Calculating the costs of a pension plan

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Abstract The present paper presents a useful algorithm for calculating the costs of a pension plan, funded or pay-as-you-go-based, that can easily be employed by everyone interested in pensions and its finance on his own pc at home using the widely available Excel tool. Exercises with the model show that the finances of a funded pension scheme are very sensitive to the economic assumptions made, like investment returns and inflation. They are far more sensitive to economic conditions than to actuarial conditions like mortality rates and other decrement rates. A pay-as-you-go-based scheme on the other hand appears to be very sensitive to population growth.

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Introduction

In determining the costs of a funded pension plan, apart from the problem of the appropriate valuation of the assets — a question of major interest but not pursued in this paper — various assumptions have to be made about a range of parameters in respect of the future economic and demographic environment affecting the growth (or decline) of the plan's liabilities. They are briefly reviewed in this section. Pension costs depend, for example, on the age and gender distribution of participants at date, their present earnings, the number of working years, the number of pension years (*longevity*) and the expected increase in earnings level, the latter being of utmost relevance when the benefit level is a function of income close to retirement date.

A forecast of future earnings includes (1) wage increases due to merit and productivity gains and (2) the rate of general price/wage inflation. Furthermore, in an inflationary environment the benefits to retirees probably will at least in part be compensated for loss of their purchasing power.

Another most critical factor in establishing the liabilities and costs of a pension plan are the investment returns, including the change of the assets' value itself.

Decrement rates in respect of mortality, disability, unemployment and withdrawal (turn-over), nuptiality rates, change in age and gender distribution and the outlays for administration are also of relevance for pension plan costs. They can be considered as the 'purely' actuarial assumptions, as opposed to the economic assumptions about rate of earnings growth and rate of return on capital.

Finally, the interest rate used for discounting future benefits is a factor of predominant

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relevance. In most countries usually a discount rate somewhat below the long-term bond yield is employed.

Calculating the costs of a funded pension plan is traditionally the domain of the actuary, who is using complex and rather mystifying models, thereby consciously or unconsciously but unnecessarily monopolising his professional skills.¹ The present approach therefore is the simplest imaginable, thereby introducing unavoidable deviations from reality. It is assumed that a pension is provided to a representative, though artificial individual. The model is easily accessible to all those interested in the field of aging and pensions who have no actuarial background and is user-friendly. It enables one to find out the impact on the contribution level of variables like capital return, rate of wage increase, indexation rate, pension result at retirement age, length of retirement period and length of contribution paying period.² A spreadsheet programme like Excel suffices.

In the next pages we describe how a funded system of financing a pension scheme works and compare it with a pay-as-you-go-based pension plan. Later in the further sections empirical evidence is shown. We only compare long-term contribution rates (steady state), leaving aside transition issues. Concluding remarks are made in the final section.

An algorithm for calculating the costs of a pension plan

Two main types of pension finance systems that exist are capital reserve or funding and the pay-as-you-go-based system. The mathematics of both is pointed out and compared.

Funding

In the case of a funded scheme employees pay part of their (labour) income into a fund of financial assets from which eventually the pensions are paid out. Cohorts or groups of individual participants break even over their lifecycle or, in other words, funded pension schemes are designed to be actuarially fair. They are intended to be nonredistributive (DC-plans)

or redistributive for a defined group only (DB-plans). The retirement pensions are paid out of current capital income, originating from investment revenues and by selling assets — more accurately, the physical or real capital goods as productive capacity hidden behind these assets — to the younger generations. A funded system relies more on the development of the international capital market as opposed to the dependency of a pay-as-you-go-based system on the current labour force.

