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The Relevance of Portfolio Management Action for Solvency Measurement

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Solvency II aims at capturing the true risk landscape of an insurer with immediate consequences for the regulatory capital required. One key element of every risk management approach is the so-called reversibility option available to the insurer's management that involves the change of a decision taken in the past in order to adapt to a modified risk landscape. This paper analyses the effects of simple strategies formulated by the insurer's management in an *ex ante* setting and carried out during the solvency assessment period on the Value at Risk (VaR) of a two-stock portfolio. We show that the effect of even simple strategies is non-negligible for the portfolio's VaR and hence for the required capital when using an internal model under Solvency II. The issue discussed affects both Pillars I and II of Solvency II and will therefore be one focal point of discussion between insurers and regulators when reviewing internal models.
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Introduction

The Solvency II project to create a solvency regime for European Union–domiciled (re)insurance companies based on economic principles is gathering pace. The draft directive was released on 10 July 2007 and is now in the process of discussion. The basic principle in terms of capital requirement will involve insurance companies having to keep an amount of capital sufficient to absorb a negative result, which corresponds to the 1-in-200-years event. The time horizon over which this solvency calculation is assessed is 1 year. If insurers take up the opportunity to develop an internal model, they will have to develop probability distributions for all sources of uncertainty affecting their undertaking over a 1-year horizon. They have to address, therefore, both the investment as well as the underwriting side, identify dependencies within the different risk categories, and analyse the overall correlation structure in order to properly capture the true economic risk situation. Based on the assertion that the investment side of an insurance can be interpreted as "a leveraged investment fund",[1] where assets are regularly recalibrated and reinvested, it can be argued that insurers

[1] Scotti (2005, p. 11).

should include pre-defined (asset) management action to some extent. This paper explains why and how, in an economic setting, portfolio management actions on the asset side of the insurance business should be included in the equation, in order to create a more complete model of the option space of an insurance undertaking.

This paper is organised along the following lines: First, the reason for including the effect of management actions in the calculation of solvency requirements is given in the following section. The next section deals with the notion of economic risk and the importance of time. In order to give our argument for the asset side more formal structure, we propose in the subsequent section a simple model for the behaviour of stock prices through time and derive the Value at Risk (VaR) for a model portfolio. The next section defines a simple portfolio management strategy and its application to a stock portfolio, showing the implications to the undertaking's risk situation of including a strategy. The penultimate section will discuss the limitations of modelling such asset-side strategies, expanding beyond the asset side. To conclude, the final section discusses the consequences of our findings for risk management in general and for the Solvency II discussion in particular.

Management action and Solvency II

The risk (or capital) measure mostly advocated in the current Solvency II discussion is the VaR at the above mentioned confidence level of 99.5 percent over 1 year.[2] In the banking industry VaR is normally used as a means of producing limits for trading departments or individual traders in order to keep the bank solvent. The most interesting aspect of the differences between banking and insurance in this respect lies in the incorporation of the time dimension in the economic reasoning. Banks calculate VaR figures normally for a time horizon of between two and 10 working days, whereas insurance undertakings are supposed to extend this timeframe in their risk modelling to 1 year in the Solvency II context. It is interesting to analyse the differences in these time restrictions from an economic perspective.

From our perspective, the reason for these differences is to be found in the relation of risk and reversibility.[3] Assuming a position with unsure future economic outcome is in the nature of the business. Yet in many cases it may be possible to describe the quantitative and probabilistic characteristics of the possible future outcomes. Risk management may not support the influencing of *ex ante* risk properties by prevention or mitigation, but it does allow uncertain outcomes to be replaced by certain outcomes at certain points in time (for a price). This ability to replace or reverse a risk position is described as reversibility. If reversibility is given, then

[2] The VaR attracts some criticism as it does not satisfy the requirements of the so-called "coherence" of a risk measure, as given. For an introduction into the concept of coherence see Artzner *et al.* (1999). An alternative risk measure satisfying the coherence requirements is the so-called Tail Value at Risk (TVaR) which – simply put – does not only take into account the probability of a shortfall but also the amount of the shortfall. The TVaR at a confidence level of 99 percent is the risk measure that is to be used in the Swiss Solvency Test; see Swiss Federal Office of Private Insurance (2004, p. 13). For a discussion from an industry perspective on the suitability of the risk measure, see CEA (2006).

[3] Bernstein (1999) discusses comprehensively on the topic risk and reversibility.

management is able to react to new information. It may adapt to new economic conditions, thereby altering the risk landscape and thus changing the probability of bankruptcy for the undertaking (provided the measures taken are sensible from an economic *ex ante* perspective). However, being in a position to respond radically changes the risk management task for management. Management action is a very powerful instrument.

Looking at an insurance undertaking, insurance contracts are normally concluded for (at least) 1 year.[4] The insurance company is not free to withdraw from a policy before expiration without mutual consent. Once the policy is concluded, the contractual obligations are fixed. The insurance undertaking can, however, use traditional and innovative risk mitigation instruments such as reinsurance and insurance-linked securities in order to pass on some of the risk to a reinsurer or the capital market. Again, the insurer and, consequently, the risk taker are bound for the time specified in the supporting legal documentation. Therefore, looking at market practice and accounting requirements, from a purely technical perspective it makes a lot of sense to base a solvency regime on a 1-year time horizon as management will have the opportunity to act and restructure the risk profile after 1 year's time.

However, this reasoning does not apply to the investment side of the insurance business. The bulk of an insurer's assets is invested in liquid securities of which the undertaking can dispose in deep and liquid markets normally without incurring negative market repercussions. It can do this at any point in time when markets are open for trading, that is, virtually at any moment.[5] This option to reverse an economic decision once taken has a significant impact on the requirements that any risk measurement framework reflecting the true economic situation of an insurer has to satisfy. The evidence in support of our statement is overwhelming and applies to nearly any situation in which human beings are required to manage risk. Nobody remains static in a dynamic environment.[6] A portfolio manager will act according to market movements, that is, his or her behaviour may depend on the history of the market prices and, therefore, be conditional on what has happened before. If the market decreases, the proportion of equity could be reduced or increased depending on management strategy. In any case, these management actions must be incorporated in any risk measurement model that claims to be economically meaningful.

However, these aspects of the value of management action as such in the intra-periodic context, for example within the 1-year time horizon, have – as far as the authors see it – found their way into the Solvency II discussions only indirectly via Pillar II on risk management processes but not yet into Pillar I – though risk models

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[4] Contract periods, i.e. cancellation points in time, are mostly annual and aligned to the external publication periods.

[5] See the section on the limitations for inclusion of management action for a discussion and limitation of this assumption.

