Original Article

How many managers? The impact of manager selection skill and fund size

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ABSTRACT This article measures the *diversification* benefits of increasing the *number of managers in a hedge fund portfolio* in three different historical periods. It shows that not only are returns and volatility markedly different in these different periods, but the usual relationship of higher risk with higher returns is sometimes reversed. Irrespective of the environment, this article posits a practical constraint for allocators to use in deciding the number of hedge funds in a portfolio using a measure of average allocator opportunity. The ‘average’ allocator constraint is then replaced by a *skilled* allocator and it shows that an increase in skill, all else being equal, allows more funds to be included in the portfolio, without decreasing the opportunity to outperform the average. The ‘allocator opportunity’ is used to compare the diversification benefit (trade-off) of adding managers (reducing opportunity) in portfolios of *small managers* versus large managers. It shows that in constructing portfolios with a given

allocator opportunity, an allocator could allocate to a larger number of managers if picking from the universe of small managers rather than large managers.

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INTRODUCTION

It is widely accepted that the standard deviation and drawdown characteristics of a portfolio can be reduced through an increase in the number of securities. However, the marginal reduction of portfolio risk decreases as the number of securities increases. In fact, most of the benefits of diversification, given random security selection, can be achieved with fewer than 15 stocks (Copeland and Weston, 1992). In a stock portfolio, diversification allows the investor to eliminate the idiosyncratic risk associated with individual securities but not the systematic risk of the market itself.

As the hedge fund and fund of funds industries have grown, there have been various attempts to apply similar analyses to determine the appropriate number of hedge funds in a hedge fund of funds portfolio (Wintner, 2001; L'habitant and Learned, 2002; Amo *et al*, 2007; Patel, 2008). These studies have found that, as with individual securities, the expected standard deviation of a portfolio can be reduced with an increased number of allocations and that marginal diversification benefits decrease with an increased number of allocations.

In the 'Random Sampling' section, we add to this by noting that as the number of hedge fund allocations to a portfolio increase, the dispersion of potential portfolio returns decreases. Consequently, a fund of funds manager's ability to add value decreases. When determining the appropriate number of investments, a fund of

funds manager must consider this trade-off between the 'Allocator Opportunity' and diversification benefits. We use a random sampling technique to quantify the probability distribution of that trade-off.

In the 'Introduction of Hedge Fund Selection Skill' section, the previous analysis is extended to examine the impact of allocator skill. Previous studies considered only random selections. This portion of the study introduces an allocator skill variable (λ) to the selection process. We quantify the trade-off between allocator opportunity and diversification benefits by evaluating return distributions as a function of N and λ .

In the section 'Dependence on Fund Size', we extend the analysis for skilled and unskilled sampling to consider the impact of hedge fund size on the framework for determining the appropriate numbers of managers. We quantify the impact of manager size on the trade-off between allocator opportunity and diversification benefits.

THE RELEVANT LITERATURE

Wintner (2001) examines the theoretical relationship of the volatility of a fund of funds portfolio to the number of managers. This study examines idealized portfolios with 10 funds or less, each with homogenous volatility and correlation characteristics. Wintner determines that the correlation between the funds within a portfolio and the volatility of the individual funds

are important to consider together with the number of funds.

L’habitant and Learned (2002) move to a real-world data set of hedge funds to investigate the appropriate number of hedge funds to achieve diversification. Here, the heterogeneous real-world historical correlations and volatilities come into the picture with all their nuances and variations over time embedded in the data. L’habitant and Learned examine diversification measures other than volatility and strategies other than naïve $1/n$ diversification as well. Interestingly, they find that although diversification increases with the number of funds in a portfolio, so does the resultant portfolio’s correlation to market indices. This then is an additional practical constraint to consider while increasing the number of managers in pursuit of lower volatility.

French *et al* (2005) find that most funds of funds hold 10–30 hedge funds with an average of 20. They study the reduction of standard deviation within strategy buckets and find results that further substantiate the findings of L’habitant and Learned.

Amo *et al* (2007) examine methodologies of investigating risk and diversification in funds of funds and lay out the benefits of simulation-based analysis versus time series covariance methods that fall short owing to their inherent normality assumptions. We use a simulation method in this article.

Patel (2008) examines the number of hedge funds in a fund of funds in the context of reaching a benchmark return. The mean return in this study of hedge funds was *higher* than the chosen benchmark for the period in question. This naturally led to the conclusion that a portfolio with a larger number of funds with a

return more likely closer to the mean would have a better chance of reaching the benchmark. We extend this idea in the manager skill section to encompass the case that if the mean return is *lower* than the desired return, then a lower number of managers has a better chance of producing a portfolio with the required return.

RANDOM SAMPLING

Data set

The Hedge Fund Research database was used for manager data over the time period 2002–2011 in the universe. Because markets experienced dramatic dislocations in 2007 and 2008, we chose to examine periods that both included and excluded that period, to investigate the extent to which results would be altered. Three separate universes were created for two 5-year periods (2002–2006 and 2007–2011) and one full 10-year period (2002–2011).

