Derivatives Use

A synthetic protective put strategy for phased investment in projects without an outright deferral

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Practical applications

If the industry in which a business enterprise is operating is extremely competitive, an outright deferral of an expansion project may not be strategically feasible. Perhaps a very relevant example is a chain of restaurants that is considering expanding to emerging population centres by opening a number of outlets. Given the heavy competitive pressures usually faced by the restaurant business, in many cases it would have to go ahead and invest immediately in the new outlets lest its immediate competition steal a head start. In doing so, however, it would expose itself to a downside risk in the event that business conditions turned out to be economically unfavourable in these untested locations. In such a case, one strategy that would certainly hold financial merit is that of using an investment insurance on the lines of a protective put. This paper develops and demonstrates a straightforward and practically applicable computational model by which a synthetic real call option could be engineered to replicate the pay-off from a protective put type of downside protection for expansion projects.

Abstract

This paper proposes and computationally demonstrates a synthetic protective put strategy for real options. Specifically, it deals with the problem of deferral option when an outright

deferral is not permissible owing to competitive pressures. It is demonstrated that, in such a situation, an appropriate strategy would be to invest in the new project in phases rather than doing it all at once. The replicating portfolio for

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a protective put on the project is obtained by setting the owner's equity in the project equal to the price of a call option on the value of the project. This is a logical extension of the financial protective put to the real options scenario and is relatively simple and practicable for businesses to adopt and apply.

DEFINITION OF PROBLEM

In the parlance of real options, a deferral option comprises a right to defer the start of a business unit or commencement of a particular project. These are quite realistic in situations where competition is not a crucial issue, eg when the business enterprise in question enjoys a certain degree of monopoly power in the market. With an option to defer the start of a project, say for a year, a business enterprise would choose to invest in that project only if conditions turned out to be favourable at the end of the year, otherwise it would choose not to. The cost of developing the project could be viewed as the exercise price of the embedded call option on the value of the project, and the risk associated with the nature of returns from that particular type of a project could be viewed as the underlying source of uncertainty. Following the marketed asset disclaimer (MAD) convention in real options analysis (ROA), the present value of the project without an option to defer may be used as the initial value of the underlying risky asset when pricing the deferral option.¹

In an arbitrage-free world, the deferral option can be priced as any other real option using a risk neutral valuation approach. Moreover, all the theoretical underpinnings

of the basic pricing scheme for financial options can be also applied to real options, as the value that an investor receives from a risky project can follow a random walk (a Gauss–Wiener stochastic process), even though project cash flows may not follow a random walk, as demonstrated by Bhattacharya.² This is because, when the current value of a project incorporates all information about expected future cash flows from the project, any changes in the current value result from changes in the expectations of future cash flows that are necessarily random; following Samuelson's argument.³ Thus random shocks to the expected cash flows will be reflected as random movements in the investor's wealth and in the returns from the project. Since the value of the project may be shown to follow a Gauss–Wiener stochastic process, it can be modelled as a recombining binomial tree. As the number of binomial trials per period tends to infinity, the binomial process approaches the Gauss–Wiener stochastic process as the limiting distribution.

Mathematically, it is possible to prove that the net present value (NPV) of a project that can be deferred, when valued using ROA, is greater than the maximum of the alternative values that are obtained using the traditional discounted cash flow (DCF) techniques.⁴

That is to say; a traditional DCF technique, if used to evaluate the project with an option to defer, would systematically undervalue its NPV, as it would fail to take into account the element of managerial flexibility afforded by the deferral option. The price of the embedded call option gives a better measure of the

project's NPV, as it gives the requisite value weighting to this element of managerial flexibility, ie the right to choose to invest in the project only when it is economically favourable to do so.⁵

What happens, however, when a deferral is too much of a luxury to afford, given that the industry in which the business enterprise is operating is extremely competitive? One of the most appropriate examples is the restaurant industry. The business enterprise would have to go ahead and invest immediately lest it frittered away an opportunity of staying ahead of the competition. In doing so, however, it would expose itself to a downside risk in the event that conditions turned out to be economically unfavourable for the project. In such a case, one strategy that would certainly hold financial merit is that of using a protective put.

In its simplest form, the business enterprise could try and borrow at the lowest possible borrowing rate of interest and 'buy' a protective put on the project with the borrowings. In real terms, the protective put would give the business enterprise a right to hand over the project to a potential buyer ('writer' of the put option) at cost in case it eventually turned out to be an uneconomical investment.

