Original Article

Multidimensional Unfolding and the use of alternative input scales

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ABSTRACT Non-metric Multidimensional Unfolding (MDU) techniques have been misused for some time due to some degeneracy problems. Recently, the PREFSCAL algorithm has been proposed which deals properly with degenerate solutions. However, and in spite of the interest of MDU outputs – perceptual maps depicting two groups of entities and illustrating their proximities – there are still some open issues regarding MDU applications. In particular, one may ask whether different input scales can have significant impact on MDU solutions. Although this subject has been approached in Multidimensional Scaling (MDS) quite a long time ago, it seems that it is now pertinent to address it in the context of MDU. The main goal of the present study is to analyse the association between MDU input, concerning the use of alternative scales, and MDU performance. Simulated data with uniform distribution are randomly generated, different scales with different numbers of response categories are considered and MDU analysis is performed using PREFSCAL. The results are evaluated based on the proportion of variance on disparities accounted for by the corresponding distances. Inferential results indicate that there are significant differences in MDU model adjustment originated by the use of alternative input scales. An application referred to the use of alternative communication technologies is presented which illustrates the association between the use of different MDU input scales and MDU performance. The obtained results agree with the ones provided by the simulation study. Furthermore, they add some insights concerning the relationship between Portuguese individuals and alternative communication channels.

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INTRODUCTION

Non-metric Multidimensional Unfolding (MDU) is a method for analysing two-way, two-mode ordinal data, which commonly refers to the preferences of a set of subjects over a set of objects. The goal of MDU is to obtain a low-dimensional spatial representation of the data, where both subjects and objects are represented in a way that the more an object is preferred, the closer the point representing this object is located to the (ideal) point representing the subject. MDU is commonly used to portray market structures using graphical representations of consumers (or consumer segments) preferences over a set of products.¹⁻⁴ The use of MDU as a data visualisation tool, to support decision-making, regards two-dimensional solutions which are specifically considered in the present work.

The unfolding model was first proposed by Coombs⁵ and Bennet and Hays.⁶ Several different developments have been made on the original non-metric unfolding models. A variety of proposals include metric models, external models and three-way models (for a review of these models, see Borg and Groenen⁷). Although, referring to applications, non-metric MDU is the most popular model, having the advantage of requiring no assumptions concerning data characteristics. In spite of this, non-metric MDU has been previously associated with degeneracy problems.^{3,7-9} Recently, the PREFSCAL algorithm¹⁰ has been proposed trying to overcome the MDU degeneracy problems offering a new alternative tool to non-metric MDU analysis.

Commonly, MDU input data refer to the ratings of C objects or stimulus by each of the S subjects. However, there are several scales formats¹¹ and, to authors knowledge, no attention has been paid to the effects of using alternative scales (with different numbers of response categories) in MDU results. There is a great deal of research concerning the number of categories in preferences scales in what concerns questionnaire design. It is focused on the respondents' perspective and on reliability and validity issues.^{12,13} In this context a balance is needed between the desired information given by scales with more categories and the respondents'

additional effort demanded by these scales.¹³ Specifically, scales with less than five response categories generally perform poorly in what concerns reliability and validity.¹⁴

The impact of input scales in MDS results has been addressed in non-metric Multidimensional Scaling (MDS) by Green and Rao¹⁵ and Green and Wind.¹⁶ Both works involve the ability for recovering bi-dimensional configurations defined by one set of points (one-mode, two-way data).

Green and Rao¹⁵ attempt to analyse the number of response categories of attributes' rating scales. They assume that inter-brand proximity measures are obtained through the data consisting on the brand ratings of a set of individuals on a given number of attributes. By analysing results of two configurations, a systematic and a random one, it is found that, generally, the degree of recovering the configuration increases with the number of scales' response categories. In addition, the authors also conclude that the use of two or three response categories leads to rather poor recoveries of the original configurations.