For this study, only managers with monthly data throughout the stated time period were considered. Clearly, this selection process introduces survivorship bias. The impact of survivorship bias has been well documented. However, the goal of this study is to understand the impact of the number of managers on the probability distributions of returns, standard deviation and other metrics rather than the exact shape of the distributions. We do not believe that an inclusion of ‘dead’ funds would impact the eventual conclusions. The database was edited to eliminate duplicate funds run by the same manager in similar strategies that differed only in leverage, domicile, base currency or accreditation standard. Roughly $1/3$ of managers were eliminated because of duplication. Table 1 shows the number of eliminated funds as

Table 1: Manager universe characteristics

	Period 1	Period 2	Period 3
Years	5	5	10
Time period	2002–2006	2007–2011	2002–2011
Initial manager #	2255	2477	929
Duplicates	627	832	210
Final unique funds	1628	1645	719
Fund avg. return	11.80%	4.20%	8.00%
Fund avg. standard deviation	11.40%	16.60%	14.60%
Fund avg. largest drawdown	–15.00%	–31.30%	–33.40%

well as returns, standard deviations and largest drawdowns for each of the three periods.

Sampling methodology

In the ‘Random Sampling’ section, a random sampling technique was utilized to create a set of N manager portfolios where N varies from 10 to 60 in steps of five managers. All portfolios were ‘buy and hold’ with monthly rebalancing. For each N , 5000 portfolios were created and results were tabulated. This process was repeated for all three time periods.

The statistical distributions were determined for the portfolios simulated for each time period and for N . We recorded the 5th, 25th, 50th, 75th, 95th percentile and mean of the distribution for the returns, standard deviations, Sharpe ratios, Var@95%, Var@99% and maximum drawdowns for each portfolio.

As expected, there were clear differences in the performance statistics between Period 1 and Period 2. Figure 1 shows a scatter plot of risk/return characteristics of each universe. It shows that managers were mostly rewarded for increased risk levels as measured by standard

Figure 1: Comparison of risk/reward between periods 1 and 2.

deviation between 2002 and 2006, and punished for increased risk levels between 2007 and 2011.

Results

Figure 2 shows that for each time period, the mean and median annualized return is independent of the number of managers. As N increases, the marginal changes in the 5th, 25th, 75th and 95th percentiles of the distribution decrease. The dispersion of returns, which defines an allocator’s ability to add value, decreases as the number of managers increases. In addition, the incremental decline in dispersion of returns decreases with the number of managers. The absolute returns were highest in Period 1 and lowest in Period 2. Period 3 tended to generate results that were intermediate between those of the two 5-year periods. While the absolute returns were different for each time period, the return distributions showed qualitative similarities.

Figure 3 shows the probability distributions for standard deviation. In each time period, the mean

Figure 2: Return distribution for periods 1, 2 and 3.

standard deviation declines as N increases, and the marginal decline in mean standard deviation decreases with N . The average standard deviation for each fund in the universe was 11.4, 16.6 and 14.6 per cent, respectively, for each period while 10 manager portfolios have mean standard deviations of 6.1, 9.8 and 8.8 per cent. Therefore, it is clear that multi-manager portfolios provide considerably lower mean standard deviations and that the majority of potential diversification benefits occur by increasing from $N=1$ to $N=10$. However, there are remaining diversification benefits as N is increased further.

In 2007–2011, the simulated portfolios experienced lower mean returns, broader distribution of returns, higher mean standard deviations and broader distribution of standard

deviations when compared with either Period 1 or Period 3.

Defined metrics

1. Allocator Opportunity 1 ‘AllOp1’ is the percentage difference in returns between the 75th and 25th percentiles.
2. Allocator Opportunity 2 ‘AllOp2’ is the percentage difference between the 95th and 5th percentiles.
3. Mean SD Diff is the difference in standard deviation between a portfolio with N funds and a portfolio with 10 funds.
4. Standard Deviation Dispersion 1 ‘SDD1’ is the percentage difference in standard deviation between the 75th and 25th percentiles.

Figure 3: Standard deviation for periods 1, 2 and 3.

5. Standard Deviation Dispersion2 ‘SDD2’ is the percentage difference between the 95th and 5th percentiles.
6. Sharpe Ratio Opportunity 1 ‘SharpeOp1’ is the percentage difference in standard deviation between the 75th and 25th percentiles.
7. Sharpe Ratio Opportunity 2 ‘SharpeOp2’ is the percentage difference between the 95th and 5th percentiles.
8. VaR 99 per cent Opportunity 1 ‘Var99Op1’ is the percentage difference in Var@99% between the 75th and 25th percentiles.
9. VaR 99 per cent Opportunity 2 ‘Var99Op2’ is the percentage difference in Var@99% between the 95th and 5th percentiles.

