Autocorrelation, bias and fat tails: Are hedge funds really attractive investments?

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Practical applications

This article provides a framework for evaluating hedge fund performance in consideration of autocorrelation, bias, and fat tails. For this purpose we develop an adjusted version of the modified Sharpe ratio presented by Gregoriou and Gueyie (2003). Investors might use this adjusted modified Sharpe ratio to measure the performance of their hedge fund investments. In a practical application of the proposed measure we find that funds, which follow the Equity Market Neutral strategy provide a superior risk return profile, but that many other hedge fund strategies lose their attraction.

Abstract

Hedge funds have become an increasingly popular investment tool in the past decade, owing to their general lack of correlation with stock and bond markets. When evaluated using the Markowitz portfolio selection theory, hedge funds appear to offer a remarkable opportunity. Yet use of the Markowitz theory neglects three important qualities of hedge funds: the existence of significant autocorrelation, bias and fat tails. Each of these three issues has been studied individually, but no literature exists in which their combined effect is considered. The purpose of the research reported here is to evaluate hedge fund performance incorporating these combined effects. The results indicate that hedge funds lose most of their attractiveness when the existence of autocorrelation, bias and fat tails is taken into account.

INTRODUCTION

Hedge funds have been subject of much research since the mid-1990s. In the literature, hedge fund performance is often evaluated by Markowitz's portfolio selection theory and by classical performance measures such as the Sharpe ratio, under which hedge funds appear to be very attractive investments.[1] Recent

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research, however, has pointed out three problems concerning hedge fund returns, thus making their attractiveness less certain.[2] When hedge fund returns are compared with those of traditional investments, they exhibit a significant extent of autocorrelation (the autocorrelation problem), contain systematic estimation errors (the bias problem), and tend to stronger deviations from normally distributed returns (the fat tail problem).

Each of these problems has been analysed in the literature, but only in isolation: Kat and Lu[3] and Getmansky *et al.*[4] examine the statistic characteristics of hedge fund returns and show the possibility of integrating the autocorrelation of returns in the performance measurement. Christansen *et al.*,[5] Cappocci and Huebner[6] and Ammann and Moerth[7] investigate hedge fund performance using a multifactor model and give a detailed bias analysis. Favre and Galeano[8] use a modified VaR for hedge fund evaluation with consideration of the higher moments of return distribution, whereas Agarwal and Naik[9] incorporate the fat tail problem by choosing a mean-conditional VaR framework.

In addition, there are many new performance measures that try to integrate the higher moments of return distribution by considering the risk of loss,[10–12] but all these measures similarly concentrate on one problem area only. Amenc *et al.*[13] and Kouwenberg1[14] both analyse the impact of survivorship bias and non-normal returns on hedge fund performance, but do not account for the autocorrelation of returns. Thus, the basic question for investors is still unanswered: Jointly considering these three problems, do hedge funds actually represent attractive investments? The purpose of this paper is to answer this question.

First, classical hedge fund performance measurement methods are discussed and their inherent problems are pointed out. Then, ways of integrating the three above defined problems in hedge fund performance measurement are shown. Finally, the implications for the evaluation of hedge funds are presented, by integrating all problems in one common framework, the results of which allow the basic question to be answered: Are hedge funds really attractive investments?

HEDGE FUND DATA AND STRATEGIES

In the empirical investigation, monthly returns of the Credit Suisse First Boston/Tremont (CSFB) hedge fund indices are examined over the period January 1994–December 2004.[15] Various hedge fund strategies are reflected in the hedge fund indices. Credit Suisse First Boston/Tremont places all the hedge funds in three strategy groups, depending on their risk characteristics. In order of increasing return volatility, these strategies are: market neutral, event driven and opportunistic. A total of nine individual strategies can be differentiated within the strategy groups. In Table 1, the individual strategies are sorted into the CSFB strategy groups and a brief description of each is provided.

In addition to the nine strategies, an aggregated index (CSFB Hedge Fund Index) comprising the performance of all the strategies is considered. This broadly

Table 1: Hedge fund strategies

Strategy group	*Strategy*	*Description*
Market Neutral	Fixed Income Arbitrage	Identification of mispricings between similar fixed income securities; speculation on price convergence of these securities
	Convertible Arbitrage	Purchase of undervalued convertible bonds and short selling of the underlying stocks; speculation on removal of the undervaluation
	Equity Market Neutral	Exploiting short-term price differences in equity trading; speculation on price convergence for equity portfolios with a similar structure
Event Driven	Distressed	Investing in companies that are in financial or operational difficulties; speculation on the continuation of business operations
	Risk Arbitrage	Purchase of takeover candidates' shares and short selling of the bidding company shares; speculation on the realisation of the takeover
Opportunistic	Global Macro	Top-down approach; speculation on a fundamental change of the direction in prices of specific asset classes worldwide
	Dedicated Short Bias	Short selling of overvalued securities; speculation on buying back the securities at a lower price later
	Emerging Markets	Investing in emerging market countries; speculation on positive economic development in these countries
	Long/Short Equity	Bottom-up approach; speculation on increasing prices of undervalued stocks and declining prices of overvalued stocks

diversified index is treated as the tenth strategy. The hedge fund indices are compared with four market indices; two of them measure equity performance, the other two measure bond performance. Standard & Poor's 500 (S&P500) and Morgan Stanley Capital International (MSCI) World are used as equity indices and J.P. Morgan (JPM) Global Government Bond and Lehman Brothers (LB) Government/Corporate Bond are the bond indices. Hence, the study considers two world indices (MSCI World, JPM Global Government Bond) and two indices with a focus on the US capital market (S&P500, LB Government/Corporate Bond). As all indices were calculated on a USD basis, the perspective of a US investor is modelled. To measure returns from price changes and dividends, performance indices are considered. The data were collected from the Datastream database.

Table 2: Performance measurement results (Sharpe ratio)

Group	*Index*	*Mean monthly return (%)* (r_{id})	*Standard deviation of monthly returns (%)* (σ_i)	*Sharpe ratio* (SR_i)
CSFB indices				
Aggregated	Hedge Fund	0.90	2.35	0.23
Market Neutral	Fixed Income Arbitrage	0.56	1.11	0.19
	Convertible Arbitrage	0.78	1.35	0.32
	Equity Market Neutral	0.82	0.87	0.54
Event Driven	Distressed	1.09	1.94	0.38
	Risk Arbitrage	0.66	1.25	0.25
Opportunistic	Global Macro	1.15	3.35	0.24
	Dedicated Short Bias	–0.18	5.10	–0.10
	Emerging Markets	0.73	4.92	0.08
	Long/Short Equity	1.00	3.06	0.21
Market indices				
Stocks	S&P500	0.97	4.40	0.14
	MSCI World	0.75	4.12	0.10
Bonds	JPM Global Government Bond	0.59	1.84	0.13
	LB Government/Corporate Bond	0.52	0.99	0.17