[6] A risk management framework is called static if it does not allow for management intervention but requires a portfolio to be formed and kept in the same composition without changes. See Brohm and König (2004).

do exist that explicitly model such management actions. The academic literature has picked up the issue of the implementation of management strategies into a model of the insurance firm at different times and to various degrees,[7] and with the increase in computing power it found its way into commercial risk management products.[8] Most recently the papers of Eling *et al.*[9] discuss the implementation of management strategies into a 5-year model where decisions about parameters affecting both the insurance as well as the investment side of the business in a high-level approach can be made at the beginning of each year.[10] The authors assess the influence of dynamic decision-making on absolute and relative return measures as well as on risk and performance measures. However, intra-periodic decision-making, which is vital to the understanding of the dynamics driving capital requirements over a Solvency II–relevant horizon of 1 year, is not considered in their approach. In contrast to Eling *et al.*, we are discussing intra-periodic decision making based on the history of events since the beginning of the year (but before the assessment period of 1 year is over) in order to gain an understanding of how risk capital requirements over a Solvency II–compatible 1-year horizon react to management decisions affecting the investment side.

Having said that, we consider that there is an economic analogon to the situation described here that is well-established in discussions on the new Solvency II framework: In life insurance, there are products with "with-profit" characteristics where – as a general rule – the insured will participate in excess profits the insurer incurs relative to the pricing parameters fixed in the policy. However, the insurance undertaking has the possibility to lower the amount of future benefits to be paid to the policyholders in a situation of economic distress. The Committee of European Insurance and Occupational Pension Supervisors (CEIOPS)[11] captured this option of the insurer in the QIS 3 study via the so-called "KC factor"[12] that models management behaviour in this respect. Economically, the KC factor is nothing more than a description of discretionary management action and the option to adapt, that is, reverse an economic decision taken in the past. From a structural point of view, this is equivalent to a portfolio manager's option of readjusting the risk profile of the insurance undertaking's asset portfolio at any point in time (and not only after 1 full year has passed).

From a modelling perspective, the challenge is how to incorporate an approach of this kind into an economic solvency framework aspired to by Solvency II and how to

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[7] Brohm (2002) compiles an overview on various generations of insurance models.

[8] See recently Niering (2006) or Liebwein (2006); for the underlying economic concepts see, for example, Brohm (2002, pp. 134 ff) and cited references or Brohm and König (2004).

[9] See Eling *et al.* (2006, 2007) for details.

[10] Eling *et al.* are using a parameter α_t denoting the portion of high-risk investment in time period t and a parameter β_t describing the company's portion of the market volume in time period t. Management decisions are then reflected by increasing or decreasing both α_t and β_t at the beginning of each model period. See Eling *et al.* (2006, p. 4).

[11] CEIOPS, a body of cooperation between European supervision authorities that actively shapes the discussion of the future European insurance regulation.

[12] The KC factor aims at taking into account the risk absorption ability of future profit sharing. See CEIOPS (2007, p. 24).

build this phenomenon into an internal model. We will address these issues in this paper in greater detail. However, it is quite evident that discretionary management action has to be modelled in an *ex ante* setting (i.e. before running any model). The question to be answered is: how will management react if, at a certain point in time, the insurer's risk or capital position changes to such a degree that the intended quality of the insurance coverage can no longer be met?

From our perspective, there is some modelling evidence to suggest that a well-defined risk mitigation strategy will considerably lower the stand-alone capital requirements under Solvency II. It goes without saying that such a portfolio management strategy is a Pillar II issue to be discussed with the appropriate insurance regulator. However, in our opinion, it goes far beyond a pure Pillar II issue. It constitutes the combination of Pillars I and II and actually bridges the gap between the two pillars because a portfolio management strategy, like any other risk mitigation strategy with clear quantitative impact on the economic position (approved by the regulator) will have to be implemented in any internal model under Pillar I for the calculation of the Solvency Capital Requirement (SCR).[13]

The focus of our analysis lies on the impact of management actions on the asset side: how will management react if certain thresholds in the market price developments are triggered? The potential impact of management action on the underwriting side is only discussed in the context of the extension of the time horizon for risk and solvency measurement beyond the 1-year framework proposed in the Solvency II project.

Definition of risk, time and normalisation

For our further consideration, it is necessary to dwell on the notion of economic risk that we will use throughout the paper. We have already labelled, somewhat imprecisely, the VaR as a risk measure, and the reasoning for doing this requires further explanation. The notion of risk is characterised by two conditions. Firstly, risk refers to the uncertainty of future economic states of the world. All decision-making is exposed to the fact that the decision-maker lacks knowledge about the exact future state of the world and about its impact on the decision-maker's economic position. A second condition for risk is the existence of a threshold separating favourable from unfavourable results. If the success of any economic decision-making is measured in monetary terms, then each realisation of a state of the world leading to a monetary position above the threshold is favourable *per se*, and consequently does not constitute materialisation of risk. If, however, the monetary position falls short of the threshold, the risk can be said to have materialised. This approach to measuring risk is called the shortfall approach. Clearly, the type of threshold set depends on the objective of risk measurement.

Accepting, albeit with theoretical shortcomings, the ε-percentile as a risk measure and using the corresponding probability of ruin as the yardstick by which to steer the

[13] SCR which, according to the preliminary calibration of the supervisory model, corresponds to the 0.5 percentile of the insurer's result distribution.

risk and capital situation of the bank or insurer, we can show that the physical capital available to the undertaking (labelled here as Capital at Risk CaR_t^{ε}) must correspond (at least) to the negative VaR of the relevant result distribution at time t in order to reduce the probability of ruin to the desired confidence level ε. Therefore, we have $CaR_t^{\varepsilon}=-VaR_t^{\varepsilon}$.

Note that our argument so far does not involve any interest considerations. However, the notion is obviously on capital requirements at time t, which denotes a future state, and not the departing state because the physical capital to absorb losses needs only to be available at the time horizon t of the risk and solvency analysis. Knowing which – modelled – physical capital amount is needed in t, we can infer how much capital is required at time t_0, since the management needs to decide on the physical capital base at time t_0. We therefore need to link the capital amounts at time t_0 and t. To ensure that at inception all capital requirements are met implies that the capital available for covering the downside risk would be held as cash capital throughout the period under consideration. This is not a reasonable assumption, as the potential of the capital to produce returns in itself is ignored. Therefore, the physical risk capital itself will be invested and will provide return characteristics.

If it is invested risk-free, then the risk capital itself has not added to the risk landscape, but it will provide a risk-free return $r_{RF}(0;0,t)$, that is, the investment yield known in $t_0=0$ for the period $(0,t)$. If it is invested in risky assets, then the risk capital will add a certain component to the overall risk landscape. The investment yield at inception can be expressed as an expected return $E[r_R(0;0,t)]$, but the eventual realisation will of course differ from the expected value.

How can the Capital at Risk CaR_t^{ε} be transformed into a capital requirement at inception? What is the CaR_0^{ε} that ensures that at time t the probability of ruin ε has the desired confidence level ε?