Figures 1–3 show the trade-off between an allocator’s opportunity to add value and diversification benefits. An allocator might strive to form portfolios that would meet the 75th percentile over time with occasional periods near the 95th percentile. The difference in annualized return between the mean and the 75th percentile ($0.5 \times \text{AllOp1}$) for a 25-manager portfolio was 1.34, 1.13 and 0.85 per cent for the three time periods, respectively. For a 40-manager portfolio, the differences declined to 1.07, 0.92 and 0.68 per cent, respectively. As N is increased, even the difference between the 95th percentile return and the mean ($0.5 \times \text{AllOp2}$) is no longer extremely significant; only 3.32, 2.86 and 2.05 per cent, respectively for

$N=25$ and 2.59, 2.27 and 1.63 per cent, respectively, for $N=40$.

The figures also show that as N is increased from 10 to 60, the marginal differences created from each step in N decreases until there are minimal differences between the respective probability distributions for $N=55$ and $N=60$. As a result, we conclude that $N=60$ effectively represents an asymptotic limit for each time period.

DISCUSSION

The data presented above illustrate the trade-off between an allocator's opportunity to add value and diversification benefits. As N is increased from 10 to 25 managers, allocators in the 75th and 95th percentiles outperform the mean by a maximum of 134 bps and 332 bps, respectively. With that increase in manager number, the allocator gives up more than 1/3 of its opportunity to add value in exchange for a 70 bps reduction in average annualized standard deviation. For a further increase to 40 managers, the 75th percentile implies an outperformance of a maximum of 107 bps and the 95th percentile implies a maximum of 259 bps in exchange for a 100 bps reduction in standard deviation.

Patel (2008) determined that approximately 40 is an appropriate number of underlying funds for a fund of funds whose objective is to maximize the probability of outperforming a Treasury bill benchmark. While many portable alpha programs do have such a target, most funds of funds strive to reach the top quartile of the funds of funds universe in returns and/or Sharpe ratio. In 2005, French *et al* (2005) found that funds of funds allocated to a median of 22 underlying funds. Therefore, a fund of funds that allocated to more than 22 underlying funds would have less than

a 25 per cent probability of performing in the top quartile, and that probability would decrease as N increases further if random sampling were an accurate description of manager selection. In the sections 'Introduction of Hedge Fund Selection Skill' and 'Dependence on Fund Size', we address the question assuming the selection process may not be random.

INTRODUCTION OF HEDGE FUND SELECTION SKILL

Methodology

In this section, the simulations were extended to directly consider skill in hedge fund selection. In the 'Random Sampling' section, manager selection was completely random. Thus, an allocator's skill would show as a propensity to achieve returns and Sharpe ratios that are consistently above the sample mean. In the 'Introduction of hedge fund selection skill' section, we define skill as an increased probability of selecting managers that will have top quartile performance over the time period. A skill parameter, λ , is introduced for skilled selection simulation. In the 'Random Sampling' section, an allocator had a 25 per cent probability of picking a top quartile fund. For these simulations, the probability increases with λ as shown in Table 2.

Table 2: Probability (P) of selecting a top quartile fund

λ (%)	P (%)
0	25
20	40
40	55
60	70

Aside from the introduction of the skill parameter, the simulation methodology was identical to that in the 'Random Sampling' section. These simulations used the same universes, time periods and range of values of N .

Results

Figure 4 shows the probability distribution of annualized returns as the selection skill is varied from 0 to 60 per cent. As expected, the $\lambda = 0$ case yields probability distributions, which default to those in the 'Random Sampling' section. Also as expected, the mean returns increase with skill level. The first observation is that the average return for $\lambda = 0$ in Period 1 is actually higher than the average return for $\lambda = 60$ in Period 2, which illustrates the magnitude of the difference in the market regimes. The shapes of the probability distributions distort as skill is introduced. In each case, the probability distributions became broader (representing increased allocator opportunity) and skewed as skill is increased.

Several characteristics of these distributions persisted across the time periods. For example, the mean exceeded the median for $\lambda = 20$ and 40 per cent. However, the median exceeded the mean for $\lambda = 60$ per cent. For $\lambda = 20$ per cent, the 75th percentile curves did not shift as dramatically as the 95th percentile and the return implied by the 75th percentile continued to decline with N . However for $\lambda = 40$ and 60 per cent, the 75th percentile curve shifts toward the 95th percentile.

Figure 5 shows the probability distribution of annualized standard deviation with variation in selection skill. As expected, the mean standard deviation declines with increasing N . A comparison of time periods shows that the mean standard deviation increases with skill

during 2002–2006 while it decreases with skill during 2007–2011. Such a change is actually not surprising as skill was defined in terms of return. In the more well-behaved market environment of 2002–2006, managers were generally rewarded for taking additional risk. From 2007 to 2011, the markets proved more likely to punish additional risk-taking. Therefore, in 2002–2006, more volatile managers were likely to be in the top performance quartile, while the opposite was true in the subsequent 5-year period.

