Defined benefit pension funds: Identifying, quantifying and managing asset-liability risks

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Andrew Barrie
is a mathematics graduate with more than 20 years' industry experience in both life insurance and investment banking. Before setting up Barrie & Hibbert, he was responsible for one of the UK's top-rated equity derivatives operations at a major investment bank.

Craig Turnbull
joined Barrie & Hibbert in 2000 after graduating in mathematics and finance the year before. He leads the asset liability management (ALM) team's risk and capital management consultancy and research work, with particular focus on the UK and European life assurance industry and the defined benefit pension industry. He is a member of the Barrie & Hibbert Management Committee. He qualified as a Fellow of the Institute of Actuaries in June 2003 and is a member of the Institute of Actuaries' Stochastic Modelling Guidance Working Party.

Craig McCulloch
gained a first-class honours degree in mathematics and statistics and then spent six years working for a major UK life office in a variety of product development, finance and other actuarial roles, in both the UK and Republic of Ireland. He joined Barrie & Hibbert in 2004 and is a Fellow of the Faculty of Actuaries.

Abstract This paper looks at issues relating to defined benefit pension plans. The paper presents a risk management framework for the identification, measurement and management of the asset-liability risks inherent in defined benefit pension plans. This is a flexible and transparent platform that can provide immediate insights into the key risk drivers of a defined benefit (DB) pension plan, and how these risks interact. Some of these insights are obvious — if a pension plan holds equity assets to back liability cashflows that are, at best, indirectly related to equity returns, the pension plan will be exposed to falls in equity values. However, the case study discussed in this report provides some more subtle insights into the quantification and interaction of the risks being run by a 'typical' DB pension plan:

- Diversification of risk across assets and liabilities is hugely important when the risk/reward management of DB pension plans is considered. Historically, this factor has been neglected by many — opting for equities as the sole 'return' generator.
- Typical DB pension plans are exposed to falls in equity values and falls in interest rates. Standard assumptions about the relationship between changes in equities and interest rates imply that these two exposures provide a limited amount of natural offset. That is, when equities fall, interest rates tend to rise, providing some relief in the form of a reduction in the liability value. This has some interesting implications from a risk management perspective. The consequence is that hedging interest rate risk without managing equity risk will have little effect on the total risk facing the pensions plan — even where the liability duration risk is very significant.
- Given the size of asset/liability mismatches typically present in a DB pension plan, the marginal contribution to risk of mortality risk is very low. This is not because

Craig Turnbull
Barrie & Hibbert Limited
48 Melville Street
Edinburgh EH3 7HF, UK
Tel: +44 131 625 0208;
Fax: +44 131 625 0215;
email:
craig.turnbull@barrhibb.com

significant mortality uncertainty does not exist. Rather, it is because it is uncorrelated with the large asset/liability risks that are typically present in a DB pension plan. If and when the asset/liability mismatches are mitigated, the impact of mortality risk will be significantly greater. This undermines the principle that risks can be totally removed from pension fund management.

Keywords: *defined benefits; asset liability management; stochastic asset-liability modelling; asset allocation; risk management; hedging*

Introduction

Over the past few years, there has been a growing awareness of the risks associated with defined benefit (DB) pension plans. The combination of volatile equity returns, low bond yields and fast-improving pensioner longevity has significantly impacted on the economic wellbeing of many defined benefit pension plans (both in the UK and overseas). These difficulties have perhaps been further exposed by the inexorable trend towards market-based measures of solvency, value and profit which have been embraced by accountants, regulators and parent sponsors and have shone a spotlight on these economic challenges.

This has by no means been an experience unique to DB pension plans. The insurance industry has undergone a similar experience as financial reporting, regulatory solvency, and rating agency capital adequacy assessment have all embraced a more market-based approach to risk, capital, profit and value. This has significantly impacted on the way insurers manage their business — from product pricing to capital management. The pension fund industry, however, has, to date, been slower to adopt these new approaches and use them to inform management decision-making.

One of the difficulties for pension fund practitioners is the significant number of factors to consider in the management of a DB pension plan. Further, these factors often interact in some manner that further confuses understanding. Actuarial asset-liability modelling (ALM) studies often do little more than show that the combination of all these factors makes funds risky.

It is believed that a process that decomposes the drivers of risk and return is a key analytical support for those interested in the optimal management of risk and return. This analysis leads to the concept of risk budgeting — a process where the pension fund and its sponsor decides on the level of risk appropriate and then allocates these risks to sources that give the best combined return. This paper will develop a market-risk-based risk budgeting framework for trustees and sponsors. This framework can provide transparent identification of the key risks facing a pension fund, how these risks combine and interact, and what they contribute to expected returns. This is illustrated by a case study presented in the second section of this paper. As will be seen in the third section, the insights from this analysis can help to inform management approaches to the identification and appraisal of candidate strategies for the management of the pension plan's risk/reward profile.