In Green and Wind¹⁶ the authors use synthetic data (consisting of a two-dimensional MDS configuration with 25 points) to study the effects in MDS results produced by the classification of original (input) distances into a small set of categories. It is found that the classification into nine levels still allows one to almost perfectly recover the original configuration and, even, with the use of a classification into only two levels, the original configuration is roughly recovered. A primary approach to ties is used.

The main objective of the present work is to study the association between alternative input scales and MDU results, filling a gap in the literature, aiming to give the analysts practical knowledge which can be used when performing MDU analysis.

The proposed approach relies on simulated data, recoded into alternative scales where different numbers of points are considered. Specifically, four alternative scales are considered: 0-100, 1-20, 1-9 and 1-5 scales. The 0-100 scale, naturally, is the one which contains more information. In this work we want to analyse the impact on MDU due to the loss of information

regarding the reduction of the input scales' number of points. The MDU results are evaluated using R^2 , the proportion of disparities' variance which is accounted for by the corresponding distances.

Data from an application are also used to illustrate the association between MDU input scales and MDU results, R^2 in particular: the proposed approach is similar to the above referred, except that it relies on real data that are referred to individuals' levels of exposure to different communication channels.

The association between MDU input and R^2 results is discussed both based on randomly generated data and on real data.

THE MDU PROCEDURE

Typical MDU applications refer to the preferences of S subjects (judges) over C stimulus or objects. The preference measures can be transformed into non-preference (or dislike) measures, which can be seen as the dissimilarities between subjects and objects – δ_{sc} referring to subject s ($s=1, \dots, S$) and object c ($c=1, \dots, C$) – the higher value designating the lesser amount of preference.

MDU's main goal is to find a configuration (commonly bi-dimensional in a Euclidean space) in a way that the distances between points, representing both subjects and objects, match as closely as possible the dissimilarities. Let us denote the resulting coordinates of the (ideal) point representing a subject s by x_{sh} , with $h=1, 2$, representing the space' dimensions. Similarly, the coordinates of the point representing object c are y_{ch} . Let d_{sc} represent the Euclidean distance between subject s and object c .

In MDU, the initial non-preferences (dissimilarities) data are transformed by a monotonically increasing function – and the resulting values $\hat{d}_{sc}=f(d_{sc})$ are called disparities, fitted distances or pseudo-distances. Different models proposals depend on the transformation f that is considered. The non-metric unconditional models, in particular, refer to ordinal data, where a single monotone function is used for transforming all initial dissimilarities' values.

A solution to MDU problem refers to an optimisation problem involving the approximation

of transformed dissimilarities, $\hat{d}_{sc}$, by distances, d_{sc} , among points in a (bi-dimensional) space.

There are several alternative algorithms for performing an MDU analysis.^{17,18} In fact, ordinary MDS procedures allowing the use of weights and missing data can be used to perform an MDU analysis.¹⁹ The problem with this approach is the occurrence of degenerate solutions. These solutions have a nominally perfect *STRESS* value, but the resulting configuration is very difficult to interpret because a great number of adjusted distances are equal. In the present work, MDU analysis is performed using PREFSCAL procedure.¹⁰ The PREFSCAL algorithm uses a process based on iterative majorisation, to minimise a loss function known as Penalised *STRESS* – *PSTRESS*. This goal function innovates by including a penalty part, which aims to avoid degenerate solutions, by penalising solutions with 'small' variation coefficients for disparities.

COMPARING MDU SOLUTIONS FOR ALTERNATIVE INPUT SCALES

The R^2 measure is an easy-to-use and interpret measure that is convenient for evaluating MDU results¹⁰:

$$R^2 = \frac{\sum_{s=1}^S \sum_{c=1}^C (d_{sc} - \bar{d})^2}{\sum_{s=1}^S \sum_{c=1}^C (\hat{d}_{sc} - \bar{\hat{d}})^2}$$

with $\bar{d}$ and $\bar{\hat{d}}$ representing the average of distances and disparities, respectively. The R^2 is a goodness-of-fit measure which ranges from 0 to 1. When R^2 is 0, there is no association between the disparities ($\hat{d}_{sc}$) and the fitted distances (d_{sc}). When R^2 is 1, a perfect fit is reached meaning that the variation of the disparities is completely explained by the variation of the distances, indicating that there is no loss of information in the model.

