Numeric decision support to find optimal balance between customer acquisition and retention spending

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Abstract This paper focuses on optimisation procedures for customer acquisition and retention costs. It compares a recent contribution by Pfeifer (2005) to the original Blattberg and Deighton (1996) model and gives generic formulations for both optimisation procedures. It extends the model and the optimisation procedures from the rather narrow ‘lost for good’ customer dynamic behaviour scheme to the more general ‘always a share’ behaviour model. Numeric optimisation algorithms used to build iso-curves help prove that the ratio between optimal retention and acquisition costs is fundamentally a matter of customer versus prospect responsiveness to marketing efforts, and produce decision support to find the optimal ratio between customer retention and acquisition spending. Empirical data adapted from existing literature on the subject are used to test and illustrate the optimisation models and methods that are introduced.

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INTRODUCTION

Balancing customer acquisition and retention efforts is an important CRM problem. Blattberg and Deighton,¹ using a simple customer dynamic behaviour model, determine the optimal amount of acquisition spending and retention spending independently. Berger and Nasr-Bechwati² extend this approach to a constrained allocation of promotional spending between acquisition and retention. The latter models are applied by Berger and Bernstein³ to real-world applications in the diagnostic self-testing industry. All these optimisation procedures use customer lifetime value (CLV) and the derived concept of customer equity (CE)^{4–6} as the basis.

CLV has been fundamental for many years in direct and database marketing as mentioned by Dwyer⁷ and discussed by Keane and Wang⁸ in the publishing industry.

Berger and Nasr⁹ examined a series of models to compute CLV for different situations in terms of revenue streams, costs and probabilities. Following papers added modified CLV formulas.^{10–12}

Jain and Singh,¹³ in their frequently cited review of the CLV literature, identify three main research directions. The first is the development of CLV *calculation models* that focus on the revenue stream from customers, and on acquisition, retention and other marketing costs in order to facilitate calculations, resource allocation and optimisation of CLV. The second research stream, described as *customer base analysis*, concentrates on methods that analytically predict the probabilistic value of customer transactions

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Table 1: Attrition and discount effects on future gains

t	Gain g	Discount $1/(1+d)^t$	Survival r^t	Joint $(r/(1+d))^t$
0	1.00	1.00	1.00	1.00
1	1.00	0.83	0.80	0.67
2	1.00	0.69	0.64	0.44
3	1.00	0.58	0.51	0.30
4	1.00	0.48	0.41	0.20
5	1.00	0.40	0.33	0.13
6	1.00	0.33	0.26	0.09
7	1.00	0.28	0.21	0.06
8	1.00	0.23	0.17	0.04
9	1.00	0.19	0.13	0.03

from the existing customer base. The last direction uses analytical models in a normative way and it analyses *CLV implications for managerial decisions*.

This paper focuses on optimisation procedures for customer acquisition and retention costs. It compares a recent contribution by Pfeifer¹⁴ to the original Blattberg and Deighton (BD)¹⁵ model and gives generic formulations for both optimisation procedures. It extends the model and the optimisation procedures from the rather narrow ‘lost for good’ customer dynamic behaviour scheme to the more general ‘always a share’ behaviour model. Powerful numeric optimisation algorithms are used in order to systematically compare the two alternative optimisation procedures, to perform sensitivity analysis (using iso-curves) and to derive and discuss rules that help guide managerial decisions. Empirical data adapted from existing literature on the subject are used to test and illustrate the optimisation models and methods that are introduced.

THE BD MODEL

Blattberg and Deighton proposed a model for helping managers determine the optimal balance between acquisition and retention spending. They based their reasoning on CLV calculations, using a simple model of dynamic customer behaviour known in the more recent literature as the retention model. This dynamic behaviour, found, according to Dwyer,¹⁶ in many industries like banking and insurance services, press subscriptions, etc supposes that repeat buying defines an active customer. In other words, when a customer does

not renew his purchase or contract, he is ‘lost for good’, and when he comes back after one or several periods of absence, he is treated as a new customer.¹⁷ The *retention model* relies upon the conventional assumption that after customers are lost, they do not return.¹⁸ The evaluation of customer potential in a relation of this kind will only take into account the customer’s *probability to remain active* from one period to another or what Blattberg *et al.*¹⁹ call the customer’s *survival probability*. The customer lifetime corresponds to the number of successive periods during which the customer is and remains active.

It can be shown that at constant retention rate and constant gain per purchase, an initial 1€ gain from a customer is worth $(1+d)/(1+d-r)$ € in the long term, where r and d are retention and discount rates. This means, for example, that the lifetime value (or rather long-term value) of a customer initially worth 1€ is 3€ if retention rate is 80 per cent and discount rate is 20 per cent. This formula captures two essential phenomena to be taken into account when valuing a customer. One is the attrition that occurs during the whole lifetime of a cohort and produces a survival rate after t periods worth r^t . The other one is the need to discount future revenues. A 1€ gain obtained after t years is worth no more than $1/(1+d)^t$, where d is the discount rate, a measure that is near but with an inverse impact to the interest rate. Table 1 and Figure 1 illustrate the logic guiding lifetime value calculation.