Creating a framework for managing risk budgets

In deciding how to allocate risk budget, a method of assessing the efficacy of alternative risk management strategies is needed. In particular, a framework is required to assess where the scheme is currently exposed to risk, which measures the magnitude of these risks; in turn, this allows a detailed analysis of the drivers influencing these risks, and so suggests possible risk mitigation strategies; finally it allows the impact on risk and return of any chosen strategy to be assessed. Barrie & Hibbert Ltd (B&H) have developed a powerful, flexible methodology to do all of this. This is considered via a case study of a sample UK pension scheme.

Risk? What risk?

First, it is necessary to be clear on the questions that need to be answered. In particular, it must be decided how risk will be defined and measured. The answer to this question inevitably depends on the strategic purpose behind the risk budgeting approach — very different approaches may be necessary when considering the impact on an FRS17 balance sheet basis compared with an approach designed to measure actuarial solvency (funding basis). Is the one-year balance sheet volatility of key interest? Or is the impact on the longer-term solvency position of the plan more relevant? More often than not, of course, a number of different risk measures are likely to be of interest to the decision-maker.

This analysis focuses on the specific question of how the solvency of the pension plan may vary over a one-year horizon. Solvency is defined by the liability basis — the framework could consider FRS17 solvency, a 'pure' market-consistent solvency measure, or a more traditional actuarial funding approach, or, indeed, something in the middle. The case study below uses the 'something in the middle approach'. Risk-free rates for the basis of the liability discounting, but an additional 'expected return premium' of 225 basis points is added to the discount curve. While this valuation is not therefore 'market-consistent', it will have the same risk dynamics (ie sensitivity to interest rates) as the market-consistent valuation approach.

The one-year time horizon of the analysis keeps the discussion focused and self-contained. There is no reason, other than the wish to avoid writing a 50-page tome, that the analysis could not be extended to a multi-year projection. The variability of the one-year solvency measure has also become a popular approach to analysing risk and assessing capital requirements in the life sector over the last couple of years.

One might ask how a one-year projection basis can effectively describe the risks of funding liabilities that extend beyond the next 50 years. This is one of the advantages of a market-based approach to liability valuation when assessing solvency — (one-year) changes in market values represent changes in future (50-year) expectations. Thus, the one-year variability can provide an efficient insight into the very long-term risks that drive the pension plan's ability to fund liability cashflows.

The approach

The approach is based on an analysis of the behaviour of asset and liability values over a one-year time horizon. Projected changes in asset and liability values over this period are simulated, using economic scenarios generated from B&H's Economic Scenario Generator[1] (with an end-December 2004 'best-estimate' calibration).

This allows the probability distribution of the end-year difference between the values of assets and liabilities to be estimated. Here, the key measure risk

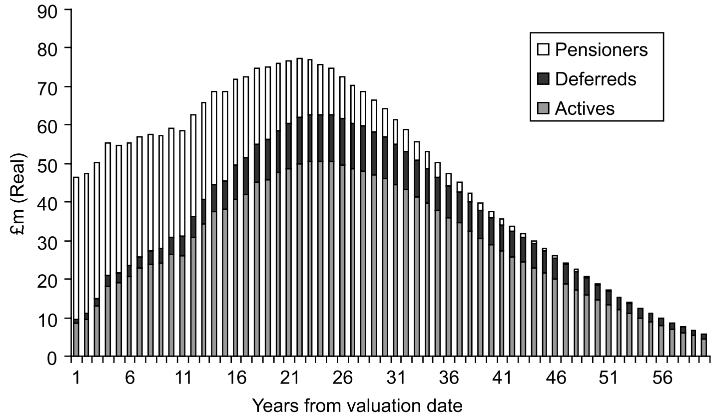

Figure 1: Expected real scheme cashflows (£m)

used is the standard deviation of end-year scheme surplus/deficit. Other statistics could equally be used — eg the 99th percentile of the end-year projected surplus would be relevant to a plan that was particularly concerned with limiting downside risk to a tolerable level. Expected end-year surplus (on the liability valuation basis chosen) is used to measure expected return.

These measures of risk and return are then further decomposed into component elements attributable to each risk driver.[2] This allows decomposition of the end-year surplus risk and return into an element for each driver, while simultaneously allowing the identification of any diversification benefits arising between assets and liabilities. This information is vital to the identification and appraisal of candidate investment management solutions.

This is all best illustrated through the case study discussed below. More detail on the modelling assumptions used in the case study is provided in Appendix A.

