Original Article

Dual directional structured products

Received (in revised form): 7th May 2014

Geng Deng
is the Director of Research at Securities Litigation & Consulting Group, Inc. He specializes in the fields of derivatives, credit modeling, structured finance, mutual fund and hedge fund investment analysis. He holds a PhD in Operations Research, an MA in Mathematics and an MS in Statistics, all from the University of Wisconsin – Madison. He also holds a BS in Applied Mathematics from Tsinghua University in China. He was the recipient of several best paper awards when he was in graduate school. He holds a Financial Risk Manager (FRM) designation and is a CFA charterholder.

Tim Dulaney
is a Senior Financial Economist at Securities Litigation & Consulting Group. He has coauthored several papers involving the modeling and valuation of complex securities, bank deposits and insurance products. He earned a PhD in Theoretical Physics from the California Institute of Technology and graduated Summa Cum Laude from the University of Maryland with a BS in Physics and BS in Mathematics.

Tim Husson
is a Senior Financial Economist at Securities Litigation & Consulting Group. He has experience analyzing and valuing complex financial products including annuities, trusts, swaps, synthetic and funded CDOs, structured products, REITs, volatility derivatives, hedge funds, and complex exchange-traded products. He holds a BA (with Honors) and PhD in Computational Neuroscience from the University of Chicago. His doctoral work focused on the development and applications of a novel neural imaging system.

Craig McCann
is experienced in securities class action litigation, financial analysis, investment management and valuation disputes. He has taught graduate investment management at Georgetown University and at the University of Maryland, College Park. Prior to founding Securities Litigation and Consulting Group, he was the Director at LECG and Managing Director, Securities Litigation at KPMG. He was a senior financial economist at the Securities and Exchange Commission where he focused on investment management issues and contributed financial analysis to numerous investigations involving alleged insider trading, securities fraud, personal trading abuses and broker–dealer misconduct.

Correspondence: Craig McCann, 3998 Fair Ridge Drive Suite 250, Fairfax, VA 22033, USA
E-mail: craigmccann@slcg.com

ABSTRACT **We analyze and value dual directional structured products – or simply dual directionals (DDs) – which have been issued in large amounts since the beginning of 2012. DDs evolved out of another type of structured product called absolute return barrier notes; however, DDs lack principal protection and have different embedded options positions, which are yet to be described in the literature. We find that DDs can be broadly organized into two categories: single observation dual directionals and knock-out dual directionals. We determine the appropriate option decomposition for these categories and provide analytical formulas for their valuation. We confirm our analytic results using Monte Carlo simulation and use both techniques to value a large sample of DDs registered with the**

Securities and Exchange Commission up to December 2012. Our results indicate that like many types of structured products, DDs tend to be priced at a significant premium to present value across issuers and underlying securities and that the present value of the decomposition is smaller than the face value net of commissions. We find that DDs with embedded leverage or a single observation feature tend to be worth less than products either without leverage or with a knock-out option.

Journal of Derivatives & Hedge Funds (2014) **20,** 99–112. doi:10.1057/jdhf.2014.12;
published online 5 June 2014

Keywords: structured products; structured notes; dual directional; valuation; Monte Carlo

INTRODUCTION

Structured products are complex debt securities whose payoffs are linked to the value of reference stocks, indices, commodity prices, interest rates or exchange rates. While structured product issuances have grown over the past decade, they have also become more complex (McCann and Luo, 2006; Henderson and Pearson, 2010), especially by exposing investors to all, some or none of the underlying security's downside risk. Structured products often have discontinuous payoffs that would typically only be available to derivatives traders and other sophisticated investors trading complex options.

Dual directional structured products – or simply dual directionals (DDs) – can yield positive payoffs if the underlying security's return over the life of the note is positive or if it is negative within a certain range. However, if the gains or losses are outside of that range, the investor's gains are capped on the upside and the investor can lose much or all of his/her investment on the downside.

DDs embed what is known to options traders as a 'straddle' position on the underlying security. Straddle positions are taken by buying both an at-the-money (ATM) call and an ATM put. An options trader with a straddle position then achieves a payout that is a function of the absolute value of the return – essentially profiting from a deviation from current levels, whether positive or negative. Straddles do not reflect a directional bet on future price movements, but instead are a bet on future *volatility*. DDs differ from pure straddle positions in that gains in the value of the underlying security above the range of the straddle payoff are generally capped, and losses in the value of the underlying security beyond that range lead to losses for the DD investor, including perhaps a total loss of principal.

DDs are similar to another type of structured product, absolute return barrier notes (ARBNs), and occasionally still carry this brand name. The structure of ARBNs before 2010 featured principal protection and returned the the absolute value of the underlying asset return within a certain range. While widely issued from 2007 to 2009, ARBNs disappeared from the market for 18 months in 2010–2011. When they returned in late 2011, they had a new payout structure that combined some aspects of the conventional structure with features of buffered leveraged products – for example, buffered PLUSes. The new structure dropped the principal protection and capped the upside returns while keeping the exposure to the absolute value of the underlying return, making ARBNs of today identical in structure to DDs.

The literature has shown that structured products are generally overpriced even beyond the high commissions disclosed in the filings. Deng *et al* (2010) report that the average fair value of reverse convertibles was just 93 per cent of the offering price. These findings are consistent with those of Hernández *et al* (2007), Szymanowska *et al* (2009) and Henderson and Pearson (2010). Similarly, our analysis of ARBNs in Deng *et al* (2011) finds that the average fair value of ARBNs in our sample was approximately 95 per cent of the offering price.

