are used in all cases.

Abbreviations: AIC, Akaike Information Criteria; SIC, Schwartz Information Criteria.

by considering the bias (real minus predicted prices) and the root mean squared error, which are shown in Table 3.

The three-factor model

This model was proposed by Cortazar and Schwartz⁵ and is an extension of the two-factor model presented above. In this model, it is assumed that the log-spot price is the sum of three components (or factors): two short-term factors (χ_{1t} and χ_{2t}) that follow an Ornstein-Uhlenbeck process and one long-term factor (ε_t) that follows a standard Brownian motion.

Using the same data set and estimation procedure as in the two-factor model case, the model parameter estimates for the three commodities are obtained and shown in Table 4. In-sample predictive power is shown in Table 5. As expected, the root mean squared errors obtained using the three-factor model are lower than those obtained using the two-factor model (Table 3). The relative performances of the two- and three-factor models can also be analyzed using the Schwartz and Akaike Information Criteria (SIC and AIC, respectively). As expected, the values for both measures are higher with the three-factor model (Table 4) than with the two-factor one (Table 2). It is easy to see from Figure 1 that this model volatility fits better than does the two-factor one.

We can conclude that since the three-factor model has more structure, more factors and more parameters, its goodness of fit is better than in the previous case. Therefore, in each case we have to decide between the two models (the two-factor and the three-factor model), taking into account that although the three-factor model fits better with the data, the two-factor one is simpler, and it is therefore easier to estimate its parameters; the significance of each stochastic factor is more clear, and it needs less data estimation.

Cortazar and Naranjo⁶ proved that for WTI crude oil data, the three-factor model works well

Table 3: In-sample predictive ability of the two-factor model

<i>Brent</i>			<i>Henry Hub</i>			<i>WTI</i>		
<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>	<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>	<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>
F1	0.0035	0.0428	F1	0.0175	0.0945	F1	0.0025	0.0457
F4	-0.0031	0.0323	F5	-0.0159	0.0870	F4	-0.0026	0.0333
F7	-0.0027	0.0282	F9	-0.0161	0.0838	F7	-0.0019	0.0291
F10	-0.0008	0.0261	F13	-0.0066	0.0676	F10	-0.0006	0.0265
F12	0.0006	0.0249	F17	0.0012	0.0772	F13	0.0006	0.0240
F17	0.0021	0.0231	F21	0.0044	0.0749	F16	0.0014	0.0220
F23	0.0021	0.0219	F25	0.0087	0.0717	F19	0.0016	0.0203
F34	-0.0017	0.0256	F29	0.0104	0.0720	F22	0.0009	0.0197
—	—	—	F33	0.0085	0.0706	F25	-0.0003	0.0207
—	—	—	F37	0.0057	0.0725	F28	-0.0017	0.0232
—	—	—	F41	0.0007	0.0709	—	—	—
—	—	—	F45	-0.0054	0.0749	—	—	—
—	—	—	F48	-0.0127	0.0844	—	—	—

The table shows the mean error (real minus predicted) and the root mean squared error (RMSE) obtained with the two-factor model using the parameter estimates in Table 2.

enough; there is little improvement derived from adding more factors to create a four-factor model.

INVESTMENT UNDER UNCERTAINTY: THE OPTIMAL CONTRACT DETERMINATION

Buy-sell contracts

As is said above, when a company is planning to develop a crude oil or natural gas field, the initial investment is significant, usually reaching billions of dollars, and production usually lasts many years (between 20 and 30). However, the main investment has to be made previously to get any return; consequently, the company that faces up the investment needs a sell contract to guarantee the recovery of its investment.