Table 1: Asset allocation at 31st December 2004

Asset Value	(£m)
European equities	420
Other overseas equities	424
Property	154
Risk-free nominal bonds	11
Risk-free index-linked bonds	21
Nominal corporate bonds (A-rated)	193
Total	1,223

Case study: Expected liability cashflows

The following analysis uses data that are representative of a 'typical', large, relatively mature UK defined benefit pension plan. In this pension plan, all pensions are assumed to be inflation-linked.

The liability cashflows in Figure 1 are shown in real terms (ie in today's money, before allowing for future inflation increases).

These cashflows have duration of around 20 years — while the scheme is reasonably mature, the liability duration is still very long relative to the duration of the most liquid bond assets. This is an important topic that will be revisited later in the report.

On the valuation basis used (risk-free plus 2.25 per cent), these liabilities have a value of £1,237m as at 31st December, 2004. This implies an initial deficit of £15m, given a market value of assets of £1,223m at the end of 2004.

Case study: Current asset profile

Table 1 shows the current asset mix of the scheme. In this example, the pension

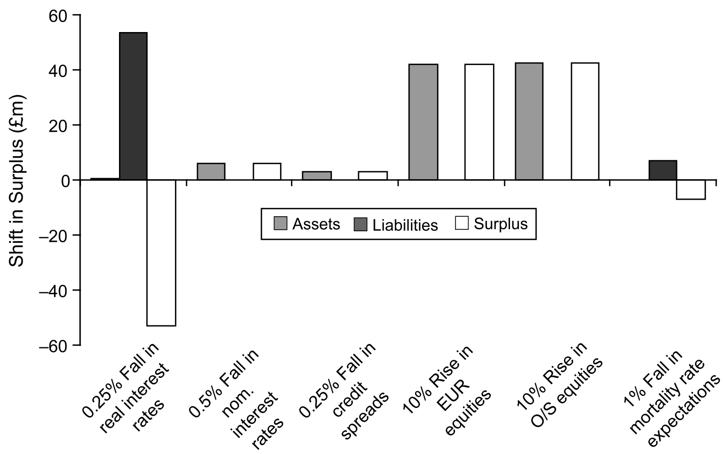

Figure 2: Some asset and liability sensitivities

plan holds risk-free (nominal) bonds, corporate (nominal) bonds and risk-free index-linked bonds. The corporate bond portfolio is assumed to be composed of A-rated bonds. The total bond portfolio duration is eight years.

Case study: Sensitivity of asset and liability values to market risk

Next to consider is the net impact of the asset and liability positions outlined in the previous two case studies. As a first indication of the nature of risks facing the pension plan, Figure 2 illustrates the sensitivity of the value of assets and liabilities to various changes in the major market risk factors that may impact on the plan's solvency.

As might be expected, given the inflation-linked nature of the pension liabilities, the liability value is heavily exposed to falls in the real yield curve. The index-linked bond holdings do little to mitigate this risk — they are of insufficient volume and duration to offset this exposure to falls in real yields.

On the asset side of the balance sheet, the large holdings of equities will generate significant asset volatility that is not directly offset by changes in liability values.

Figure 2 suggests that these asset and liability risks are, in general, not well matched from a market-risk perspective. The following section analyses what this means for the variability of the end-year surplus/deficit.

Case study: Projecting the one-year change in surplus/deficit

Figures 3–5 show the distribution of the end year value of assets, liabilities and surplus (assets less liabilities), respectively, as produced using the methodology described above. Note that, on the valuation basis used, the starting position is a deficit of £15m.

Broadly, Figures 3–5 attach a statistical likelihood to the joint behaviour of interest rates, inflation, asset returns and mortality assumptions. This allows the probability of asset, liability and, subsequently, surplus changes over the next year to be quantified.

The end-year surplus distribution has an average mean end-year deficit of £12m — a small increase on the start-year position. This increase is a result of several elements:

- An expected positive contribution arising from an expected risk premium

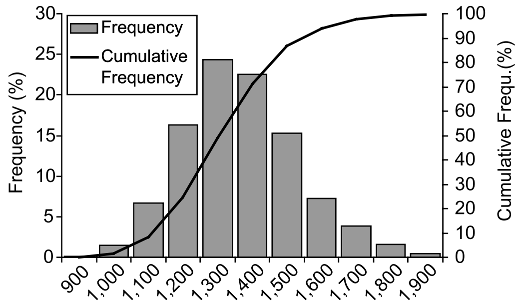

Figure 3: End-year asset value distribution

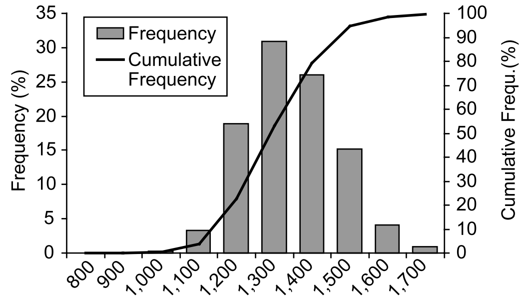

Figure 4: End-year liability value distribution

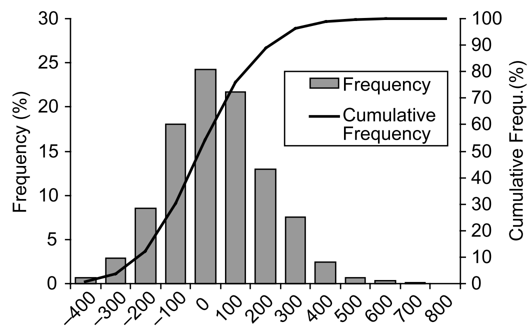

Figure 5: End-year surplus distribution

on equity, property and credit-risky bond assets.

- An offsetting negative element arising from a valuation basis which already takes credit for some of these risk premia — in particular the additional 2.25 per cent discount rate will 'unwind' each year, leading to a decrease in expected surplus, relative to the risk-free position where scheme assets and liabilities are perfectly matched.
- An expected decrease in surplus/increase in deficit as a result of the existing deficit rolling up — this deficit can be thought of as a net borrowing position, and so will increase as a result of interest costs in servicing this debt.

The end-year surplus exhibits a significant amount of variability — the modelling suggests that there is a 1 per cent chance of the deficit increasing to over £300m at the end of the year (in the context of starting asset and liability

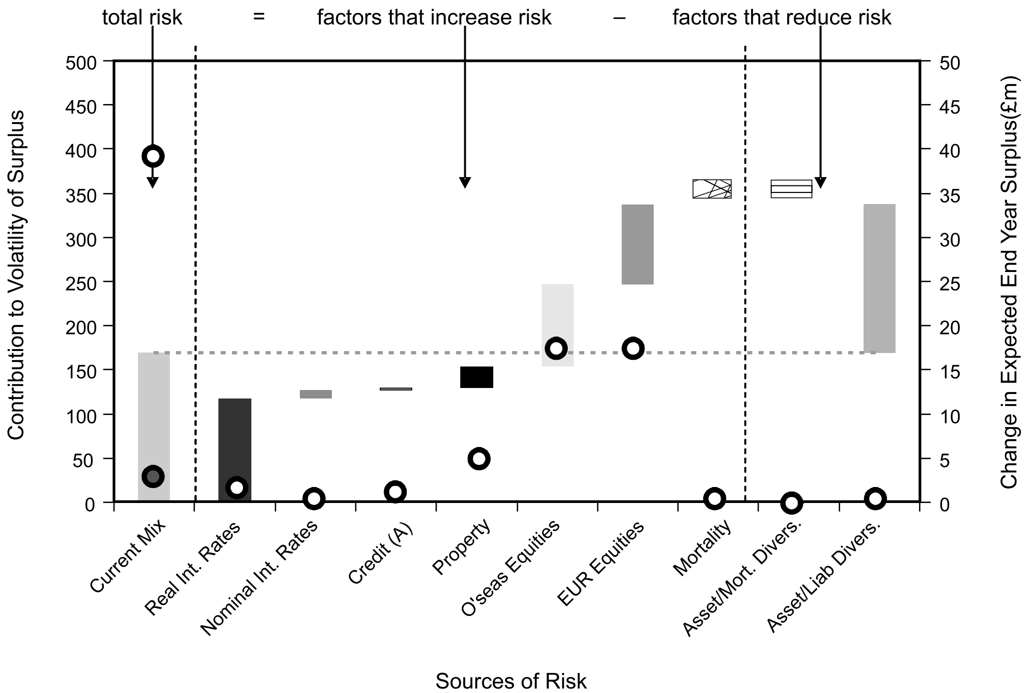

Figure 6: Decomposition of end-year change in surplus: base asset mix

values of £1.2bn). And remember, this is a deficit that already takes credit for virtually all of the expected return that can be expected from the current risky asset mix.

Figure 6 shows the breakdown of this surplus volatility into its component risk drivers. These are illustrated by the bars in the exhibit. The figure also shows the contribution to the change in the average end-year surplus arising from each of these risk exposures. These are represented by the white circles on the chart (the red circle shows the overall expected change in the surplus).

Case study: Decomposing risk and return

The previous case study was mildly interesting — however it failed the 'so what' test. It demonstrated that the assets and liabilities are volatile and gave an idea of the overall risk faced in terms of prospective surplus. It did not really help to understand how to set about managing the pension fund in a more optimal way. So, what questions must be answered to move understanding forward?

- What are the main drivers of risk and reward?
- Can some of the risks be managed?
- What effect does this have on overall risk/reward?

The next few sections will address these questions.