DDs were first issued in May 2008 and continued to be released sporadically over the next few years. In early 2012, new issues of DDs increased significantly and it is this renewed interest that motivates this study. In this article, we analyze and value more than 100 DDs issued up to December 2012. We find that, like many varieties of structured products, the present value of DDs tends to be significantly below par, and is critically dependent on a variety of risk factors that may not be transparent or easily understood by retail investors.

FEATURES OF DDs

General features of structured products

Structured products are debt securities issued primarily by banks and sold through brokers to retail or institutional investors.[1] Structured products have complex payouts that are linked to the return of a particular security, index or basket of securities. There are now a wide variety of structured product types, but most can be thought of as combinations of option positions. Structured products can be customized in size, maturity, underlying security and a variety of other features. These products often pay substantial commissions, which has helped make them a US$100 billion global industry.[2]

As debt securities, structured products are subject to the credit risk of the issuer. Also, most structured products are linked to price appreciation in their underlying assets, but not total returns, essentially forgoing dividend payments. For most structured products there is no public secondary market, and liquidity can be low over very long terms to maturity. Structured products' complexity makes valuation difficult, but closed-form solutions exist for many structured product types and many others can be valued with other approaches, such as Monte Carlo simulation (Deng *et al*, 2012).

Dual directional structured products

DDs are structured products that do not issue coupons and whose value derives from the payout at maturity dependent on an underlying security or index. Investors in DDs participate in the returns of the underlying asset at some level if the asset increases in value. DDs typically have a return that corresponds to leveraged or unleveraged exposure to the positive return of an underlying asset, subject to a cap. Investors also realize a positive return if the underlying asset decreases in value within a prespecified range – up to the barrier level – and lose principal if the asset decreases in value outside of that range. The payout diagram for a typical unleveraged DD is shown in Figure 1a and for a typical leveraged DD in Figure 1b .

DDs can be generally categorized as single observation (SODDs) or knock-out (KODDs). SODDs have a payout that depends only on the underlying asset's value at the final valuation date – usually less than 2 weeks before the maturity of the notes. Figure 1c offers a graphical depiction of such a final payout. SODDs sometimes offer a buffered exposure to downside returns such that returns beyond the lower barrier are not applied to the principal investment on a one-to-one basis.

KODDs, on the other hand, have a trigger feature such that depreciation of the underlying asset beyond the barrier level removes the possibility of positive returns on the note if the asset has depreciated in value as of the final observation date.[3] Within our sample, no KODDs offer buffered exposure to negative returns of the underlying asset. KODDs will be less valuable than SODDs with otherwise identical product features.

Importantly, the V-shaped region in DD payoff diagrams represents exposure to the absolute value of returns within that narrow range – essentially, the investor receives a payment that is dependent on the deviation of the underlying price from its initial value, also known as its 'realized volatility' over the term of the note. This volatility dependence could be advantageous for investors who are neither particularly bearish nor bullish on the underlying asset.

The market for DDs

We have collected all of the DD issuances before December 2012 from their 424B2 filings with the Securities and Exchange Commission. We performed a full-text search of all 424B2s using the third-party service EDGAR Pro (pro.edgar-online.com/). We searched for filings that contained the strings 'Dual Directional' or 'Absolute Value', which were not autocallable and did not protect principal. This search resulted in more than 500 filings that met the full-text search criteria. After removing preliminary filings and false positives from this set, we were left with 125 DDs issued between May 2008 and December 2012.[4]

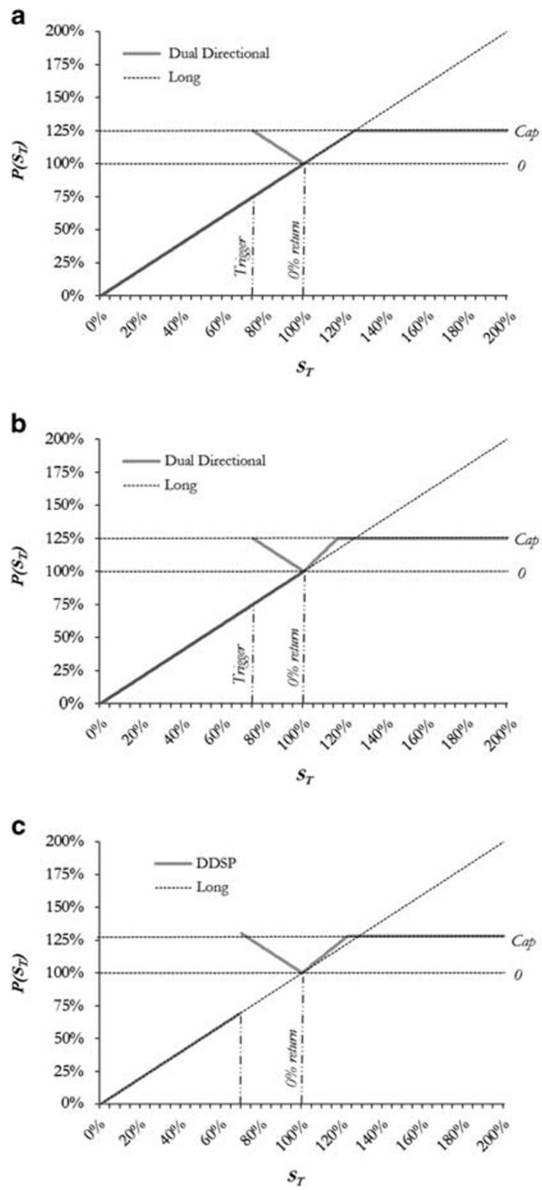

Figure 1: Payout diagrams for an unleveraged DD, a 1.5 leveraged DD and a buffered DD; the DDs have a cap of 125 per cent and a trigger of 75 per cent. (**a**) Unleveraged DD payout diagram; (**b**) leveraged DD payout diagram; (**c**) single observation with barrier.