Figure 1: Attrition and discount effects on future gains

t	0	1	2	...	∞
Gain by period	$-A/a+m$	$-R/r+m$	$-R/r+m$		$-R/r+m$
	acquired prospect-->	just acquired customer-->	new Cust.-->		existing Cust.-->

Figure 2: Timeline of gains generated by a customer/prospect

Any future gain generated by a customer will be jointly affected by attrition and discount rate and needs to be corrected by a factor of $(r/(1+d))^t$. If this gain g is constant during each period of the customer's lifetime, the discounted sum of these gains in the long term or the *CLV* is

$$\sum_{t=0}^{\infty} g \left(\frac{r}{1+d} \right)^t = g \frac{1+d}{1+d-r} \quad (1)$$

If we exclude the initial phase $t=0$, which can be seen as the acquisition phase, the value or the sum of gains of a newly acquired customer is

$$\sum_{t=1}^{\infty} g \left(\frac{r}{1+d} \right)^t = g \left(\frac{1+d}{1+d-r} - 1 \right) = g \frac{r}{1+d-r} \quad (2)$$

The gains generated by a customer in a simplified retention model during each period are $(m-R/r)$ (see Figure 2), where m is the constant margin, R the retention cost per targeted customer and r the constant retention rate. R is the cost of the marketing effort that is necessary in order to maintain a constant retention rate of r . $(m-R/r)$ measures customer profitability at each period. It indicates that margin (m) is collected from customers remaining after a retention campaign, while retention costs (R) apply to all targeted customers at the beginning of each campaign.

At $t>0$ R are incurred for all targeted customers present in a cohort (at time t), while m is collected only from those customers who will have responded, meaning those customers who will still be there at the end of the period or at the beginning of the next period (at time $t+1$). R/r is therefore the *cost per retained customer*. As the amount and the value of such periodic gain

diminishes due to customer attrition and the need to discount future revenues by a discount rate d , in order to compute the long-term value they must be multiplied by the deflated sum of customer survival rates $\sum_{t=1}^{\infty} (r/(1+d))^t = r/(1+d-r)$. The lifetime value of a just-retained customer, or what Blattberg *et al.*²⁰ tend to call Retention equity,²¹ is then

$$CLV = (m - R/r) \frac{r}{1+d-r} \quad (3)$$

In order to get the complete image of the customer relationship profitability, the customer acquisition phase when prospects are transformed into customers cannot be ignored. Using the analogy of retention gains, acquisition gain becomes $m-A/a$, which is the m minus customer acquisition cost (A/a). A sequential, chronological representation of all gains of a customer's (lifetime) allows for better understanding of the long-term values of a prospect, a new customer, a just acquired customer or an existing customer (Figure 2).

According to the stage the prospect/customer is in at the moment of calculation, several formulations of his Expected Lifetime Value (or long-term value) are possible (see Table 2).

They differ essentially in whether they include the initial margin or not.

In this paper we use the *CLV* notation (formula 3) for the just-acquired *CLV*, which does not include the initial margin. The other formulations in Table 2, the new and existing *CLV*, then become $(m + CLV)$. As these formulations do not include acquisition costs, Blattberg and Deighton²³ introduce the term *CE*.

$$CE = (m + CLV) - A/a \quad (4)$$

Table 2: Calculations of the Expected Lifetime Value according to the stage the prospect/customer is in (adapted from Pfeifer *et al.*²²)

Perspective	Expected lifetime value	ELV in terms of CLV
Prospect	$a \left[-A/a + m + (m - R/r) \frac{r}{1 + a - r} \right]$	$a(-A/a + m + CLV)$
New ($t=0$) or existing ($t>0$) customer	$m + (m - R/r) \frac{r}{1 + a - r}$	$m + CLV$
Just acquired customer ($t=0$)	$(m - R/r) \frac{r}{1 + d - r}$	CLV

In order to find optimal acquisition spending, this concept needed to be put in a prospect perspective to compute what could be called prospect lifetime value (PLV), or as Pfeifer²⁴ suggests, EPLV (expected PLV).

$$\begin{aligned}
 PLV &= aCE = a(-A/a + m + CLV) \\
 &= a \left[-A/a + m + (m - R/r) \frac{r}{1 + d - r} \right]
 \end{aligned}
 \tag{5}$$

Thus, the lifetime value of a prospect who is going to become a customer is the acquisition rate times the expected value of a new customer minus the acquisition spending.