Consider Figure 6. This shows how the overall surplus risk and return produced by the analysis has been built up. Risk (as measured by volatility) is linked to the left y-axis; return is linked to the right y-axis. The total volatility of the end-year surplus is shown by the bar on the far left. This is then decomposed into the marginal contribution[3] to the volatility that arises as a result of each risk driver.

The drivers of risk/return considered are:

- real interest rates;
- nominal interest rates;
- credit;
- property;
- European and overseas equities;
- mortality uncertainty.

Other factors such as manager selection or salary inflation could also have been considered.

To interpret the results, the paper will first look at risk, then return.

Risk analysis

Figure 6 permits some key insights into how the risks have arisen.

First, it can be seen that the single largest risk is the real interest rate risk. In particular, there is a significant exposure to *falls* in real interest rates, which increase the value of the (index-linked) liabilities, but are not matched by a corresponding asset exposure (either by volume or duration). This is considered in more detail in the section 'Identifying and appraising candidate solutions'. It is worth noting that, unlike equity risks, there is little or no associated expected return earned by taking this risk exposure.

Secondly, there are significant (UK and overseas) equity risks. Liabilities are backed by volatile equity assets, but the liability cashflows are not directly linked with changes in equity assets.[4] For any pension plan, equity risk is taken in the anticipation of an equity risk premium. The figure shows that the equity exposure makes a significant positive contribution to the expected change in the surplus.

When all these contributors to risk are added up, however, the answer is far bigger than the estimated total risk — why is this? The answer, simply, is diversification — the various risk drivers are not, in fact, perfectly correlated (so, while the plan is exposed to falls in interest rates and falls in equity values, the fact that these events will not necessarily occur at the same time means that the total risk is less than the simple sum of individual risks). These diversification benefits are captured in the last two factors of Figure 6. The figure decomposes this into diversification between mortality and the various asset risks, and between each of the asset risks.

This highlights the interesting effect that mortality risk has on the variability of the end-year surplus, as shown by the checked and cross-hatched bars. The checked bar shows the marginal contribution to total risk that mortality risk makes, *when considered in the absence of other risks*. But the cross-hatched bar shows that, *in the presence of the significant asset-liability mismatches implied by the current asset mix*, most of this mortality risk is diversified away. That is, in the presence of the significant market risks that the plan is currently exposed to, the marginal contribution to total risk of mortality risk is actually almost negligible. This arises because mortality risk is assumed to be independent of the asset risk drivers — it assumed that scheme mortality rates are not affected by the underlying economic drivers.

Finally, and perhaps most significantly, there are considerable diversification benefits arising between each of the asset and liability risks. In particular, interest rate risk (exposure to falls in real interest rates) and equity risks (exposure to falls in equity values) are assumed to be negatively correlated (a correlation of –0.3 was assumed between one-year equity returns and shifts in the real interest rate yield curve). This correlation assumption gives rise to a highly significant diversification benefit, shown in Figure 6 by the bar on the far right. Why does this correlation have such a significant impact on the diversification benefit? Well, the negative correlation means that the plan's two biggest risks provide a natural offset to each other. If equity returns fall, real yields are assumed to typically rise, providing some offsetting relief to the impact on the end-year surplus. Mitigating either of these risks will inevitably result in a significant fall in this diversification benefit. As will be seen, this can have a significant bearing on the appraisal of alternative investment strategies.[5]

Reward analysis

The expected change in end year surplus (shown by the white circle in the 'Current Mix' column), and the contribution to this change in return resulting from each of the individual risk drivers, was also set out in Figure 6.

In summary, the most significant contributors to expected return are the two equity classes, with a smaller contribution from property. All the other factors add little in terms of expected performance enhancement.

There is a slight complication that needs further comment. The discount rate being used is gilts +2.25 per cent. This means that assets need to outperform, in aggregate, risk-free rates by more than 2 per cent for this measure of pension plan solvency to remain the same. This 'unwinding' of the discount rate means that the expected surplus enhancement will not lie at the white circle, but at the lower grey circle.

Some tentative observations

The previous section has presented some further insights into the constitution of the pension fund. It is even possible to make some postulations as to how the management of the fund could be optimised. For example, it is not unreasonable to hypothesise:

> A large amount of risk comes from real interest rates.
>
> And
>
> No reward for taking this risk is expected.
>
> Therefore
>
> All real interest rate risk should be hedged out. This would leave a lower risk arrangement that has the same expected performance.

Indeed, this is what many pension funds are being persuaded to do by investment banks keen to sell fixed income hedge solutions. But does this simple argument hold true? This is considered in the next section.

Identifying and appraising candidate solutions

This section expands the analysis of the current risk position of the pension plan to consider alternative investment strategies that may improve the trade-off between risk and return. A number of possibilities are considered here, but this analysis is intended to be illustrative of how the framework developed in the section 'Creating a framework for managing risk budgets' can be used as a risk management tool, rather than an exhaustive study of the investment strategy options open to the pension plan.