To analyse the sensitivity of actuarial calculations to the choice of the real rate of return and the lengths of the contribution paying period and of the retirement period, a simplified model of financing a pension plan is used. An 'average' or representative participant to the plan is taken into account. At age a , say 25 years of age, he enters the plan and after T working years, during which contributions are paid, he retires when, for example, 65 years old. Finally he dies when $65 + p$ years of age. Earnings at age a , denoted by S_a , are expected to grow at a constant exponential rate g with no merit increases. Income at retirement age (eg 65 years), hence, is given by $S_{65} = S_a e^{gT}$. Contributions are accumulated in a pension fund, which earns investment income at a constant rate r . Future income is discounted at the same rate. On retirement age a pension equal to a per cent of last preretirement income becomes payable, for instance 60 to 70 per cent of last income. Pension benefits are indexed to prices (w). The parameter a primarily depends on the yearly accrual rate and the length of service time. The values of T and p reflect the mortality assumptions over the lifetime.

The problem is to find the contribution rate Pr , which will finance such a scheme.

Premium calculation under funding

Define S_{65} to be the last preretirement income of an average participant to the scheme, S_a the income of a worker joining the scheme and a the pension outcome as a percentage of final income (0.6 to 0.7).

At any time the actuarial liability (required capital) in respect of the representative retired

participant, $65 + t$ years of age, amounts to K_{65+t} ($0 \leq t \leq p$).

Now,

$$\frac{dK_{65+t}}{dt} = K_{65+t} \cdot r - \alpha S_{65} e^{wt} \quad (1)$$

The solution of this first-order differential equation is

$$K_{65+t} = C \cdot e^{rt} - e^{rt} \cdot \int \alpha \cdot S_{65} e^{wt} \cdot e^{-rt} dt \quad (2)$$

The constant C follows from $K_{65+p} = 0$ at the end of life.

Hence, the required capital at age $65 + t$ amounts to

$$K_{65+t} = C \cdot e^{rt} - \frac{\alpha S_{65} \cdot e^{wt}}{w - r} \quad (3)$$

Now it follows that

$$K_{65} = \frac{\alpha S_{65}}{w - r} \{e^{(w-r)p} - 1\} \quad (4)$$

Accumulation of capital K_t after t years of contribution payment results in

$$K_t = \int_0^t Pr \cdot S_a \cdot e^{gu} \cdot e^{r(t-u)} du = \frac{Pr \cdot S_a \cdot e^{rt}}{g - r} \{e^{(g-r)t} - 1\} \quad (5)$$

Note that $S_{65} = S_a \cdot e^{gT}$

Hence, the accumulation of capital at age 65 after T years of contribution payment is

$$K_T = \frac{Pr \cdot S_{65}}{g - r} (1 - e^{(r-g)T}) \quad (6)$$

Equating both expressions (4) and (6) gives as a result the funding contribution rate $Pr^{funding}$,

$$Pr^{funding} = Pr = \alpha \cdot \left(\frac{g - r}{w - r} \right) \cdot \frac{e^{(w-r)p} - 1}{1 - e^{(r-g)T}}, \quad w \neq r \neq g \quad (7)$$

This is a very useful and interesting expression, enabling one to find out rather easily the basic relationships between contribution rate ($Pr^{funding}$)

on the one hand and the rate of interest (r), rate of wage increase (g), indexation rate of pensions (w), pension result at retirement age (α), length of retirement period (p) and length of contribution paying period (T) on the other hand.

The effect of variations in the length of the retirement period, the length of the working period and the real rate of interest

Empirical evidence obtained from contribution formula (7) is shown in Tables 1 and 2. To comment on the sensitivity of actuarial premium calculations to the choice of the real interest rate ($r-g$) and the length of the retirement period (p), expression (7) is used to compute $Pr^{funding}$ for various values of ($r-g$) and p . Results are given in Table 1. Table 1 is divided into five sections with length of the retirement period of 10, 13, 15, 20 and 30 years, respectively. The retirement age varies from 60 to 72 years and the real interest rate deviates from a very low -1 per cent to a comfortable 6 per cent. The pension result is always a generous 70 per cent of final income. It should be emphasised that no one believes that future capital return, earnings growth, price inflation, disability and withdrawal can be predicted with much confidence, for we live in a dynamic world. Therefore, any kind of projections should generally be made with much caution.