JP Morgan Chase sold the first DD in May 2008 and followed this offering with nearly a dozen more amounting to nearly $74 million in DDs in the summer of 2008. Soon after several other issuers began offering DDs. In our sample, the major issuers of DDs were Goldman Sachs, JP Morgan Chase, Morgan Stanley and Credit Suisse. Together these issuers account for more the 80 per cent of the aggregate issue size for all DDs. Figure 2 includes a more detailed summary of the market segmented by issuer.

The assets underlying the DDs have varied over time, but the primary underlying assets have been the S&P 500 index and Apple stock (AAPL). Figure 3 includes a more detailed summary of the market segmented by underlying asset.

We can further break down the issuance of DDs into the types of DDs offered by these issuers. Table 1 categorizes the number of Single Observation DDs by issuer and gives the average issue size for single observation DDs.

Fewer knock-out DDs have been issued than single observation DDs – 47 knock-out DDs were issued compared with 78 single observation DDs. Table 2 summarizes the issuance of these structured products before December 2012.

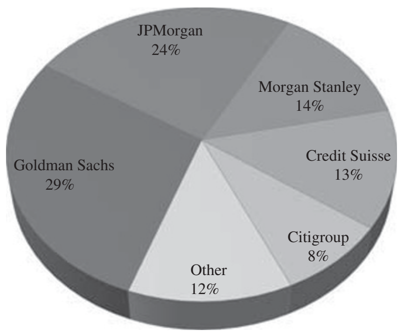

Figure 2: DD market summary by issuer based on issue size.

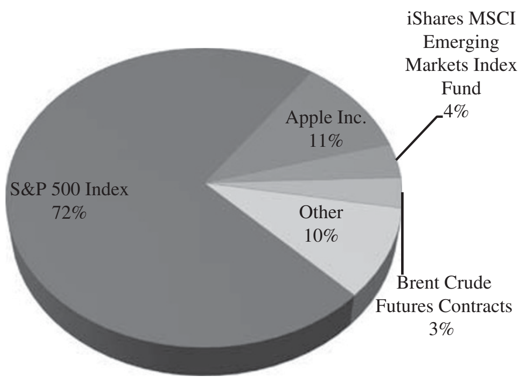

Figure 3: DD market summary by underlying index based on issue size.

Table 1: Number and average issue size of single observation DDs within our sample

Issuer	*DDs with leverage*	*DDs without leverage*	*Average issue size (in million dollars)*
JP Morgan Chase	15	9	7.1
Morgan Stanley	14	5	8.2
Credit Suisse	7	4	11.8
Citigroup	10	0	9.6
Other issuers	8	6	6.8

Table 2: Number and average issue size of knock-out DDs within our sample

Issuer	*DDs with leverage*	*DDs without leverage*	*Average issue size (in million dollars)*
Goldman Sachs	3	8	29.5
JP Morgan Chase	0	16	6.5
Credit Suisse	3	5	2.3
Other issuers	1	11	3.9

Table 3: Average parameter values within our sample of DDs

Parameter	*SODDs*		*KODDs*	
	Leveraged	*Unleveraged*	*Leveraged*	*Unleveraged*
Commission	1.6%	1.4%	1.5%	1.3%
Maximum Return	24%	21%	50%	24%
Lower barrier	−16%	−21%	−31%	−28%
Issue size (millions)	$7.2	$10.7	$3.3	$11.8
Term (years)	1.5	1.4	2.0	1.3

Finally, Table 3 summarizes the average parameter values for each category of DDs issued before December 2012. The leveraged version of DDs generally have a higher commission and a longer term in years when compared with their unleveraged counterparts. Unleveraged DDs tend to be issued in larger sizes.

OPTION DECOMPOSITION OF DDs

Like most structured products, DDs can be decomposed into zero-coupon bonds, barrier options, binary options and European options. The precise options required depend on the particular product features, but broadly speaking the decomposition falls into one of two categories: KODDs and SODDs.

Knock-out dual directionals

KODDs combine a zero-coupon bond with the same maturity as the structured product with at least four option positions. Although the choice of decomposition is not unique, one choice is a (i) long an ATM put, (ii) short two knock-in puts, (iii) long an ATM call and (iv) short an out-of-the-money (OTM) call. Figure 4 graphically depicts this decomposition. The knock-in level of the puts sets the lower barrier and the strike price of the OTM call sets the upper trigger. If the DD leverages the positive returns of the underlying below the maximum return, then the number of call options should be increased proportionally.