DISCUSSION

The results for the Allocator Opportunity and Standard Deviation show that a skilled allocator can employ a larger number of funds while maintaining a greater ability to add value. Consider the following two portfolios for each time period:

Portfolio 1: $N = 40$, $\lambda = 20$

Portfolio 2: $N = 25$, $\lambda = 0$

As shown in Table 3, a skilled allocator for which $\lambda = 20$ can still have a greater ability to impact returns and Sharpe ratio with 40 funds than an unskilled allocator would have with 25 funds.

DEPENDENCE ON FUND SIZE

Data set and methodology

In this section, the universes were divided into large and small funds to test the hypothesis that a fund of funds allocator can have greater opportunity to impact results by selecting among smaller managers. We define large funds as those with an Assets under management greater than US\$500 million at the start of the period.

For this portion of the study, we analyzed only the two 5-year periods because the 10-year

Figure 4: Annualized returns with N and λ . (a) January 2002 – December 2006 universe; (b) January 2007 – December 2011 universe; (c) January 2002 – December 2011 universe.

Figure 4: Continued.

universe had a prohibitively small number of managers. We repeated the simulations of forming 5000 portfolios at each value of N and λ for large and small cap managers (Table 4).

Results

Figure 6 shows the annualized returns for large and small managers in 2002–2006. Figure 7 shows the annualized returns for 2007–2011. The data for large funds show a deviation from prior patterns when N equals or exceeds 40 because the universe size was 241 and 201, respectively, and the supply of top quartile funds becomes nearly exhausted as N is increased. With the exception of large manager portfolios for which N is 40 or greater, the average annualized returns are roughly independent of N for each λ and manager size. For the remainder of this

section, only portfolios with 40 or fewer funds are considered in analyses.

Figures 6 and 7 show that portfolios of large funds outperformed in the 2002–2006 period while the smaller funds outperformed in 2007–2011. For each time period, an incremental step in skill level has greater impact on mean returns for small managers than for large managers as shown in Table 5. The probability distributions for large and small managers show many qualitative similarities. As in the section on introduction of hedge fund selection skill, the mean exceeds the median for $\lambda = 20$ or 40 while the median exceeds the mean for $\lambda = 60$.

The annualized standard deviations for each time period are shown in Figures 8 and 9. For large and small funds, the standard deviation decreases with N for both time periods. A comparison of the time periods shows that for

Figure 5: Standard deviation for varying skill levels. (a) January 2007 – December 2011 universe; (b) January 2002 – December 2006 universe; (c) January 2002 – December 2011 universe.

Figure 5: Continued.

Table 3: Comparison of two portfolios ($N=40, \lambda=20$ versus $N=25, \lambda=0$)

Time period	2002–2006	2002–2006	2007–2011	2007–2011	2002–2011	2002–2011
Portfolio	$N=40$	$N=25$	$N=40$	$N=25$	$N=40$	$N=25$
	$\lambda=20$	$\lambda=0$	$\lambda=20$	$\lambda=0$	$\lambda=20$	$\lambda=0$
Mean return (%)	15.12	12.33	7.66	5.55	10.77	9.04
Mean SD (%)	5.93	5.43	8.43	9.07	8.71	8.15
AllOp1 (%)	3.51	2.68	2.94	2.27	2.09	1.69
AllOp2 (%)	17.19	6.65	13.18	5.73	10.36	4.42
Mean Sharpe Ratio	2.09	1.86	0.84	0.5	1.01	0.9
SharpeOp1	0.64	0.48	0.45	0.31	0.29	0.23
SharpeOp2	1.47	1.19	2.19	0.8	0.72	0.56

both large and small managers, standard deviation increased with skill in the first time period and decreased with skill in the second. In each time period, portfolios of smaller managers had higher mean standard deviations for a given N and λ .

Table 6 shows that the mean standard deviations for smaller managers were higher by a range from 78 to 139 bps from 2002 to 2006 and by 31 to 95 bps from 2007 to 2011 – fairly small increases. Table 4 shows that the average small

Table 4: Small cap versus large cap universes

	<i>Group 1</i>	<i>Group 2</i>	<i>Group 3</i>	<i>Group 4</i>
Time period	2002–2006	2002–2006	2007–2011	2007–2011
Manager size	Large	Small	Large	Small
# of managers	241	1302	201	1297
Fund avg. ann. return (%)	13.60	11.50	3.70	4.00
Final avg. SD (%)	9.40	11.70	14.00	17.00
Fund avg. Sharpe Ratio	1.4	0.9	0.3	0.34
Fund avg. largest drawdown (%)	–11.00	–15.60	–29.70	–31.80

fund experienced standard deviation that was 2.3 per cent (11.7 versus 9.4 per cent) higher than for the average large manager in the first time period and 3.0 per cent (17.0 versus 14.0 per cent) higher in the second time period. As shown in Figures 8 and 9, small manager portfolios with $N=10$ have already offset a large portion of the excess in standard deviation. Table 7 shows that portfolios of smaller managers have greater diversification benefits as N is increased from a baseline of 10 funds. Therefore, the additional standard deviation experienced by a portfolio of smaller managers declines with N .