For the sake of simplicity the rate of wage increase is assumed to be equal to the indexation rate of current pensions ($w=g$). From the table we observe in the third section that — given a retirement age of 65 years, a life expectancy of 80 years, hence a retirement period of 15 years, reflecting the current situation in most western countries — a rate of return on capital 2 percentage points higher than the rate of growth of wages, thus $(r-g)=2$, gives rise to a required contribution rate of 14.8 per cent. When the difference is 1 per cent, the contribution rate increases to 19.8 per cent. When the difference is zero, the contribution rate will be about 26.3 per cent and so forth.

It appears that a real rate of return only 1 percentage point different from the one assumed (whatever that might be) has considerable consequences for the resulting contribution level.

Table 1: The effect of variations in the length of retirement period and the real rate of interest on the contribution rate under funding (standard male participant, varying age of retirement)

Age of retirement	Length of retirement period (years; p)	Contribution rate $Pr^{funding}(\%)$						
		Real interest rate: $(r-g)$	-1%	0%	1%	2%	4%	6%
60	10		24.9	20.0	15.9	12.5	7.6	4.4
65			22.3	17.5	13.5	10.4	5.8	3.2
70			20.3	15.6	11.7	8.7	4.6	2.3
62	13		31.4	24.6	19.1	14.6	8.4	4.6
67			28.3	21.7	16.3	12.2	6.5	3.3
72			25.9	19.4	14.2	10.3	5.1	2.4
60	15 (current situation)		38.4	30.0	23.3	17.9	10.3	5.8
65			34.4	26.3	19.8	14.8	8.0	4.1
70			31.3	23.3	17.2	12.4	6.3	3.0
60	20		52.5	40.0	30.3	22.8	12.6	6.8
65			47.0	35.0	25.8	18.8	9.8	4.9
70			42.8	31.1	22.3	15.8	7.6	3.5
60	30		82.9	60.0	43.3	31.2	16.0	8.2
65			74.8	52.5	36.9	25.8	12.4	5.8
70			67.6	46.7	31.9	21.6	9.7	4.2

Note: assuming a pension result of 70 per cent of final income, full indexation ($w=g$) and entry age 25 years.

Table 2: The effect of variations in the length of retirement period and the real rate of interest on the contribution rate under funding (standard male participant, retirement age 65 years)

Length of retirement period (years; p)	Real interest rate $(r-g)$ (%)	Contribution rate $Pr^{funding}(\%)$ with and without indexation				
		Fully indexed ($w=g$)	No indexation ($w=0$)			
	1		$r=5\%, g=4\%$	$r=4\%, g=3\%$	$r=3\%, g=2\%$	$r=2\%, g=1\%$
10		13.5	11.2	11.7	12.3	12.9
15		19.8	15.0	16.1	17.2	18.4
20		25.8	18.0	19.6	21.4	23.5
30		36.9	22.1	24.9	28.2	32.1
	2		$r=6\%, g=4\%$	$r=5\%, g=3\%$	$r=4\%, g=2\%$	$r=3\%, g=1\%$
10		10.4	8.6	9.0	9.4	9.9
15		14.8	11.3	12.1	12.9	13.8
20		18.8	13.3	14.4	15.7	17.2
30		25.8	15.9	17.7	20.0	22.6
	4		$r=8\%, g=4\%$	$r=7\%, g=3\%$	$r=6\%, g=2\%$	$r=5\%, g=1\%$
10		5.8	4.9	5.1	5.3	5.6
15		8.0	6.2	6.6	7.0	7.5
20		9.8	7.1	7.6	8.2	9.0
30		12.4	8.1	8.9	9.9	11.0

Note: assuming 40 years of contribution paying from 25 to 65 years of age and a pension result of 70 per cent of final income.