The most reasonable way to define the sell price is through a contract price quoted in a liquid international market (like NYMEX or the ICE). To choose the proper quotation variables, geographical location and product quality have to be taken into account. However, if the sell price is linked with the international one in a linear way, it will not be possible to recover the investment when the commodity price goes down. For that reason, these buy-sell contracts used to be designed with clauses that forced the buyer to pay a minimum amount independently of international quotations. To compensate the buyer for this clause, and to hedge him against unexpected and steeper price increases, it is also common to introduce other clauses to allow the buyer to pay at most a maximum amount independently of international quotations.

Table 4: Three-factor model parameter estimates

<i>Contract</i>	<i>Brent</i>	<i>Henry Hub</i>	<i>WTI</i>
μ_{ξ}	0.1346*** (0.0252)	0.1598*** (0.0308)	0.1766*** (0.0317)
k_1	0.8477*** (0.0119)	0.7001*** (0.0424)	0.8348*** (0.0288)
k_2	2.2362*** (0.0286)	6.2955*** (0.0000)	1.0800*** (0.0328)
σ_{ξ}	0.1300*** (0.0026)	0.1726*** (0.0086)	0.1515*** (0.0037)
σ_{χ_1}	0.2963*** (0.0069)	0.5333*** (0.0216)	0.8987*** (0.1946)
σ_{χ_2}	0.2826*** (0.0074)	0.8514*** (0.0000)	0.9187*** (0.1956)
$\rho_{\xi\chi_1}$	-0.1497*** (0.0289)	-0.4790*** (0.0607)	-0.3921*** (0.0314)
$\rho_{\xi\chi_2}$	0.1951*** (0.0288)	0.3197*** (0.0531)	0.3763*** (0.0313)
$\rho_{\chi_1\chi_2}$	-0.4867*** (0.0276)	-0.3741*** (0.0487)	-0.9553*** (0.0195)
μ_{ξ}^*	0.0032*** (0.0009)	-0.0458*** (0.0018)	0.0041*** (0.0015)
λ_{χ_1}	0.1961*** (0.0548)	0.3781*** (0.1061)	0.1424 (0.1686)
λ_{χ_2}	-0.1042** (0.0521)	-0.4271** (0.1858)	-0.0849 (0.1778)
σ_{η}	0.0045*** (0.0000)	0.0684*** (0.0005)	0.0055*** (0.0000)
Log-likelihood	35 575	18 106	57 240
AIC	35 549	18 080	57 214
SIC	35 493	18 031	57 156
Number obs.	537	330	654

The table shows the parameter estimates of the three-factor model. Standard errors in parentheses. The estimated values are reported with *denoting significance at 10 per cent, **denoting significance at 5 per cent and ***denoting significance at 1 per cent. *Abbreviations:* AIC, Akaike Information Criteria; SIC, Schwartz Information Criteria.

As a result, the typical buy-sell commodity contract is a long-term contract, lasting between 20 and 30 years, and is designed in the following way: in each period, the exchange price is linked with an international quotation in a linear way with two main clauses, one to guarantee investment recovery to the seller by stipulating a minimum price and other to protect the buyer from price increases by stipulating a maximum price. It is easy to see that in each liquidation period, these clauses are a put option bought by the seller and a call option bought by the buyer, respectively.

In these kind of contracts there are two highly related main issues that are interesting to consider: determining the put and call options' value and determining the maximum price that, given a minimum price, makes the value of the put options, on average, equal to that of the call options (in order to achieve, on average, a contract value equal to that of the contract without clauses). This second issue is a particular problem when it is decided that the buyer options have to be worth the same amount as the seller ones.

As these contracts contain many call and put options, some of them with maturity over a long period of time, the chosen model for carrying out the valuation, especially its volatility assumptions, is essential to the final result. The valuation of the contract using two different models, especially two different models with highly different assumptions about volatility, could create variations that differ by hundreds of millions of dollars.