Consider the following example: Assume that a 1-year business plan is modelled excluding the capital requirements completely. It shows that a loss of 100 (monetary) units mark the 1 percent percentile of worst results which we would like to cover with capital. How much should we have in $t_0=0$ to have 100 available at year-end to cover the 1 percent-percentile shortfall? A hundred units in cash, a risk-free zero bond, a stock investment that promises to be worth on average a hundred at year-end, but will contribute itself inherently shortfall risks, or a stock investment that promises to be worth at least a hundred in the 1 percent worst cases.[14]

While we do not discuss the appropriateness of an approach, it has to be stated that the addition of risk into the example above, by investing the risk capital in risky assets, will imply that in a more complex setting the required capital at inception cannot be derived in a closed form, but its calculation resembles an iterative convergence process, like the *tâtonnement* process in the market clearing process. We include in our example in the next section some options of how the capital requirement is established and how the transformation to the inception can be achieved, but will focus on the relative impact of management actions on the VaR_t^{ε}.

[14] This reasoning assumes a discrete time setting where only two points in time exist.

A simple model for the behaviour of stock prices

The univariate case: stochastic law of stock prices and returns over time and its VaR

In order to make quantitative statements about the magnitude of risk of a particular investment strategy, we need to model the behaviour of the assets that are part of our portfolio in a stochastic setting. As we do not have the intention to complicate matters more than necessary, we will concentrate on a simple model for stock prices at time t, S_t. In accordance with extensive literature on the behaviour of stock prices,[15] we assume that the stock price to be modelled in a continuous-time setting follows a so-called geometric Brownian motion with drift μ and diffusion σ^2. Based on these assumptions, the VaR can be derived as $VaR_t^\varepsilon = S_{t_0}\exp((\mu - \frac{1}{2}\sigma^2)(t - t_0) + \sigma\sqrt{t - t_0}\Phi_\varepsilon)$.[16]

Time and reversibility matters in the univariate case: a numerical example

We are now able to analyse the effect of the time elapsed between the start of the process and the assessment of the risk capital position. If we assume that ε is small, then Φ_ε is negative (e.g. $-2,5758$ for ε=0.5 percent). This negative effect grows with the square root of time but is compensated by the positive instantaneous drift rate. However, we are particularly interested in the distinction between the time frame normally used in the banking industry (between 2 and 10 days) and the proposed Solvency II time horizon (1 year).

Let us illustrate the issue with a numerical example. Suppose the stock price at inception of the analysed period is given by $S_{t_0} = 150$, the instantaneous drift rate amounts to μ=12.72 percent and the instantaneous diffusion component is given by σ=20.87 percent. With a confidence level of ε=0.5 percent and a horizon of 1 year, the VaR amounts to $VaR_1^{0.005}$=97.366.

This value can be interpreted as the threshold, being 52.634 lower than the start value of 150, under which the stock price falls within 1 year only with a probability of 0.5 percent, which is equivalent to the statement that the stock price in 1 year's time will exceed the value of 97.366 with a probability of 99.5 percent. The inherent assumption of this analysis is that the portfolio composition does not change, that is, that it is completely static and not affected by any portfolio management action.

In contrast, if we choose a time horizon of just 2 days, that is, 0.005479 years, then the corresponding VaR for the same confidence level amounts to $VaR_{0.005479}^{0.005}$=144.232. In this model, the probability of the investor losing more than 5.768 within 2 days is 0.5 percent. This value can be interpreted in the context of trade limits in banks, as the assumption that the portfolio remains static does not seem to be unreasonable within the time horizon of 2 days, implying a reversibility period of 2 days. However, extending the time horizon of the risk analysis while retaining the assumptions that portfolio management will not have and taking up an option to act according to

[15] Black and Scholes (1973), for example, assume this model for the behaviour of underlying stock prices.
[16] Φ_ε denotes the ε percentile of the standard normal distribution.

market conditions within a year will inevitably produce unrealistic results for the risk assessment because the economic relationship between risk, time and reversibility will have been violated.

Translation of VaR into capital requirements and normalisation: the univariate case

In the above calculations, we compared with $VaR_1^{0.005}$=97.366 and $VaR_{0.005479}^{0.005}$= 144.232 two different valuation points in time, that is, t=1 and t=0.00549. The figure $VaR_{0.005479}^{0.005}$=144.232 relates to a distribution in t=0.00549 with the expected value $E(S_{0.00549})=S_0\exp(\mu(t-t_0))=150\times\exp(0.1272\times 0.00549)=150.105$ whereas the figure $VaR_1^{0.005}$=97.366 relates to a distribution in t=1 with the expected value $E(S_1)$= 170.347.

In order to assess the value differences coherently, we have to normalise the two values, that is, we need to refer the values to the same point in time. The discount rate to be applied constitutes the core issue to be solved. The drift μ=12.72 percent, a risk-free rate, or any other value could be applied for this purpose. Only by normalising the capital requirement $CaR_t^{0.005}$ resulting from the $VaR_t^{0.005}$ to the inception date will we be able to create full transparency as to how much capital is required at the start of the observation period t_0=0 to ensure that the survival probability of the company at the end of the observation period – in t=1 for Solvency II – amounts to 99.5 percent.

This capital requirement at inception is defined as ${}_{t=0}^{r(0;0,t)}CaR_t^{0.005}$ with $r(0;0,t)$ being the discount rate known in t_0=0 to be used for the normalisation onto t_0=0. Due to the very short horizon in the banking context, the drift of the process representing the expected time value of money can be neglected. The only concern which matters and, consequently, the focus of discussions in the banking industry, is the diffusion component, that is, the inherent uncertainty of the underlying security. However, in the context of the insurance industry's 1-year horizon, this assertion induces a systematic distortion of capital requirements. The capital requirement calculated in this way will be too conservative, as the solvency capital will not be held in cash but will itself be invested at least on a risk-free and continuously reversible basis.

As the solvency capital itself gets invested, we have to define the proper discount rate for the capital as defined by the 0.5 percent percentile at the end of the observation period. We will not offer a generic solution for the proper discount rate, but an economically reasonable upper and lower boundary for the discount rate. The risk-free interest rate as stated in the model serves as the lower boundary. The expected return of the portfolio as given by the underlying return assumptions serves as the upper boundary. In the latter case, it is assumed that the capital itself would not contribute to the asset risk, which is, of course, not a sensible assumption.

In the short-term observation horizon of the banking sector, the two discounting factors can be calculated as follows: for the lower boundary we use an annualised risk-free rate of 3.39 percent which leads to $r_{RF}(0;0,0.00549)=\exp(0.00339\times 0.00549)-1=0.000186$ if we use exponential interest calculation. The upper bound is calculated using the drift of the stock and amounts consequently to $r_\mu(0;0,0.00549)$= $\exp(0.1272\times 0.00549)-1=0.00069723$.

Our further consideration starts with the nominal VaR value at t=0.00549. Recall that in our example we calculated ${}^{0}V_aR^{0.005}_{0.00549}={}_{t=0.00549}{}^{0}V_aR^{0.005}_{0.00549}=144.232$. The further main characteristic of the result distribution at the end of the observation period is the expected value $E_{0.00549}=150.105$ versus the starting value of 150. Based on this, we now have to decide which reference value to choose in the definition of capital requirements. If the reference position is the expected value at the end of the observation period, in t=0.00549 we require $CaR^{0.005}_{0.00549}:=150.105-144.232=5.873$.