Single observation dual directionals

SODDs combine a zero-coupon bond with at least five option positions. One choice is (i) long an ATM put, (ii) short asset-or-nothing (AON) puts, (iii) short OTM puts, (iv) long an ATM call and (v) short an OTM call. Figure 5 graphically depicts this decomposition. The AON binary puts give the discontinuity around the lower barrier and the strike price of the OTM call sets the upper trigger. By changing the number of put options, a buffer can be included on the negative returns beyond the trigger level. In an identical fashion to the KODDs, leverage on positive returns can be included by increasing the number of call options proportionally. Leveraging the upside in the straddle effectively makes the product look more like a strap than a straddle.

VALUATION OF DDs

Model

Our valuation of DDs follows the Black–Scholes model with risk-neutral

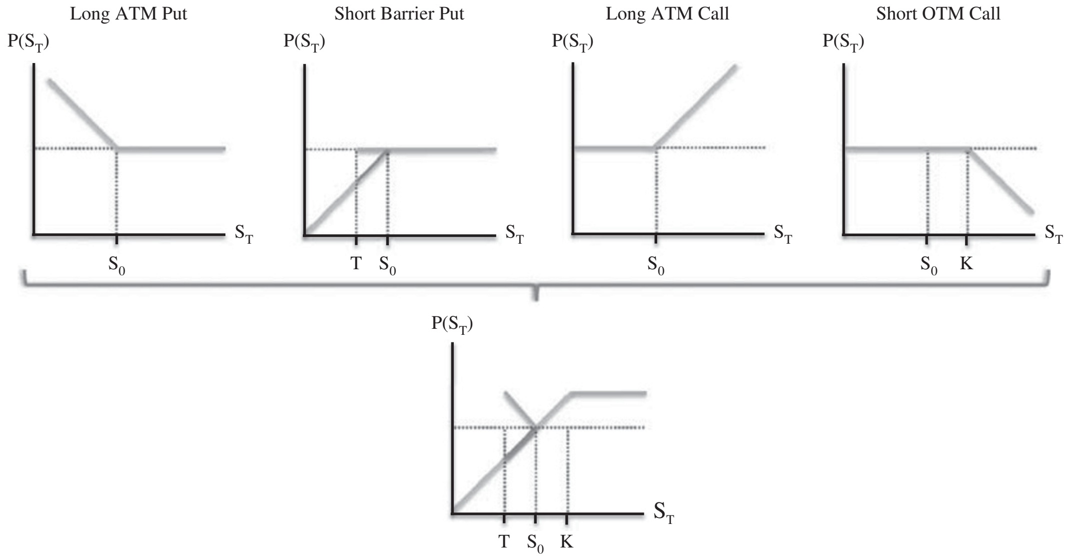

Figure 4: Option decomposition of a KODD.

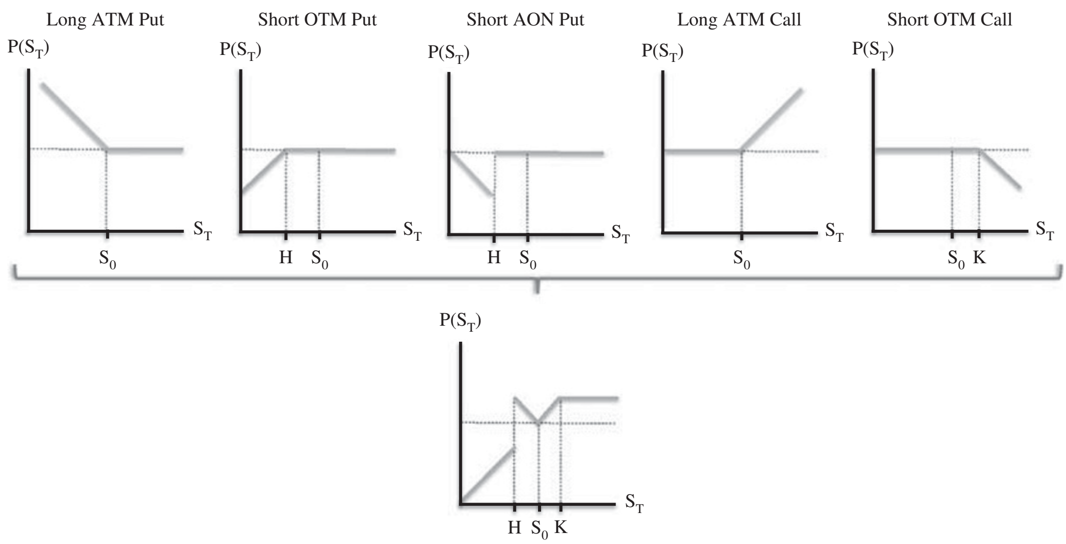

Figure 5: Option decomposition of a SODD.

assumptions (Black and Scholes, 1973; Merton, 1973). The reference asset's price is a generalized Brownian motion

$$dS_t = (r - q)S_t dt + \sigma S_t dW_t \quad (1)$$

where r is the risk-free rate, q is the dividend yield and σ is the volatility of the price process.[5] If we assume the price of the structured product $V(S, t)$ is a function of time $t \in [0, T]$ and the reference asset's price $S \in [0, \infty)$, the Black–Scholes formula implies that a structured product's dynamic value can be expressed as the following partial differential equation (PDE):

