INTRODUCTION

by Karl BORCH *

1. This issue of the *Geneva Papers* contains three survey articles covering three very different aspects of insurance economics. The articles are short and self-contained, and as complete as one can expect under the severe limitation of space. There is no need to make this introduction a survey of surveys, but it may be appropriate to indicate, under the same limitations, that all the three articles in some way fit into the same general theoretical framework.

2. In the first of the three papers Professor FARNY gives an outline of the theories of business management which have been developed in the German-speaking countries, and he indicates how these theories can be applied to insurance companies. Business management (Betriebswirtschaft) was developed relatively early as an academic discipline in German universities. On the other hand, the so-called " neo-classical " economic theory (Jevons and Walras) made its impact relatively late in the German-speaking world. In most other countries this theory was firmly entrenched when the first systematic attempts were made to develop a theory of business economics. Thus German research had a different starting point, and developed approaches which were quite different from those followed when the subject later was taken up in other countries.

3. The central concept in the neo-classical theory is the market. The model often referred to as the " equilibrium of pure exchange ", explains the formation of prices in a market, given the preferences of the participants, and the initial allocation of goods. The crowning achievement of the theory was to show that the reallocation of goods which results from trading at equilibrium prices, is what today's economists call Pareto optimal.

It is not diffcult to bring production into this model. One usually assumes that there are " production functions " which make it possible to transform the goods given initially, into other goods, and one obtains a fairly general theory of prices, production and markets. In this theory there is however no room for entrepreneurs or firms which are engaged exclusively in bringing goods from producers to consumers. This is an obvious weakness of the theory, which generations of economists have struggled to overcome — so far, apparently without being entirely successful.

The German pioneers in business management were not so preoccupied with market equilibrium as their colleagues in other countries were a few decades later. Hence there was no particular reason for making sharp distinctions between producing firms and firms engaged in distributive trade.

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4. The neo-classical theory was not very successful in its first attempts to deal with risk and uncertainty, and it is only during the last two decades that economists have been able to develop satisfactory models for market equilibrium under uncertainty. Most of these models are markets for pure exchange of risk, and their intended applications are in the capital market. One can show under suitable assumptions that perfect markets for trading bonds, shares and other securities, will lead to a Pareto optimal distribution of the risks inherent in economic activity.

These models do in a sense contain the insurance market as a special case, but they leave no room for insurance companies. If a market of exchange functions perfectly, there is no need to create firms to carry out the exchanges, and it seems artificial to consider such intermediaries as producers of goods and services.

5. The German literature on business management is often very abstract and theoretical. It is therefore worth noting that its starting point was quite pragmatic. The German pioneers observed that insurance companies existed and evidently were in business. It was then natural to assume that a general theory of business management would be applicable, also to insurance companies, possibly with some minor modifications. The stumbling blocks created by the neo-classical economic theory were no serious obstacle in the approach taken by the German School.

6. Professor FERRY in the second article discusses the demand for life insurance. He surveys the literature, and refers to 21 articles. All of these are fairly recent — the older being published in 1965 — so it may be useful to point out that the problems have been studied earlier. They have a long history, going back at least to Irving Fisher's work on capital and interest. This work was in the spirit of the neo-classical theory, and its aim was to determine the rate of interest which would make the supply of savings equal to the demand for capital for investment and consumption.

7. Economists have usually referred to the problem in question as that of determining the optimal "saving-consumption plan" or "life-time saving plan". In the models set up to analyse this problem, it is usually assumed that the person's income stream $Y(t)$ is given, and that one seeks the optimal stream $C(t)$ for his expenditure for consumption. It is natural to assume that this problem has to be solved under the condition:

$$(1) \qquad e^{\delta T}\int_0^T e^{-\delta t}\,[Y(t)-C(t)]\,dt \geqslant 0$$

where δ is the force of interest, and T is the person's planning horizon or life-time. The condition simply says that the present (or compounded) value of consumption expenditure cannot exceed the present value of income, i.e. the person cannot be insolvent at time T. The condition is clearly based on an assumption that the person can save and borrow at the same rate of interest. It is not difficult to write down the corresponding condition under more realistic assumptions about the interest rates.