This capital will ensure that only in 0.5 percent of the cases a negative result will occur at the end of the observation period. Result in this context means "deviation from the expected value". In this economic setting, every negative deviation from the expected value will be interpreted as a realisation of risk. If in this situation 5.873 units of the invested capital at the end of the observation period were labelled as "risk capital" and consequently set aside, then this amount would be sufficient to cover all the cases in which the return within the observation period does not reach the expected value with the exception of the worst 0.5 percent cases.

If the drift component is neglected, that is, the initial position in $t_0=0$ is considered as reference value for distribution in t=0.00549 and the investment return is thus non-existing, then we have $CaR^{0.005}_{0.00549}:=150-144.232=5.768$, which indicates the capital required relative to the initial position to ensure that the 99.5 percent confidence level is surpassed. The short observation period will prevent a large deviation of the approaches, but as a matter of consistency we will use the economically more correct one as reference.

Into what capital requirement at inception does this translate? The upper boundary for the required capital in values at the inception point in time $t_0=0$ can be calculated as ${}^{0.000186}_{t=0}CaR^{0.005}_{0.00549}:=150.105-144.232/\exp(0.0339\times0.00549)=5.872$, whereas the lower boundary amounts to ${}^{0.000697}_{t=0}CaR^{0.005}_{0.00549}:=150.105-144.232/\exp(0.1272\times0.00549)=5.869$.

Obviously, the differences in capital requirements at inception are very small and can therefore be neglected in the univariate case for the normal banking observation period relative to nominal values at the end of the observation period.

We must expect bigger differences for the insurance case with an observation period of 1 year. The discounting factors are $r_{LB}(0;0,1)=\exp(0.0339\times1)-1=0.034481$ for the lower bound and $r_{UB}(0;0,1)=\exp(0.1229\times1)-1=0.135644$ for the upper bound using the drift of the stock.

In order to calculate the respective capital requirements, the two scenarios are applied to the nominal value of the VaR at the time horizon t=1 $VaR^{0.005}_{1}={}^{0}_{t=1}VaR^{0.005}_{1}=97.366$.

Table 1 summarises the approximate capital requirements for the banking and the insurance case. Obviously the differences cannot be neglected any longer over an observation period of 1 year. They are driven by applying different assumptions regarding the discounting that can equally be interpreted as investment scenarios for discounting purposes. The capital requirements at inception in principle also depend on – assumed or implemented – investment behaviour defined *ex ante* and – in this case – kept static over time. As discussed in the preceding sections we will not further dwell on the issue of which valuation point and reference capital are to be used, but will concentrate on the relative impacts, starting off with the multivariate case in a

Table 1 Comparison of approximate risk capital requirements

(Start reference value 150)	*"Banking case" (time horizon 2 days)*	*"Insurance case" (time horizon 1 year)*
Upper boundary of CaR (risk free return on CaR)	${}^{0.000186}_{t=0}CaR^{0.005}_{0.00549} = 5.872$	${}^{0.003448}_{t=0}CaR^{0.005}_{1} = 70.548$
Lower boundary of CaR (portfolio return on CaR)	${}^{0.000897}_{t=0}CaR^{0.005}_{0.00549} = 5.869$	${}^{0.135644}_{t=0}CaR^{0.005}_{1} = 64.263$

continuous-time setting, then expanding into discrete-time modelling and ending with the impact of asset management strategies on the result distribution at the end of the observation period.

The multivariate case: stochastic law of stock prices, returns over time and the VaR

We will now extend the mathematical description to a multivariate setting and assume, therefore, that the vector of the continuously compounded rate of returns of the stock prices $\mathbf{R}=(R_1, R_2, \ldots, R_n)^T$ follows a multivariate normal distribution. The portfolio value at the time horizon P_t is given by $P_t = P_{t_0}\exp(R_P(t-t_0))$ and its VaR can be shown to be[17]

$$VaR_t^{\varepsilon}(P) = P_{t_0}\exp\left(\sum_{i=1}^{n} x_i(\mu_i - \tfrac{1}{2}\sigma_i^2)(t-t_0) + \sqrt{\sum_{i=1}^{n} x_i^2\sigma_i^2}\sqrt{t-t_0}\,\Phi_\varepsilon\right).$$

Time, diversification and reversibility matters in the multivariate case: a numerical example

For the numerical example in Table 2, we will assume a bivariate case, in which we combine two risky assets (stocks) in one portfolio. We will assume stochastic characteristics for the different stocks in question (Table 2).[18]

The portfolio value at inception is then given by $P_{t_0} = 150$, which allows us to compare the results with the univariate case and show the effect of diversification in absolute terms. The expected value of the continuously compounded rate of return amounts to $\mu_P = 12.72$ percent as in the univariate case. However, the standard deviation of the continuously compounded rate of return on an annual basis decreases

[17] Obtained by a straightforward formal derivation omitted for space and readability reasons.

[18] The stochastic properties of the stocks were defined in such a way that the characteristics of the univariate and bivariate case are comparable in terms of the expected return. Furthermore, the standard deviations are chosen in such a way that a higher expected return is coupled with a higher standard deviation for the individual stocks for both cases. The portfolio formed in the bivariate case profits from the positive diversification effects between the two stocks due to the lack of correlation.

Table 2 Properties of stock portfolio in the bivariate case

	Stock 1	*Stock 2*
μ_i	12.58%	12.93%
σ_i	19.03%	23.36%
$S_{t_0,i}$	150	150
x_i	60%	40%

from σ_P=20.87 percent in the univariate case to just σ_P=14.75 percent in the bivariate case as we assume for the time being that the two stocks are uncorrelated.

For the VaR calculations, we choose again the confidence level at ε=0.5 percent. The VaR at the time horizon of 1 year is calculated as $VaR_1^{0.005}(P)$=113.980, whereas the corresponding value in the univariate case amounted to $VaR_1^{0.005}$=97.366, which again shows the impact of diversification. The corresponding value for a time period of only 2 days is given in the bivariate case by $VaR_{0.005479}^{0.005}(P)$=145.923 versus $VaR_{0.005479}^{0.005}$=144.232 in the univariate case. Clearly the effect of the assumed independence between the two stocks can be seen.

The bivariate case – though benefiting from diversification – underlines the necessity already shown in the univariate case to include the management's option to act according to current market conditions in any meaningful internal modelling under Pillar I of the Solvency II framework, in order to reflect the risk situation of the insurance undertaking in a truly economic setting and shape the capital requirements accordingly. The next section defines a simple portfolio management strategy and analyses its effect over a time horizon of 1 year.

Impact of simple asset management strategies on the SCR as modelled in an internal risk model

Introduction of a stochastic simulation in a discrete-time setting

So far the discussion has been based on analytical mathematical formulae in a continuous-time setting and on the pure impact of lapsed time on the VaR calculation in a static environment, ignoring any portfolio management action. In other words, the undertaking's management did not avail itself of the option to reverse a decision once taken in respect to the portfolio composition construed at the beginning of the observation period. In this section, we would like to introduce very basic and coarse-grained management strategies in a 1-year horizon setting and visualise the impact of these on the result distribution and consequently on the risk capital requirements.