To compare MDU solutions due to the use of alternative input scales, a descriptive analysis is provided. In addition, inferential analysis is carried out, specifically using the non-parametric Friedman test.²⁰ The Friedman two-way analysis

of variance by ranks tests the null hypothesis that k samples have been drawn from the same population or populations with the same median. Let θ_j be the median of the j th population (R^2 median values observed for each scale), then the null hypothesis can be stated as $H_0: \theta_1 = \theta_2 = \dots = \theta_k$. The alternative hypothesis is $H_1: \theta_i \neq \theta_j$ for at least one pair of medians.²⁰ For the purpose of multiple comparisons, the hypothesis $H_0: \theta_i = \theta_j$ ($i, j = 1, \dots, k$) is also tested.

SIMULATION APPROACH

The main goal of the present work is to study the impact of using alternative input scales (with different number of response categories) on non-metric MDU solutions. An approach based on randomly generated data is proposed. In the data generation process, five alternative scenarios are considered concerning the dimensionality of the input data matrix. The initial data are recoded into four alternative scales with different number of points. MDU solutions are obtained using PREFSCAL algorithm and the R^2 goodness-of-fit measure is considered for their evaluation. Finally, both descriptive and inferential tools are used to analyse the association between MDU input scales and MDU outputs.

Data generation and input scales for MDU

In this study input MDU data are randomly generated. Each element δ_{sc} is obtained by considering the nearest integer of a random number from a uniform distribution on the interval $[0, 100]$.

Following a procedure suggested by Green and Wind,¹⁶ the initial values, δ_{sc} (in the 0–100 scale) are then recoded into three other different scales with 20, 9 and 5 points. This process can be described as follows: the amplitude of the initial scale is divided into a number of intervals (with the same amplitude) corresponding to the number of categories of the new scale, then the same value is assigned to all initial δ_{sc} falling in the same interval.

Several combinations of S and C are also considered, which are compatible with practical applications aiming for data visualisation:

$(S = 5, C = 10)$, $(S = 5, C = 15)$, $(S = 5, C = 20)$, $(S = 10, C = 20)$ and $(S = 10, C = 30)$. For each one of these cases a set of 25 data matrices is analysed.

In order to adequately perform comparisons related to alternative input scales, an additional condition is imposed: degenerate solutions are discarded. To that end, the R^2 as well as the Shepard Index¹⁰ and the variation coefficient of disparities are taken into account. The Shepard Index is defined as the ratio between the number of distinct distances and the total number of distances. A sufficient number of replications are generated in order to guarantee that all the 25 replications considered, for each S and C combination, correspond to non-degenerated solutions in every of the four different scales. For example, for $S = 5$ and $C = 10$, 65 matrices are generated in order to obtain 25 non-degenerate solutions.

Using the PREFSCAL algorithm

Non-metric MDU unconditional analysis is performed using the PREFSCAL algorithm¹⁰ for each input matrix. The Penalised *STRESS* formula with default parameters is used:

$$PSTRESS = \left[\left(\frac{\sum_{s=1}^S \sum_{c=1}^C (\hat{d}_{sc} - d_{sc})^2}{\sum_{s=1}^S \sum_{c=1}^C d_{sc}^2} \right)^{1/2} \left(1 + \left(\frac{cv(\delta)}{cv(\hat{d})} \right)^2 \right) \right]^{1/2}$$

where d_{sc} represents the Euclidean distance between subject s and object c and, $cv(\delta)$ and $cv(\hat{d})$ represent the variation coefficient of initial data and of disparities, respectively.

An ordinal transformation of original data is used, with the condition of keeping ties tied. For reducing the possibility that the algorithm reaches a local minimum, a hundred random starting configurations are used and the solution with lowest *PSTRESS* value is selected. The algorithm stops when one of the following (default) criteria is reached: the maximum number of iterations (5000), the minimum *PSTRESS* value (0.0001) or the difference in consecutive *PSTRESS* values is less or equal to 0.000001.