CLASSIC PERFORMANCE MEASUREMENT AND PORTFOLIO OPTIMISATION

Hedge fund performance measurement

Under the concept of risk-adjusted performance measurement, the return is related to a suitable risk measure. In hedge fund performance analysis, the Sharpe ratio is often chosen as the performance measure and a comparison is made with the Sharpe ratios of other funds or market indices.[16] The Sharpe ratio uses the mean excess return over the risk-free interest rate as a measure of the return and the standard deviation of the returns as a measure of risk. Using historical monthly returns r_{i1}, …, r_{iT} for security i, the Sharpe ratio (SR) can be calculated as follows

$$SR_i = \frac{r_i^d - r_f}{\sigma_i} \tag{1}$$

$r_i^d = (r_{i1} + \ldots + r_{iT})/T$ represents the average monthly return for security i, r_f the risk-free monthly interest rate, and $\sigma_i = (((r_{i1} - r_i^d)^2 + \ldots + (r_{iT} - r_i^d)^2)/(T-1))^{0.5}$ the estimated standard deviation of the monthly return generated by security i. The arithmetic mean of discrete returns is employed so that these data can be used as input parameters in the following portfolio optimisation and value-at-risk (VaR) determination. The question of computing arithmetic or geometric averages as well as discrete or continuously compounded returns is discussed in the literature with some controversy.[17] The returns are calculated at the end of each month. A constant risk-free interest rate of 0.35 per cent per month is used. This corresponds to the interest on ten-year US treasury bonds as of 30th December, 2004 (4.28 per cent per annum). Alternatively, a rolling interest rate, an average interest rate for the period under consideration or the interest rate at the beginning of the investigation period could be used, which yields almost identical results. The performance measurement results on basis of the Sharpe ratio are shown in Table 2.

On a Sharpe ratio basis, hedge funds yield a better performance than do traditional investments; the performance of the aggregated CSFB Hedge Fund Index (0.23) is higher than the maximum performance of the traditional investments (0.17, for the LB Government/Corporate Bond Index).[18] Market-neutral and event-driven hedge funds achieve a higher performance than do stocks and bonds. The Equity Market Neutral strategy offers by far the best performance. Apart from Global Macro and Long/Short Equity, opportunistic hedge funds show a smaller performance than the other strategy groups do — Dedicated Short Bias even has a negative Sharpe ratio. Thus, on the basis of the Sharpe ratio, it is concluded that many hedge fund indices exhibit a better performance than do traditional investment indices.

Hedge fund portfolio optimisation

To examine the portfolio context, the correlations of the indices' returns are needed. Table 3 shows the Bravais/Pearson correlation coefficient of the hedge fund returns among themselves as well as compared with stock and bond returns.

With the exception of funds using the Dedicated Short Bias strategy, all hedge funds show small positive correlated returns to stocks and bonds (the arithmetic mean in the lower-left quadrant of the correlation matrix is 0.14). With the Dedicated Short Bias strategy, the correlation with stock markets is negative. Hedge fund returns also show small positive correlations among themselves (the arithmetic mean in the upper-left quadrant is 0.24). Owing to the low correlations, the integration of hedge funds into portfolios of traditional investments seems promising.

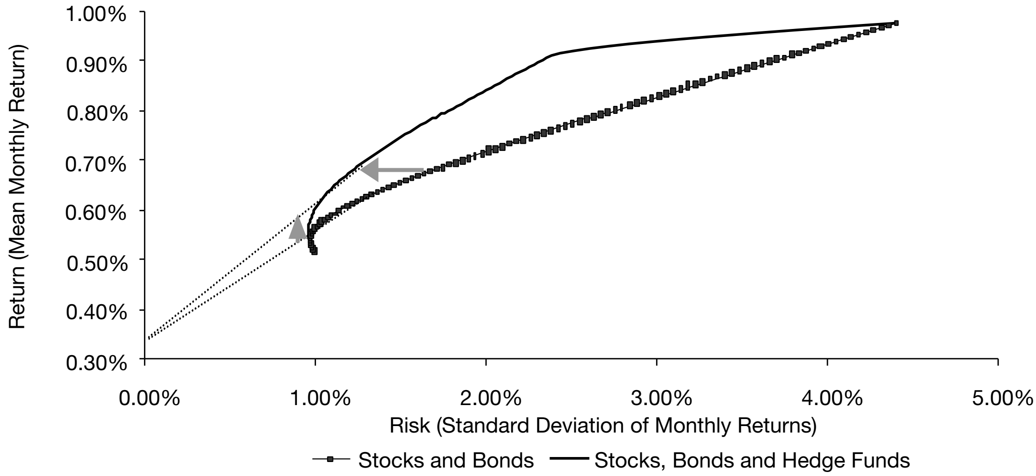
To see the influence of hedge funds on a portfolio of traditional investments, one can determine portfolio optimisation on the basis of the standard deviation, which is the classical Markowitz approach.[19] Figure 1 shows risk, return and efficient portfolios calculated following the classical Markowitz approach. The right curve is a portfolio of stocks and bonds. The left curve is a portfolio of stocks, bonds and hedge funds (using, as an example, the CSFB Hedge Fund Index).

Comparing the right and the left curves shows that integrating hedge funds in a portfolio of traditional investments results in a reduction in risk and an improvement in portfolio performance. Each expected return is achieved with smaller risk. For example, if a return of 0.65 per cent per month is

Table 3: Bravais/Pearson correlation coefficient (hedge funds, stocks, and bonds)

Index	*Hedge Fund*	*Fixed Income Arbitrage*	*Convertible Arbitrage*	*Equity Market Neutral*	*Distressed*	*Risk Arbitrage*	*Global Macro*	*Dedicated Short Bias*	*Emerging Markets*	*Long/Short Equity*	*S&P500*	*MSCI World*	*JPM Global Gov. Bond*	*LB Govern./ Corp. Bond*
Hedge Fund														
Fixed Income Arbitrage	0.45													
Convertible Arbitrage	0.40	0.53												
Equity Market Neutral	0.33	0.07	0.32											
Distressed	0.57	0.31	0.50	0.33										
Risk Arbitrage	0.39	0.13	0.40	0.30	0.56									
Global Macro	0.86	0.45	0.29	0.21	0.31	0.13								
Dedicated Short Bias	–0.48	–0.08	–0.23	–0.33	–0.63	–0.50	–0.13							
Emerging Markets	0.65	0.29	0.31	0.22	0.59	0.42	0.41	–0.57						
Long/short Equity	0.78	0.20	0.26	0.34	0.58	0.50	0.42	–0.72	0.59					
S&P500	0.48	0.03	0.13	0.39	0.55	0.45	0.23	–0.76	0.48	0.59				
MSCI World	0.47	0.03	0.10	0.35	0.57	0.46	0.18	–0.76	0.53	0.61	0.94			
JPM Global Government Bond	–0.07	–0.10	–0.10	0.06	–0.06	–0.04	–0.10	0.02	–0.17	0.06	–0.01	0.07		
LB Government/ Corporate Bond	0.18	0.12	0.12	0.15	0.03	–0.07	0.26	0.08	–0.06	0.09	0.00	–0.07	0.60	

Figure 1: Optimisation results (CSFB Hedge Fund Index, standard deviation)



desired, the portfolio risk can be reduced by 23.92 per cent (upper arrow).

