Invited Editorial

Option pricing: Very simple formulas

Received (in revised form): 9th April 2014

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ABSTRACT We provide simple formulas for pricing both the European and American options.

Journal of Derivatives & Hedge Funds (2014) **20**, 71–73. doi:10.1057/jdhf.2014.11

Keywords: American options; European options; simple option pricing; investment; portfolio

INTRODUCTION

The limitations of the Black–Scholes model, such as smiles, smirks and term–structure patterns, are well known. In response to these limitations, Duffie *et al* (2000), Eraker *et al* (2003) and Eraker (2004) introduced discontinuous stochastic volatility models. However, this increases the number of the parameters to be estimated. Eberlein and Keller (1995) and Eberlein *et al* (1998) used inverse Gaussian distribution. Carr and Wu (2003) introduced an asymmetric stable distribution model. Edwards (2005) presented a switching regime model.

However, as Epps (2009, p. 357) puts it ‘As yet, however, no model passes the test of providing good fits over time ... This is the challenge of future researchers’. The existing methods of option pricing adopt strong assumptions. Furthermore, they usually use advanced mathematics to produce controversial pricing methods. The use of mathematically

advanced models does not necessarily produce better results than using simple formulas. To the contrary, the mathematically advanced models require simplifying assumptions that yield controversial results (for a discussion, see, for example, Cvitanic and Zapatero (1994), Epps (2009), and Alghalith (2012)).

In response to these limitations, this note provides simple methods of pricing both the European and American options. In so doing, we utilize the relationship between the volatility, time, upper bound and the option price.

It is well known that the price of the option increases in both volatility and time to maturity. Therefore, the main innovation of this article is to present simple formulas that depict the positive relationship between the option price and both the volatility and time to maturity. This method is based on the relative volatility of the underlying asset and maturity time (the asset volatility relative to the volatilities of the other

assets in the market). The formulas are derived in a way to insure that the upper bound (the maximum option price) coincides with both the maximum volatility and the maximum time to maturity, and that the option price is increasing in both volatility and time to maturity.

To summarize, our formulas are based on these stylized (accepted) assumptions:

- The price of the option increases in volatility.
- The price of the option increases in time to maturity.
- The maximum price of a European call cannot exceed the price of the underlying stock.

PRICING FORMULAS

The European call: For a non-dividend paying and newly written European call option, the option price E_t , at the current time t , is given by

$$E_t = \frac{T + \sigma_t}{T_{\max} + \sigma_{\max}} s_t; T \leq T_{\max}, \sigma_t \leq \sigma_{\max},$$

where T is time to maturity, $T_{\max}$ is the market's maximum possible time to maturity (usually 36 months), s_t is the current price of the underlying stock, σ_t is its current volatility and $\sigma_{\max}$ is the market's maximum current volatility (the volatility of the most volatile stock in the market). We know that, for a European call, the upper bound should not exceed the price of the underlying asset, and thus this formula guarantees this. The upper bound is obtained when $T = T_{\max}$ and $\sigma_t = \sigma_{\max}$. The advantage of this approach is that the option price increases in both volatility and time to maturity, and that the upper bound coincides with $T + \sigma_t = T_{\max} + \sigma_{\max}$. We also note that, for a newly written option, the intrinsic value ($s_t - K$) can be ignored, where K is the strike price.

Example 1: If the option matures in 4 months ($T = 4$), $s_t = \$30$, $\sigma_t = 20$ per cent and $\sigma_{\max} = 300$ per cent, the option price is

$$E_t = \frac{4 + 20}{36 + 300} (30) = \$2.14.$$

Evidently, the option price increases with T , s_t and σ_t , as it should. The put-call parity formula can be used to calculate the price of the equivalent put.

American options: For a non-dividend-paying and newly written American put, we use the following formula

$$A_t = \left(1 + \frac{T + \sigma_t}{T_{\max} + \sigma_{\max}} \right) E_t,$$

where E_t is the current price of the equivalent European option and A_t is the current price of the American option. This formula guarantees that the price of the European option cannot exceed the price of the equivalent American option, and that the price difference between the American option and the European option increases in both σ_t and T .

Example 2: The American put option matures in 12 months $T = 12$, $E_t = \$5$, $\sigma_t = 30$ per cent and $\sigma_{\max} = 300$ per cent then

$$A_t = \left(1 + \frac{12 + 30}{36 + 300} \right) 5 = \$5.47.$$

We note that the price difference between the American option and the European option increases in σ_t and T , as it should.

REAL MARKET DATA

As of 13 December 2013, the stock of Emmis Corp. was rated as the most volatile stock ($\sigma_t = 418$ per cent, thus $\sigma_{\max} = 418$ per cent).

Table 1: HP option price

<i>Expiry date</i>	<i>The formula price</i>	<i>Market price</i>
January 2014	2.29	2.65
February 2014	2.35	2.07
May 2014	2.53	2.74
January 2015	3.00	3.20
January 2016	3.71	3.50
Average	2.78	2.83

Table 2: IBM option price

<i>Expiry date</i>	<i>The formula price</i>	<i>Market price</i>
January 2014	7.61	8.95
February 2014	7.99	7.51
April 2014	8.75	9.50
July 2014	9.89	9.50
January 2015	12.18	13.03
Average	9.28	9.69

The price of the HP stock is US\$26.770 ($s_t = 26.77$), its volatility is 38 per cent ($\sigma_t = 38$). The price of the IBM stock is \$172.80 ($s_t = 172.80$), its volatility is 19 per cent ($\sigma_t = 19$ per cent). We used our formula to calculate the

price of the (near-the-money) European call option for each corporation for different maturities, and compared them with real market prices (see Tables 1 and 2). Our formula yielded prices that are astonishingly close to the market prices, especially the average values.

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