Note that — starting from a retirement age of 65 and a real rate of return of 1 per cent — a tremendous extension of life expectancy by five years (from 15 to 20 years of length of retirement period) increases the contribution level from 19.8

to about 25.8 per cent, slightly less than the impact 1 percentage point underestimate of the real interest rate has on the premium level. The premium level rises from 19.8 to 26.3 per cent. A 2 percentage point further underestimate of the

real rate of return is equivalent to almost a doubling of the retirement period: 34.4 per cent vis-à-vis 36.9 per cent contribution rate. In other words, it appears that contribution levels under funding are much more sensitive to capital returns than to longevity or the length of the retirement period. The increase in the contribution rate with life expectancy appears to be surprisingly small.

Clearly with high retirement age (about 70 years) combined with short periods of retirement (about 10 years), contribution rates — even with minor real interest rates — are quite affordable. A higher retirement age matters: retiring at the age of 70 years given a real rate of return of 2 per cent yields a pleasant contribution rate of 12.4 per cent. Otherwise, a low retirement age (about 60 years) together with long periods of retirement (20 to 30 years) — given a modest real interest rate equal to or less than 2 per cent — gives rise to unsustainable contribution rates of about 40 per cent of workers' income or more. With high real capital returns (4 per cent or more) even the aggregate of low retirement age and long pension period are far from problematic at least from a pension finance point of view.

Other variants are obtained by considering different rates of capital return, income increase and indexation rate of current pension payments, entry age, retirement age and concomitant different lengths of the contribution paying period and retirement period.

Indexation of current pensions

In the Introduction it is assumed that current pensions are fully indexed. Indexation is not provided without considerable costs being incurred. Now indexation is released. Table 2 is composed of three parts with real interest rates of 1, 2 and 4 per cent, respectively. The length of the retirement period (p) varies from 10 to 30 years.

Holding the real rate of interest ($r-g$) at the level of the given 1, 2 and 4 per cent, respectively, Table 2 shows the values of the contribution rate necessary to finance nonindexed pensions ($w=0$) from the age of 65 onwards for various combinations of r and g and length of retirement period.

It appears that in all three main scenarios — with given values for ($r-g$) of 1, 2 and 4 per cent,

respectively — when the rate of earnings growth is higher, the more the contribution rates can be decreased if current pensions are not indexed. In the first scenario (1 per cent real interest rate and 15 years spent in retirement) the contribution rate falls from 19.8 per cent (full indexed pension) to values between 18.4 and 15.0 per cent, respectively for a nonindexed pension, if earnings increase from 1 to 4 per cent.

When the retirement period doubles to 30 years the contribution rate in the same way decreases from 36.9 per cent to values between 32.1 and 22.1 per cent, whose results likely can be considered as unsustainable. On the other hand, with higher levels of the real rate of interest (up to 4 per cent) contribution rates are becoming more and more affordable irrespective of the length of the retirement period.

Special case

When the interest rate is equal to the rate wage increase, which also equals the indexation rate ($r=g=w$), somewhat simpler expressions for the actuarial liability (required capital) and the accumulated capital at the age of retirement are obtained. Analogously we find,
Required capital at the age of 65 + t ,

$$K_{65+t} = C \cdot e^{\alpha t} - e^{\alpha t} \cdot \int \alpha \cdot S_{65} dt = C \cdot e^{\alpha t} - e^{\alpha t} \cdot \alpha \cdot S_{65} \cdot t \quad (8)$$

As $K_{65+p}=0$, it follows that $C=\alpha \cdot S_{65} \cdot p$
Hence,

$$K_{65+t} = \alpha \cdot S_{65} \cdot e^{\alpha t} \cdot (p-t) \quad (9)$$

Accumulated capital after t years of contribution payments:

$$K_t = Pr \cdot t_a \cdot \int_0^t e^{\alpha u} du + C = Pr \cdot S_{65} \cdot e^{-gT} \cdot e^{\alpha t} \cdot t \quad (10)$$

Similarly at age 65 after T years of contribution payments,

$$K_T = Pr \cdot S_{65} \cdot T \quad (11)$$

Table 3: The effect of variations in the length of retirement period, the length of the working period and the growth rate of population on the contribution rate under payg (standard male participant)