The improvement of portfolio performance is represented by the gradient of the tangent from the risk-free interest rate (0.35 per cent per month) to the efficiency curve (lower arrow). The gradient of this tangent corresponds to the value of the Sharpe ratio. A comparison of the Sharpe ratios of portfolios with and without hedge funds can quantify the influence of hedge funds. In this example, the portfolio performance can be increased from 0.22 to 0.27, and thus by 23.23 per cent.

Table 4 shows the improvement of portfolio performance for all hedge fund strategies. Therefore, the optimisation shown in the CSFB Hedge Fund Index example was also accomplished for each of the other nine strategies.

In eight of the ten hedge fund strategies, portfolio performance can be increased by more than 10 per cent. The largest improvement results from the use of the Equity Market Neutral strategy (150.39 per cent). Owing to its negative average monthly return, the Dedicated Short Bias strategy does not increase portfolio performance, despite the small correlation of returns. Thus, from the viewpoint of classical portfolio selection theory, hedge funds seem to be very attractive investments.[20]

PROBLEMS OF CLASSIC PERFORMANCE MEASUREMENT

The argumentation set out in the previous section can be found in many science and practice publications. Recent literature, however, has pointed out that there are several problems with hedge fund performance measurement: The returns of the hedge funds are autocorrelated, systematically distorted, and deviate from normally distributed returns. The following provides a short overview of each of these

Table 4: Improvement in portfolio performance (Sharpe ratio)

Index	*Hedge Fund*	*Fixed Income Arbitrage*	*Convertible Arbitrage*	*Equity Market Neutral*	*Distressed*	*Risk Arbitrage*	*Global Macro*	*Dedicated Short Bias*	*Emerging Markets*	*Long/Short Equity*
Maximum Sharpe ratio	0.27	0.27	0.36	0.55	0.41	0.31	0.28	0.22	0.22	0.24
Improvement in performance (%)	23.23	24.77	65.31	150.39	87.27	42.46	27.33	0.00	0.52	10.06

Table 5: Autocorrelation and higher moments of return distribution

Index	*Hedge Fund*	*Fixed Income Arbitrage*	*Convertible Arbitrage*	*Equity Market Neutral*	*Distressed*	*Risk Arbitrage*	*Global Macro*	*Dedicated Short Bias*	*Emerging Markets*	*Long/Short Equity*	*S&P500*	*MSCI World*	*JPM Global Gov. Bond*	*LB Govern./ Corp. Bond*
Part A: Autocorrelation														
Autocorrelation (ρ_i)	0.12	0.39	0.56	0.29	0.29	0.28	0.06	0.10	0.30	0.17	0.00	0.03	0.20	0.16
Ljung–Box statistic (LB_i)	2.03	20.56***	41.90***	11.55***	11.62***	10.22***	0.44	1.46	12.36***	3.94*	0.00	0.10	5.15**	3.42*
Part B: Higher moments of return distribution														
Skewness (S_i)	0.10	–3.17	–1.41	0.28	–2.77	–1.28	–0.00	0.89	–0.60	0.22	–0.60	-0.59	0.27	–0.39
Excess (E_i)	1.85	16.06	3.52	0.17	16.46	5.92	2.17	1.98	3.89	3.94	0.37	0.48	0.27	0.52
Jarque–Bera statistic (JB_i)	19.03***	1639.9***	112.1***	1.88	1658.2***	228.8***	25.96***	39.20***	91.36***	64.95***	8.61**	9.01**	1.97	4.80*

* (**; ***): Significant at 10 per cent (5 per cent; 1 per cent).

problems, beginning with the autocorrelation problem.

Autocorrelation results from difficulties in the monthly valuation of the investments. If, for example, a valuation is impossible because of illiquid positions, the hedge fund manager takes the return of the last month or an estimation of the market value.[21] Table 5 (Part A) gives the first-order autocorrelation value and the Ljung–Box[22] statistic, which is used to check the statistical significance of the autocorrelation values.

For six hedge fund indices, the returns are positively autocorrelated at the 1 per cent significance level. The bond indices' returns are also autocorrelated, but to a smaller extent than the hedge fund returns. What are the consequences of this autocorrelation for performance measurement? Autocorrelation leads to an underestimation of the standard deviation of returns.[23] Thus, the Sharpe ratio is overestimated.[24]

The database of the hedge fund indices exhibits systematic distortions (the so-called bias problem), which can affect the measurement result in the sense that index returns are too high.[25] Two forms of this distortion can be distinguished: the survivorship bias and the backfilling bias.[26]

Survivorship bias arises because an index only considers viable funds. Unsuccessful funds that have been discontinued, perhaps owing to poor performance, and removed from the database are not considered. Thus, the database gives an unrealistically positive picture.

Backfilling bias exists because many hedge fund data providers integrate the past returns of new funds into their databases. Only successful funds, however, have an incentive to report past performance. Thus, this backfilling again leads to an unrealistically positive representation. It should be noted that CSFB does not backfill, so this sort of bias is not a feature of the CSFB indices.

The fact that hedge funds use derivative instruments leads to an asymmetric return distribution and fat tails. Thus one cannot assume that hedge fund returns are normally distributed. Returns are not normally distributed if the higher moments (skewness and excess) deviate from zero. For a risk-averse investor, negative skewness and positive excess kurtosis are unattractive, because they generally indicate a higher probability of large losses than in the case of normally distributed returns.[27] The Jarque–Bera[28] statistic is used to check whether the observed values of skewness and excess are consistent with the normal distribution assumption. The values of skewness, excess and the Jarque–Bera statistic are shown in Part B of Table 5.[28]

The returns of six of the ten hedge fund indices display the unattractive combination of negative skewness and positive excess kurtosis. This combination also occurs for three of the four market indices, but their values for skewness and excess kurtosis are less extreme than those shown for the hedge funds. On the basis of the Jarque–Bera statistic, the assumption of normally distributed hedge fund returns is valid only for the Equity Market Neutral strategy. It is not only the hedge fund indices that display these characteristics, however; the monthly returns of the S&P500 and MSCI World also fail to display a normal distribution.

The higher moments of the return distribution are not considered in the Sharpe ratio or in the Markowitz approach. Thus, the higher probability of large losses is faded out for some hedge funds and their risk is possibly underestimated.

INTEGRATING THE PROBLEMS IN THE PERFORMANCE MEASUREMENT

Approaches are now presented to integrate the above-described performance measurement problems, again starting with the autocorrelation problem. An easy way of integrating autocorrelation is to calculate the standard deviation, not on basis of monthly returns but on the basis of quarterly returns.[29] Afterwards, the monthly and quarterly values are annualised in order to compare them.[30] Table 6 (Part A) shows the results.

Without autocorrelation, the standard deviation should remain unchanged. But, instead, it rises for some hedge fund strategies (eg Convertible Arbitrage (+37.68 per cent) or Emerging Markets (+29.57 per cent)). In addition, the standard deviation also rises for the traditional indices (eg MSCI World (+18.50 per cent)).