Valuing contracts

To illustrate this fact, we propose three fictitious contracts, one for a WTI crude oil field, another for a Brent crude oil field and a third for a natural gas field located in Henry Hub, which

Table 5: In-sample predictive ability of the three-factor model

<i>Brent</i>			<i>Henry Hub</i>			<i>WTI</i>		
<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>	<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>	<i>Contract</i>	<i>Mean</i>	<i>RMSE</i>
F1	0.0007	0.0395	F1	0.0163	0.0816	F1	0.0002	0.0406
F4	-0.0020	0.0321	F5	0.0120	0.0768	F4	-0.0007	0.0328
F7	-0.0009	0.0276	F9	-0.0004	0.0823	F7	0.0002	0.0270
F10	0.0001	0.0249	F13	-0.0010	0.0710	F10	0.0003	0.0241
F12	0.0006	0.0235	F17	0.0006	0.0695	F13	0.0001	0.0222
F17	-0.0003	0.0222	F21	0.0005	0.0740	F16	0.0000	0.0210
F23	-0.0014	0.0215	F25	0.0033	0.0697	F19	-0.0001	0.0199
F34	0.0003	0.0198	F29	0.0052	0.0746	F22	-0.0003	0.0190
—	—	—	F33	0.0045	0.0690	F25	-0.0002	0.0185
—	—	—	F37	0.0037	0.0682	F28	0.0003	0.0185
—	—	—	F41	0.0016	0.0755	—	—	—
—	—	—	F45	-0.0010	0.0684	—	—	—
—	—	—	F48	-0.0055	0.0718	—	—	—

The table shows the mean error (real minus predicted) and the root mean squared error (RMSE) obtained with the three-factor model using the parameter estimates in Table 4.

are defined in the following way: the contracts lasting 20 years, from 1/1/2009 to 12/31/2028; the amount of crude oil or natural gas exchanged is the same in each period; liquidation is monthly; and contract prices are the averages of the first-month WTI, Brent and Henry Hub futures prices traded at NYMEX and ICE, respectively.

If the seller wants to include a clause that guarantees a minimum price to recover the initial investment, there are two questions to consider: what is the value of the options and what should be the maximum price that the buyer should include so as not to lose money. In addition, if the buyer wants to introduce another clause to guarantee a maximum price, there emerges an additional question: what is its value?

We are going to answer these questions using the three models presented above. In all cases, the valuation date is 3/24/2008, and the assumed risk-free interest rate for the whole period is 5 per cent. As there are no forward curves for any commodity that covers the whole contract period (from 1/1/2009 to 12/31/2028), a forward curve has been estimated for the three commodities based on the observed forward curve at the validation date (3/24/2008). The estimate has been carried out, assuming in all cases that there is a long-term forward price and that the observed futures prices converge to the long-term one exponentially.

In Figure 2, one can see the minimum price clause value as a function of strike prices