Capturing a dynamic model in a dynamic setting using continuous-time, closed-form approaches for an integration of management strategies results in complex mathematics.[19] Therefore, in reality only a discrete-time, stochastic simulation model allows the integration of asset-side-driven management strategies. In order to include

[19] For more details and examples refer, for example, to Brohm (2002).

those strategies in a model framework, it is required that such a model divide the period into discrete sub-periods. At each modelled point-in-time an assessment of the risk and capital position is possible, which then may or may not trigger an asset rebalancing strategy.[20]

The underlying discrete-time, stochastic setting is a one-period model,[21] with four equidistant sub-periods, that is, a year or four-quarters. At each quarter end, a fresh assessment of the overall risk and capital position is made and, based on its result, the initial asset allocation may be revised based on a pre-defined asset management strategy that has been formulated by management from an *ex ante* perspective. Quarterly checkpoints provide only a very sparse granularity. The use of quarterly instead of weekly or daily checkpoints clearly is an assumption that is not truly authentic, as adverse events such as market crashes do not necessarily happen at the end of the quarter. However, if asset-side strategies already produce an impact even in as simplified a setting as this, then the value of including asset-driven management actions in a more "continuous model setting" is obvious. In this sense, all our considerations and findings can be interpreted as conservative approximations of economic reality, as the option for management to rebalance asset portfolios more frequently will result in more pronounced changes relative to the static portfolio situation.

The underlying economic assumptions

The available investment capital is provided via a pre-set amount of physical capital, generated by the underlying insurance business, but it could also be given as available equity capital.[22] In total the capital is set at 150 available for investment at inception. As no insurance losses are modelled, 150 would be profit at year-end and, in addition, all investment income would come on top.[23] The exclusion of insurance risk for this discussion is for purely illustrative purposes. Including the insurance risk will alter the picture, but will not impact the underlying economic concept.

The set-up is chosen in order to establish a simplistic environment to focus on the effects of asset-management strategies. This is because the impact analysis is not driven by the stochastic model but by the threshold chosen, which will trigger a pre-defined management action.

Comparison of a simulative and analytical solution

As stated above, the purpose of the first discrete time simulation is to compare the analytically derived solution with a simulative solution, given that the same stochastic properties introduced in Table 2 in the sub-section on time, diversification, and reversibility matters in the multivariate case apply to both settings. Using these two

[20] See Liebwein (2006).
[21] The model used was developed by Swiss Re, see Niering (2006).
[22] The modelled insurance company is simplified to focus on the asset management strategy and thus reduced to a mono-line insurance whose insurance risk is reduced to a negligible deterministic setting as it is not in the focus of the analysis.
[23] The Appendix summarises all quantitative model assumptions.

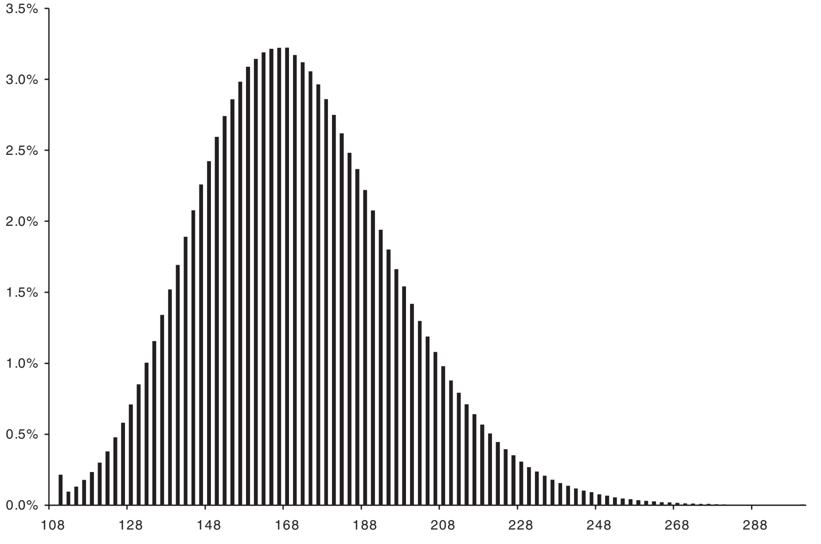

Figure 1. Simulated probability distribution of two stock portfolio value with zero correlation.

stocks and their properties, notably their correlation of zero, the analytical results were (Figure 1):

$$VaR_1^{0.005}(P) = 113.980$$

$$\begin{aligned} E(P_1) &= P_{t_0} \exp(\mu_p(t - t_0)) \\ &= 150 \times \exp(0.1272 \times 1) = 170.347 \end{aligned}$$

Turning to the simulation solution, with five million simulation runs, the results are (Table 3):

Table 3 Simulation results for the zero correlation case[a]

Simulation result	*Value*
$\hat{V}aR_1^{0.005}(P)$	115.045
$\hat{E}(P_1)$	170.310
$\hat{\sigma}(P_1)$	25.400
$\gamma(P_1)$	0.455

[a]The hat "^" denotes estimated moments and percentiles of the distribution based on the simulation in contrast to calculated figures based on the parameters of the log-normal distribution.

Relative to the starting value of 150, 115 is the 0.5 percent-percentile value. Therefore, 35 units of risk capital[24] will be sufficient in order to reduce the probability of not being able to pay out the initial value of 150 in $t=1$ invested in $t=0$ into the said stock portfolio to 0.5 percent.

The deviation of the simulated result for the VaR amounts to 0.93 percent whereas the corresponding figure for the expected value is only in the order of -0.02 percent. In addition to the visual inspection, we performed a Kolmogorov-Smirnow goodness of fit test, where we clustered the simulated portfolio values into 30 classes and calculated the relative frequencies of the simulated values. The comparison between the empirical and the hypothetical distribution – taking into account the absolute difference between both the upper and the lower class border on the one hand and the hypothetical value on the other hand – yielded a highest absolute difference of 0.0087 for the 30 classes. The test statistic for a level of significance of 1 percent beyond which the null hypothesis that the distribution is the desired lognormal distribution and is to be rejected is available in tabulated format and amounts to 0.290. Hence, the null hypothesis cannot be rejected and therefore we will assume that our random number generator produces sufficiently good results.

Having ascertained this, we can now turn to a more refined economic setting with the assumed correlation of 0.81,[25] which will serve as a basis for the simulation and later implementation of asset strategies to be described in the next section.

Definition of a pre-defined management strategy

Asset rebalancing could be driven by different reasons but all are geared towards safeguarding the fulfilment of regulatory requirements, solvency, or liquidity. In our considerations we will focus on the capital position as such. Hence, a pre-defined management action is invoked upon a certain fall in the capital position.[26] In this simple setting, the capital position is identical to the asset position or the value of the stock portfolio. A fall is induced in this simple setting by a decrease in the overall value of the two-stock asset portfolio. This impacts the capital position of the insurance undertaking in such a way that the quality of the insurance product can no longer be guaranteed unless the overall risk position is adapted – reduced – to reflect the lower capital position available.