Age of retirement	Length of retirement period (years; p)	Contribution rate $Pr^{payg}(\%)$			
	Growth rate population:	2%	1%	0%	−1%
60	10	12.5	15.9	20.0	24.9
65		10.4	13.5	17.5	22.3
70		8.7	11.7	15.5	20.3
62	13	14.6	19.1	24.6	31.4
67		12.2	16.3	21.7	28.3
72		10.3	14.2	19.4	25.9
60	15 (current situation)	17.9	23.3	30.0	38.4
65		14.8	19.8	26.3	34.4
70		12.4	17.2	23.3	31.3
60	20	22.8	30.3	40.0	52.5
65		18.8	25.8	35.0	47.0
70		15.8	22.3	31.1	42.8
60	30	31.2	43.3	60.0	82.9
65		25.8	36.9	52.5	74.3
70		21.6	31.9	46.7	67.6

Note: assuming a pension result of 70 per cent of final income ($w=g$); entry age 25 years.

Equating $K_{65} = K_T$ yields

$$Pr^{funding} = Pr = \alpha \cdot p / T, \quad w = r = g \quad (12)$$

where α is the replacement rate, p is the number of retirement years and T years is the length of the working period. Empirical results of the case $w=r=g$ are given in Table 1, column where $(r-g)=0$ per cent.

Pay-as-you-go financing

Under a pay-as-you-go system pensions are paid out from contribution payments (predominantly out of domestic labour income) only. Instead of the lifecycle reallocation of a cohort's (or cohorts') aggregated income, there is a cross-sectional income redistribution from workers to pensioners on the basis of an implicit social contract: each generation finances the pension income of the previous generation on the understanding that its own pension income will be financed by the next generation, with the inherent risk that future generations can break, at least in part, the contract. Pay-as-you-go schemes

can bring about systematic income redistribution or, in other words, individual people generally will not break even over their lifecycle.

Pay-as-you-go-based pension plans are normally implemented through the state's power to levy contributions.

Premium calculation under pay-as-you-go

To find the pay-as-you-go contribution level, we equate the amount of required pension payments to the amount of contributions to be paid in period $[t, t+1)$.

Define $\bar{S}_{65.plus}^{ave}$ to be the average income in the last preretirement year of all pensioners of age 65 and over, $\bar{S}_w^{ave}$ the corresponding average income of all workers and $\bar{S}_{w,t}^{ave}$ the average workers' income in year t . By L_t^p we denote the number of pensioners at time t that is growing at rate κ and L_t^a is the number of workers with growth rate λ .

Total amount of pension payments in $[t, t+1)$,

$$L_t^p \cdot \alpha \cdot \frac{\bar{S}_{65.plus}^{ave}}{\bar{S}_w^{ave}} \cdot \bar{S}_{w,t}^{ave}; \quad L_t^p = L_0^p \cdot e^{\kappa t} \quad (g = w) \quad (13)$$

Total amount of contributions in $[t, t+1)$,

$$Pr_t^{payg} \cdot L_t^a \cdot \bar{S}_{w,t}^{ave}, \quad L_t^a = L_0^a \cdot e^{\lambda t} \quad (14)$$

Hence, the contribution rate under pay-as-you-go Pr^{payg} ,

$$Pr_t^{payg} = \alpha \cdot \frac{\bar{S}_{65,plus}^{ave}}{\bar{S}_w^{ave}} \cdot \frac{L_0^p}{L_0^a} \cdot e^{(\kappa-\lambda)t} \quad (15)$$

Assuming $\bar{S}_{65}^{ave}/\bar{S}_w^{ave} = 1$ — in all periods average retirement income equals average workers' income — and assuming a stable demography — both the working population and the retired population grow at rate n — it can be shown that

$$\frac{L_t^p}{L_t^a} = \frac{\int_0^p e^{nt} dt}{\int_p^{p+T} e^{nt} dt} = \frac{e^{np} - 1}{e^{np}(e^{nT} - 1)} \quad (16)$$