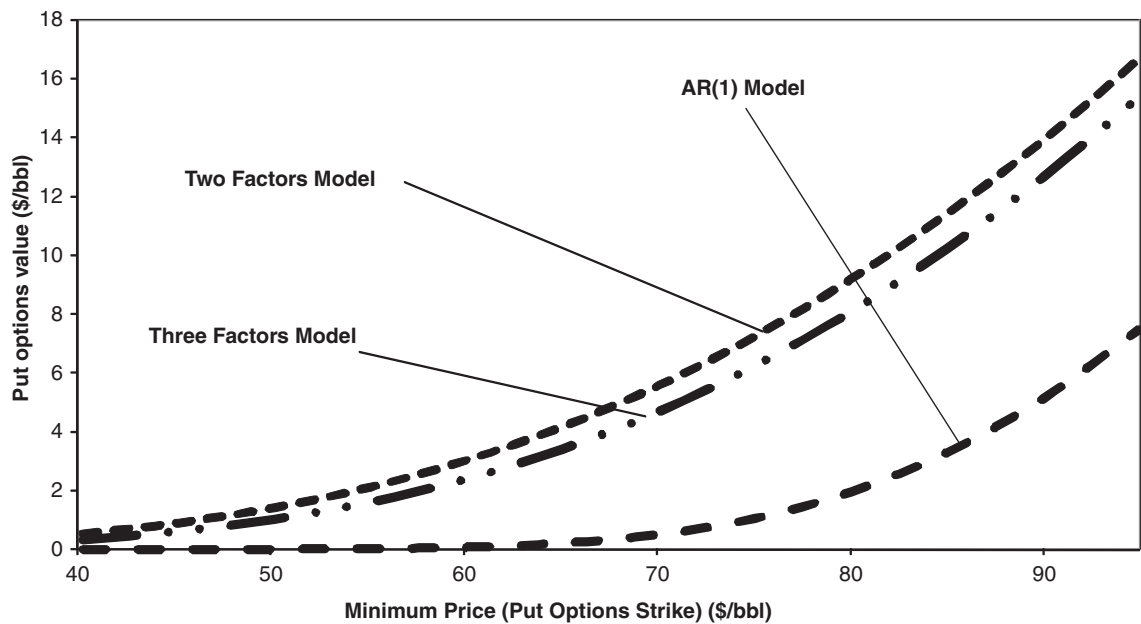


Figure 2: Put options value (Brent).

(the minimum price) for Brent crude oil. Figure 3 contains the maximum price clause valuation for natural gas from Henry Hub, and in Figure 4, one sees the maximum price estimate that the buyer should include so as not to lose money depending on the minimum price for WTI crude oil (for brevity's sake, we are only going to present the results for one commodity in each case; however, the results for the other ones are available from the authors on request). As can be understood from the figures, the results are presented in \$/bbl or \$/MMBtu so that they can be compared with the commodity average price during the contract period, which is 95.97 \$/bbl in the case of WTI crude oil, 96.14 \$/bbl in the case of Brent crude oil and 8.56 \$/MMBtu in the case of natural gas from the Henry Hub field. Comparing the clauses' values in \$/bbl or in \$/MMBtu with the average prices is equivalent to comparing the clauses' value with the whole contract value, as the

amount exchanged in each period of time is the same.

Table 6 lists the differences in valuations using the three-factor model versus using the other two models as a percentage of the average price, which is equal to the differences in valuation as a percentage of the whole contract value. Table 7 shows the differences between the maximum price that the buyer should include so as not to lose money when the seller includes a minimum price using the three-factor model or using the other two models as a percentage of the result obtained using the three-factor model.

The first issue to highlight is the fact that the results obtained are basically the same for all commodities, especially natural gas, no matter whether we use the two-factor or the three-factor model. This again brings to the fore what has been said regarding both models: as the two-factor model is a particular instance of the