Formula (5) is essential to the BD model and a starting point in the BD approach to balance the trade-off between acquisition and retention spending. The BD approach has been recently challenged by Pfeifer.²⁵ In this paper we thoroughly compare the two approaches, analyse their differences and suggest further extensions.

OPTIMISATION PROCEDURES BALANCING ACQUISITION AND RETENTION COSTS COMPARED

The BD model expresses the retention probability as a function of the marketing effort (R) directed towards the customers being targeted. A modified exponential function of the retention budget is

used whose form is consistent with an assumption of strictly diminishing returns to retention spending.

$$r = c_r [1 - \exp(-k_r R)] \tag{6}$$

The ceiling c_r and the sensibility of a customer to retention effort k_r can be estimated by decision calculus using subjective response estimations from managers.²⁶

Parameter c_r , the retention ceiling rate, is the manager's direct assessment of the maximum proportion of targeted customers who would be retained if there were no limit to spending.

Parameter k_r can next be determined once the manager gives the current retention spending level and the current retention rate as $\log(c_r/(c_r - r))/R$.

The fact that the retention curve goes through the origin is seen as a weakness of the BD model by Pfeifer²⁷ if the firm expects to retain some percentage of customers even in the absence of retention spending. We show that this 'weakness' can be easily overcome without influencing the fundamental results of the model, by introducing an additional parameter in the response function that does not affect marginal retention cost.

$$r = f_r + (c_r - f_r)[1 - \exp(-k_r R)] \tag{7}$$

Parameter f_r , the retention floor rate, is the manager's direct assessment of the proportion of targeted customers who would be retained in absence of retention spending, and can be seen as a measure of customer inertia.

By introducing (7) or (6) in (3), the retention costs that maximise CLV can be found. As there is no closed-form expression in order to find the optimal retention spending R^* , Blattberg and Deighton suggest finding it graphically.

Similar to the retention response function, the model assumes that acquisition rate a is a decelerating function of acquisition spending per prospect (A):

$$a = c_a [1 - \exp(-k_a A)] \quad (8)$$

where c_a is the ceiling and k_a is the sensibility of prospects to acquisition efforts.

To find the optimal balance between customer acquisition and retention spending, Blattberg and Deighton²⁸ suggest optimising both separately. A two-step optimisation procedure can be derived from their approach, which is given in formula 9.

$$\underbrace{a(A)(m - A/a(A))}_{\max(1) \rightarrow a'} + \underbrace{a'CLV(R)}_{\max(2)} \quad (9)$$

It starts with maximising what we call the prospect acquisition value $a(m - A/a)$, using formula (8) to express the prospect's acquisition response as a function of marketing efforts. It then uses the resulting optimal acquisition rate a' to find the optimal retention equity in a prospect's perspective (see Table 3). Their approach is criticised by Pfeifer,²⁹ who considers it 'unnecessarily myopic'. In his view, acquisition expenses are 'suboptimal' when considering the smaller immediate gain (the m produced by an acquired customer) instead of taking into account the bigger long-term value of a new customer ($m + CLV$) (see Table 3). It is the *long-term value of a prospect to be acquired* $a(m + CLV) - A$ and not that prospect's *acquisition value* ($am - A$) that should be maximised. Without changing the original formulation of the model (equation [5]) he reverses the above mentioned two-step

optimisation procedure by first finding the retention spending (R^*) that maximises the new customer's lifetime value and uses it in the next step to find the optimal acquisition spending (A^*), as in formula 10.

$$\underbrace{a(A)(m + CLV(R)) - A}_{\substack{\max(1) \\ \max(2)}} \quad (10)$$

Table 3 tries to summarise the differences between the two optimisation procedures.

In our opinion, both optimisation procedures have merits. The BD method looks for complete separation between the acquisition and retention stage, and seems to adopt a risk-averse attitude to acquisition spending optimisation. This is a short-term view that results in lower acquisition rate and spending, as compared to the long-term view suggested by Pfeifer.³² The latter, by adopting a risk-neutral attitude, regroups the two stages and integrates expected customer value in the calculation, which aims to optimise acquisition spending. By replacing the margin with the lifetime value, the latter method invests more in customer acquisition, and potentially results in higher CE and lower optimal ratios of customer retention to acquisition.

Although we adopt Pfeifer's approach in our subsequent developments, we do not reject the original BD model, and do not consider their optimisation procedure as being wrong. We prefer to regard it as an alternative that tries to separate reality that can be measured (acquisition value) from the future that can be only be guessed or estimated (retention value).³³

By taking Blattberg and Deighton's original numeric example, the comparison between the two procedures can be illustrated graphically, as in Figure 3.