The systematic distortion of the database (bias problem) cannot be eliminated retrospectively. To consider it, nevertheless, the results from investigation of the bias problem are used to estimate the distortion of the database. Estimations of survivorship bias range from 0.01 to 0.36 percentage points and are on average about 0.18 percentage points per month.[31] Liang[32] points out, however, that the estimated bias values differ within different hedge fund databases. The average survivorship bias in six investigations of the CSFB database amounts to 0.21 percentage points per month.[32] The estimations of backfilling bias range from 0.00 to 0.12 percentage points and are, on average, about 0.08 percentage points per month.[33] As there is no backfilling bias for CSFB, only the survivorship bias must be considered in the investigation.[34,35]

To integrate the fat tail problem in the performance measurement, a risk measure that shows the skewness and excess of the return distribution is needed. Such a measure is the modified VaR presented by Favre and Galeano.[8] Therefore, in the well-known formula for the standard VaR (w denotes the value of the investment)

$$VaR_i = -(z_\alpha \sigma_i + r_i^{\mathrm{d}})w \tag{2}$$

the alpha-quantile of the standard normal distribution z_α is replaced by the value of the Cornish–Fisher expansion z_{CF}

$$MVaR_i = -(z_{\mathrm{CF}i} \sigma_i + r_i^{\mathrm{d}})w \tag{3}$$

The value of the Cornish–Fisher expansion is calculated as the alpha-quantile of the standard normal distribution plus some terms that adjust for skewness and excess ($z_{\mathrm{CF}i} = z_\alpha + 1/6(z_\alpha^2 - 1)S_i + 1/24(z_\alpha^3 - 3z_\alpha) E_i - 1/36(2z_\alpha^3 - 5z_\alpha)S_i^2$). Next, we follow Gregoriou and Gueyie[12] and calculate a modified Sharpe ratio (MSR), in which the standard deviation is replaced by the modified VaR[36]

$$MSR_i = \frac{r_i^{\mathrm{d}} - r_{\mathrm{f}}}{MVaR_i} \tag{4}$$

The results of the standard VaR, the modified VaR and the modified Sharpe ratio

Table 6: Annual standard deviation and modified Sharpe ratio

Index	*Hedge Fund*	*Fixed Income Arbitrage*	*Convertible Arbitrage*	*Equity Market Neutral*	*Distressed*	*Risk Arbitrage*	*Global Macro*	*Dedicated Short Bias*	*Emerging Markets*	*Long/Short Equity*	*S&P500*	*MSCI World*	*JPM Global Gov. Bond*	*LB Govern./Corp. Bond*
Part A: Annual standard deviation														
Annual σ (monthly) (%)	9.00	4.08	5.09	3.30	7.57	4.67	13.18	17.44	18.58	11.68	17.05	15.55	6.81	3.63
Annual σ (quarterly) (%)	9.27	4.41	6.90	3.98	9.00	5.50	13.15	19.55	23.94	13.45	18.80	18.02	7.79	3.84
Part B: Modified Sharpe ratio														
Value at risk (VaR_i)	4.57	2.02	2.35	1.20	3.42	2.25	6.64	12.04	10.71	6.12	9.40	8.98	3.69	1.79
Modified value at risk ($MVaR_i$)	5.42	4.56	3.84	1.03	9.24	4.39	8.34	9.52	16.69	8.01	11.11	10.86	3.40	2.14
($MVaR_i/VaR_i$) (%)	–118.45	125.9	63.60	–14.08	170.1	95.07	25.61	–20.97	55.87	30.89	18.25	21.02	–7.91	19.38
Modified Sharpe ratio (MSR_i)	0.10	0.05	0.11	0.46	0.08	0.07	0.10	–0.06	0.02	0.08	0.04	0.02	0.07	0.08

are given in Part B of Table 6, where the VaR is calculated for a confidence level of 1 per cent ($z_\alpha = -2{,}326$) and $w = 100$ USD. The change in risk is also shown by a comparison of the VaR in the standard and the modified versions.

The risk of the hedge funds is much higher with the modified VaR. For the Fixed Income Arbitrage strategy, the risk increases by 126 per cent; the Distressed strategy incurs a risk increase of 170 per cent. In contrast, risk rises only moderately for the market indices. The modified Sharpe ratio relativises the outperformance of hedge funds in relation to stocks and bonds. For example, the Distressed strategy is not in second place now, but has dropped to being only the fifth best Sharpe ratio out of the 14 indices. Nevertheless, hedge funds still obtain a higher performance than stocks and bonds. The modified Sharpe ratio of the aggregated hedge fund index amounts to 0.10, in comparison with 0.08, the maximum for the traditional investments.[37]

IMPLICATIONS FOR THE EVALUATION OF HEDGE FUNDS

Adjusted hedge fund performance measurement

The three problems — autocorrelation, bias and fat tails — have to date only been considered in isolation. Thus, it still is not clear whether hedge funds are attractive investments, considering all three problems together. To answer this question, all three problems are now examined in one common framework.

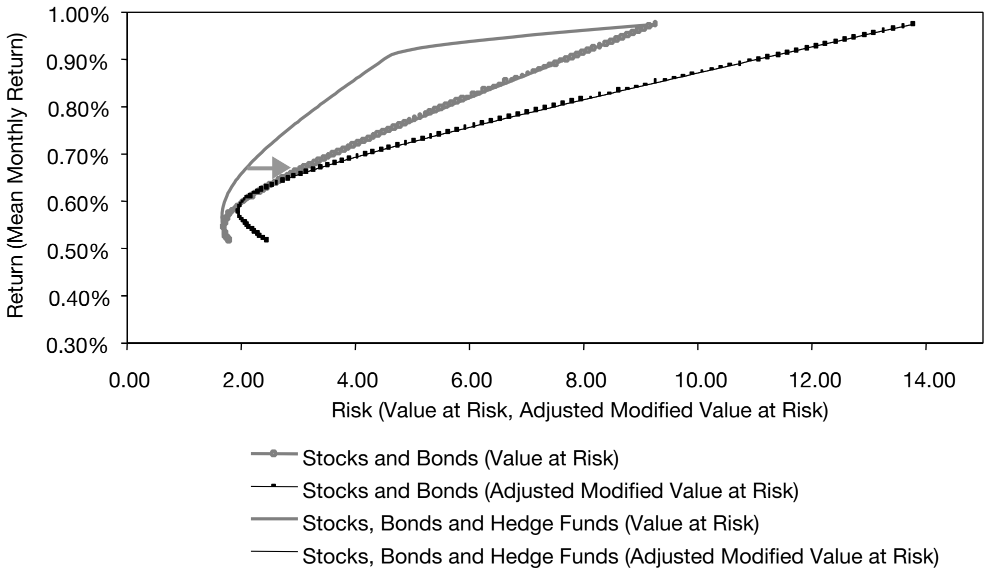
A three-step approach is used. First, the autocorrelation problem is mitigated using the standard deviation based on quarterly returns instead of monthly returns. The recalculated version of the annual standard deviation of quarterly returns on a monthly basis is called the adjusted standard deviation (σ_{Ai}). Therefore, the annual standard deviation (on a quarterly basis) is divided by the root of 12. Second, the bias problem is dealt with by reducing the hedge fund returns using the estimated bias adjustment of 0.21 percentage points per month. The reduced monthly returns are denoted as adjusted monthly returns (r^d_{Ai}). As an intermediate step, one can now calculate an adjusted Sharpe ratio, given as $ASR_i = (r^d_{Ai} - r_f)/\sigma_{Ai}$, based on the adjusted monthly returns and their standard deviation, which incorporates autocorrelation and bias in the hedge fund performance measurement. Finally, the fat tail problem is integrated by calculating the modified Sharpe ratio on the basis of the adjusted monthly returns and their standard deviation. This ratio is called the adjusted modified Sharpe ratio and is calculated as $AMSR_i = (r^d_{Ai} - r_f)/AMVaR_i$, with $AMVaR_i = -(z_{CFi}\sigma_{Ai} + r^d_{Ai})w$. The results of this adjusted performance measurement are shown in Table 7.