8. Any consumption plan $C(t)$ which satisfies (1) is called feasible. The problem then is to select the best, or the optimal among the feasible plans. To solve it one must clearly know the preferences of the person, and this is an empirical question.

If the person can die before time T, and that the income stream then will cease, it is natural to strengthen (1) to

$$\int_0^t e^{-\delta s}[Y(s)-C(s)]\,ds \geqslant 0 \qquad \text{for all } t \leqslant T. \tag{2}$$

This condition implies that the person is not allowed (by his creditor) to consume so that he may die insolvent. The stronger condition (2) evidently imposes severe restrictions on the set of feasible consumption plans.

9. At this point the person under consideration may find life insurance useful as a means of extending his field of choice over consumption plans. His creditors can allow him to be in debt at any time, if the debt is covered by life insurance.

Let $B(t)$ be the amount which the insurer will pay if the person should die at time t. Let $P(t)$ be the stream of premiums which the person pays to the insurer. Condition (2) can then be replaced by

$$e^{\delta t}\int_0^t e^{-\delta s}[Y(s)-C(s)-P(s)]\,ds \geqslant -B(t) \tag{3}$$

which of course reduces to (2) for $P(t) = B(t) = 0$.

10. The relationship between the functions $P(t)$ and $B(t)$ is given by the actuarial principle of equivalence:

$$\int_0^\infty P(t)\,e^{-\delta t}\frac{\ell_{x+t}}{\ell_x}\,dt = \int_0^\infty B(t)\,e^{-\delta t}\frac{\ell_{x+t}}{\ell_x}\,\mu_{x+t}\,dt \tag{4}$$

where ℓ_x represents the mortality table, and $\mu_x = -\ell_x'/\ell_x$ is the force of mortality.

Any pair of functions $P(t)$ and $B(t)$ which satisfy (4) do in a sense define a feasible life insurance contract. In practice there are however some other conditions which must be fulfilled.

Firstly we must have

$$B(t) \geqslant 0 \qquad \text{for all } t. \tag{5}$$

This condition rules out any insurance contract which requires a payment to the insurer upon the death of the insured person. The practical considerations behind this conditions lead to the general requirement that the premium reserve of the contract must be non-negative at any time, i.e. that

$$\int_0^t P(s)\,e^{-\delta s}\frac{\ell_{x+s}}{\ell_x}\,ds \geqslant \int_0^t B(s)\,e^{-\delta s}\frac{\ell_{x+s}}{\ell_x}\,\mu_{x+s}\,ds \text{ for all } t. \tag{6}$$

11. The conditions (5) and (6) seem natural in the institutional framework assumed by most authors who have studied the problem. Under other assumptions these conditions may be revised, or omitted.

Condition (4) ignores the " loading " of the insurance premiums, which is necessary to cover the transaction costs, which are ignored also in condition (1). In neither case it is difficult to take these elements into consideration, and to formulate more realistic conditions.

The conditions (3), (4) and (6) will define the set of feasible consumption plans. To select the best or optimal among these plans, one has to make some assumptions about preferences.

The usual assumption is that the person will seek the consumption plan which maximizes the expression of the type.

$$\int_0^\infty e^{-\alpha t} u\,[C(t)]\,dt$$

or

$$\int_0^\infty \left\{ \int_0^t e^{-\delta s} u\,[C(s)]\,ds \right\} \frac{\ell_{x+t}}{\ell_x}\,dt$$

which can be interpreted as the expected utility of the consumption stream.

12. If a person uses these models to determine his optimal consumption plan, he will usually leave no assets at his death. The optimal plan may imply that he is in debt at the time of death, but condition (3) implies that this debt is covered by life insurance.

Irving Fisher suggested that a person would want to leave a bequest at his death for reasons of " family affections ". He will then seek a consumption plan which maximizes the combined utility of the consumption and the bequest. The utility of a bequest will presumably depend on the composition of the family which the person leaves at his death. These ideas open the way to a rational theory of life insurance, and Professor Ferry's article demonstrates that the ideas have been taken up by a number of authors during the last decade.