Table 1: Simulation results: Descriptive statistics for R^2

S	C	Statistics	Scales			
			0–100	1–20	1–9	1–5
5	10	Mean	0.883	0.876	0.861	0.847
		Median	0.886	0.884	0.872	0.844
		Standard deviation	0.043	0.034	0.032	0.036
5	15	Mean	0.820	0.815	0.801	0.792
		Median	0.830	0.814	0.800	0.791
		Standard deviation	0.058	0.053	0.038	0.037
5	20	Mean	0.792	0.785	0.786	0.757
		Median	0.805	0.786	0.787	0.757
		Standard deviation	0.053	0.046	0.039	0.027
10	20	Mean	0.508	0.513	0.520	0.517
		Median	0.499	0.505	0.513	0.515
		Standard deviation	0.065	0.066	0.039	0.026
10	30	Mean	0.447	0.446	0.454	0.468
		Median	0.443	0.444	0.454	0.459
		Standard deviation	0.031	0.033	0.028	0.031

Figure 1: Simulation results: Boxplots for R^2 .

Simulation results

Results referred to simulation – R^2 values – are presented in Table 1 and pictured in Figure 1. Generally, for 15, 20 and 25 items, with the reduction of scales range, the mean, the median

and the standard deviation of R^2 get lower values: scales range reduction turns the MDU fit poorer. However, this relationship is not clear when considering a larger number of points (30 and 40). Furthermore, in these cases, the values

Figure 2: Results of multiple comparison tests for 15 and 20 items.

of R^2 are low (smaller than 70 per cent), which means that the model does not recover well the original data. One then concludes that for more than 30 points, the MDU task turns to be harder.

Generally, the more the input items, the worst the model fitting (reduction of R^2 means and medians). This observation is consistent with the previous known results for MDS analysis.^{21–24}

For comparing the distribution of R^2 among the alternative input scales, the Friedman test is performed. Five tests are computed, one for each combination of S and C . Friedman tests' results show that the medians of R^2 for the alternative scales are significantly different (with significance level of 0.05), when 15, 20, 25 or 40 items are considered. For $C + S = 30$ the corresponding P -value is 0.054. The results of multiple comparisons are identical when 15 and 20 items are considered (see Figure 2). In what concerns results for 25 or 40 items they are similar to the ones presented in Figure 2, except for the difference between the 0–100 and 1–9 scales which is not significant. These results show that, generally, the use of alternative scales has a significant impact on MDU results. The Friedman test indicates that the results for R^2 , a model adjustment measure, yield significant differences among alternative scales for almost all tested scenarios.

When the number of items (subjects and objects) in the model is not high, 15 or 20, it is clear that the reduction of scales' points corresponds to a reduction in the quality of MDU adjustment. The 0–100 scale presents significant differences, on R^2 results, when

compared to 1–9 or 1–5 scales. Also, the 1–20 scale adjustment results are significantly different from those obtained for the 1–5 scale.

When 25 or 40 items are considered in the model, the 0–100 scale and the 1–5 scale, as well as the 1–20 and 1–5 scales, present significant differences on the model adjustment.

MAPPING THE USE OF COMMUNICATIONS TECHNOLOGIES

The use of randomly generated input data allowed for conclusions concerning the existence of significant differences on MDU performance resulting from the use of alternative scales. An application illustrating this relationship complements the proposed approach. The application refers to the use of communications technologies by Portuguese individuals.

The proliferation of alternative communications channels and media platforms as well as the ways in which consumers are engaging with these alternatives is continuously changing. Marketers face new challenges involving the knowledge of how consumers use the communication technologies.²⁵

In this context, emerges the importance of developing new tools to support a multimedia planning, relying on multi-source approach.^{25,26} In this work, an approach is proposed where MDU is used to picture the consumers' exposure to alternative communication media.