Table 7 shows that the adjusted Sharpe ratio (ie considering the autocorrelation and bias problem) leads to much lower outperformance of hedge funds compared with traditional investments. For example, there are only three strategies that obtain a higher performance than stocks and bonds (Equity Market Neutral, Distressed, Global Macro versus LB Government/Corporate Bond). This effect is heightened when

Table 7: Performance measurement results (adjusted modified Sharpe ratio)

Group	*Index*	*Adjusted mean monthly return (%) (r_{Aid})*	*Adjusted standard deviation of monthly returns (%) (σ_{Ai})*	*Adjusted Sharpe ratio (ASR_i)*	*Adjusted modified Sharpe ratio ($AMSR_i$)*
CSFB indices					
Aggregated	Hedge Fund	0.69	2.67	0.13	0.05
	Fixed Income Arbitrage	0.34	1.27	–0.01	0.00
Market Neutral	Convertible Arbitrage	0.57	1.99	0.11	0.03
	Equity Market Neutral	0.61	1.15	0.23	0.14
Event Driven	Distressed	0.87	2.60	0.20	0.04
	Risk Arbitrage	0.45	1.59	0.06	0.02
	Global Macro	0.93	3.80	0.15	0.06
	Dedicated Short Bias	–0.39	5.64	-0.13	–0.07
Opportunistic	Emerging Markets	0.52	6.91	0.02	0.01
	Long/Short Equity	0.79	3.88	0.11	0.04
Market indices					
Stocks	S&P500	0.97	5.43	0.12	0.05
	MSCI World	0.75	5.20	0.08	0.03
Bonds	JPM Global Gov.Bond	0.59	2.25	0.11	0.06
	LB Govern./Corp. Bond	0.52	1.11	0.15	0.07

Figure 2: Optimisation results (CSFB Hedge Fund Index adjusted modified value at risk)



additionally considering the fat tail problem, thus viewing the adjusted modified Sharpe ratio, as the aggregated CSFB Hedge Fund Index no longer exceeds the maximum of traditional investments (0.07). Furthermore, Equity Market Neutral is the only strategy that obtains a higher performance than traditional investments do. Thus, for most strategies, the largest part of the original outperformance disappears when considering autocorrelation, bias and fat tails.[38]

Adjusted hedge fund portfolio optimisation

To transfer these adjustments to the portfolio framework, a portfolio optimisation is performed on the basis of an adjusted modified VaR. The adjusted modified VaR results from the modified VaR calculated with the adjusted returns and the adjusted standard deviations. To compare the new optimisation with the results of the classical portfolio optimisation, a portfolio optimisation is performed on the basis of the standard VaR in the first step. The second step is then an optimisation on the basis of the adjusted modified VaR. Therefore, the classical Markowitz objective function (minimise the portfolio standard deviation) is replaced by minimisation of the portfolio VaR (first step) and the portfolio adjusted modified VaR (second step).[39] The results of the first optimisation are almost identical to the results of classical portfolio optimisation.[40] In particular, none of the problem areas described is taken into consideration. The second optimisation (based on the adjusted modified VaR), however, integrates autocorrelation, bias and fat tails.[41]

This procedure emphasises two aspects of hedge fund performance. On the one hand, comparison of the efficiency curves, one based on the standard VaR and the other on the adjusted modified VaR, shows the change in portfolio risk that is due to the three problems of hedge fund performance measurement. On the other hand, comparison of the efficiency curves based on the adjusted modified VaR with and without hedge funds addresses the question of whether the performance of a traditional investment portfolio will still be improved by the addition of hedge funds even after taking into account autocorrelation, bias and fat tails. Figure 2 shows the efficiency curves of portfolios consisting of stocks and bonds and portfolios consisting of stocks, bonds and hedge funds (again using as an example the CSFB Hedge Fund Index).

The efficiency curves that result from portfolio optimisation based on the adjusted modified VaR run congruently and lie to the right of the efficiency curves based on the standard VaR. This has two important implications. First, portfolio risk increases when autocorrelation, bias and fat tails are taken into account. With an expected return of 0.65 per cent, the risk of the stock and bond portfolio increases about 3.38 per cent. In contrast, the risk of the portfolio containing hedge funds rises about 46.74 per cent (see arrow). Second, integrating hedge funds into this portfolio does not result in a reduction in portfolio risk and does not improve portfolio performance, as the efficiency curve remains unchanged in the adjusted framework.

To quantify the influence of hedge funds on the portfolio, the adjusted modified Sharpe ratio of portfolios with and without hedge funds is compared. This comparison shows that the performance of the stock and bond portfolio (0.12) cannot be improved by adding hedge funds to it. Thus, the original outperformance of the portfolio with hedge funds compared with the portfolio without hedge funds disappears when autocorrelation, bias and fat tails are taken into account.

Similar to Table 4, Table 8 shows the improvement in portfolio performance on the basis of the adjusted modified Sharpe ratio for all ten hedge fund strategies. The last row of the table compares the improvement in portfolio performance to portfolio optimisation on basis of the standard deviation.

Using the adjusted modified Sharpe ratio again leads to a relativisation of hedge fund outperformance. For nine strategies, the improvement in portfolio performance is reduced. Only two strategies (Equity Market Neutral and Distressed) can improve performance by more than 10 per cent. Five strategies (Hedge Fund, Fixed Income Arbitrage, Risk Arbitrage, Dedicated Short Bias, Emerging Markets) have no considerable effect on the efficiency curve. The Equity Market Neutral strategy is the only exception to these findings: it can increase portfolio performance by about 35.22 per cent. Thus, in general, the positive influence of hedge funds in traditional investment portfolios is narrowed after taking autocorrelation, bias and fat tails into account.