The risk position is rebalanced by divesting the shares and investing the proceeds in assets of a lower risk class, that is, the asset management strategy employed foresees a

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[24] *Prima facie* this risk capital would have to be put aside in a risk-free investment, otherwise any risky investment would itself contribute to the riskiness of the whole system and thus change again the VaR figures. See also the section entitled "Definition of risk, time, and normalisation".

[25] This correlation avoids the creation of excessive diversification benefits, respectively under-estimations of capital requirements.

[26] Typical management strategies for modelling the asset management rules can be classified into four classes: (i) equity strategies (e.g. change of asset management behaviour triggered by a certain depletion of the available risk capital), (ii) pricing strategies (e.g. buy or sell instruments depending on price movements), (iii) cash-flow strategies (invest or divest free net cash flow from underwriting activities), and (iv) frame strategies (e.g. to reflect strategic asset allocation or regulatory boundaries).

100 percent divestiture of shares and an investment of the proceeds into money market papers once the trigger condition has been met. In the example to come, we will sell the stocks and switch to the money market option if the stock portfolio value falls below the threshold of 130. This threshold is completely arbitrary and could be set at whatever level. From a structural perspective, our findings would not change. However, in order to produce a pronounced effect on the distribution, a certain risk tolerance is necessary. Moreover, a loss of 10 percent to 15 percent relative to the current valuation seems not too unrealistic for practical purposes. Note that it is less the rebalancing which is important than the proper threshold setting. If the capital is reduced by a certain percentage to be chosen by the user in relation to either the capital position at inception or to the expected capital position at the end of the period, then a pre-defined management action is taken.

Management strategies: comparison of result distributions and VaR figures

Once we have defined the underlying setting and the asset-driven strategy, the simulation is started, run once with the strategy inactive as the baseline scenario, and then once again with the activated strategy in order to assess the impact.

There should not be any case of ruin in this set-up as no insurance risk is included, and neither the share prices nor the value of the money-market investments can fall below zero. So in essence the worst case to be expected is an almost 100 percent crash of the stock market. Therefore, the probability of ruin should be zero. However, the key question in the context of our analysis of the economic validity of the envisaged Solvency II regulation in respect to market risk is: will the inclusion of a management strategy that shifts investments to a relatively lower risky asset produce a positive effect in terms of capital requirements?

As the tails of the distributions are under scrutiny, a high number of simulation runs are necessary to provide a stable tail. Hence, the simulation is run 5,000,000 times, which leads to the simulated probability distributions of the annual results. This means that the tail for a 99.5 percent security level consists of the 25,000 worst cases that occurred during the simulation.

The result distribution at year-end in the case without asset-driven management strategies is shown in Figure 2. The simulated average capital at year-end, about 170, reflects the expectation that the capital base will grow by an average of approximately 20 a year due to the positive drift of the stocks if no strategy is employed. As expected, no ruin cases have occurred, as the stock price, which is the only source of uncertainty, follows a logarithmic normal distribution, which means that it cannot fall below zero. The distribution shows the classical shape of a geometric Brownian motion with drift.

The simulation results are provided in Table 4.

Given that the fundamental items of information are consistent with the expected results, our interest is in the calculated CaR (percentile or VaR), which identifies how much additional capital is required at year-end. Essentially it can be seen that the 0.5 percent-percentile value is 100.55 which means that by holding 49.45 units of risk capital, the probability of not being able to pay

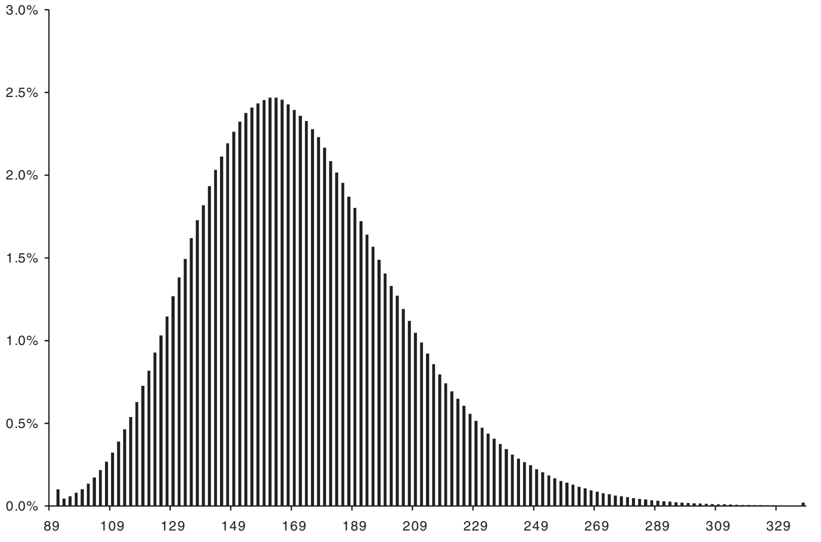

Figure 2. Simulated probability distribution of two stock portfolio value with correlation 0.81.

Table 4 Simulation results for the 0.81 correlation case

Simulation result	*Value*
$\hat{V}aR_1^{0.005}(P)$	100.550
$\hat{E}(P_1)$	170.308
$\hat{\sigma}(P_1)$	33.997
$\gamma(P_1)$	0.613

out the initial value of 150 will be reduced to 0.5 percent. Note that we are referring to values in $t=1$.[27]

Having set the baseline, the alternative scenario to be tested is the activated asset management strategy. On each simulated path, if the net asset position at quarter end triggers the threshold as defined in the strategy, the shares must be sold and the proceeds invested in money-market papers. As the expected returns from the money market are lower, the overall expected return must be lower too. This can clearly be seen in the result, as the average year-end position is only about 169 (compared to 170 in the baseline). Again there are no cases of ruin as the limit of the capital consumption is unattainable zero units.

[27] Note also that although we selected a setting depicting the nominal values as valuated at the end of the horizon, it would also be possible to translate these into values at inception, without any loss of information but with a loss of clarity for the reader.

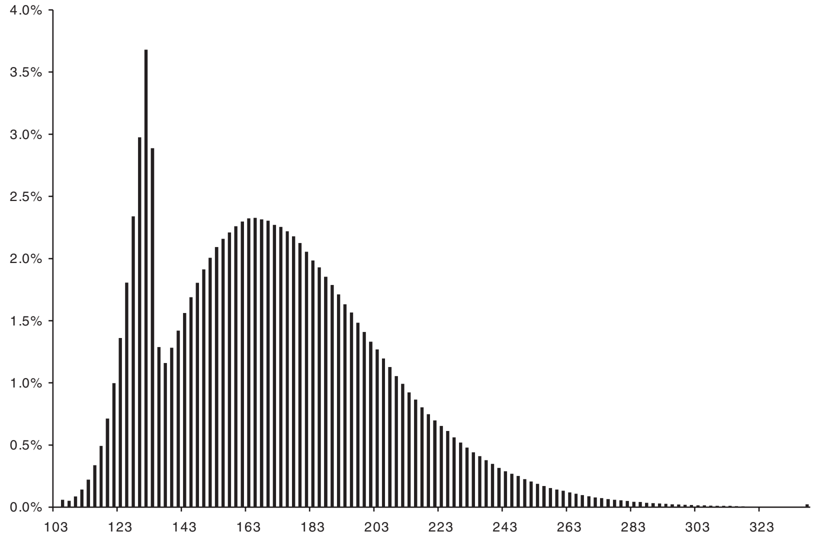

Figure 3. Simulated probability distribution taking into account a simple asset management strategy.