The mathematical problem of determining the optimal consumption plan — or insurance plan — can be solved when the preferences — represented by a utility function — are known. This should open the field for empirical work and practical applications. There are however a number of complications, and these are brought out in the articles referred to by Professor Ferry.

If one examines the insurance arrangements made by people in the real world, one will often find that these arrangements are far from optimal — under the preferences assumed in most theoretical models. This presents a challenge, and the task is obviously to reconcile theory and practice.

13. In the third article, Professor ROSA surveys the literature on the demand for non-life insurance. Apparently he finds the present state of the theory very unsatisfactory. His references are all to fairly recent literature. It would have been useful if he had mentioned some older work, such as the classical book by Willet and the

work by the Austrian School to show that at least 2 or 3 generations of economists have found it a frustrating task to develop a satisfactory theory of insurance.

The most promising way to a satisfactory theory seems to be the fashionable "Portfolio Theory". An insurance company will usually hold one portfolio of insurance contracts, and one portfolio of securities. The ultimate payoff from both portfolios is of stochastic nature. On the former, the portfolio of insurance contracts, payoff will be negative, and of the latter it can be assumed to be positive. The problem of the company is therefore to determine the optimal pair of portfolios.

14. The demand for insurance arises out of the individual customers' efforts to solve their portfolio problems. Hence it appears that the situation can be analysed neatly in the framework of the neo-classical theory, once this has been extended to include equilibrium under uncertainty. The individuals and the insurance companies constitute a market, and they all seek to obtain the most preferred among the feasible portfolios.

The key to the economics of uncertainty is the expected utility theorem. One usually assumes that the utility function $u(x)$ which represents the preference ordering is concave, i.e. that $u''(x) > 0$. This assumption, which is really necessary to explain the existence of insurance, does however lead to contradictions between theory and actual observations.

15. As an illustration consider a person with an "initial wealth" S, and assume that this includes an asset worth A, which can be lost with a probability p. It is natural to assume that he can obtain insurance against the loss of the asset in the following form: If he pays a premium kP to an insurance company, he will receive a compensation kA if the asset is lost. The problem is then to determine the optimal value of k.

The expected utility for an arbitrary value of k is

$$U(k) = (1 - p)\, u\,(S - kP) + pu\,(S - kP - A + kA).$$

The first derivative of $U(k)$ is:

$$U'(k) = -(1 - p)\, Pu'\,(S - kP) + p\,(A - P)u'\,(S - kP - A + kA),$$

and the second:

$$U''(k) = (1 - p)\, P^2u''\,(S - kP) + p\,(A - P)\, u''\,(S - kP - A + kA).$$

As $U''(x) < 0$, it follows that $U''(k) < 0$. Hence if the equation $U'(k) = 0$ has a solution, it will give the value of k which maximizes expected utility.

Normally the solution cannot be $k = 1$, since

$$U'(1) = -(1 - p)\, Pu'\,(S - P) + p\,(A - P)\, u'\,(S - P) = [pA - P]\, u'\,(S - P).$$

As $u'(x) > 0$ it follows that $U'(1) < 0$ provided that $P > pA$.

The latter inequality simply says that the premium which the person pays to the insurance company is greater than the expected value of the compensation he will receive under the insurance contract. In practice this will always be the case, so one must conclude that it is not optimal to buy full insurance cover.

16. The conclusion above seems to be contradicted by observations from real life. If a person considers insuring an asset (his house, yacht, baggage or automobile), he will normally either insure it for its full value, or he may decide not to insure it at all. It will be surprising if he should insure the asset for say 60 % of its value, and even a casual examination of insurance statistics will show that people do not behave in this way.

Such observations tend to throw some doubt on the validity of the expected utility theorem, and they pose the question if people really are rational when they make decisions about insurance. If they are not, one may have to re-examine also the other rationality assumptions made in economic theory.

17. The three articles together demonstrate that important advances have been made in the economics of insurance over the last decades. They should also remind a reader of the old saying that a great beautiful theory can be ruined by one nasty little fact. The almost trivial conclusion may be that in the economics of insurance — as in most other fields — it is imperative that theoretical studies and empirical work should advance together.