CONCLUSION

A true evaluation of hedge fund performance requires consideration of autocorrelation, bias and fat tails. Such an evaluation is provided and it is discovered that the majority of

Table 8: Improvement in portfolio performance (adjusted modified Sharpe ratio)

Index	*Hedge Fund*	*Fixed Income Arbitrage*	*Convertible Arbitrage*	*Equity Market Neutral*	*Distressed*	*Risk Arbitrage*	*Global Macro*	*Dedicated Short Bias*	*Emerging Markets*	*Long/Short Equity*
Maximum AMSRi	0.12	0.12	0.13	0.17	0.14	0.12	0.13	0.12	0.12	0.12
Improvement in performance (%)	0.00	0.00	2.29	35.22	12.65	0.05	3.25	0.00	0.11	0.00
Change from Table 4 (% points)	–23.23	–24.77	–63.02	–115.17	–74.62	–42.42	–24.09	0.00	–0.41	–10.06

hedge funds lose their attractiveness. This is illustrated by comparing the classical Sharpe ratio with an adjusted version of the modified Sharpe ratio proposed by Gregoriou and Gueyie.[12] An exception is the Equity Market Neutral strategy, which exhibits a high performance even after addressing all three of these qualities. Therefore, only few hedge fund strategies appear to be attractive investment options.

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References and Notes

1 See, for example, Crerend, W. J. (1998) *Fundamentals of Hedge Fund Investing*, New York, McGraw-Hill; Ackermann, C., McEnally, R. and Ravenscraft, D. (1999) 'The Performance of Hedge Funds: Risk, Return, and Incentives', *Journal of Finance*, Vol. 54, No. 3, pp. 833–874; Liang, B. (1999) 'On the Performance of Hedge Funds', *Financial Analysts Journal*, Vol. 55, No. 4, pp. 72–85; Cottier, P. (2000) *Hedge Funds and Managed Futures*, 3rd edn, Bern, Stuttgart and Wien, Haupt; Gregoriou, G. N. and Rouah, F. (2002) 'The Role of Hedge Funds in Pension Fund Portfolios: Buying Protection in Bear Markets', *Journal of Pensions Management*, Vol. 7, No. 3, pp. 237–245; Nicholas, J. G. (2004) *Hedge Fund of Funds Investing, Princeton*, NJ, Bloomberg.

2 See, for example, Asness, C., Krail, R. and Liew, J. (2001) 'Do Hedge Funds Hedge?' *Journal of Portfolio Management*, Vol. 28, No. 1, pp. 6–19; Lo, A. W. (2001) 'Risk Management for Hedge Funds: Introduction and Overview', *Financial Analysts Journal*, Vol. 57, No. 6, pp. 16–33; Favre, L. and Galeano, J.-A. (2002) 'Mean-Modified Value-at-Risk Optimization with Hedge Funds', *Journal of Alternative Investments*, Vol. 5, No. 2, pp. 21–25; Fung, W. and Hsieh, D. A. (2002) 'Hedge-Fund Benchmarks: Information Content and Biases', *Financial Analysts Journal*, Vol. 58, No. 1, pp. 22–34; Amin, G. S. and Kat, H. M. (2003) 'Stocks, Bonds, and Hedge Funds', *Journal of Portfolio Management*, Vol. 30, No. 4, pp. 113–119; Gregoriou, G. N. (2002) 'Hedge Fund Survival Lifetimes', *Journal of Asset Management*, Vol. 3, No. 3, pp. 237–252; Kouwenberg, R. (2003) 'Do Hedge Funds Add Value to a Passive Portfolio? Correcting for Non-normal Returns and Disappearing Funds', *Journal of Asset Management*, Vol. 3, No. 4, pp. 361–382; Amenc, N., Martellini, L. and Vaissié, M. (2003) 'Benefits and Risks of Alternative Investment Strategies', *Journal of Asset Management*, Vol. 4, No. 2, pp. 96–118.

3 See Kat, H. M. and Lu, S. (2002) 'An Excursion into the Statistical Properties of Hedge Fund Returns', Working Paper 0016, Alternative Investment Research Centre, Cass Business School, City University London.

4 See Getmansky, M., Lo, A. W. and Makarov, I. (2004) 'An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns', *Journal of Financial Economics*, Vol. 74, No. 3, pp. 529–609.

5 See Christiansen, C. B., Madsen, P. B. and Christensen, M. (2003) 'Further Evidence on Hedge Fund Performance', Working Paper, Department of Finance, Aarhus School of Business.

6 See Capocci, D. and Hübner, G. (2004) 'Analysis of Hedge Fund Performance', *Journal of Empirical Finance*, Vol. 11, No. 1, pp. 55–89.

7 See Ammann, M. and Moerth, P. (2005) 'Impact of Fund Size on Hedge Fund Performance', *Journal of Asset Management*, Vol. 6, No. 3, pp. 219–238.

8 See Favre and Galeano, ref. 2 above.

9 See Agarwal, V. and Naik, N. Y. (2004) 'Risk and Portfolio Decisions Involving Hedge Funds', *Review of Financial Studies*, Vol. 17, No. 1, pp. 63–98.

10 See Sortino, F. A. and van der Meer, R. (1991) 'Downside Risk', *Journal of Portfolio Management*, Vol. 17, No. 4, pp. 27–31.

11 See Shadwick, W. F. and Keating, C. (2002) 'A Universal Performance Measure', *Journal of Performance Measurement*, Vol. 6, No. 3, pp. 59–84.

12 See Gregoriou, G. N. and Gueyie, J.-P. (2003) 'Risk-Adjusted Performance of Funds of Hedge Funds Using a Modified Sharpe Ratio', *Journal of Alternative Investments*, Vol. 6, No. 3, pp. 77–83.

13 See Amenc *et al.* ref. 2 above.

14 See Kouwenberg, ref. 2 above.

15 Since 1999, CSFB has also published investable

indices, which cover exclusively open funds. Owing to the longer time series, however, we choose the indices that contain both closed and open funds.

16 See, for example, Ackermann *et al*. ref. 1 above; Edwards, F. R. and Liew, J. (1999) 'Hedge Funds Versus Managed Futures as Asset Classes', *Journal of Derivatives*, Vol. 6, No. 4, pp. 45–64; Liang, ref. 1 above; Schneeweis, T., Kazemi, H. and Martin, G. (2002) 'Understanding Hedge Fund Performance: Research Issues Revisited — Part I', *Journal of Alternative Investments*, Vol. 5, No. 3, pp. 6–22.

17 See, for example, Ibbotson, R. G. and Sinquefield, R. A. (1979) 'Stocks, Bonds, Bills and Inflation: Updates', *Financial Analysts Journal*, Vol. 35, No. 4, pp. 40–44, for the reasoning behind the choice of the arithmetic mean. See Dorfleitner, G. (2002) 'Stetige versus diskrete Renditen: Überlegungen zur richtigen Verwendung beider Begriffe in Theorie und Praxis', *Kredit und Kapital*, Vol. 35, No. 2, pp. 216–241, for the reasoning behind the use of discrete returns.

18 We cannot examine the statistic significance in the differences of the Sharpe ratios on the basis of the widespread Jobson and Korkie statistic, as this test assumes normally distributed and not autocorrelated returns. See Jobson, D. and Korkie, B. (1981) 'Performance Hypothesis Testing with the Sharpe and Treynor Measures', *Journal of Finance*, Vol. 36, No. 4, pp. 888–908. As shown in the following, both conditions usually are not present in the case of hedge funds.