Data analysis

For understanding the role of internet in the communications practices in Portugal, a national survey was conducted between March and July 2003. A questionnaire was used on a sample of 2500 individuals representing continental population with 15 or more years old.²⁷ In the proposed questionnaire each individual answers about the average daily time spent on internet, watching television, listening to radio, reading newspapers and using the mobile phone. A subsample of 165 respondents to the questionnaire is used for MDU analysis. It corresponds to all the individuals which use all communications media on a daily basis.

Table 2: Descriptive statistics for R^2 provided by application data

S	C	Statistics	Scales			
			0–100	1–20	1–9	1–5
10	5	Mean	0.919	0.918	0.908	0.892
		Median	0.914	0.918	0.907	0.894
		Standard deviation	0.031	0.30	0.043	0.051
15	5	Mean	0.898	0.880	0.859	0.858
		Median	0.889	0.875	0.865	0.854
		Standard deviation	0.035	0.028	0.029	0.026
20	5	Mean	0.838	0.866	0.833	0.818
		Median	0.837	0.862	0.840	0.817
		Standard deviation	0.063	0.039	0.034	0.038

Figure 3: Boxplots for R^2 provided by application data.

An MDU analysis of the referred data allows us to obtain a map where the distances between the points representing each individual and each communication medium illustrate the individual's exposure to the specific medium: the lower the distance, the more is the time spent by the individual using the specific communication medium.

For analysing the impact of using alternative scales in MDU results, an approach similar to the one described in the simulation study is used. Thus, conclusions obtained by means of using random input data can be compared with the results obtained using real data referred to individuals' exposure to different communication media.

There are five communication media: $C=5$. Random samples with $S=10$, $S=15$ and $S=20$ individuals are collected from the initial data base.

Each one of the collected samples originates an MDU input matrix. In order to consider the input data in a 0–100 scale, input values (time spent in internet, television, radio, newspapers and mobile phone) are standardised (by subtracting the minimum value, dividing for the maximum and multiplying by 100). Three other alternative scales are considered: scales 1–20, 1–9 and 1–5 (the procedure is described before for the simulation study). For each sample and each alternative input scale, an MDU analysis is performed using the PREFSCAL algorithm, with a parameterisation similar to the one considered in the simulation study.

The use of alternative scales

Degenerate solutions are discarded (as in the previous simulation study). For 10, 15 and 20

Figure 4: MDU resulting map. The communication channels are represented by * and the individuals are represented by o.

Figure 5: MDU resulting map with individuals' age: Δ 15–24 years; o 25–34 years; ▼ 35–54 years; + 55 and more years. The communication channels are represented by *.

individuals, the numbers of input MDU matrices considered are 13, 10 and 8, respectively. The descriptive results of R^2 obtained are presented in Table 2 and represented in Figure 3. Note that (as expected) R^2 values are higher than in the simulation study where data did not exhibit a specific structure. This conclusion is expected, as the simulation study refers to random data

with no structure. Although descriptive results do not present a clear relationship with the ones obtained in the simulation study, inference analysis provides similar results, as described next.

For each sampling case considered, $S=10$, $S=15$ and $S=20$, the Friedman test is performed in order to compare the distribution of R^2 among alternative scales. When samples with 10 or 15 individuals are considered, the test result indicates the rejection of the null hypotheses for a significance level of 0.05. When $S=20$, the null hypotheses of Friedman test is not rejected. Thus, in cases where $S=10$ or $S=15$, results agree with the ones obtained in the simulation study, indicating that the use of alternative scales yields significant differences in the distribution of R^2 . Generally, multiple comparison tests' results referred to the application are similar and agree with the simulation study results presented in Figure 2. An exception occurs when $S=15$, for which the test indicates no significant differences between MDU results referred to input scales 1–20 and 1–5.

The communications media picture

The data concerning communications technologies enables an overall MDU analysis which can provide a map depicting individuals and communication media. This map illustrates the exposure of the individuals to the alternative communication channels.

Using the 165 individuals an MDU analysis is performed using PREFSCAL procedure. For the starting configuration, 200 random starts are considered, as well as, the classical scaling configuration (with the initial rectangular matrix completed by using the triangular inequality for imputation of the values between rows and between columns). The selected solution is the one with lower $PSTRESS$ value and is obtained by using a random starting configuration. The algorithm stopped after 5000 iterations with a difference in consecutive penalised stress values of $1,33e-013$.