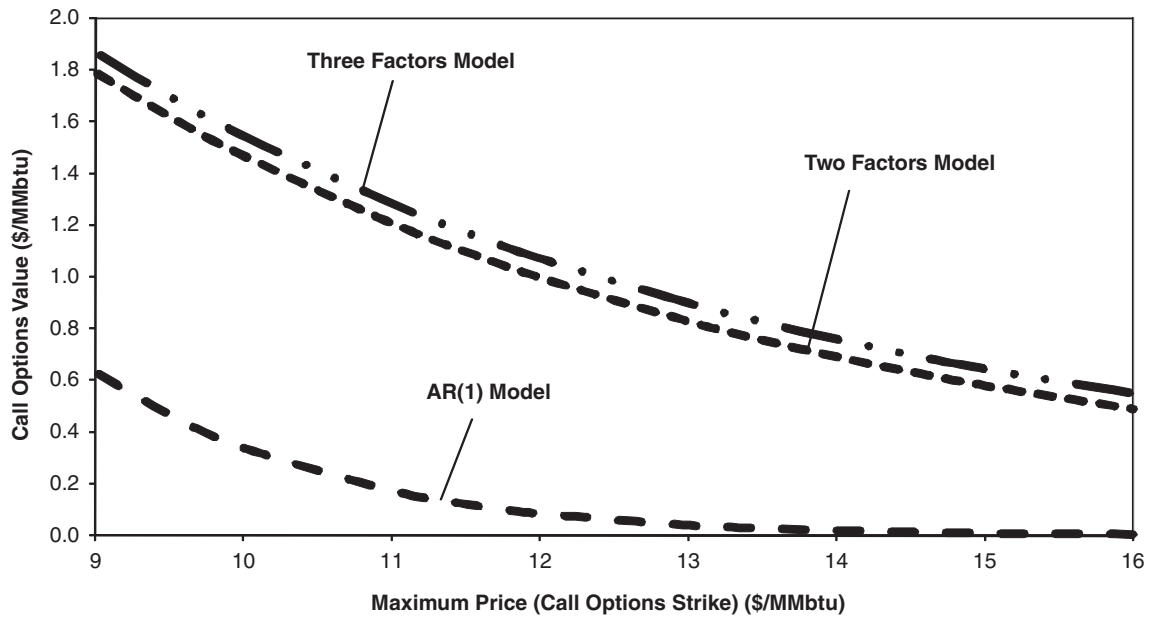


Figure 3: Call options value (HH).

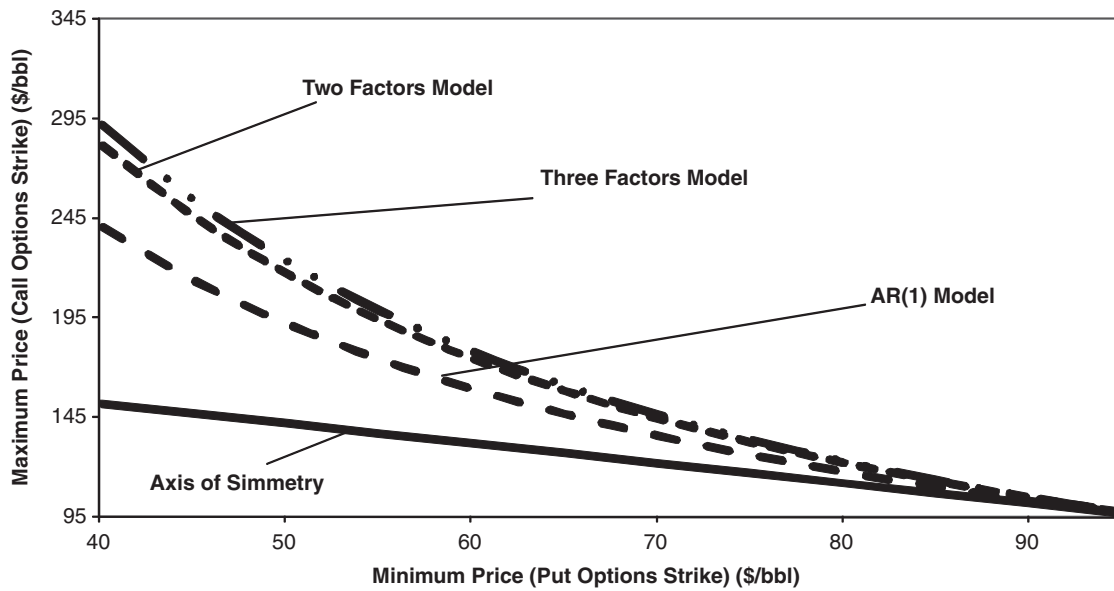


Figure 4: Maximum price as function of the minimum one (WTI).

three-factor one, the three-factor model gets more accurate estimates; however, the two-factor one is simpler and easier to deal with. Therefore, depending on the problem, the

optimal choice should be accuracy or simplicity. In this case, as billions of dollars are involved, it sounds reasonable to choose accuracy instead of simplicity.