19 See, for example, Crerend, ref. 1 above; Cottier, see ref. 1 above; Könberg, M. and Lindberg, M. (2001) 'Hedge Funds: A Review of Historical Performance', *Journal of Alternative Investments*, Vol. 4, No. 1, pp. 21–31. Formally, the optimisation result is as follows. Minimise $\sigma_P = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_i \sigma_j k_{i,j}}$, under $r_P = \sum_{i=1}^{n} x_i r_i^d$, $\sum_{n=1}^{n} x_i = 1$ and $x_i \geq 0$. Thereby σ_P denotes the standard deviation of monthly portfolio returns, r_P the monthly portfolio return, n the number of securities, k_{ij} the correlation of security i and j, and x_i the fraction of security i in the portfolio. See Markowitz, H. M. (1952) 'Portfolio Selection', *Journal of Finance*, Vol. 7, No. 1, pp. 77–91.

20 One could assume that the improvement in portfolio performance is caused particularly by the restriction to three asset classes (stocks, bonds, hedge funds). The positive influence of hedge funds on a portfolio of traditional investments also remains, however, when further asset classes are examined. For example, when considering a portfolio of stocks, bonds, a money market index (JPM US Cash 3 Month), and a real estate index (Global Property Research General Property Share Index), the inclusion of the CSFB Hedge Fund Index results in an improvement of about 18.88 per cent as opposed to 23.23 per cent in the three-security case presented here. Therefore, we continue to use only the three asset classes. The monthly returns of the money market and the real estate performance indices (time series on USD basis from January 1994 to December 2004) were collected from the Datastream database.

21 See Kat, H. M. (2002) 'Some Facts about Hedge Funds', *World Economics*, Vol. 3, No. 2, pp. 93–123.

22 The first-order autocorrelation (ρ_i) of security i is calculated as $\rho_i = \sum_{t=2}^{T}(r_{it} - r_i^d)(r_{it-1} - r_i^d) / \sum_{t=1}^{T}(r_{it} - r_i^d)^2$. The Ljung–Box statistic (LB_i) of security i is given by: $LB_i = [T \times (T+2)] / (T-1) \times \rho_i^2$. LB_i is χ^2-distributed with one degree of freedom. See Ljung, G. M. and Box, G. E. P. (1978) 'On a Measure of Lack of Fit in Time Series Models', *Biometrika*, Vol. 65, No. 2, pp. 297–303. Another statistic used for examining autocorrelation coefficients is the variance ratio test following Lo, A. W. and MacKinlay, A. C. (1998) 'Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test', *Review of Financial Studies*, Vol. 1, No. 1, pp. 41–66, which leads to almost identical results in our case: Apart from Dedicated Short Bias and the Emerging Markets strategies, all hedge fund strategies exhibit statistically significant test values. Also, the returns of the bond indices are autocorrelated.

23 See, for example, Asness *et al*., ref. 2 above.

24 See Lo, A. W. (2002) 'The Statistics of Sharpe Ratios', *Financial Analysts Journal*, Vol. 58, No. 4, pp. 36–52.

25 See Ackermann *et al*., ref. 1 above.

26 Additionally, there are three further forms of bias (selection, liquidation and double counting) that are not considered here because they could not be quantified in a bias investigation yet. See Lhabitant, F.-S. (2002) *Hedge Funds: Myths and Limits*, Chichester, Wiley, pp. 133–136.

27 See Kat, H. M. (2003) '10 Things that Investors Should Know about Hedge Funds, *Journal of Wealth Management*, Vol. 5, No. 4, pp. 72–81, and Favre, L. and Signer, A. (2002) 'The Difficulties of Measuring the Benefits of Hedge Funds', *Journal of Alternative Investments*, Vol. 5, No. 1, pp. 31–42.

28 The skewness (S_i) and excess (E_i) of security i are given by and $S_i = (1/T\Sigma_{t=1}^{T}(r_{it} - r_t^d)^3)/\sigma_i^3$ and $E_i = (1/T\Sigma_{t=1}^{T}(r_{it} - r_i^d)^4)/\sigma_i^4 - 3$. The Jarque–Bera statistic (JB_i) of security i is $JB_i = T/6\,(S_i^2 + 1/4E_i^2)$. See Jarque, C. M. and Bera, A. K. (1987) 'A Test for Normality of Observations and Regression Residuals', *International Statistical Review*, Vol. 55, No. 2, pp. 163–172. JB_i is χ^2-distributed with two degrees of freedom. Again, a second test was consulted — the modified Jarque–Bera statistic, following Urzua, C. M. (1996) 'On the Correct Use of Omnibus Tests for Normality', *Economic Letters*, Vol. 53, No. 3, pp. 247–251. After this test, only the returns of Equity Market Neutral and the JPM Global Government Bond index are compatible with a normal distribution assumption.

29 See, for example, Asness *et al.*, ref. 2 above. A further approach for considering autocorrelation is the unsmoothing of the returns. See Kat and Lu, ref. 3 above, for this approach, which leads to almost identical results in the sample. Apart from the Long/Short Equity strategy (–31.03 per cent), the standard deviation of all hedge fund indices rises (eg Emerging Markets (+34.49 per cent)). In comparison, the standard deviation of the traditional indices increases only moderately.

30 The annual standard deviation of security i is calculated by: $\sigma_i^\tau = \sqrt{[(1 + r_i^d)^2 + \sigma_i^2]^\tau - (1 + r_i^d)^{2\tau}}$. See Dorfleitner, ref. 17 above. τ denotes the number of considered time intervals (with monthly returns (quarterly returns) $\tau = 12$ (4)). To avoid an estimation error, the calculations were also performed with continuously compounded returns. These show the same results, however: with exception of the Long/Short Equity strategy (–36.12 per cent), the standard deviation rises with all indices similarly to that shown in Table 6 (eg Emerging Markets (+27.99 per cent)). Therefore, the estimation error is probably negligible in this investigation.