DISCUSSION

Previous studies have concluded that small managers outperform large managers in aggregate as well as the converse. This study shows that, in fact, such conclusions are period-dependent. Isolated studies of either 5-year period would have reached opposing conclusions. However, this study does show that, over the 10-year period at a given N and λ , a fund of funds allocator has greater ability to impact performance when selecting among smaller

managers as compared to larger managers, with no exceptions.

The study also shows that an allocator that selects among smaller managers can allocate to a higher number of managers while maintaining a greater ability to impact results than an allocator that picks a smaller number of large managers.

Consider the following portfolios at the same skill level:

Portfolio 1: $N=35$ and $\lambda=0$, selecting small funds

Portfolio 2: $N=25$ and $\lambda=0$, selecting large funds

Portfolio 3: $N=35$ and $\lambda=20$, selecting small funds

Portfolio 4: $N=25$ and $\lambda=20$, selecting large funds

Table 8 shows a comparison of Portfolio 1 and Portfolio 2. From 2002 to 2006, an allocator had greater ability to impact returns by selecting 35 smaller funds than an allocator selecting 25 larger funds. From 2007 to 2011, the AllOp1, AllOp2, SharpeOp1 and SharpeOp2 are comparable for 35 smaller funds and 25 large funds.

Table 9 shows a comparison of Portfolio 3 and Portfolio 4. Once again, an allocator had greater ability to impact returns by selecting 35 smaller

Figure 6: 2002–2006 annualized returns for small and large managers. (a) January 2002 – December 2006 universe quartile small managers (<500MM); (b) January 2002 – December 2006 universe quartile large managers (>500MM).

Figure 7: 2007–2011 annualized returns for small and large managers. (a) January 2007 – December 2011 universe quartile small managers (<500MM). (b) January 2007 – December 2011 universe quartile large managers (>500MM).

Table 5: Change in mean annualized return from incremental skill

N	λ : 0–20% Large	λ : 0–20% Small	λ : 20–40% Large	λ : 20–40% Small	λ : 40–60% Large	λ : 40–60% Small
<i>Increase in mean return from one higher skill level (2002–2006)</i>						
10	2.41	2.84	2.34	2.84	2.39	2.63
15	2.41	2.82	2.36	2.70	2.43	2.79
20	2.56	2.86	2.18	2.78	2.43	2.67
25	2.40	2.89	2.56	2.84	2.34	2.53
30	2.35	2.91	2.51	2.60	2.26	2.80
35	2.53	2.81	2.24	2.58	2.56	2.86
40	2.37	2.85	2.41	2.91	2.43	2.50
<i>Increase in mean return from one higher skill level (2007–2011)</i>						
10	1.70	2.09	1.73	1.92	1.76	1.99
15	1.88	2.05	1.63	1.87	1.81	2.11
20	1.79	1.94	1.71	1.85	1.80	2.10
25	1.85	2.06	1.66	1.84	1.76	2.06
30	1.84	2.01	1.66	1.97	1.76	1.85
35	1.76	2.10	1.81	1.87	1.67	1.98
40	1.84	2.04	1.67	2.06	1.75	1.88

funds than an allocator selecting 25 larger funds from 2002 to 2006 and comparable ability from 2007 to 2011.

CAVEATS

The actual portfolio returns in the simulations performed for this article could not have actually been realized. Although we used real-world hedge fund returns, we did not account for real-world costs and constraints involved in hedge fund investing. Examples of these include but are not limited to liquidity, operations, capacity and monitoring. For example, liquidity constraints might have affected the ability to rebalance between funds and, if allowed, might have changed the realized returns. Operationally in the

real world, the timing of redemption payouts is variable, leading to underinvestment below 100 per cent or, at times, the use of leverage to match cash flows with subscriptions. Therefore, the practically possible return could be different from the theoretically modeled return. Capacity is periodically shut off by hedge fund managers, and this complicates rebalancing as well as the initial choice of hedge funds in creating our theoretical portfolios for our simulation. Pre-investment due diligence as well as ongoing monitoring costs for hedge funds add up as each involves numerous meetings, background checks and data analysis. These costs could change an allocator's decision about the number of hedge funds appropriate in a portfolio in ways not captured in this article. Finally, as we saw in the 'Random Sampling'

Figure 8: Standard deviation 2002–2006. (a) January 2002 – December 2006 universe quartile small managers (<500MM); (b) January 2002 – December 2006 universe quartile large managers (>500MM).

Figure 9: Standard deviation 2007–2011. (a) January 2007 – December 2011 universe quartile small managers (<500MM); (b) January 2007 – December 2011 universe quartile large managers (>500MM).