With the protective put in place, in the worst-case scenario, the business enterprise would end up exercising the put option and closing off the debt with the proceeds. In the best-case scenario, the put option would not be exercised, and it would expire worthless. The business enterprise would be able to harvest the excess value of the project after paying off the debt.

The obvious problem, however, would be finding a suitable potential buyer for the project, ie someone who is willing to 'write' such an option in exchange for an acceptable 'premium'. This paper develops and demonstrates a simple computational strategy by which a synthetic call option could be engineered to replicate the pay-off from the protective put, along very similar lines to those one would take in replicating a protective put on a financial asset such as a stock or a future.

COMPUTATIONAL METHODOLOGY

The proposed computational approach of synthetically designing a *real* protective put rests fundamentally on the assumption that the put option is European. Being such, a hypothetical portfolio, consisting of a borrowing at the riskless rate and investment of the proceeds in the project and a put option thereon, would, owing to the application of the put-call parity, have a pay-off the same as that of a European call option on the value of the project (ie the project with an embedded option to defer). Mathematically, this boils down to the following equation

$$(V_t + P_t) - Xe^{\{-r(T-t)\}} = C_t$$

Here, V_t is the NPV of the project (without an option to defer), P_t is the price of the put option at time t , X is the exercise price (ie cost of the project), r is the riskless rate of return, T is the terminal date on which the business enterprise must take the decision as to whether or not it wants to invest in the project, and C_t is the

price of the call option on the value of the project (ie the project with an embedded option to defer).

In the worst-case scenario, on the LHS, the business enterprise would end up exercising the put and closing off the debt with the proceeds. On the RHS, the call would expire useless. In the best-case scenario, on the LHS the put would expire worthless and, on the RHS, the call would have a pay-off equal to the value of the project minus the debt payback.⁶

In terms of a project where deferral is allowed, the usual NPV evaluation procedure using ROA simply calculates the fair price of the embedded call option on the value of the project. This involves only the RHS of the put-call relationship. The same analytical approach, however, may be reverse-engineered to work from the LHS of the put-call parity relationship as well, and may offer a particularly interesting and reasonably practicable solution in cases where a complete deferral is not possible owing to extremely intense competition. It is assumed that there is an opportunity of starting off small and scaling up when conditions are favourable and scaling down when conditions are unfavourable. As already observed, a restaurant chain is probably an appropriate example.

The proposed solution approach is surprisingly simple, as it is a straightforward application of the LHS of the put-call parity relationship. The LHS of the put-call parity relationship may actually be viewed as a synthetic protective put. If viewed as such, it is the same as investing in the project and at the same time holding a put

option thereon akin to a standard portfolio insurance strategy involving a financial asset such as a stock portfolio.

In the context of a capital budgeting decision, the solution then is simply to have an amount of equity in the project that equals its NPV, ie equal to the fair price of the call option. If the option value goes up, the project will have to be upscaled; if it goes down, the project will have to be downscaled. This investment strategy would work (albeit to a limited extent) even when there are 'limits' to the extent to which a project can be upscaled or downscaled. It is, however, implicitly assumed that the firm can procure funds for the project (when upscaling) and reapply funds elsewhere (when downscaling) at the same rate, ie the firm's weighted average cost of capital equals the project's opportunity cost of capital.

The cost of borrowings should theoretically be the risk-free rate. In real life, however, it is not possible for a business enterprise to borrow at a 'risk-free' rate, given that it would employ the borrowed funds in an inherently risky business project. But a protective put portfolio consisting of an investment in the project as well as a put thereon is, by definition, protected from downside risk. Therefore, if the lending institution can be adequately satisfied that the borrowed funds would be employed in 'acquiring' a portfolio that is theoretically risk free, the business enterprise should certainly be able to procure the borrowings at the lowest cost.

A discrete time model with origin time

$t = 0$ and terminal time $T = 1$ is formulated. The possibility of taking a bank loan B_t (equivalent to issuing in a zero-coupon bond) is assumed. The interest rate is $r \geq 0$, ie $B_1 = B_0 e^{(rT)} = B_0 e^r$. The project has only one *node* at time 0 with present value V_0 , and two nodes at time 1, with present values $V_1^{(+)}$ and $V_1^{(-)}$ such that $V_1^{(+)} > V_1^{(-)}$. There are probabilities p and $(1 - p)$ for the PV of the project to move to $V_1^{(+)}$ or $V_1^{(-)}$, respectively. It is further assumed that $e^{(-rT)} V_1^{(-)} < V_0 < e^{(-rT)} V_1^{(+)}$. If this relationship did not hold, it would imply an arbitrage opportunity which cannot exist in a perfect market.