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+(r-q)S\frac{\partial V}{\partial S}-(r+\bar{c})V = 0 \qquad (2)$$

where $\bar{c}$ is the credit default swap (CDS) spread on the contemporaneously issued debt by the issuer with a similar tenor to the structured product. Structured products are unsecured debt securities, and hence lose value if the issuer defaults. It is therefore essential to include the issuer's credit risk $\bar{c}$ in the PDE to calculate the structured product's present value. Jarrow and Turnbull (1995), Baule and Tallau (2011) and Hull (2011) discuss the adjustment of derivative valuations for counterparty credit risk. Credit risk affects the prices of structured products as it does other forms of corporate debt. This is reflected anecdotally by the fact that prices of Bear Stearns debt dropped significantly in March 2008, and bounced back when JP Morgan Chase announced its acquisition.[6]

Option decomposition of DDs

Let $P_{di}(S_0; H, K, T)$ represent the value of a down-and-in put option with strike K and barrier H expiring T years from now on an asset with initial price S_0. Similarly let $P_c(S_0; K, T)$ be a European call option and $P_p(S_0; K, T)$ be a European put with strike K expiring in T years on an asset with initial price S_0.

Assume the KODD matures in T years and that a positive return during the term of the note is leveraged by a factor L.[7] The value of a \$1000 face-value KODD with cap K, lower trigger price H and valuation date asset value S_0 is given by

$$\begin{aligned} V(S_0;\, K, H,\, L,\, T) &= \$\,1000\, e^{-(r+\bar{c})T} + L\frac{\$\,1000}{S_0}e^{-\bar{c}T} \\ &\quad (P_c(S_0;\, S_0,\, T) - P_c(S_0;\, K,\, T)) \\ &\quad + \frac{\$\,1000}{S_0}e^{-\bar{c}T}\big(P_p(S_0;\, S_0,\, T) \\ &\quad - 2P_{di}(S_0;\, H,\, S_0,\, T)\big) \end{aligned} \qquad (3)$$

where $\bar{c}$ is the CDS rate of the issuer and the strike of the down-and-in put option is S_0. For a similar decomposition of a reverse exchangeable, see Hernández *et al* (2007).

The option decomposition of SODDs follows a very similar form to KODDs. The structure of a SODD is identical to a KODD if the linked asset appreciates in value during the term of the note. On the downside, as there is no knock-out feature the decomposition of SODDs involves binary options instead of barrier options.

Assume the SODD matures in T years and that a positive return during the term of the note is leveraged by a factor L. Assume that note is buffered such that depreciation beyond the lower trigger is leveraged by a factor D against the principal investment. The value of a \$1000 face-value SODD with cap K, lower trigger price H and valuation date asset value S_0 is given by

$$\begin{aligned} V_{S_0}(S_0; K,\, H,\, L,\, T) &= \$\,1000\, e^{-(r+\bar{c})T} + L\frac{\$\,1000}{S_0}e^{-\bar{c}T} \\ &\quad (P_c(S_0;\, S_0,\, T) - P_c(S_0;\, K,\, T)) \\ &\quad + \frac{\$\,1000}{S_0}e^{-\bar{c}T}\Bigg(P_p(S_0;\, S_0,\, T) \\ &\quad - \frac{2S_0}{H}P_p(S_0;\, H,\, T) + \left((D+1) - \frac{2S_0}{H}\right) \\ &\quad P_{AoNP}(S_0;\, H,\, T)\Bigg) \end{aligned} \qquad (4)$$

where P_{AoNP} is an AON (binary) put option that delivers one unit of the underlying asset if the asset's spot is below the strike at maturity.

Option valuation

DDs contain either a barrier option or a binary option. Barrier options can can be valued using the reflection lemma in Carr and Chou (1997a). In fact, Carr and Chou (1997b) give a formula for down-and-out contingent claims in terms of an equivalent payoff involving European options.[8]

We have written the value of KODDs in terms of a single barrier option – a down-and-in put option. Let $P_{di}(S_0; H, K, T)$ represent the value of a down-and-in put option with strike K, barrier H expiring T years from now on an asset with current price S_0. Assume that the underlying asset has a log-normal return distribution with mean $\mu = r - q - \sigma^2/2$ and variance σ^2. We assume that the risk-free rate and dividend yield are constant and continuously compounded and that the variance is constant as well. The option pays off like a vanilla put option if the barrier H has been hit before the option's expiration.

Following Hull (2011), the present value of a down-and-in put option is given by

$$\begin{aligned}
&P_{di}(S_0; H,\ K,\ T) \\
&\quad = Ke^{-rT}\Phi\left(\frac{\log\left(\frac{H}{S_0}\right) - \mu T}{\sigma\sqrt{T}}\right) \\
&\quad - S_0 e^{-qT}\Phi\left(\frac{\log\left(\frac{H}{S_0}\right) - (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right) \\
&\quad + S_0 e^{-qT}\left(\frac{H}{S_0}\right)^{2\frac{\mu+\sigma^2}{\sigma^2}} \\
&\quad \left[\Phi\left(\frac{\log\left(\frac{H^2}{S_0 K}\right) + (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right)\right. \\
&\quad \left. - \Phi\left(\frac{\log\left(\frac{H}{S_0}\right) + (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right)\right] \\
&\quad - Ke^{-rT}\left(\frac{H}{S_0}\right)^{2\frac{\mu}{\sigma^2}}\left[\Phi\left(\frac{\log\left(\frac{H^2}{S_0 K}\right) + \mu T}{\sigma\sqrt{T}}\right)\right. \\
&\quad \left. - \Phi\left(\frac{\log\left(\frac{H}{S_0}\right) + \mu T}{\sigma\sqrt{T}}\right)\right] \qquad (5)
\end{aligned}$$

where Φ is the standard normal cumulative distribution function assuming that the knock-in level is lower than the strike price. For the down-and-in put options embedded in DDs, the strike is always equal to the initial asset price and the knock-in level is less than the strike ($H < K = S_0$).