The obtained MDU solution has $R^2=0.766$ and the resulting map is illustrated in Figure 4. The configuration, although referring to a larger

Figure 6: MDU resulting map with individuals' gender: • Females; ○ Males. The communication channels are represented by *.

sample than usual in MDU applications, correctly identifies the media most used for 71 per cent of the respondents, and the second media most used for 43 per cent of the individuals. The map also enables to conclude that the majority of the individuals spend more daily time watching television than using the other communication media. Despite this, there are several individuals exhibiting small distances to other media (small distance illustrating a higher usage frequency), newspapers being the only exception. In addition, in the lower side of the map we can identify a group of individuals that are more prone to use new technologies, namely they do often use internet and mobile phone.

The individuals' age and gender are pictured in Figures 5 and 6. There are few individuals over 55 years, using all the communication media, internet in particular, on a daily basis. Most of the individuals that use more the mobile phone and internet are less than 35 years old. In Figure 6, one can observe that the majority of women seem to select the television as the most used communication medium. On the other hand, most of the individuals that exhibit lower distances from internet and mobile phone (bottom of the map) are males.

CONCLUSIONS AND PERSPECTIVES

Input data for MDU analysis typically refer to the ratings of a set of objects or stimulus, given by a group of individuals. Different alternative scales can be used. In particular, a wide variety of alternatives can be considered regarding the number of response categories. The main goal of this work is to study the impact of alternative measurement scales (with different number of response categories) on MDU results.

Literature contributions regarding the study of alternative input scales in the context of MDS are scarce. To authors' knowledge, there are only two studies addressing this issue.^{15,16} Both works refer to non-metric MDS solutions. They involve the ability for recovering bi-dimensional configurations defined by one set of points (one-mode, two-way data).

In the present work, an approach is presented which deals with the association between input scales and MDU performance. Initially, simulated data with a uniform distribution are randomly generated in a 0–100 scale. Data are then recoded into alternative 1–20, 1–9 and 1–5 scales using a procedure suggested by Green and Wind¹⁶ in an MDS study.

An application referred to the use of alternative communication technologies is also presented. MDU analysis enables to picture the individuals' habits concerning the use of alternative communication media (television, radio, internet, newspapers and mobile phone). The application illustrates the association between input preference scales and MDU results and complements the initial approach.

MDU analysis is performed using the PREFSCAL algorithm, both for uniform randomly generated data and random samples of real data, referring to different levels of exposure to alternative communications' media. The results are evaluated using R^2 , the proportion of variance on disparities accounted for by corresponding distances. This measure is an easier to interpret alternative to *STRESS* functions (used in alternative forms as objective functions by the different MDU algorithms).

An inferential analysis is performed (Friedman test and multiple comparisons tests) in order to

compare the distributions of R^2 among results of MDU originated by different input scales. Generally, there are significant differences on the R^2 distribution originated by the use of alternative scales. Specifically, one can state that when the number of items (subjects and objects) is smaller than 30, the larger the scale range, the better the quality of MDU adjustment. In addition, since there are no significant differences between the results referred to the 0–100 and 0–20 scales, one can recommend a 0–20 scale (an easier task for the respondent). When the number of items is 30 or 40 (considered in the present study), one finds that the two dimensions do not recover well the original data (R^2 values are low).

This study results can be a helpful tool for the analysts' decision making. When using MDU analysis, the authors suggest careful selection of the number of response categories considered in the input scales, as it has impact on MDU results.

A limitation in the present study (using revealed preferences data) concerns the problem of scale usage bias (occurring in the context of stated preference data) which was not addressed. Thus, further research should take this issue into account, especially when scales with fewer categories are considered.

Further research should also concern the impact of using alternative preference scales (with different number of response categories) in other MDU evaluation measures. In addition, the effect of the reduction of scales' points in the resulting MDU maps may be evaluated by means of Procrustes analysis.

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