The business enterprise is able to employ the borrowed funds in the project and thereby acquires a portfolio whose value is given by $I_0 = kV_0 + k'B_0$, which means that, at time $t = 0$, it invests amount k in the project and has a bank loan to the tune of k' . The vector $[k, k']$ is the strategy vector at time $t = 0$. Then, at time $T = 1$ the portfolio will cost $I_1 = kV_1 + k'B_1$, which equals $kV_1^{(+)} + k'B_0 e^r$ with probability p , and $kV_1^{(-)} + k'B_0 e^r$ with probability $1 - p$. The expected value of this random variable is given by the following equation⁷

$$E(I_1) = k\{V_1^{(+)}p + V_1^{(-)}(1 - p)\} + k'B_0 e^r$$

Now, to imitate the call option, one needs to construct a portfolio which has the same pay-off as the call option on the value of the project, ie the portfolio will mimic or replicate the call. A strike price $X = \text{cost of project} > 0$ is fixed for the call option. The exercise price must satisfy the relationship $V_1^{(-)} < X < V_1^{(+)}$. The expiry is at $t = T$.

The problem is to find a vector (k, k') such that the pay-off of this portfolio is $(V_1 - X)$. If such a vector is found, it will constitute a synthetic call, which will replicate the pay-off from the call option on the value of the project if it had an embedded deferral option. Hence, their prices coincide, ie one gets $C_0 = I_0$.

To find the strategy vector (k, k') , the Cox-Ross-Rubinstein one-step model⁸ requires the following system of linear equations to be solved

$$\begin{aligned} kV_1^{(+)} + k'B_0 e^r V_1^{(+)} - X; \text{ and} \\ kV_1^{(-)} + k'B_0 e^r = 0 \end{aligned}$$

The solution to the above system gives the synthetic call which is equivalent to the one-period call option on the project. The value of the call option at time $t = 0$ is $C_0 = I_0 = kV_0 + k'B_0$, with the strategy vector (k, k') . Solving the above system yields (k, k') as $k = [V_1^{(+)} - X]/[V_1^{(+)} - V_1^{(-)}]$ and $k' = [-kV_1^{(-)}]/B_0 e^r$.

Then, a dynamic synthetic real protective put strategy would simply involve making an investment equal to C_0 at the onset. The call option pay-off would have to be synthetically replicated by $I_0 = kV_0 + k'B_0$, with kV_0 representing the part of the total project cost that has to be invested at the onset and $(X_0 - kV_0)$ representing the part that is 'deferred'. At $t = T_j$, the whole process is re-enacted with the new exercise price being $X_1 = (X_0 - C_0)$. The routine is repeated until such time as $V_j < X_j$ and terminates when $V_j \leq X_j$. At that point, $\sum_{j=1} (X_j) < X_0$. The next section gives a numerical illustration of the proposed computational approach.

Table 1: Input data ($j=0$)

Present value of whole project	\$1.43
Best-case value	\$2.00
Worst-case value	\$1.00
Exercise price	\$1.50
Lowest-cost borrowing rate	5%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

NUMERICAL ILLUSTRATION

The computational scheme

Step 1: Compute the NPV of the project as the price of a call option on the value of the project at $t = 0$ using the Cox–Ross–Rubinstein one-step binomial process. Use the NPV of the project calculated by the traditional DCF technique as V_0 and the variance of the returns from the project as the volatility. The cost of development of the project is the exercise price X_0 and $T - t = 1$. The risk-free rate is proxied by the lowest-cost borrowing rate available to the business.

Step 2: C_0 is the NPV of the project (price of the call option on the value of the project) as calculated in the previous step. Conversely, since a portfolio with a current dollar value of $C_0 e^r$ at $T - t = 1$ will decay to a value of C_0 at $T - t = 0$, $C_0 = I_0 = kV_0 + k'B_0$ is multiplied by e^r on both LHS and RHS to get the actual amount to be invested in

the synthetic put portfolio at $t = 0$. Therefore, the actual dollar amount to be invested in the project at $t = 0$ is $C_0 e^r$, out of which $kV_0 e^r$ is the ‘current equity’ and $(X - kV_0 e^r)$ is the ‘deferred equity’.⁹ The actual dollar debt amount at $t = 0$ is $k'B_0 e^r$. The ‘deferred’ investment is to the tune of $(X_0 - C_0 e^r)$.