For completeness, the present value of a European call option is given by the conventional formula

$$\begin{aligned}
P_c(S_0;\ K,\ T) &= S_0 e^{-qT}\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right) \\
&\quad - Ke^{-rT}\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + \mu T}{\sigma\sqrt{T}}\right) \qquad (6)
\end{aligned}$$

while the present value of a European put option has the following conventional formula

$$\begin{aligned}
P_p(S_0;\ K,\ T) &= Ke^{-rT}\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - \mu T}{\sigma\sqrt{T}}\right) \\
&\quad - S_0 e^{-qT}\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right) \qquad (7)
\end{aligned}$$

The present value of a European AON put option is given by

$$\begin{aligned}
&P_{AoNP}(S_0; K,\ T) \\
&\quad = S_0 e^{-qT}\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - (\mu + \sigma^2)T}{\sigma\sqrt{T}}\right) \qquad (8)
\end{aligned}$$

With Equations (5)–(8) and the decomposition in Equation (3) for KODDs and in Equation (4) for SODDs, we have an analytic formula to value

DDs. In the next section, we will compute the value of a few example DDs using parameters from 424B2s filed with the Securities and Exchange Commission (SEC). Extending this framework to include stochastic volatility – such as in the variance gamma model – is trivial given our previous work on the subject Deng *et al* (2013), and would not change the qualitative results.

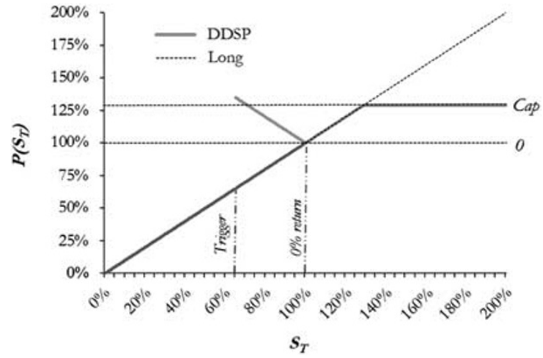

Figure 6: Unleveraged KODD issued by JP Morgan Chase on 21 November 2012 linked to iShares MSCI emerging markets index fund.

EXAMPLE VALUATIONS OF DDs

KODD without leverage

On 21 November 2012, JP Morgan Chase issued an unleveraged \$8.4 million DD linked to iShares MSCI Emerging Markets Index Fund (CUSIP: 48125VCS7) (www.sec.gov/Archives/edgar/data/19617/000119312511317247/d258311d424b2.htm). The lower barrier becomes effective when the underlying asset (ticker: EEM) experiences a 35 per cent decrease in value and the cap on the upside return becomes effective at 29 per cent. The underwriting fee on this note was 3 per cent. The final observation date for the note is 16 May 2013. The payout diagram for this example product is shown in Figure 6.

As of the pricing date, EEM's dividend yield was 2.15 per cent, its price was \$39.39, and the 1-year put implied volatility of the fund was 35 per cent. The 1-year swap rate (USSW1) was 0.72 per cent and the 2-year swap rate (USSW2) was 0.76 per cent. We linearly interpolated these rates to produce a risk-free rate of 0.74 per cent. JP Morgan Chase's 1-year CDS rate as of the pricing date was 0.95 per cent.

We find that these EEM-linked DD notes were worth \$943.85 per \$1000 when issued by JP Morgan Chase. The net proceeds to the issuer was \$970.00, and therefore the premium JP Morgan Chase charged for the product was \$26.15 (\$970.00−\$943.85). As a result of this issuance, JP Morgan Chase priced in a gross profit of \$219 625.07.[9]

KODD with leverage

On 21 August 2012, Credit Suisse issued a leveraged knock-out \$1 million DD (CUSIP: 22546TXB2) linked to the S&P 500 Index (www.sec.gov/Archives/edgar/data/1053092/000095010312004245/dp32342_424b2-u696.htm). The lower barrier becomes effective when the S&P 500 Index experiences a 30 per cent decrease in value and the cap on the upside returns becomes effective at 17.5 per cent. The leverage factor is 1.5 (resulting in a 26.25 per cent maximum upside return) and the underwriting fee on this note was 3 per cent. The final observation date for the note is 18 February 2015. The payout diagram for this example product is shown in Figure 7.

As of the issue date, the S&P 500's dividend yield was 2.1 per cent, its level was 1415.51 and the 2-year put implied volatility of the S&P 500

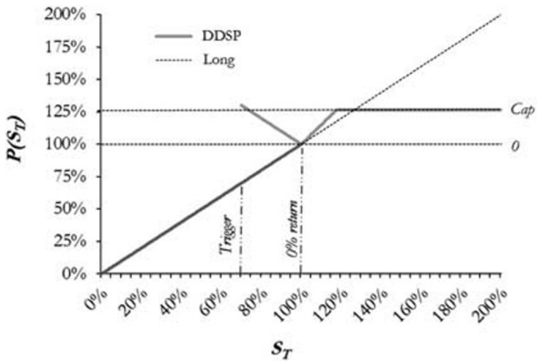

Figure 7: Leveraged KODD issued by Credit Suisse on 21 August 2012 linked to the S&P 500 index.