Hence,

$$Pr^{payg} = \alpha \cdot 1 \cdot \frac{e^{np} - 1}{e^{np} \cdot (e^{nT} - 1)}, \quad n \neq 0 \quad (17)$$

Special case: if population growth is zero ($n=0$), then

$$Pr^{payg} = \alpha \cdot p / T \quad (18)$$

Note that in this stationary demography (zero growth) the contribution rate under pay-as-you-go (18) equals the contribution rate under funding (12), where $w=r=g$. Furthermore, it can be shown that in a stable demography (constant, nonzero growth) the contribution rates under funding and pay-as-you-go are identical if $r+n=g$, reflecting the well-known Aaron condition.

It should be observed that the Aaron condition holds from the view of the individual participant to the scheme, but not necessarily so from a

macro point of view. This question will be pursued in a forthcoming paper.

The effect of variations in the length of the retirement period, the length of the working period and the growth rate of population. Empirical evidence obtained from contribution formula (17) is shown in Table 3. To comment on the sensitivity of pay-as-you-go premium calculations to the choice of the length of the contribution paying period (T), the length of the retirement period (p) and the growth rate of population (n), expression (17) is used to compute Pr^{payg} for various values of T , p and n . Note that under a pay-as-you-go scheme an identical linkage exists between the contribution rate and the average change in population growth under equilibrium conditions. The rate of population growth functionally replaces the real rate of return under a funding system. Results are given in Table 3.

Table 3 is also divided into five sections with lengths of the retirement period of 10, 13, 15, 20 and 30 years, respectively. The retirement age varies again from 60 to 72 years and the growth rate of population differs from -1 to 2 per cent.

Changes in population growth alter the ratio of the number of retirees and contribution payers and thereby the required contribution rate. Thus, whenever the growth in population is 1 per cent yearly — and together with a retirement age of 65 years and a length of the retirement period of 15 years, reflecting the current situation in the US and Canada, but not in most European countries — the contribution rate under a pay-as-you-go system is also about 19.8 per cent, equal to the contribution rate under funding and a real interest rate of 1 per cent (see Tables 1 and 2). When there is zero population growth, the premium rate is 26.3 per cent and when the population growth is -1 per cent, the pay-as-you-go contribution rate will be 34.4 per cent and so forth. The pay-as-you-go contribution rate, hence, appears to be highly sensitive to the growth rate of population (n). The lower the n , the higher the ratio of retired beneficiaries to the contribution paying working population and, hence, the higher the required contribution rate.

Clearly a stationary or even shrinking population retiring at young ages (about 60 years) combined with very long periods of retirement (about 25–30 years) results in very high and allegedly unsustainable levels of pay-as-you-go contributions, amounting to 50 per cent and more of workers' income. This will *ceteris paribus* happen in some southern European countries in the coming 15–20 years. Those results can be compared with contribution rates under funding with disastrously low capital returns.

Other variants are obtained by considering varying rates of population growth, income increase and replacement rate, entry age, retirement age and concomitant different lengths of the contribution paying period and retirement period.

Concluding remarks

The contribution formulas under funding (7) and pay-as-you-go (17) are derived under stylised conditions, deviating from reality in different ways. Its charming simplicity and clear usefulness, however, remain, enabling everyone to probe the impact of the economy with population aging on the costs of pensions. At a rather simple level the predominance of economic assumptions over the

actuarial ones in contribution calculations could be clearly established. The costs of a funded pension scheme appear to be very sensitive to the set of economic assumptions chosen (in particular capital returns), far more sensitive to these than to the actuarial assumptions made (eg longevity).

When the real rate of return at any intermediate age of the 'average' participant is to be assessed 1 per cent point higher than assumed on entry age, the accumulated contributions or accrued benefits at date — in other words, the actuarial or past-service liability — will increase by one percentage point too. Consequently, an additional increase in contribution rate will be necessary, reaching a highest value just before retirement age. An alternative response might be reducing the quality of benefits, for example, by deleting or mitigating the indexation provision or adjusting the pension contract more directly in some way.

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