Step 3: If $C_0 < 0$, the ‘put option’ is exercised, ie the project is liquidated, and the debt is paid off with the proceeds. If $C_0 > 0$ and the present value of the project is greater than the exercise price, the process terminates with the balance amount being invested in full in the project, ie the project is implemented 100 per cent. Else if $C_0 > 0$ and the present value of the project is less than the exercise price, Step 1 is re-run with X_1 as the exercise price after making adjustments to the project value and volatility estimates if necessitated by changed business conditions. The steps are repeated so long as $X_j > V_j$. The process terminates when $X_j \leq V_j$. At that point the balance funds are invested into the project and $\sum_{j=1} (X_j) = X_0$.

NUMERICAL RESULTS

The proposed computational scheme is illustrated with a simple numerical example. The basic input data for the problem are supplied in Table 1.

The expected value of the project was obtained as (Best-case value $\times$ 0.5) +

Table 2: Iteration $j = 0$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	$\Delta equity$
At $T - t = 0$	0.75	0.50	0.25	—
Best-case at $T - t = 0$	0.95	0.47	0.48	0.23
Worst-case at $T - t = 0$	0.48	0.48	0.00	-0.25
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	$\Delta Equity$	
Best-case at $T - t = 0$	1.50	1.90	0.40	
Worst-case at $T - t = 0$	1.50	0.95	-0.55	

Table 3: Input data ($j = 1$)

Present value of project	\$0.71
Best-case	\$1.00
Worst-case	\$0.50
Exercise price	\$0.75
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.5
Probability of worst-case scenario	0.5

Table 5: Input data ($j = 2$)

Present value of project	\$0.36
Best-case	\$0.50
Worst-case	\$0.25
Exercise price	0.38
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.5
Probability of worst-case scenario	0.5

Table 4: Iteration $j=1$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	$\Delta equity$
At $T - t = 1$	0.38	0.25	0.13	—
Best-case at $T - t = 0$	0.48	0.24	0.24	0.11
Worst-case at $T - t = 0$	0.24	0.24	0.00	-0.13
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	$\Delta equity$	
Best-case at $T - t = 0$	0.75	0.95	0.20	
Worst-case at $T - t = 0$	0.75	0.48	-0.27	

Table 6: Iteration $j = 2$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	$\Delta equity$
At $T - t = 1$	0.19	0.13	0.06	—
Best-case at $T - t = 0$	0.24	0.12	0.12	0.06
Worst-case at $T - t = 0$	0.12	0.12	0.00	-0.06
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	$\Delta equity$	
Best-case at $T - t = 0$	0.38	0.48	0.10	
Worst-case at $T - t = 0$	0.38	0.24	-0.14	

Table 7: Input data ($j = 3$)

Present value of project	\$0.18
Best-case	\$0.25
Worst-case	\$0.13
Exercise price	0.19
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.5
Probability of worst-case scenario	0.5

Table 9: Input data ($j = 4$)

Present value of project	\$0.08
Best-case	\$0.13
Worst-case	\$0.06
Exercise price	\$0.09
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.5
Probability of worst-case scenario	0.5

Table 8: Iteration $j = 3$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	$\Delta equity$
At $T - t = 1$	0.09	0.06	0.03	—
Best-case at $T - t = 0$	0.12	0.06	0.06	0.03
Worst-case at $T - t = 0$	0.06	0.06	0.00	-0.03
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	$\Delta equity$	
Best-case at $T - t = 0$	0.19	0.24	0.05	
Worst-case at $T - t = 0$	0.19	0.12	-0.07	

Table 10: Iteration $j = 4$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	Δ_{equity}
At $T - t = 1$	0.05	0.03	0.02	—
Best-case at $T - t = 0$	0.06	0.03	0.03	0.01
Worst-case at $T - t = 0$	0.03	0.03	0.00	-0.02
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	Δ_{equity}	
Best-case at $T - t = 0$	0.09	0.12	0.03	
Worst-case at $T - t = 0$	0.09	0.06	-0.03	

Table 11: Input data ($j = 5$)

Present value of project	\$0.04
Best-case	\$0.06
Worst-case	\$0.03
Exercise price	\$0.05
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.5
Probability of worst-case scenario	0.5

Table 13: Input data ($j = 6$)

Present value of project	\$0.01
Best-case	\$0.03
Worst-case	\$0.02
Exercise price	\$0.02
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table 12: Iteration $j = 5$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	Δ_{equity}
At $T - t = 1$	0.03	0.02	0.01	—
Best-case at $T - t = 0$	0.03	0.01	0.02	0.01
Worst-case at $T - t = 0$	0.01	0.01	0.00	-0.01
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	Δ_{equity}	
Best-case at $T - t = 0$	0.05	0.06	0.01	
Worst-case at $T - t = 0$	0.05	0.03	-0.02	