31 This average value results from the arithmetic mean of the estimated values from 16 investigations of the survivorship bias problem. We used: Ackermann *et al.*, ref. 1 above (0.01 percentage points per month); Ammann and Moerth, ref. 7 above (0.20); Amin, G. S. and Kat, H. M. (2003) 'Welcome to the Dark Side — Hedge Fund Attrition and Survivorship Bias Over the Period 1994–2001', *Journal of Alternative Investments*, Vol. 6, No. 1, pp. 57–73 (0.17); Baquero, G., Ter Horst, J. and Verbeek, M. (2004) 'Survival, Look-Ahead Bias and the Performance of Hedge Funds', Working Paper, Department of Financial Management and Econometric Institute, Erasmus University Rotterdam (0.17); Barés, P.-A., Gibson, R. and Gyger, S. (2003) 'Performance in the Hedge Funds Industry: An Analysis of Short and Long-Term Performance', *Journal of Alternative Investments*, Vol. 6, No. 3, pp. 25–41 (0.11); Barry, R. (2003) 'Hedge Funds: A Walk Through the Graveyard', Research Paper No. 25, Applied Finance Centre, Macquarie University, North Ryde Sydney (0.31); Brown, S. J., Goetzmann, W. N. and Ibbotson, R. G. (1999) 'Offshore Hedge Funds: Survival and Performance 1989–1995', *Journal of Business*, Vol. 72, No. 1, pp. 91–117 (0.25); Capocci and Hübner, ref. 6 above (0.36); Edwards and Caglayan, M. O. (2001) 'Hedge Fund Performance and Manager Skill', *Journal of Futures Markets*, Vol. 21, No. 11, pp. 1003–1028 (0.15); Edwards and Liew, ref. 16 above (0.16); Fung, W. and Hsieh, D. A. (2004) 'Performance Characteristics of Hedge Funds and Commodity Funds: Natural vs. Spurious Biases', *Journal of Financial and Quantitative Analysis*, Vol. 35, No. 3, pp. 2000, 291–307 (0.25); Liang, ref. 1 above (0.07); Liang, B. (2000) 'Hedge Funds: The Living and the Dead', *Journal of Financial and Quantitative Analysis*, Vol. 35, No. 3, pp. 309–326 (0.05 and 0.18); Liang, B. (2001) 'Hedge Fund Performance: 1990-1999', *Financial Analysts Journal*, Vol. 57, No. 1, pp. 11–18 (0.20); Liang, B. (2003) 'On the Performance of Alternative Investments: CTAs, Hedge Funds, and Funds of Funds', Working Paper, Isenberg School of Management, University of Massachusetts, Amherst (0.19); Schneeweis *et al.*, ref. 16 above (0.18). In these investigations, the survivorship bias is partially estimated on the basis of continuously compounded returns instead of discrete returns and partially on a yearly basis instead of on a monthly basis. Using logarithm and annualisation, however, all values were transferred into discrete monthly returns.

32 See Ammann and Moerth, ref. 7 above (0.20 percentage points per month); Amin and Kat, ref. 31 above (0.17); Baquero *et al.*, ref. 31 above (0.17); Barry, ref. 31 above (0.31); Fung, W. and Hsieh, ref. 31 above (0.25); Liang (2000), ref. 31 above (0.18), and Liang (2001), ref. 31 above (0.20).

33 This average value results from the arithmetic mean of the estimated values from five investigations of the backfilling bias problem. We used Ackermann *et al.*, ref. 1 above (0.00 percentage points per month); Barry, ref. 31

above (0.12); Capocci and Hübner, ref. 6 above (0.07); Edwards and Caglayan, ref. 31 above (0.10); and Fung and Hsieh, ref. 31 above (0.12). Again, all values were transferred into discrete monthly returns by logarithm and annualisation.

34 See Amenc *et al.*, ref. 2 above, and Christiansen *et al.*, ref. 5 above, who correct the hedge fund returns by about 0.21 and 0.25 percentage points per month. Liang (2000), ref. 31 above, and Edwards and Caglayan, ref. 31 above, point out that the distortion can differ between different hedge fund strategies. In addition, Ammann and Moerth, ref. 7 above, show that the distortion can differ between small and large funds. A documentation of the distortion for different strategies or fund size is not possible here, however, owing to missing data.

35 Also, a distortion of the traditional mutual funds might occur, as Brown, S. J. and Goetzmann, W. N. (1995) 'Performance Persistence', *Journal of Finance*, Vol. 50, No. 2, pp. 679–698, Brown, S. J., Goetzmann, W. N., Ibbotson, R. G. and Ross, S. A. (1992) 'Survivorship Bias in Performance Studies', *Review of Financial Studies*, Vol. 5, No. 4, pp. 553–580, and Grinblatt, M. and Titman, S. (1989) 'Mutual Fund Performance: An Analysis of Quarterly Portfolio Holdings', *Journal of Business*, Vol. 62, No. 3, pp. 393–416, determine a survivorship bias of on average 0.06 percentage points per month with traditional mutual funds. A distortion of traditional indices should be even smaller, however, as the annual mortality rate is generally smaller than that of mutual funds that is fewer securities are excluded from an index than mutual funds from a database. See Lhabitant, F.-S. (2004) *Hedge Funds: Quantitative Insights*, Chichester, Wiley, p. 91. Therefore, a distortion of traditional indices is not considered here.

36 Since the average monthly return enters the denominator of the modified Sharpe ratio, the modified Sharpe ratio can lead to another sequence in the evaluation of different investments from the Sharpe ratio (also with normally distributed returns). Hence, both numbers are very similar, but not directly transferable.

37 These results depend on the given confidence level, since the confidence level determines (over the Cornish–Fisher expansion) the influence of the higher moments on the modified VaR. If the confidence level is reduced, eg from 99 per cent to 97.5 per cent (95 per cent), the difference in the modified Sharpe ratio of the CSFB Hedge Fund Index to the LB Government/Corporate Bond Index expands from 0.14–0.10 (0.19–0.14). In the literature, the modified VaR is evaluated only for a confidence level of 95 per cent or 99 per cent. See Favre and Galeano, ref. 2 above; Favre, L. and Signer A. (2002) 'The Difficulties of Measuring the Benefits of Hedge Funds', *Journal of Alternative Investments*, Vol. 5, No. 1, pp. 31–42; Gregoriou and Gueyie, ref. 12 above; Gregoriou, G. N. (2004) Performance of Canadian Hedge Funds Using a Modified Sharpe Ratio', *Derivatives Use, Trading & Regulation*, Vol. 10, No. 2, pp. 149–155. An analysis of lower confidence levels is generally not meaningful, as the higher moments of the return distribution then usually only cause small changes in the VaR.

38 The results again depend on the given confidence level. If the confidence level is reduced from 99 per cent to 97.5 per cent (95 per cent), the adjusted modified Sharpe ratio of the CSFB Hedge Fund Index and of the LB Government/Corporate Bond Index increases to 0.07 (0.09) and 0.09 (0.12). In no case, however, is an outperformance of hedge funds against stocks and bonds observed.

39 Contrary to the classical Markowitz optimisation, the portfolio VaR cannot be determined directly from the VaR and the correlation of the individual securities. Instead, we first calculate portfolio returns depending on the security fractions x_i for each point of time ($t = 1, \ldots, T$) and then calculate the VaR of this portfolio return time series, which must be minimised. The minimum adjusted modified VaR is therefore calculated by: $AMVaR = -(z_{CF_P}\sigma_{AP} + r_{AP})w \rightarrow \text{Min!}$, under $r_{AP} = \sum_{i=1}^{n} x_i r_{Ai}^{d}$, $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0$. Thereby z_{CF_P} denotes the value of the Cornish–Fisher expansion of the portfolio, σ_{AP} is the portfolio standard deviation, r_{AP} the portfolio return, n the number of securities, and x_i the portfolio fraction of security i.

40 The first optimisation is a transformation of the classical Markowitz optimisation into a dimension uniform with the second optimisation. This does not offer additional information, but allows one to compare the results of both calculations. The results are almost identical as, with the VaR, the returns of the securities are considered. See equation (2).

41 See, for this procedure, Signer, A. (2003) *Generieren Hedge Funds einen Mehrwert?* Bern, Stuttgart, and Wien, Haupt, pp. 107–114, and Amenc *et al.*, ref. 2 above, who only integrate the fat tail problem into the performance evaluation.