The parameters defining retention response behaviour are inferred from managerial experience by decision calculus. The response ceiling $c_r = 70$ per cent is directly assessed, while the responsiveness parameter k_r is calculated knowing the retention rate $r = 40$ per cent obtained for a given retention effort per customer

Table 3: Comparison of two procedures to optimally balance customer acquisition and retention costs

Compared elements	Blattberg et Deighton ³⁰	Pfeifer ³¹
Perspective regarding the customer acquisition stage	Short term	Long term
Gains used to optimise acquisition value	Margin	CLV of a new customer
Calculation perspective	Prospect	Prospect
Customer value components (isolated)	Prospect acquisition value + just acquired CLV in a prospect perspective	Prospect Lifetime Value – Acquisition Cost
Formulation (same formula, different maximisation order)	$\underbrace{a(A)(m - A/a(A))}_{\max(1) \rightarrow a'} + \underbrace{a'(m - R/r)}_{\max(2)} \frac{1}{1 + d - r}$	$\underbrace{a(A) \left[\underbrace{m + (m - R/r) \frac{r}{1 + d - r}}_{\max(1)} \right]}_{\max(2)} - A$
Stage 1	Find acquisition spending A^* that maximises acquisition value of a prospect (on the short term) $a(m - A/a)$ and consider the optimal rate a' obtained in this way	Find retention spending R^* that maximises CLV
Stage 2	Find retention spending R^* that maximises CLV	Find acquisition spending A^* that maximises PLV

Table 4: Summary of numerical example results (lost for good behaviour)

	Acquisition(BD)	Acquisition(Pfeifer)	Retention
Ceiling (c_a, c_r)	0.400	0.400	0.700
Responsiveness(k_a, k_r)	0.139	0.139	0.085
<i>Optimal:</i>			
Spending (A', A'', R^*)	7.356	9.489	16.319
Response rate (a', a'', r^*)	0.256	0.293	0.524
Avg. Cost ($A'/a', A''/a'', r^*/R^*$)	28.765	32.423	31.121
Value (AV^*, PLV^*, CLV^*)	5.430	10.177	17.198
Margin=50, $d=0.10$, $CE=AV^*+a'CLV^*=9.83$, $PLV^*=10.177$			

$R=10\$$. It results in $k_r = \log(c_r/(c_r - 0.4))/10 = 0.0847$. A summary of the numerical examples used is given in Table 4.

Figure 3 shows how the retention spending per targeted customer determines the response curve of the retention rate, and indicates graphically the optimal spending R^* that produces the highest CLV. Notice that this curve, which is represented on the negative scale, starts at the origin,³⁴ goes through the point $(\$10, 0.4)$ and increases asymptotically to the ceiling rate (c_r). At optimal spending of $R^* = 16.32\$$, the firm retains 52.4 per cent of its customers at an average (or cost per retained customer) of $\$16.32/52.4$ per cent = $\$30.1$ to achieve a maximum CLV^* of $\$17.2$ for a

just-acquired customer or $\$67.2$ for a new customer if acquisition margin ($m=50$) is included.

Acquisition behaviour is similarly inferred in the original BD example, by directly assessing the acquisition response ceiling $c_a = 40$ per cent and by computing the responsiveness parameter k_a using the response rate $a = 20$ per cent for a given acquisition spending $A = 5\$$ to be $k_a = \log(c_a/(c_a - 0.2))/5 = 0.13863$.

Figure 3 shows how acquisition spending per prospect determines acquisition rates and can indicate the optimal acquisition spending graphically. Here the original BD procedure differs from Pfeifer's. By separating an acquisition

Figure 3: Two procedures to find optimal balance between acquisition and retention spending compared (lost for good behaviour). r =retention rate on a negative scale, a =acquisition rate on a negative scale, PLV=expected prospect Lifetime value, CLV=Lifetime value of a just acquired customer, AV=Acquisition value of a prospect, CE*=optimal customer equity in a prospect perspective, R^* =optimal retention spending, r^* =optimal retention rate, A' =optimal acquisition spending (short term view as in Blattberg & Deighton), A'' =optimal acquisition spending (long term view as in Pfeifer), a' =optimal acquisition rate (short term view), a'' =optimal acquisition rate (long term view)

value that ignores future gains from retention value, it finds smaller optimal acquisition spending. By spending an optimal $A' = (1/k_a) \ln(k_a c_a m) = 7.356\$$ a prospect acquisition rate of $a' = 25.57$ per cent is obtained at an average cost (or cost per acquired customer) of $7.356\$/25.57$ per cent = $49\$$ leading to a maximum prospect acquisition value $a(m - A/a)$ of 5.43 . By adding the just acquired CLV in a prospect's perspective ($a' \text{ CLV}^*$) of 25.57 per cent $\times 17.198\$ = 4.4\$$, optimal CE in a prospect's perspective is obtained ($9.83\$$).