Table 5 Simulation results when applying a simple asset management strategy

Simulation result	*Value*
$\hat{V}aR_1^{0.005}(P)$	112.100
$\hat{E}(P_1)$	169.403
$\hat{\sigma}(P_1)$	34.318
$\gamma(P_1)$	0.686

The distribution is clearly storted, an effect that shows that the employed strategy has been invoked in a number of simulation runs. The result distribution also shows that there are cases where the sparse checkpoints meant that a continuing slide of the stock market was not detected early on, either because the slide started just after a checkpoint or because the slide was very steep. Nevertheless, in many cases the strategy was triggered by a capital position just below the threshold of 130, resulting in a cluster of simulation runs now transferring the assets from shares into the money market. Due to the low nature of the risk, those runs will not show the more likely upside nor less likely downside for the remainder of the simulation run, but lump together around the trigger value. This will result technically in a higher volatility of the storted distribution but economically and measurably in a less dangerous tail of the distribution when compared to the original case and measured by VaR (Figure 3).

Table 5 shows the main characteristics of the simulated distribution.

Again, given that the fundamental information is consistent with what was expected, our interest focuses on the calculated CaR (percentile or VaR), which identifies how much additional capital is required at year-end. Essentially, it can be seen that the value of the portfolio falls below 112.1 only with probability

0.5 percent, that is, 37.9 units of risk capital are sufficient for the defined security level of 99.5 percent. Eventually, the management strategy can be transformed into a potential capital relief of 11.55 (or 23.4 percent) relative to the situation where no management strategies are applied. Note that we still are referring to values in $t=1$.

Before interpreting the results and drawing some conclusions, it has to be stated that the results will not be altered to any material degree by increasing the number of simulation runs, as the tails show satisfactory convergence. More elaborate strategies or more complex types of investment potentially require more simulation runs.

Management strategies: interpretation and conclusions

As seen in the results of the simulations above, even coarse use of checkpoints alters the result distribution at the end of the observation period and has a significant impact on the $VaR_1^{0.005}$. A higher checkpoint granularity will not lead to a complete disappearance of the impact of stock market crashes on the asset position, but will make the extreme negative return cases far less likely. Nevertheless, even or especially in a more complex setting, an easy, explicit strategy for a small portfolio, which could be a sub-portfolio, will have a significant impact.

Beyond the asset strategy driven by the capital position which we used here to demonstrate our case, other triggers are possible such as market movements, crashes or interest rate shifts, liquidity constraints, or even overarching asset allocation restrictions, for example regulatory constraints. Their impact is essentially of the same nature: they can be used to dampen the uncertainty funnel resulting in a lower capital requirement. As opposed to banking solvency models, asset management strategies in general are more effective in relative terms within insurance operations where other key risk exposure stems from insurance risk. Therefore, assessments of capital requirements with internal risk models in insurance operations point at modelling key asset management strategies as the realistic risk mitigant.

The introduction of management strategies obviously affects the risk landscape. It may be argued that it creates a less conservative risk assessment, thus affecting the quality of the insurance product. On the other hand, there is the excess capital that has to be remunerated if an overly conservative risk assessment is done by excluding management activities from a model. The quest to provide return for the inadequate capital base will require management to search for risk premiums, and this will, if not included into the model, also lessen the quality of the insurance product. Additionally, it might be favourable from a prudential perspective, if there are some quantifiable incentives regarding risk mitigating sub-annual investment strategies, which in turn is a key issue of Pillar II.

Limitations to inclusion of management actions

Limitations to intra-periodic management actions on the asset side

As we have shown in the preceding sections, the decision not to model the behaviour of the management is not economically adequate for the asset risk management purposes of an insurance with a 1-year context. The very existence of management options has an intrinsic value, and no management can afford to be inactive if the company's position moves in an adverse direction. The question to be answered is how many, and which actions, can and should be modelled.

Basically, including management strategies as a part of a solvency model and of internal risk capital models assumes that:

(a) monitoring of the risk and capital position is done on an ongoing basis during the solvency assessment period,
(b) contingent actions that are planned in an *ex ante* setting can realistically be implemented instantaneously or within the modelled time delay, and
(c) contingent actions are executed the moment a pre-defined threshold has been triggered.

While the first two assumptions seem fair, the third could theoretically be automated, thus ensuring adherence. Alternatively, proof of execution of an action or contingency plans "coded" in corporate governance or other guidelines has to be given to the supervisory body. However, from the perspective of economic reality, the second element is the crucial one. Is the management actually in a position to execute the promised actions as the scope available is often narrowed considerably in difficult economic situations? Even if such management actions could be carried out in the market place, the costs involved in executing an action may vary depending on the market situation. In a market crash situation, the liquidity required to rebalance the books may not be attainable at all in the market or only at far lower asset values, implying high costs to the organisation. If portfolio rebalancing is automated and widespread in the market, then sell and/or buy orders may lead to a sort of vicious circle, an issue that shows the limitations of the portfolio insurance investment concept in distress market situations.

Therefore, the validity of asset-related management actions depends on the firm willingness and ability to indeed execute it and on how the other market participants are behaving. Including this factor in a model increases its complexity significantly. Assuming that an insurance undertaking is a pure price-taker on the capital market is not a fair assumption in difficult market situations where supply would skyrocket and demand would dry up. From the theoretical perspective, the geometric Brownian motion model for stock prices would no longer hold true. In fact, the tails of the distribution would be much fatter, especially on the adverse side of the curve. In a breakdown situation, no market-clearing prices can be attained at all, which in this case restricts management from carrying out any transactions. This means, economically, that the reversibility option no longer exists and that risk management ceases to exist in situations where the market as a whole is affected.[28]

One remedy would be to impose penalties on the sale of assets, for example only 50 percent of the market value is realisable in a blowout sale. If such a management action on the asset side is not triggered by an overall market movement but by an

[28] Proof of this assessment could be observed in the credit crisis affecting asset-backed securities where the market dried up completely.

idiosyncratic situation of an insurance company, then the price-taker assumption is a more realistic concept, as demand may not be affected at all.