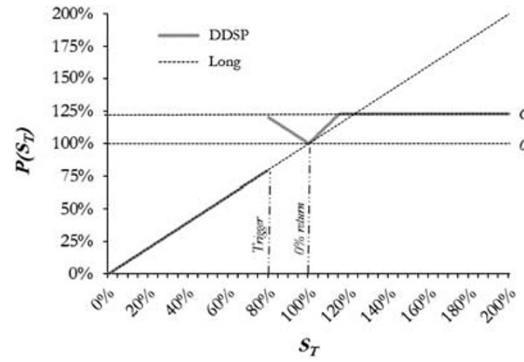

Figure 8: Leveraged SODD issued by Citigroup on 2 October 2012 linked to the Financial Select Sector SPDR Fund.

was 21.9 per cent. The 2-year swap rate (USSW2) was 0.50 per cent and the 3-year swap rate (USSW3) was 0.62 per cent. We linearly interpolated these rates to produce a risk-free rate of 0.56 per cent. Credit Suisse's 2-year CDS rate as of the issue date was 1 per cent.

We find that the notes were worth \$944.11 per \$1000 when issued by Credit Suisse. The net proceeds to the issuer was \$970 and therefore the premium Credit Suisse charged for the product was \$25.89 (\$970−\$944.11), or nearly 3 per cent of the face value. As a result of this issuance, Credit Suisse priced in a gross profit of \$25 890.95.

Single observation dual directional

On 2 October 2012, Citigroup issued \$4.125 million in Dual Directional Trigger PLUS notes (CUSIP: 17318Q582) linked to the Financial Select Sector SPDR Fund (XLF) (www.sec.gov/Archives/edgar/data/831001/000095010312005071/dp33215_424b2-xlf.htm). The lower barrier becomes effective if XLF experiences a 20 per cent decrease in value and the cap on the upside returns becomes effective at 15.33 per cent. The upside leverage factor is 1.5 (resulting in a 23 per cent maximum upside return) and the underwriting fee on this note was 2.25 per cent. The final observation date for the note is 24 September 2014. The payout diagram for this example product is shown in Figure 8.

As of the issue date, XLF's dividend yield was 1.69 per cent, its price was \$15.64 and the 2-year put implied volatility of XLF shares was 26.55 per cent. The 2-year swap rate (USSW2) was 0.40 per cent. Citigroup's 2-year CDS rate as of the issue date was 103 basis points.

We find that the notes were worth \$9.37 per \$10 when issued by Citigroup. The net proceeds to the issuer were \$0.405 per \$10, and therefore the premium Citigroup charged for the product was more than 2 per cent of the face value of the notes. As a result of this issuance, Citigroup priced in a gross profit of \$167 057.64.

Summary of valuations

We valued a set of 103 DD structured products representing all DDs issued with unique CUSIPs linked to equities or equities indexes. Five of the 125 DDs in our original sample were not valued since these products were issued on Brent crude futures and the assumption of geometric

Brownian motion for these contracts may not be as well-motivated. In addition, 17 of the DDs were not valued because of data limitations – for example, no CDS rate data. As a result, we valued 103 DD structured products.

The valuation results produced according to the formulas presented in the previous section were independently verified using numerical simulations. A summary of our valuation results for all DDs can be found in Table 4.

On average, the leveraged DDs in our sample have a lower issue date valuation (95.4 per cent) than their unleveraged counterparts (97.3 per cent).[10] Although the leverage may seem to only increase the upside potential, the longer average term for these notes (1.57 years versus 1.37) contributes to a lower issue date valuation. However, both types are issued at a premium to the face value even net of commissions on average. The profit priced into the notes was comparable to the commission for unleveraged products and nearly two times the commission for leveraged products. The average KODD had a higher issue date valuation (97.3 per cent) than the average SODD (95.7 per cent).[11] Table 5 summarizes the value of DDs.

Table 4: Average DD value per $1000 invested for each issuer and underlying (number of products in each subgroup is denoted parenthetically)

Issuer	*Model value (in dollars)*	*Underlying*	*Model value (in dollars)*
Goldman Sachs (11)	965.52	SPX (59)	969.96
JP Morgan Chase (38)	974.81	AAPL (16)	955.43
Credit Suisse (19)	965.24	EEM (8)	961.81
Other (35)	948.29	Other (13)	949.23
Average	963.04	Average	963.04

Table 5: Average DD value per $1000 invested by type and leverage (number of products in each subgroup is denoted parenthetically)

	Knock-out (in dollars)	*Single observation (in dollars)*	*Average (in dollars)*
Leveraged	959.23	953.23	954.00
Unleveraged	976.70	967.40	973.41
Average	973.48	956.94	963.04

DISCUSSION

In this article, we extend our analysis and valuation of ARBNs to include a new type of product, DDs, that are also linked to the absolute return of an underlying asset within a particular range but lack principal protection. These products have yet to be fully described by the literature, and we provide a summary of the market for DDs up to December 2012 as well as a full decomposition of these notes and valuations using analytical and Monte Carlo approaches.

Having valued nearly all DD structured products registered with the SEC through 2012, our results indicate that like many types of structured products, DDs tend to be priced at a significant premium to present value across issuers and underlying securities. We find that DDs with leverage on positive returns have a lower average value than their unleveraged counterparts. Finally, we found that the average SODD tends to have an issue date valuation significantly lower than the average KODD.