Table 6: Clauses valuation

<i>Minimum price</i>	<i>Put value</i>		<i>Maximum price</i>	<i>Call value</i>	
	<i>Three-factor vs AR(1) (%)</i>	<i>Three-factor vs two-factor (%)</i>		<i>Three-factor vs AR(1) (%)</i>	<i>Three-factor vs two-factor (%)</i>
<i>WTI</i>					
45	1.0	0.3	100	9.5	1.3
55	2.3	0.5	110	9.2	1.3
65	4.2	0.8	120	8.4	1.3
75	6.4	1.0	130	7.4	1.2
85	8.3	1.2	140	6.3	1.2
95	9.4	1.3	150	5.3	1.1
<i>Brent</i>					
45	0.6	-0.3	100	8.4	-1.4
55	1.7	-0.5	110	8.0	-1.4
65	3.3	-0.8	120	7.2	-1.3
75	5.4	-1.0	130	6.1	-1.3
85	7.2	-1.2	140	5.1	-1.2
95	8.2	-1.3	150	4.1	-1.1
<i>HH</i>					
45	1.4	0.2	100	14.4	0.9
55	3.4	0.4	110	14.1	0.9
65	6.4	0.5	120	12.2	0.9
75	9.8	0.7	130	10.0	0.8
85	12.6	0.8	140	8.6	0.8
95	14.2	0.8	150	7.4	0.8

The table shows the difference in option value illustrated using the three-factor model versus using the other two models as percentages of average price, which is equal to the differences in valuation as percentages of the whole contract value.

We attain an opposite conclusion in comparing the two and three-factor models' valuation with the valuation of the AR(1) one. Just to get an idea of the amount of this difference, we can see, for example, that the

call options' value at the maximum price is 100 \$/bbl or the put options' value if the minimum price is 95 \$/bbl differs by almost 8 \$/bbl in the case of WTI crude oil and almost 9 \$/bbl in the case of Brent crude oil when it is

Table 7: Minimum prices versus maximum price

<i>Minimum price</i>	<i>Three-factor vs AR(1) (%)</i>	<i>Three-factor vs two-factor (%)</i>
<i>WTI</i>		
45	15.7	3.1
55	12.1	2.3
65	8.7	1.6
75	5.6	1.0
85	2.8	0.5
95	0.2	0.0
<i>Brent</i>		
45	13.0	-2.6
55	10.0	-2.0
65	7.2	-1.4
75	4.7	-0.9
85	2.4	-0.5
95	0.2	0.0
<i>HH</i>		
45	19.4	2.6
55	15.4	1.8
65	11.8	1.2
75	8.2	0.7
85	4.4	0.3
95	0.3	0.0

The table shows the differences in the maximum price that the buyer should include so as not to lose money if the seller includes a minimum price using the three-factor model versus using the other two models as a percentage of the result obtained with the three-factor model.

calculated using the AR(1) model as opposed to the other two. In both cases, it represents almost 10 per cent of the average price. In the case of natural gas, this difference is around

1.15 \$/MMBtu, which represents almost 15 per cent of the average price. As these projects involve enormous amounts of commodities (as noted above, its value usually reaches billions of dollars), a 10 or a 15 per cent of the whole project could represent hundreds of millions of dollars. Comparing the differences between the two- and the three-factor models we get, at a maximum, 1.3 \$/bbl in the case of crude oil and 0.07 \$/MMBtu in the case of natural gas. These figures represent less than 1.5 per cent in the case of crude oil and 1 per cent in the case of natural gas. As the amount of money included in this kind of project is so significant, these differences are also important; however, the extent of the difference is about 10 times less than that achieved previously.