Pfeifer's approach considers not only present gains (margin) but also future gains (CLV) when maximising prospect acquisition value. This results in higher optimal acquisition spending $A' = (1/k_a) \ln(k_a c_a \text{CLV}^*) = 9.49$ to convert 29.2 per cent of the prospects at an average cost of 32.42 to achieve an optimal prospect lifetime value PLV^* of 10.18 .

Besides the difference in the optimal amount to be spent on customer acquisition, in this particular numeric example the managerial recommendations differ fundamentally as to the optimal ratio between acquisition and retention spending a firm should adopt. The BD pessimistic approach, which does not give future credit to

acquisition, recommends, at optimality, spending more on customer retention than on customer acquisition. Pfeifer's long-term view gives more credit to acquisition and recommends, in this specific example, spending more on acquisition than on retention. This situation is illustrated graphically in Figure 3. Optimal average acquisition and retention costs are ratios of this spending to the response rates it achieves. Graphically they are equivalent to cotangents of the acute angles starting in the origin (0,zero) and having optimal spending as the adjacent side of the triangle and response rate as the opposite side. The bigger this ratio (or the cotangent), the smaller the angle. Optimal average retention spending is represented by the acute angle $\widehat{R^*0r^*}$. Optimal average acquisition spending is represented by angle $\widehat{A'0a'}$ for the original BD approach and by $\widehat{A''0a''}$ angle for Pfeifer's procedure. Compared to the angle indicating optimal average retention cost in Figure 3, a bigger angle corresponding to the BD approach signifies smaller optimal average acquisition cost and recommends spending more on retention than on acquisition at optimality, while a smaller angle associated to Pfeifer's approach signifies

bigger optimal average acquisition cost and recommends spending more on acquisition than on retention.

OPTIMAL ACQUISITION VERSUS RETENTION COST RATIO ANALYSIS

Analysing the optimal ratio between retention and acquisition costs provides further guidance to managers as to whether it is optimal to spend more on retention when customers are easier to retain than to acquire. There is an adage³⁵ or maxim³⁶ in marketing folklore that says it is cheaper to keep existing customers than to recruit new ones. Although this is logical, intuitively appealing and normally a reason for investment considerations behind the CLV concept, little empirical evidence has been published.³⁷ Although some academic research obtained contrary results,^{38,39} there are several arguments that support this conjecture. Acquiring a new customer requires one-time costs for advertising, promotions and the like.⁴⁰ Retained customers tend to purchase more than new customers,^{41–43} and are more efficiently dealt with. Companies often use switching costs for customers in order to obtain advantage over competitors, and offer lower prices in order to acquire customers.⁴⁴ The ratio between customer acquisition and retention costs is often fixed at five in marketing folklore, due to a much-quoted⁴⁵ study by the US Department of Consumer Affairs.⁴⁶ Additional empirical evidence⁴⁷ suggests an interval varying from 3 to 20. This ratio most probably refers to costs per retained (R/r) and acquired (A/a) customer and not to marginal costs, as theoretically, marginal retention and acquisition costs should be equal at optimality. We show that these average costs can be represented as cotangents in the graphical representation of the BD model, where optimal spending (A' , A'' or R^*) is the adjacent side and the optimal response rate (a' , a'' or r^*) is the opposite side of a triangle defining an acute angle whose size can then be visually used to evaluate the optimal ratio between retention and acquisition costs (see Figure 3).

Pfeifer's procedure, applied to the original BD example, recommends spending almost equal amounts on acquisition and retention, which contradicts real world evidence. He therefore adds a 'carefully constructed' numerical example based upon BD's original example, where the recommended optimal acquisition spending is five times bigger than retention spending as suggested by 'marketing folklore'. A summary of the two numeric examples is given in Table 5.

We try to find rules explaining such diverging situations. Cheaper to retain implies a combination of responsiveness (elasticity) and amplitude (or saturation levels) in the customer response curve, compared to the prospect's. When the amplitudes of the two response curves are identical, higher responsiveness in retention implies that it is optimal to spend more on retention, and vice versa. Amplitudes can, when higher for retention than for acquisition (which is normally the case), strengthen this ratio in favour of retention if retention responsiveness is higher, but can influence the opposite way when acquisition responsiveness is higher. This situation appeared in the original BD example and has been speculated by Pfeifer.⁴⁸ But his 'carefully constructed example' is rather extreme.⁴⁹

Instead of isolated 'carefully constructed' numerical examples, we introduce a systematic decision support tool that uses iso-curves or

Table 5: Original BD 1X average cost ratio example compared to Pfeifer's 5X ratio example