The issuance of DDs has increased significantly in 2012, and several reports have described DDs as non-directional exposure to the underlying asset (see, for example, Hampson, 2012; Robinson, 2012). However, the actual exposure these notes embed is highly complex and dependent on several factors, including the implied volatility of the underlying but also the creditworthiness of the issuer, dividend rates, trigger levels and the term of the note. We demonstrated that banks charge an average premium of approximately 2.2 per cent above their stated commission levels, suggesting DDs may not be suitable for unsophisticated investors.

NOTES

1 For an extensive introduction to structured products, see Henderson and Pearson (2010).

2 According to Bloomberg Structured Notes Brief dated 5 January 2012, $101.7 billion in structured notes were sold in 2011 on 4406 offerings amongst the top 20 global issuers. According to their 2012 Review dated 3 January 2012, $104.1 billion in structured notes were sold in 2012.

3 The barrier feature in DDs is typically triggered if the closing value of the reference asset on any trading day within its term is less than the barrier level. Intraday fluctuations therefore do not trigger the barrier feature.

4 DD products are also issued in the European structured product market, where the brand names may differ. For example, we have seen European products labeled 'Twin-Win' securities that are examples of what we call DDs.

5 Throughout this paper, we assume r, q and σ are constant and continuously compounded over the product's term $[0, T]$. For simplicity, we omit the subscript t from S_t.

6 For additional information, see, for example, Hull (2011) and Deng *et al* (2012).

7 $L=1$ for unleveraged KODDs, while $L>1$ for leveraged KODDs. Although none of the notes in our sample offer a coupon payment, such payments could easily be incorporated into this framework.

8 We write the path-dependent payoff function above in terms of an equivalent static payoff because there is evidence that the static hedge of a portfolio with path-dependent options is preferred over a dynamic hedging because of lower transaction costs (Tompkins, 2002).

9 For our Monte Carlo simulations, we followed procedures substantially similar to Glasserman (2003), using 100 000 simulations and increasing the frequency of observations to confirm convergence to the analytic result.

10 A difference of means test reveals that this difference is statistically significant ($P<0.001$).

11 A difference of means test shows that this difference is statistically significant ($P<0.005$). This is perhaps a reason that single observation DDs are more frequently issued.

References

Baule, R. and Tallau, C. (2011) The pricing of path-dependent structured financial retail products: The case of bonus certificates. *Journal of Derivatives* 18(4): 54–71.

Black, F. and Scholes, M. (1973) The pricing of options and corporate liabilities. *Journal of Political Economy* 81(3): 637–654.

Carr, P. and Chou, A. (1997a) Breaking barriers. *RISK* 10(9): 139–145.

Carr, P. and Chou, A. (1997b) Hedging Complex Barrier Options. Working paper, http://www.math.nyu.edu/research/carrp/papers/pdf/multipl3.pdf.

Deng, G., Dulaney, T. and McCann, C. (2013) Valuation of reverse convertibles in variance gamma economy. *Journal of Derivatives and Hedge Funds* 19(November): 244–258.

Deng, G., Guedj, I., Mallett, J. and McCann, C. (2011) The anatomy of absolute return barrier notes. *The Journal of Derivatives* 19(2): 61–70.

Deng, G., Husson, T. and McCann, C. (2012) Valuation of Structured Products. *The Journal of Alternative Investments* 16(4): 71–87.

Deng, G., McCann, C. and O'Neal, E. (2010) What TiVo and JP Morgan Teach us about Reverse Convertibles. SLCG working paper, http://www.slcg.com/.

Glasserman, P. (2003) *Monte Carlo Methods in Financial Engineering*. New York, NY: Springer.

Hampson, S. (2012) HSBC straddle product offers each-way bet on S&P 500, http://www.risk.net/structured-products/, accessed 28 February 2013.

Henderson, B. and Pearson, N. (2010) The dark side of financial innovation: A case study of the pricing of a retail financial product. *The Journal of Financial Economics* 100(2): 227–247.

Hernández, R., Lee, W.Y. and Liu, P. (2007) An Economic Analysis of Reverse Exchange-Able Securities: An Option-Pricing Approach. Working paper, University of Arkansas, Arkansas.

Hull, J. (2011) *Options, Futures and Other Derivatives*, 8th edn. Upper Saddle River, NJ: Prentice-Hall.

Jarrow, R.A. and Turnbull, S.M. (1995) Pricing derivatives on financial securities subject to credit risk. *The Journal of Finance* L(1): 53–85.

McCann, C. and Luo, D. (2006) Are Structured Products Suitable for Retail Investors? SLCG working paper, http://www.slcg.com/.

Merton, R. (1973) Theory of rational option pricing. *Bell Journal of Economics and Management Science* 4(1): 141–183.

Robinson, M. (2012) 'Dual Directional' notes rise to three year high, http://www.bloombergbriefs.com/, accessed 28 February 2013.

Szymanowska, M., Horst, J.T. and Veld, C. (2009) Reverse convertible bonds analyzed. *Journal of Futures Markets* 29(10): 895–919.

Tompkins, R.G. (2002) Static versus dynamic hedging of exotic options: An evaluation of hedge performance via simulation. *Journal of Risk Finance* 3(4): 6–34.