Another – more pragmatic – solution would be to assess the effectiveness of the qualified asset strategies with the internal risk model,[29] and then to agree with representatives of prudential supervision to what extent this capital discount can be built in. If the regulator feels still some uncertainties are left, a capital discount would not be adequate; if the regulator's assessment indicates a sufficient level of comfort, a 100 percent or at least a certain lower percentage of the discount could be approved. To put it in different words: Insurers would have clear incentives to assess, define, and quantify their asset management strategies in the context of risk mitigation, and regulators would have the freedom to allow for capital discounts.

Extending the discussion to include asset strategies driven by insurance-risk-triggered actions in the risk or capital position, rebalancing of the books with price-taker assumptions seems reasonable in this situation. Arguably there are cases when large events influence both the insurance as well as the asset side of the balance sheet.[30] However, such extreme scenarios can be captured less in interdependency models than in stress scenarios, which are also included in the Solvency II discussions.[31]

While there are obviously limitations to the inclusion of intra-periodic asset side management strategies, a complete exclusion of any management action is not deemed to be economically reasonable and would significantly distort the economic reality faced by insurance companies. Therefore, we are of the opinion that some simple and robust asset management strategies formulated in an *ex ante* setting should be incorporated at least in an internal risk model to be submitted to the regulator for approval in the Solvency II capital regime. This would create more incentives to insurance undertakings to move faster towards a full internal model as intended by the regulators.

Intra-periodic management actions on the insurance side

Notwithstanding the reasons given for focusing on the asset side when including management actions in solvency capital calculations, from the design point of view insurance risk can just as well be altered in the period considered by applying insurance risk strategies. Therefore, from a conceptual perspective, the reversibility option is available to management on the insurance side of the business as well. The management can focus on two aspects: alter the inward book, for example write fewer or assume other insurance covers, or redefine the outward cession, that is, the net insurance risk position to be retained by the undertaking and thus supported by capital. Given that changes in underwriting policy will only have a gradual effect on primary insurance, the main focus for swift, intra-periodic management actions can

[29] Qualified in the sense of either proven historic execution or firm willingness and ability to execute in future situations.

[30] For a hypothetical example of a notional disastrous Japanese earthquake scenario see Geman (1996, p. 21) and references given therein.

[31] After all, those extreme events are less of a concern to the percentile type of VaR definitions than they are to the TVaR concept.

only be to alter the net position by changing the outward cession. Again the question arises as to the validity of implementing such a decision. Is the change in a reinsurance structure mirroring the behaviour of the overall market or is it idiosyncratic?[32]

Conclusion: management actions and solvency models

We have provided arguments in support of certain management strategies being included in the *ex ante* calculation of SCRs. While the extent to which the effect of management strategies should be incorporated into a solvency relief can be argued, the inclusion of strategies as such should not be questioned, as this would imply that the main activity of management, which is responding immediately to external and internal influences, is not seen as an asset to the company.

We agree that the management must be trusted to implement and adhere to agreed strategies. Digression from the strategies may lower the quality of the insurance product, but other deviations have the same impact, for example changes in underwriting policy.

Our proposal to quantitatively model management action strategies on the asset side of the insurance constitutes the bridge between Pillar II of Solvency II, where risk management practices and governance frameworks are to be examined by the regulator, and Pillar I with requirements for internal models to be approved by the regulator. Why, after due diligence by the supervisory body, should the quality of the management and internal models be accepted while the inclusion of management activities in such internal models and their impact on capital requirements are not?

Furthermore, the concept of continuous risk management cannot come to full fruition if the notion of management is not considered within the 1-year horizon but only at the inception of the 1-year period. Defining the 1-year horizon conceptually as a static, single-period model is not a logical conclusion. Excluding intra-periodic management actions by disallowing the inclusion into models would distort economic reality at best. Moreover it would not be consistent with the way management options are proposed to be treated in "with-profit" life insurance products under QIS 3, where a KC-factor models the management's ability to react.

If the time horizon is to be extended beyond the 1-year framework, as required in the business plans of any insurance company, the economic arguments advocated above hold true for both the asset as well as for the insurance side. Excluding standard management tools is unreasonable. Having said that, it is fairly obvious that this does not constitute a Solvency II issue at first glance, as the time horizon for the regulatory framework is restricted to 1 year; at second glance it indicates potential material risk-mitigating effects of procedures and strategies implemented on a sub-annual scale. Eventually, this proves the strong interrelation of Pillar I and Pillar II issues. Additionally, for a more general risk management discussion, and taking into account the risk capital position in the future beyond the 1-year framework, these arguments are highly relevant.

[32] The question is whether an event like Hurricane Katrina is affecting the whole market or a single, large loss affecting only a specific insurance undertaking.

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Appendix

Parameters of underlying economic assumptions

As another means of simplification, investments in shares as discussed in the continuous-time setting are replicated, that is, the whole sum is invested in shares. Transaction costs are neglected and the capital available for investment is allocated to two stocks according to the split in Table A1.

The shares are invested in two stocks modelled using a geometric Brownian motion with a set of realistic parameters. We start off with a comparison and control case of non-correlated stocks where the correlation matrix is obviously given by the identity matrix (Table A2).

In a second step, a correlation structure between the two shares as experienced in the real market environment is introduced, which will then be used as base case for the introduction of management strategies (Table A3).

Table A1 Weights in the two-stock portfolio

No.	*Name of stock/index*	*Weight in portfolio (%)*
1	Stock 1	60%
2	Stock 2	40%

Table A2 Correlation matrix for the uncorrelated case

	Stock 1	*Stock 2*
Stock 1	1	0
Stock 2	0	1

Table A3 Correlation matrix for the correlated case

	Stock 1	*Stock 2*
Stock 1	1	0.81
Stock 2	0.81	1

Table A4 Other relevant stochastic properties

Name of stock/index	*Initial value*	*Drift (percent)*	*Diffusion (percent)*
Stock 1	1	12.58	19.03
Stock 2	1	12.93	23.36

Table A5 Stochastic properties for the term structure model

Initial value of long-term interest rate (%)	5.42
Diffusion parameter of long-term interest rate (%)	2.25
Initial value of spread (long-term/short-term) (%)	2.034
Long-term equilibrium of spread (%)	2.16
Diffusion parameter of spread (%)	1.97
Market price of risk (%)	1

The other relevant stochastic properties are summarised in Table A4.[33]

Additionally, an adequate stochastic interest rate environment is provided to allow modelling of the stochastic development of the term structure of interest rates over

[33] A certain part of the investment is returned via dividend payments, but liquidity aspects are not relevant for the purpose of this discussion, as liquidity-driven strategies are not looked at. Cost aspects for buying and selling shares are also set to zero, as this effect is not considered material to our discussion.

time. Given these items of information, the prices of zero coupon bonds over time can be derived and simulated, which allows us to consistently price all future cash flows, especially money market and other interest-bearing assets, which will serve as alternative investment opportunities with a lower risk profile than a pure stock investment (Table A5).

This setting describes the underlying economic landscape in which the insurance company can now react to changes. While it is a very simple and coarse model, it still allows us to model asset-side strategies that either depend on share prices, the interest rate environment, or on the overall risk or capital position of the insurance company at each quarter end.

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