Table 6: Additional standard deviation for small manager portfolios small manager portfolios

<i>N</i>	0%	20%	40%	60%
<i>Difference in standard deviation (Small–Large) 2002–2006</i>				
10	1.10	1.21	1.34	1.39
15	0.99	1.09	1.16	1.20
20	0.89	0.99	1.12	1.18
25	0.90	0.98	1.01	1.06
30	0.83	0.94	0.93	1.04
35	0.82	0.86	0.92	0.99
40	0.78	0.87	0.87	0.98
<i>Difference in standard deviation (Small–Large) 2007–2011</i>				
10	0.69	0.78	0.88	0.95
15	0.50	0.68	0.66	0.80
20	0.44	0.57	0.64	0.71
25	0.41	0.47	0.53	0.61
30	0.37	0.44	0.56	0.64
35	0.32	0.42	0.58	0.62
40	0.31	0.40	0.40	0.59

Table 7: Diversification benefits (small versus large benefits (small versus large))

<i>N</i>	0% <i>Large</i>	0% <i>Small</i>	20% <i>Large</i>	20% <i>Small</i>	40% <i>Large</i>	40% <i>Small</i>	60% <i>Large</i>	60% <i>Small</i>
<i>Diversification benefits (SD (N = 10) – SD (N)) 2002–2006</i>								
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.27	0.38	0.28	0.40	0.27	0.46	0.29	0.48
20	0.40	0.61	0.39	0.60	0.43	0.66	0.46	0.68
25	0.50	0.70	0.53	0.76	0.47	0.81	0.51	0.85
30	0.53	0.80	0.58	0.85	0.54	0.95	0.60	0.95
35	0.60	0.87	0.58	0.93	0.61	1.03	0.59	1.00
40	0.62	0.94	0.65	0.98	0.63	0.99	0.66	1.08
<i>Diversification benefits (SD (N = 10) – SD (N)) 2007–2011</i>								
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.24	0.42	0.29	0.39	0.26	0.48	0.34	0.48
20	0.33	0.58	0.45	0.66	0.43	0.67	0.52	0.75
25	0.46	0.74	0.51	0.82	0.55	0.90	0.63	0.97
30	0.50	0.82	0.58	0.92	0.61	0.93	0.72	1.02
35	0.52	0.89	0.61	0.98	0.70	1.00	0.78	1.11
40	0.54	0.91	0.65	1.03	0.72	1.13	0.80	1.16

Table 8: Portfolios 1 and 2

Time period	2002–2006	2002–2006	2007–2011	2007–2011
Manager size	Small	Large	Small	Large
Manager #	35	25	35	25
Skill level	0	0	0	0
Mean return %	12.05	13.29	5.39	4.39
Mean SD %	5.42	4.69	9.14	8.89
AllOp1 %	2.32	2.10	1.87	1.85
AllOp2 %	5.60	5.00	4.65	4.70
Mean Sharpe Ratio	1.81	2.26	0.46	0.36
SharpeOp1	0.41	0.36	0.25	0.24
SharpeOp2	1.01	0.91	0.63	0.59

Note: Bolded measures are referred to in the text.

Table 9: Portfolios 3 and 4

Time period	2002–2006	2002–2006	2007–2011	2007–2011
Manager size	Small	Large	Small	Large
Manager #	35	25	40	25
Skill level	20	20	20	20
Mean return %	14.86	15.70	7.49	6.24
Mean SD %	6.10	5.29	8.77	8.46
AllOp1 %	3.85	3.31	3.27	3.28
AllOp2 %	17.39	15.16	12.46	11.83
Mean Sharpe Ratio	1.99	2.4	0.78	0.66
SharpeOp1	0.65	0.55	0.47	0.44
SharpeOp2	1.52	1.19	1.98	1.81

Note: Bolded measures are referred to in the text.

section, where the risk/return trade-off fundamentally reversed in different time periods, we caution against extending the conclusions below very generally, but do believe they are helpful guides for allocators in the real world.

CONCLUSION

Naïvely the volatility of a hedge fund portfolio decreases as the number of managers increases. Besides the practical constraints outlined above,

we conclude that allocator opportunity combined with a desired return can serve as a limiting factor to the number of hedge funds used in a portfolio. Using this framework, we also conclude that *ceteris paribus*, the small manager hedge fund space provides a larger opportunity set versus the large manager universe of hedge funds. In addition, we conclude that small manager portfolios can have more managers as compared to large manager portfolios without sacrificing an allocator's opportunity to add

value. Finally we conclude that highly skilled allocators can have more managers than average allocators while still retaining the ability to outperform.

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APPENDIX

Table A1 shows the relationship between number of managers and the sample mean as well as other defined metrics for Period 3. Results for the other two 5-year periods not shown here show substantially the same relationship.

Mean return is basically invariant as the number of managers increases, which highlights that the sampling technique was adequately random. AllOp1, AllOp2, Mean SD, SDD1, SDD2, SharpeOp1, SharpeOp2, Avg Var99, Var99Op1 and Var99Op2 decrease as the number of managers increases. The rate of

decline of all these variables decreases as the number of managers increases, which shows the declining marginal benefit *vis-à-vis* these variable as the number of managers increases.