Reducing real interest rate risk

The analysis in the previous section demonstrated that one of the most

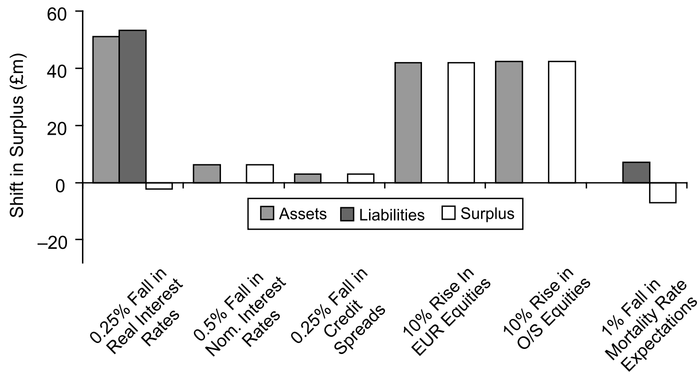

Figure 7: Asset and liability sensitivities, with real interest rate swap

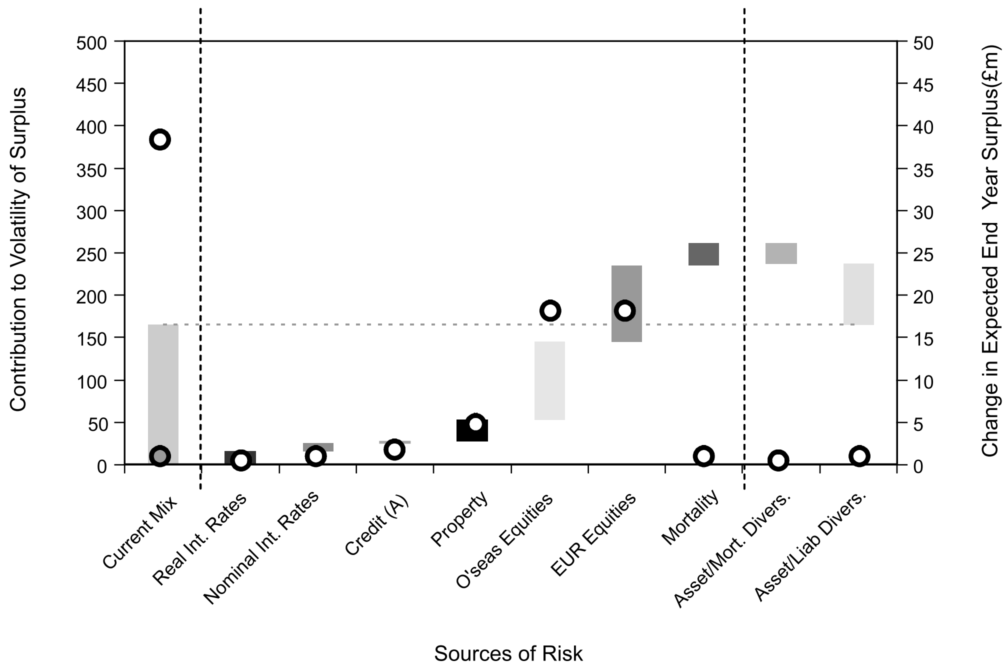

Figure 8: Decomposition of end-year change in surplus: base asset mix+25Y £1,000m receive-fixed real interest rate swap

significant risks the pension plan is exposed to is falling real interest rates. This is due to a lack of asset exposure to real interest rates — index-linked liabilities are unmatched and falls in real interest rates will increase the cost of switching to a more matched position. It is unlikely that the necessary real interest rate duration will be easily sourced from the index-linked bond market. A more efficient approach to removing this exposure is to hold a receive-fixed real interest rate swap. This can be used to generate offsetting asset value rises in the event of falls in real interest rates.

The example below shows the impact of entering into a 25-year receive-fixed real interest rate swap for £1,000m principal, while leaving the remaining asset mix unchanged.

Figure 7 shows the impact on surplus sensitivities of following such a strategy. Compare this with Figure 2. As would be expected, the surplus sensitivity to a fall in interest rates has been immunised

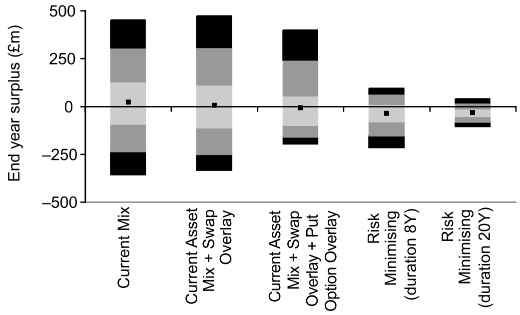

Figure 9: Summary of risk/return profile of end-year surplus distribution under the various sample asset strategies

— the rise in liabilities is now broadly offset by an equivalent rise in assets.