	Acquisition	Retention
<i>Original</i>		
Ceiling	0.400	0.700
Responsiveness	0.139	0.085
Optimal: spending	9.489	16.319
Resp.rate	0.293	0.524
Avg.Cost	32.423	31.121
Value	10.177	17.198
<i>Extreme</i>		
Ceiling	0.040	0.950
Responsiveness	0.289	0.144
Optimal: spending	2.411	22.001
Resp.rate	0.020	0.910
Avg.Cost	120.179	24.176
Value	1.074	123.699
Margin d		
50.00	0.10

Figure 4: Ratio of optimal average acquisition to retention spending for varying customer and prospect responsiveness. (a) normal response amplitudes ($c_r=0.7$ and $c_a=0.4$) (b) extreme response amplitudes ($c_r=0.95$ and $c_a=0.04$)

contour lines that clearly show how varying pairs of acquisition and retention responsiveness parameters influence the optimal ratio among the two spending alternatives (see Figure 4).

Figure 4 shows that the ratio of optimal average acquisition to retention spending for customer and prospect responsiveness varies systematically. Two situations are illustrated in which the 'normal' BD example (Figure 4a) and Pfeifer's extreme example (Figure 4b) are fixing the response amplitudes. The filled circles indicate responsiveness parameter values for acquisition and retention corresponding to each example. The optimal ratio between acquisition and retention spending that is achieved by those responsiveness parameters can be read from the contour lines on each ratio map. For the original BD example, one can see in Figure 4a that the acquisition and retention responsiveness parameters were 0.13863 and 0.0847, respectively, and that the resulting optimal ratio between acquisition and retention spending was slightly bigger than one. For the 'extreme' Pfeifer example, one can see in Figure 4b that the acquisition and retention responsiveness parameters were 0.2888 and 0.1440, respectively, and that at optimality, the cost to acquire a new customer was five times (5x) the cost to retain an

existing customer. This 5x ratio between optimal acquisition and retention costs is often mentioned in relationship marketing folklore.

In order to produce optimal ratio contour maps with the numeric solution (Figure 4a and 4b), we used numeric optimisation algorithms available in S-plus and R systems.⁵⁰

Managers can use these contour maps corroborated with a decision support tool that would allow them to estimate either objectively or subjectively (by decision calculus) their customers' acquisition and retention response curves and find optimal acquisition and retention spending like in Figure 5.

After having estimated the acquisition (a) and retention (r) response curves as in Figure 5a, associated to the original BD example or as in Figure 5b, linked to Pfeifer's example, the manager obtains optimal acquisition and retention spending from which optimal ratio between retention and acquisition spending can be derived. Graphically, the optimal ratio is also given by the cotangent of the *optimal retention angle* ($\widehat{R^*0r^*}$) divided by the cotangent of the *optimal acquisition angle* ($\widehat{A''0a''}$). One can easily see in Figure 5a that the retention angle is slightly bigger than the acquisition angle, which gives an optimal

Figure 5: Visual decision support to optimal acquisition and retention spending. (a) normal response amplitudes ($c_r=0.7$ and $c_a=0.4$) (b) extreme response amplitudes ($c_r=0.95$ and $c_a=0.04$). r =retention rate on a negative scale, a =acquisition rate on a negative scale, PLV=expected prospect Lifetime value, CLV=Lifetime value of a just acquired customer, AV=Acquisition value of a prospect, R^* =optimal retention spending, r^* =optimal retention rate, A^* =optimal acquisition spending, a^* =optimal acquisition rate

retention to acquisition ratio near to one. This result can be identified on the contour map in Figure 4a. In Figure 5b, the retention angle is much bigger than the acquisition angle. This ratio is near five and can be identified on the contour map in Figure 4b. In a segmented customer base, response to acquisition and retention efforts varies among segments, and each segment can be positioned on a contour map. According to its responsiveness to acquisition and retention efforts, the segment's optimal ratio can be read from the map. If response amplitude is similar among segments, several segments can be positioned on the same contour map. Customer acquisition and retention policy can profit from such decision support tools.

EXTENDING FROM 'LOST FOR GOOD' TO 'ALWAYS A SHARE' BEHAVIOUR

The 'always a share' behaviour is the alternate scheme to 'lost for good' in what is known as a dichotomy. CLV here comes not only from surviving customers but also from customers allowed to reactivate after a given number of inactive periods. Customers are considered 'lost for good' only after exceeding that number of successive periods of inactivity. By reducing the tolerated number of successive periods of

inactivity to zero, the 'always a share' model reduces to the 'lost for good model', which can be seen as a special case of this more general model. The main difficulty in adapting the previous calculations to this dynamic behaviour resides in the potential complexity of a customer response model integrating retention and reactivation, and of a CLV calculation based upon this response model.