Table A2 shows AllOp1 for Period 3 for varying N and λ .

The allocator opportunity increases with increasing skill. In addition, the marginal increase in AllOp1 is significantly larger for an increase in λ from 20 to 40 per cent than for a similar increase from 0 to 20 per cent. For $\lambda = 0$ per cent or 20 per cent, AllOp1 declines with increasing N while for $\lambda = 40$ per cent or 60 per cent, the AllOp1 actually increases slightly with N . In each time period, the AllOp1 for $N = 50$ and $\lambda = 20$ per cent is higher than that for $N = 20$ and $\lambda = 0$ per cent.

Table A3 shows the change in standard deviation with N and λ for period 3. In general, standard deviation increases with skill level.

The allocator opportunity metrics for portfolios of large and small managers are shown in Tables A4 and A5. At any given level of N and λ , AllOp1 and AllOp2 were greater during 2002–2006. AllOp1 and AllOp2 were greater for portfolios of small managers than for large managers. For $\lambda = 0$ or 20, AllOp1 decayed by roughly 50 per cent as N was increased from 10 to 40. For higher levels of skill, AllOp1 did not decay with N .

The Sharpe ratios behave in a similar qualitative manner for both small and large managers. Table A6 shows that Sharpe ratio Opportunity metrics (SharpeOp1 and SharpeOp2) are always higher for smaller managers.

Table A1: 2002–2011 statistics

<i>N</i>	<i>Mean</i> <i>return</i> (%)	<i>AllOp1</i> (%)	<i>AllOp2</i> (%)	<i>Mean</i> <i>SD</i> (%)	<i>SDD1</i> (%)	<i>SDD2</i> (%)	<i>Mean</i> <i>Sharpe</i>	<i>SharpeOp1</i>	<i>SharpeOp2</i>	<i>Avg.</i> <i>VarR99</i> (%)	<i>Var99Op1</i> (%)	<i>Var99Op2</i> (%)
10	9.0	2.6	6.6	8.8	2.79	6.99	0.85	0.32	0.81	−6.35	3.01	7.14
15	9.0	2.2	5.4	8.4	2.40	5.78	0.88	0.28	0.70	−6.20	2.69	6.37
20	9.1	1.9	4.7	8.3	2.03	4.83	0.89	0.24	0.61	−6.20	2.31	5.56
25	9.0	1.7	4.1	8.2	1.81	4.42	0.90	0.23	0.56	−6.18	2.11	5.26
30	9.1	1.5	3.8	8.1	1.63	4.06	0.91	0.21	0.52	−6.17	1.87	4.68
35	9.1	1.5	3.5	8.0	1.58	3.73	0.92	0.20	0.50	−6.19	1.83	4.41
40	9.1	1.4	3.3	8.0	1.43	3.44	0.92	0.19	0.46	−6.21	1.69	4.13
45	9.1	1.2	3.1	7.9	1.33	3.19	0.92	0.17	0.42	−6.19	1.61	3.94
50	9.1	1.1	2.8	7.9	1.25	3.11	0.93	0.17	0.40	−6.19	1.49	3.68
55	9.1	1.1	2.7	7.9	1.21	2.88	0.93	0.16	0.38	−6.20	1.41	3.44
60	9.1	1.1	2.6	7.9	1.18	2.78	0.93	0.15	0.37	−6.20	1.35	3.33

Table A2: AllOp1 (*N* versus λ)

<i>AllOp1 (2002–2011)</i>				
<i>N</i>	0%	20%	40.0%	60.0%
10	2.64	4.47	8.23	8.09
15	2.16	3.68	8.39	8.13
20	1.93	2.99	8.27	8.15
25	1.69	2.85	8.15	8.21
30	1.51	2.47	8.07	8.15
35	1.45	2.38	8.00	8.21
40	1.36	2.09	7.99	8.19
45	1.20	2.15	7.93	8.19
50	1.15	1.95	7.90	8.20
55	1.10	1.81	7.88	8.18
60	1.06	1.75	7.87	8.19

Table A3: Standard deviation (*N* versus λ)

<i>SD (2002–2011)</i>				
<i>N</i>	0%	20%	40.0%	60.0%
10	8.75	9.48	10.24	10.97
15	8.39	9.16	9.81	10.54
20	8.27	8.94	9.63	10.35
25	8.15	8.85	9.58	10.26
30	8.07	8.76	9.46	10.17
35	8.00	8.75	9.47	10.09
40	7.99	8.71	9.39	10.09
45	7.93	8.68	9.38	10.10
50	7.90	8.65	9.36	10.03
55	7.88	8.62	9.37	9.99
60	7.87	8.57	9.26	9.97