Figure 8 updates Figure 6 with the impact of the above real interest rate swap.

Surprisingly, the total volatility of overall end-year surplus (far left column) is barely changed by the presence of the real interest rate swap, despite the fact that it has efficiently mitigated almost all of the pension plan's very significant real interest rate risk — as expected, the marginal contribution to end-year risk from real interest rates (second column) reduces significantly, from around £120m to £15m,[6] with no significant change in overall expected end-year surplus. What is going on?

The explanation for this can be found at the last item in the chart (far right column), asset/liability diversification. When compared with Figure 6, it is clear that the diversifying element has been reduced significantly by the real interest rate swap. The assumed negative correlation between equities and real interest rates means that the exposure to falls in both equities and real interest rates creates some natural risk mitigation. As a result of this effect, managing one risk in isolation has little impact on the total risk faced by the plan. The statistics underlying this observation are further discussed in Appendix B.

In summary, the earlier tentative observation is flawed. The impact on overall risk of hedging a single risk factor depends on that factor's relationship with other important drivers of risk.

Risk and reward: Management asset strategies

The results from the previous section lead to some interesting conclusions for those interested in risk and reward management.

Simply managing down the risks of a pension fund is not trivial. The more the impact of individual drivers of risk is reduced, the greater the absolute impact of other drivers. Clearly, an approach is needed to address the joint impact of these drivers.

The results of the previous section suggest that an investment strategy that tackles both real interest *and* equity exposures is required in order to make a significant impact on the volatility of the end-year surplus. There are a number of ways that the equity risk exposure could be managed: equity put options could be purchased; a dynamic equity allocation could be pursued which reduces equity exposure as solvency deteriorates.

Diversification is a hugely powerful factor in risk management. Historically, diversification of pension fund assets has been fairly weak, with equities dominating. However, this analysis would imply that, by combining a broader range of uncorrelated assets, risk can be reduced while retaining the upside of keeping long-term costs down. Of course, such a conclusion is contingent on being able to identify diversifying asset classes that offer expected returns in excess of the risk-free rate. This is easier said than done.

Figure 9 summarises the impact of a range of candidate strategies considered for the case-study pension plan on the end-year surplus.

In Figure 9, the black dots indicate the mean of the end-year surplus, the darkest bars the 25th to 75th percentiles of the end-year surplus distribution, the middle bars the 5th to 25th and 75th to 95th percentiles, and the palest bars the 1st to 5th and 95th to 99th percentiles. The first two bars represent the current asset mix of the plan, as discussed in the section 'Creating a framework for managing risk budgets', and the real interest rate swap overlay discussed in the section 'Reducing real interest rate risk'. As discussed in the latter section, the swap overlay, in isolation, has done little to manage the total risk of the pension plan.

The third strategy considers adding an (at-the-money) equity put option to the swap overlay strategy. It can be seen from the chart that this has significantly reduced the downside surplus risk of the fund — the 99th percentile deficit has been reduced from over £300m to less than £200m. However, it has also reduced the expected change in surplus — the deficit is now expected to increase, on average, under this strategy.

The fourth strategy considers an asset strategy where 100 per cent of assets is shifted into the risk-free index-linked bond portfolio (and holding no derivatives). This strategy further reduces expected return relative to the third strategy, as equity exposure has now been reduced to zero. However, downside risk is still significant, as a significant real interest rate duration risk remains (assets are invested in eight-year duration bonds while liabilities have a duration of 20 years). An exposure to falls in real interest rates still remains as a result of this significant reinvestment risk.

Finally, the fifth strategy considers the case where 100 per cent of assets are invested in risk-free index-linked bonds with 20-year durations (in practice, this would be achieved using real interest rate swaps). This removes all market/economic risk from the pension plan's asset-liability position (note: real salary growth volatility has not been modelled in this analysis). The residual risk under this strategy is mortality risk. Note that in each of the last three strategies considered here, it is likely that the expected return reduction that results from these strategies will result in assets being insufficient to fund the eventual liability cashflows. In this example, risk mitigation is likely to necessitate additional funds being injected into the pension plan. However, one might argue that the current asset strategy merely delays the inevitable, and exposes the sponsor to significant risk of far greater future contribution requirements to fund its liabilities.

Extending the analysis

This case study barely scratches the surface of what can be achieved within this flexible framework. In particular, the analysis can be readily expanded to allow for alternative risks, for example salary inflation risks, active management risks, or indeed any other risk where the impact on scheme assets or the impact on the valuation of cashflows can be modelled. It is also possible to consider different definitions of risk —

longer-term projections, different liability valuation bases. Further, the analysis can be readily expanded to include an assessment of a diverse universe of alternative asset classes and hedging strategies: equity option strategies, investing in commodities or hedge funds, etc.