Even more importantly, comparing the two- and three-factor models in valuing the put and call options, we get a bias that is not too large and operates in the same direction in both cases; subsequently, in calculating the maximum price that the buyer should include so as not to lose money if the seller includes a minimum price in the contract, the differences between the models reach around 3 per cent. When comparing the AR(1) model with the other two, these differences reach more or less 15 per cent in the case of the crude oil and 22 per cent in the case of natural gas. As before, we can conclude that, as the amount of money involved in this type of projects is vast, the differences between the models are important in all cases; however, in comparing the AR(1) model with the other two, the differences become critical to contract determination.

These huge differences in model valuation come from the fact that the AR(1) has fenced volatility, whereas the two- and three-factor models do not. As an option contract is not symmetrical, volatility plays a central role in its

valuation (see for example Hull⁸), and as the contract lasts such a long time, the fact of volatility's being fenced or not is crucial in determining the value, as we have seen.

Another issue to highlight is the fact that, as can be understood from Figure 4, there is no symmetry between the minimum and the maximum prices. For example, with an average price of 95.97 \$/bbl for WTI crude oil, one naive manager could think that if the minimum price is fixed at 50 \$/bbl, the maximum price should be fixed at $95.97 + (95.97 - 50) = 141.97$ \$/bbl to achieve the same figure upwards as downwards. Nevertheless, as can be appreciated in the figure, this reasoning is false; with the AR(1), the two-factor model and the three-factor model, the maximum price should be 192, 217 and 223 \$/bbl, respectively. The reason behind this is again the options contract asymmetry, and the reason for getting a higher maximum price than the one defined by the symmetric axis is the fact that the put option revenue is fenced (the maximum revenue is the strike price) whereas the call option revenue is not, and so therefore higher strike prices are needed for the call options. However, as we can see from Figure 4 and Table 7, the degree of asymmetry depends on the chosen model and especially depends on the chosen model's volatility assumptions.

CONCLUSIONS

The development of a crude oil or natural gas field is highly intensive in terms of investment and lasts many years; however, the main investment has to be faced up previously to get any return. Therefore, it is common that the buy-sell contract guarantees investment recovery and hedge the buyer from unexpected

and steeper price increases through clauses that include a minimum and a maximum price. These clauses are put and call options and, as volatility is a decisive parameter in option valuation, the model chosen to characterize the commodity price dynamics is decisive, especially in terms of its volatility assumptions.

This is the reason why this article presents an empirical study of three different models (AR(1), a two-factor model and a three-factor model) that can be used to characterize the commodity price dynamics, putting a special emphasis on the volatility of futures returns estimates. We have seen that the AR(1) is not very realistic, whereas the two-factor and three-factor models' estimated volatility fits better with the empirical one than with the basic AR(1) model. Moreover, since the three-factor model has more structure, the goodness of fit is better than in the two-factor one; however, it is more complex than the two-factor one.

Once we have compared the relative performance of the models, we apply the conclusions to investment under uncertainty and particularly to valuing the buy-sell contracts described above. We have seen a hypothetical project for each commodity illustrate the significant differences in contract valuation using different models. Concretely, if we use an AR(1) model instead of the other two, the clauses; valuation can differ by between 10 and 15 per cent of the whole contract value, which could represent hundreds of millions of dollars. These large differences stem from the fact that the AR(1) model assumes that volatility is fenced in the long term and the two or three-factor models do not. If we compare two models with the same assumption about volatility (fenced or not fenced), the differences are much smaller. However, as these sort of contracts involves

billion of dollars, small differences in valuation also entails big money.

There are also significant differences in the maximum price that the buyer should include in order not to lose money if the seller includes a minimum price in the contract if we use the AR(1) model instead of the other two. The reason is the same as above: in one case, the volatility is assumed to be fenced and in the other it is not. Another interesting issue to take into account is the fact that there is no symmetry between the maximum and the minimum price. The reason behind this is the fact that the put option revenue is fenced, whereas the call option revenue is not; therefore, higher strike prices are needed in call options. Even though this issue arises independently of the model used, ones again, the model volatility assumptions are crucial in determining the degree of asymmetry between the maximum and the minimum price.

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