Table A4: AllOp1

<i>N</i>	<i>0%</i> <i>Large</i>	<i>0%</i> <i>Small</i>	<i>20%</i> <i>Large</i>	<i>20%</i> <i>Small</i>	<i>40%</i> <i>Large</i>	<i>40%</i> <i>Small</i>	<i>60%</i> <i>Large</i>	<i>60%</i> <i>Small</i>
<i>AllOp1 (2002–2006)</i>								
10	3.6	4.1	5.8	7.9	11.6	13.2	11.6	13.4
15	2.8	3.4	5.0	6.2	11.7	13.3	11.9	13.3
20	2.4	3.0	4.0	5.5	11.8	13.4	11.8	13.5
25	2.1	2.8	3.3	4.8	11.9	13.4	11.6	13.5
30	1.9	2.5	3.0	4.1	12.0	13.5	11.7	13.8
35	1.7	2.3	2.9	3.8	12.0	13.5	11.6	13.6
40	1.6	2.1	2.5	3.6	12.1	13.6	11.6	13.8
<i>AllOp1 (2007–2011)</i>								
10	2.8	3.4	5.1	5.3	8.0	9.3	7.9	9.1
15	2.3	2.8	4.4	4.5	8.1	9.4	8.2	9.1
20	2.1	2.4	3.7	4.0	8.2	9.6	8.4	9.3
25	1.9	2.3	3.3	3.6	8.4	9.6	8.5	9.4
30	1.7	2.1	2.9	3.2	8.7	9.6	8.7	9.4
35	1.6	1.9	2.5	3.3	8.7	9.6	8.7	9.4
40	1.4	1.7	2.5	2.8	8.9	9.6	8.5	9.5

Note: Bolded values highlight AllOp1 for large 25 manager portfolios compared to small 40 manager portfolios.

Table A5: AllOp2

<i>N</i>	<i>0%</i> <i>Large</i>	<i>0%</i> <i>Small</i>	<i>20%</i> <i>Large</i>	<i>20%</i> <i>Small</i>	<i>40%</i> <i>Large</i>	<i>40%</i> <i>Small</i>	<i>60%</i> <i>Large</i>	<i>60%</i> <i>Small</i>
<i>AllOp2 (2002–2006)</i>								
10	8.5	10.5	17.3	20.3	18.0	22.6	18.3	23.0
15	6.7	8.5	16.1	19.2	16.7	21.1	16.9	21.3
20	5.9	7.5	15.6	18.5	15.8	19.8	15.8	20.3
25	5.0	6.6	15.2	18.3	15.3	19.4	15.2	19.6
30	4.6	6.1	14.7	17.9	14.8	19.0	14.7	19.1
35	4.1	5.6	14.2	17.8	14.3	18.5	14.1	18.5
40	3.7	5.2	13.9	17.4	13.8	18.2	13.6	18.5
<i>AllOp2 (2007–2011)</i>								
10	8.0	8.9	13.2	15.0	15.0	15.9	15.3	16.2
15	6.0	7.2	12.8	14.3	13.6	14.7	13.7	14.9
20	5.2	6.4	12.1	13.7	12.7	14.3	12.8	14.3
25	4.7	5.7	11.8	13.5	12.0	13.8	11.9	13.7
30	4.2	5.1	11.4	13.0	11.5	13.4	11.4	13.3
35	3.9	4.7	11.0	12.7	11.0	13.0	10.9	13.2
40	3.6	4.4	10.7	12.5	10.7	12.8	10.6	12.9

Note: Bolded values highlight AllOp2 for large 25 manager portfolios compared to small 40 manager portfolios.

Table A6: SharpeOp1

<i>N</i>	0% <i>Large</i>	0% <i>Small</i>	20% <i>Large</i>	20% <i>Small</i>	40% <i>Large</i>	40% <i>Small</i>	60% <i>Large</i>	60% <i>Small</i>
<i>SharpeOp1 (2002–2006)</i>								
10	0.54	0.61	0.68	0.79	0.74	0.90	0.67	0.82
15	0.45	0.54	0.62	0.75	0.69	0.89	0.65	0.82
20	0.41	0.50	0.58	0.72	0.69	0.93	0.62	0.83
25	0.36	0.46	0.55	0.70	0.68	0.87	0.61	0.85
30	0.32	0.44	0.51	0.67	0.67	0.88	0.61	0.86
35	0.31	0.41	0.53	0.65	0.67	0.88	0.59	0.84
40	0.29	0.39	0.46	0.62	0.68	0.88	0.58	0.85
<i>SharpeOp1 (2007–2011)</i>								
10	0.36	0.42	0.66	0.74	1.22	1.15	1.23	1.23
15	0.29	0.37	0.57	0.65	1.29	1.25	1.31	1.29
20	0.26	0.32	0.49	0.59	1.34	1.31	1.35	1.39
25	0.24	0.30	0.44	0.55	1.38	1.36	1.39	1.42
30	0.22	0.28	0.39	0.48	1.41	1.37	1.42	1.42
35	0.20	0.25	0.34	0.47	1.44	1.38	1.43	1.46
40	0.19	0.24	0.34	0.41	1.46	1.43	1.43	1.47

Note: Bolded values highlight SharpeOp1 for large 25 manager portfolios compared to small 40 manager portfolios.