This powerful risk analytical framework, combined with B&H leading-edge stochastic asset modelling, provides a flexible and transparent risk management tool for sponsors and trustees as they consider the variety of management options available to them to manage the significant asset-liability challenges inherent in today's defined benefit pension plans.

Appendix A: Liability modelling assumptions

The analysis presented in the case study makes a number of important assumptions. In particular, liabilities are valued based on the discounted value of cashflows for accrued scheme benefits only. No allowance is made for future accrual or for future contributions made to fund future accrual or any current shortfall in assets over liabilities. Liability cashflows are discounted at relevant risk-free rates (either nominal or index-linked) plus 2.25 per cent.

This valuation approach is an interesting example of a combination of a market-based approach to liability valuation (ie using risk-free rates) and a 'traditional' (funding) scheme valuation (which will discount at a fixed rate which is significantly greater than the risk-free rate).This valuation approach will produce a current surplus/deficit which is similar to a typical funding approach. But the end-year liability basis that is used to assess the surplus/deficit at the end of the one-year projection will incorporate changes in risk-free yields over the one-year projection. As seen below, this can be a key driver of the variability of the one-year surplus projection. The one-year volatility in long-term risk-free rates represents the long-term risk of interest rates being insufficiently high to produce asset proceeds that are sufficient to fund liability cashflows.

Scheme liabilities are based on expected cashflow data for pensioners, deferred members and active scheme members. These cashflows are split into real (RPI-linked) and fixed elements.

Assets are valued at market values.

Stochastic mortality is implemented using B&H's stochastic mortality model.[7] The model modifies end-year expected liability cashflows to allow for changes in mortality expectations over the one-year projection.

Finally, any future contributions and liability accrual are ignored. The analysis is limited to examining changes in the current solvency position of the scheme.

Appendix B: Impact of correlation assumptions

As mentioned in the section 'Identifying and appraising candidate solutions', a rather counter-intuitive result is observed when removing interest rate risks in isolation — the expected end-year surplus can actually become more volatile than when this additional risk existed. A simple example illustrates this point.

Suppose an end year surplus which depends on two elements: interest rate exposure and equity returns. Given an end-year distribution of interest rates and equities, calculate that surplus will have an end-year volatility of 100 if equity risk is held in isolation, and 50 if interest rate risk is held in isolation.

Some elementary statistics show that the volatility of the total end year surplus is:

$$\sqrt{100^2 + 50^2 + 2\rho \times 100 \times 50}$$

where ρ is the correlation between equity and interest rate changes. It can be seen that, on the standard assumption of a –0.3 correlation, the total volatility of end-year surplus is

$$\sqrt{100^2 + 50^2 + 2 \times -0.3 \times 100 \times 50} = 97.5$$

So, were all interest rate risk to be removed, total volatility actually *increases* from 97.5 to 100. In effect, there is a natural hedge between interest rate risks and equity risks, given the assumed negative correlation. Clearly this is dependent on the correlation assumption made. A zero or positive correlation naturally reverses this impact.

References

1 The B&H ESG is the industry-standard stochastic asset model in the UK and European life assurance sectors. It is increasingly being used in the DB pension fund industry. See, for example, PricewaterhouseCoopers' December 2004 survey 'Weapons of Mass Computation' (Section 6) for more information on its use in the life industry.

2 This is done by considering the end-year surplus position where each risk factor is considered in isolation — all other drivers are 'turned off' by assuming the corresponding assets earn a risk-free rate of return.

3 Each marginal contribution to risk is calculated relative to the matched case, ie it is the volatility created by the risk in the absence of any other risk.

4 Note that this modelling does not currently include analysis of the salary-dependant nature of the active member cashflows pre-retirement. That is, this analysis assumes salary inflation is perfectly correlated with price inflation. The approach can easily be extended to include such analysis. Traditional actuarial reasoning argues that equity assets provide a reasonable match to such salary-related liabilities. The evidence for this is limited, given the historic correlation observed between wage inflation and equity returns, and the fixed nature of pension outflows from the scheme once these come into payment.

5 It is worth bearing in mind that the level of diversification benefit achieved will depend on the correlations assumed between the various assets being modelled. The calibration of the B&H ESG sets target correlations based on the analysis of the long-term historic correlations. See B&H Calibration Note 2004/021 for a detailed discussion of the equity/bond correlation calibration.

6 Some real interest rate risk remains because the swap only hedges against changes in the level of the real interest rates — it does not hedge against changes in the shape of the real interest rate yield curve. A greater number of swaps would be required to also hedge this second-order risk.

7 This stochastic mortality model has been used in risk-based capital assessment in the life sector, particularly in annuity business, and the valuation of mortality-contingent derivatives. See B&H Technical Note 2000/017 for the original discussion of this model.