Adopting a matrix approach, we define a vector of customer response probabilities declining with recency (or the number of periods since the last buy) as a function $p(x, R) = c_{x=1}(R) \exp(-k_x \cdot (x-1))$ where x is the recency that varies from 1 to the firm imposed recency limit ($x=1..reclim$) and where the response ceiling of the retention probability $c_{x=1}(R) = \max c_r (1 - \exp(1 - k_r R))$ is a growing function of retention effort (R), equivalent to formula (6). From there we build the matrix of transition probabilities that represents this dynamic behaviour:

$$\begin{bmatrix} p(R) & \text{diag}(1 - p(R)) \\ 0 & 0, \dots, 1 \end{bmatrix} \quad (11)$$

Like formula (6), it allows for subjective response estimations from managers estimated by decision calculus.

Figure 6: Two procedures to find optimal balance between acquisition and retention spending compared (always a share behaviour). r =retention rate on a negative scale, a =acquisition rate on a negative scale, PLV=expected prospect Lifetime value, CLV=Lifetime value of a just-acquired customer, AV=Acquisition value of a prospect, CE*=optimal customer equity in a prospect perspective, R^* =optimal retention spending, r^* =optimal retention rate, A' =optimal acquisition spending (short-term view as in Blattberg & Deighton, 1996), A'' =optimal acquisition spending (long-term view as in Pfeifer, 2005), a' =optimal acquisition rate (short-term view), a'' =optimal acquisition rate (long-term view)

Table 6: Summary of numerical example results (always a share behaviour)

	Acquisition(BD)	Acquisition(Pfeifer)	Retention
Ceiling (c_a, c_r)	0.200	0.200	0.36
Responsiveness(k_a, k_r)	0.139	0.139	0.36
Responsiv. to recency (k_x)			0.360
<i>Optimal:</i>			
Spendig (A', A'', R^*)	2.403	5.125	5.140
Response rate (a', a'', r^*)	0.057	0.102	0.303
Avg. Cost ($A'/a', A''/a'', r^*/R^*$)	42.06	50.02	16.963
Value (AV^*, PLV^*, CLV^*)	0.454	2.371	23.195
Margin=50, $d=0.10$, $CE^*=AV^*+a'CLV^*=1.78$, $PLV^*=2.371$			

Using matrix $\mathbf{P}(\mathbf{R})$, we compute the retention equity vector $CLV(\mathbf{R})$ that contains values of customers in cohorts of recency varying from 1 to the business controlled recency limit as in formula (12), where $\mathbf{I}$ is the identity matrix, $\mathbf{m}$ the margin vector ($m, 0, 0, \dots$) and $\mathbf{R}$ the vector of retention spending ($-R, -R, \dots$) for customers in all recency states:

$$CLV(\mathbf{R}) = [(\mathbf{I} - \mathbf{P}(\mathbf{R})/(1+d))^{-1} - \mathbf{I}](\mathbf{m} - \mathbf{R}) \quad (12)$$

By introducing (12) in the generic formulas (4) and (5), optimal retention and acquisition costs can be obtained numerically and graphically for the two optimisation procedures that are summarised in these formulas.

Calculations are inspired by Dwyer's example, which has been slightly modified in order to facilitate estimating the reduction of response probability with recency by decision calculus and is summarised in Table 6.

Figure 6 shows how the retention spending per targeted customer determines the retention rate of recency one customers, and indicates graphically the optimal spending R^* that produces the highest CLV.

At optimal spending of $R^*=5.14$ €, the firm retains 30.34 per cent of its recency one customers to achieve a maximum CLV^* of \$23.2 for a just-acquired customer or \$73.2 for a new customer if acquisition margin ($m=50$) is included. This calculation also considers reactivation probabilities for customers with

recency 2 to 4. Customers with higher recency are 'purged', as their response probabilities are lower than the prospects' ones.

The optimal acquisition spending following BD procedure is $A' = (1/k_a) \ln(k_a c_a m) = (1/0.14) \ln(0.14 \cdot 0.2 \cdot 50) = 2.403\$$ a prospect acquisition rate of $a' = 5.714$ per cent is obtained at an average cost (or cost per acquired customer) of 42.06\$ leading to a maximum prospect acquisition value $a(m - A/a)$ of \$0.45. By adding the just acquired CLV in a prospect's perspective ($a' \text{CLV}^*$), optimal CE in a prospect's perspective is obtained $(0.45\$ + 0.05714 \cdot 23.2\$ = 1.78\$)$.

Pfeifer's approach considers not only present gains (margin) but also future gains (CLV) when maximising prospect acquisition value. This results in higher optimal acquisition spending $A' = (1/k_a) \ln(k_a c_a \text{CLV}^*) = \5.13 to convert 10.24 per cent of the prospects at an average cost of \$50 to achieve an optimal prospect lifetime value PLV^* of \$2.37.