Table 14: Iteration $j=6$

<i>With protective put</i>	<i>Portfolio</i>	<i>Debt</i>	<i>Equity</i>	$\Delta equity$
At $T-t=1$	0.01	0.01	0.00	—
Best-case at $T-t=0$	0.01	0.01	0.00	0.00
Worst-case at $T-t=0$	0.01	0.01	0.00	0.00
<i>Without protective put</i>	<i>Investment</i>	<i>Equity</i>	$\Delta equity$	
Best-case at $T-t=0$	0.02	0.03	0.01	
Worst-case at $T-t=0$	0.02	0.01	-0.01	

(Worst-case value $\times 0.5$). The cost of the project is same as its expected value. The present value of the project was obtained by dividing the expected value of the project by $e^{5\%}$. The probability of best-case scenario and the probability of worst-case scenario at each step were set equal at 0.50 so as to maximise the element of uncertainty by maximising the entropy of the outcomes.¹⁰ The computational scheme

Table 15: Final position ($j=7$)

Present value of project	\$0.01
Best-case	\$0.02
Worst-case	\$0.01
Exercise price	\$0.01
Lowest borrowing/lending rate	5%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table 16: Upscaling of the project with a synthetic protective put

j	C_0	<i>Dollar investment</i>	<i>Cumulative dollar investment</i>
0	0.24	0.75	0.75
1	0.19	0.38	1.13
2	0.06	0.19	1.32
3	0.03	0.09	1.41
4	0.01	0.05	1.46
5	0.01	0.02	1.48
6	0.01	0.01	1.49
7	0.00	0.01	1.50

Figure 1: Upscaling of the project with a synthetic protective put

was implemented on an MS Excel spreadsheet running a VBA routine. The numerical results are summarised in Tables 2–15.

As the dollar investment in the project does not change from $j = 6$ to $j = 7$, the iterative process should be terminated at this point. The business enterprise should invest the balance fund in the project at this stage in order to implement it at hundred per cent level as the gap between the present value of the project and required dollar investment to complete the project has practically been reduced to zero.

CONCLUSION

Table 16 and Figure 1 show the upscaling of the project over time with a synthetic protective put.

The above numerical example illustrates that using a synthetic protective put strategy in a manner as outlined above, a business enterprise can choose to invest in a project in phases rather than doing it all at once; when it cannot defer the commencement of the project outright owing to the pressures of competition. In the numerical example, the input data were chosen so as to make the outcome consistent with the law of diminishing marginal returns on investment.

To the best of the author's knowledge, the proposed synthetic put strategy in a real options scenario is unique, although it is a commonplace risk management tool when managing a financial portfolio of stocks or futures. The computational procedure of working the hedging scheme is quite simple and applicable in practical situations.

An obvious direction of future research would be extending this portfolio insurance concept to include American options as well.

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References

- 1 Copeland, T., and Antikarov, V. (2001) 'Real Options: A Practitioner's Guide', Texere, New York.
- 2 Bhattacharya, S. (1978) 'Project Valuation with Mean- Reverting Cash Flow Streams', *Journal of Finance*, Vol. 33, pp. 1313–1331.
- 3 Samuelson, P. A. (1965) 'Proof that Properly Anticipated Prices Fluctuate Randomly', *Industrial Management Review*, Vol. 6, pp. 41–49.
- 4 Copeland, T., Weston, J. E and Shastri, K. (2005) 'Financial Theory and Corporate Policy', Pearson Addison Wesley, New York.
- 5 Dixit, A. and Pindyck, R. (1994) 'Investments under Uncertainty', Princeton University Press, Princeton, NJ.
- 6 Rubinstein, M. and Leland, H. E. (1981) 'Replicating Options with Positions in Stock and Cash', *Financial Analysts Journal*, July/August, pp. 113–121.
- 7 Rendleman R. and Bartter, B. (1979) 'Two State Option Pricing', *Journal of Finance*, Vol. 34, pp. 1092–1110.
- 8 Cox, J., Ross, S. and Rubinstein, M. (1979) 'Option Pricing: A Simplified Approach,' *Journal of Financial Economics*, Vol. 7, pp. 229–263.
- 9 Kester, W. (1984) 'Today's Options for Tomorrow's Growth', *Harvard Business Review*, Vol. 62, pp. 19–33.
- 10 Shannon, C. E. (1948) 'A Mathematical Theory of Communication', *Bell System Technical Journal*, Vol. 27, pp. 379–423.