AN ANALYTIC SOLUTION TO THE BD PROBLEM

As mentioned before, the retention response function used in the BD model (the modified exponential function) does not lead to a closed form expression for the optimal retention spending R^* . Therefore, the original authors suggest a graphic solution while other's like Pfeifer⁵¹ give numeric solutions (using Microsoft Excel's Solver). In Ayache *et al.*,⁵² we introduce a response function that allows finding the optimal retention spending analytically with a closed-form expression. The obvious advantage is quasi-instantaneous calculation of the optimal marketing spending in the BD problem. This facilitates large-scale sensitivity analysis and allows for a thorough investigation of optimal retention versus acquisition spending ratios under varying conditions.

This function applied to retention response is

$$r = c_r \left(1 - \frac{1}{1 + k_r R} \right) \quad (13)$$

Like the original retention response expression (the modified exponential function formula 6)

our function is consistent with an assumption of strictly diminishing returns to retention spending.

By introducing 13 in 3 and setting the derivative of the latter CLV expression by R equal to zero, the retention spending that achieves highest CLV is given by

$$R^* = \frac{-d - 1 + \sqrt{(1+d)c_r[(1+d-c_r)k_r m + 1]}}{k_r(d+1-c_r)} \quad (14)$$

We suggest that this closed form expression might be easily applied in sensitivity analysis. It also helps deduce some limits or border conditions that should be observed in such situations. One such limit is the need to verify that optimal spending is positive. We show that this condition is satisfied for optimal retention spending when $c_r k_r > 1/m$ or $m > 1/(c_r k_r)$. Similarly, in order to have a positive optimal acquisition spending, the following condition needs to be satisfied: $c_a k_a > 1/\text{CLV}^*$ or $\text{CLV}^* > 1/(c_a k_a)$. This is an interesting result, as it links ceiling and responsiveness parameters to profitability and value measures. To illustrate, let's take the original BD example. In order to have positive optimal retention spending, the margin must be bigger than $1/(0.7 \cdot 0.0847) = 16.87\$$, and in order to have positive optimal acquisition spending, the maximum CLV must be bigger than $1/(0.4 \cdot 0.13863) = 18\$$. An analyst can use these border conditions in several ways. He can, as already illustrated, find the minimum margin and optimum CLV needed to satisfy the border conditions, knowing the prospects' and customers' responsiveness and response ceiling. He could also calculate border values for the responsiveness coefficients while knowing response ceiling, margin and optimal CLV. And finally, he could compute response ceiling values and border values while knowing responsiveness, margin and optimal CLV.

CONCLUSIONS AND FURTHER RESEARCH

The contributions of this research focus on the BD model and its extensions. This model consists essentially of two customer and prospect response

models, a simple customer dynamic behaviour model (the 'retention model') and an optimisation procedure for balancing customer acquisition and retention spending. As such, it is a rather stylised model, well suited to explore the rationale that control marketing efforts to acquire new customers and retain existing ones. We adopt Pfeifer's (2005) modification of the BD optimisation procedure, as it seeks a sound economic formulation to balancing customer acquisition and retention efforts. We do not go so far as to consider the BD optimisation procedure wrong, and prefer to see it as a short term and pessimistic approach when finding optimal acquisition costs. We appreciate the BD model's approach to complexity reduction and separation of concerns.

Our analysis of the question of whether it is optimal to spend more on retention tries to be systematic. While Pfeifer (2005) uses two 'carefully constructed' examples, we build maps of optimal ratios between acquisition and retention spending. These enable us to show that the optimal ratios between acquisition and retention spending are determined by a combination of responsiveness and saturation level in the customer response curve, compared to the prospect's. When the amplitudes of the two response curves are identical, higher responsiveness in retention implies that it is optimal to spend more on retention, and vice versa. Amplitudes or saturation levels can, when higher for retention than for acquisition (which is normally the case), strengthen this ratio in favour of retention if retention responsiveness is higher, but can influence the opposite way when acquisition responsiveness is higher.

As for the original response functions in the BD model, there is no closed-form expression to compute optimal retention spending. Therefore, we used numeric optimisation algorithms available in the R/S-plus statistic systems in order to compute the optimal customer acquisition versus retention spending ratio. Maps could be drawn representing ratio values corresponding to all combinations of retention versus acquisition responsiveness parameters under given response amplitudes. In this way, the rules linking response amplitude and

responsiveness to optimal ratio of retention to acquisition spending are visually revealed and can be used by managers and analysts.

Visual decision support artefacts such as representing average retention and acquisition spending as cotangents of triangles that are added to the BD graphical solution can also be considered as contributions of this paper.

Recognising that the 'lost for good' dynamic behaviour scheme can be seen as a special case of the more general 'always a share' model, we extended the BD model to this alternative scheme in this dichotomy of customer dynamic behaviour.

Some of the artefacts discussed or developed here in customer cohort context might be extended to the larger customer base and customer portfolio context.

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