Original Article

Empirical competitiveness of deterministic option pricing models: Evidences from the recent waves of financial upheavals in India

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ABSTRACT It is acknowledged by the leading researchers that the fundamental force behind the emergence of advanced option pricing models was the result of abundant empirical research analysis, leading to the fact that the asset return distribution is non-log-normal. It obviously resulted in strong moneyness and maturity pricing biases of Black–Scholes (BS). The concept defined the assertive distributional assumptions for Nifty (benchmark index of India), because the stated view supported the idea and its implications majorly focused on establishing the well-protected alternative model. The core concept of almost all the option pricing models was substantially indicating towards the flexible distributional structure. Such behaviour not only correlated underlying stock returns and its volatility but also subdued the level of skewness and kurtosis. Therefore, this article endeavours to investigate empirically the out-of-sample moneyness–maturity forecasting performance of deterministic volatility option pricing models during the recent waves of economic imbalance. It forecasts S&P CNX Nifty 50 index options of India against significant BS option pricing model and also attempts to find out forecasting performances of a family of deterministic volatility option pricing models, namely, Practitioner Black–Scholes (PBS), Gram–Charlier (GC) and Constant Elasticity of Variance (CEV). The accuracy of models' prices testify to the relative market price of Nifty index options using error metrics. For the purpose of this research, the prices are compared analytically through continuously

updating the parameters of the models using cross-sectional option data, almost on a daily basis. The underlying focus of the article emphasizes on how to investigate the parameters of various models, pertaining specifically to a period of turbulence in the Indian economy. Eventually, while giving verdict to the results deduced, we measure out that the PBS model has smaller out-of-sample valuation errors in pricing Nifty Index options than the CEV and GC. However, it remains challenging to root out price bias completely out of all models we have framed out and applied.

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INTRODUCTION

Path-breaking publication of Black–Scholes (1973) paved the way for the advanced and realistic option pricing models. The host of researchers divided the models majorly into two categories, deterministic and stochastic. Although the stochastic models are more advanced compared with their deterministic counterparts, they are managed in providing critical theoretical insights. Simultaneously, their analytical tractability is poor when compared with deterministic models, resisting their practical applicability. Further, the theoretical differences in their fundamental assumptions enforce traders and practitioners to test the effectiveness of models empirically before its real-world usages. The overall objective contributes to determine the most suitable option model for practical uses.

Empirically, option pricing in the financial industry has been one of the most exciting and researched areas over the last four decades. Black–Scholes' (BS) seminal paper on option pricing ignited the world of finance and set the foundation of Financial Engineering. Their formula provided closed-form solution for pricing stock/index call options of European type. In their model, they assumed that the

volatility of asset returns remains constant till maturity of options. However, it was found that when the model calibrated to price accurately at-the-money options, the model often misprices deep-out-of-the-money and deep-in-the-money options (Backus *et al*, 1997). This model-anomalous behaviour was named 'volatility smiles/smirk' (Derman and Kani, 1994a, b). Figure 1 depicts Nifty index option volatility smile/smirk skewed pattern resulting from calculating implied volatilities across a range of moneyness and maturity.

The BS model does not conceptualize volatility smile phenomenon as depicted above. It arises mainly owing to the parsimonious assumptions (asset return normal distribution and constant volatility) used to derive the BS formula. As per the concept of BS, the graphical structure of plot of implied volatility versus strike price (X) or moneyness (X/S) normally results in a neutral facial expression (Dumas *et al*, 1998). Exploring further, it was delved out that it falls into smile/smirk shape. Since the origin of BS formula, correctness of constant volatility assumption has been questioned by many, namely, Black (1976), Cox *et al* (1979) and Rubinstein (1994). Empirical analysis provides evidence that the

Figure 1: Moneyness–maturity smile of BS implied volatility.

(Data of 24 April 2008 is used to figure out smile/smirk pattern, index price: 4999.85, volatility: 25 per cent, risk-free rate: 7.43 per cent)

constant volatility assumption of BS is inappropriate in the real-market situations, and hence it is rejected. Whether or not the formula provides the good estimate of the market, a matter of empirical debate, it can be inversely used to estimate the value of volatility, coined implied volatility, which is a good forecast of future volatility (Day and Craig, 1992; Edey and Elliot, 1992; Canina and Figlewski, 1993; Christensen and Prabhala, 1998; Ederington and Guan, 2002).

The BS deficiencies evoke researchers to pursue the development of more realistic models that can incorporate asset price volatility and interest rates as stochastic processes. Over the past four decades, practitioners are constantly trying to develop models to price options with non-constant asset price volatility and they have put the models into two categories: deterministic and stochastic volatility models. Deterministic models assume that volatility is determined by some variables observable in the market, whereas

stochastic volatility models assume that volatility follows a stochastic process, and parameters are not directly observable in the market. However, historical analysis shows that volatilities are often stochastic and correlated with the underlying asset price (Dumas *et al*, 1998). Though stochastic volatility models are a more realistic choice for valuing derivative securities and modelling asset price dynamics. However, only few models (Hull and White, 1987; Heston, 1993; Heston and Nandi, 2000) retain enough analytical tractability. The enacting fact is that the calibration of stochastic models is extremely burdensome and time consuming (Bakshi *et al*, 1997). Therefore, for the acute purpose of this research, we have mapped only deterministic volatility models that are less complex to implement than the class of stochastic models.

Deterministic volatility models other than BS include models of Cox *et al* (1979) derived from European option prices under various alternatives, including the absolute diffusion,

pure jump and square root Constant Elasticity of Variance (CEV) models. The CEV model of Cox (1996) expressed the volatility as a function of the underlying asset price. Practitioner Black–Scholes (PBS) model of Christoffersen and Jacobs (2004) and Dumas *et al* (1998) describes the parabolic shape of the volatility smile, and Gram–Charlier (GC) model of Backus *et al* (1997) incorporates skewness and kurtosis of asset returns, not included in the classic BS model. As we are dealing with options (derivative assets), emphasis of this empirical work is on finding the consistency of time-series properties of the underlying asset price with option price.

Like other renowned stock exchanges, stock exchange of India, NSE (National stock exchange) is also using the classic BS model for fixing the base price of Nifty 50 index options. Thus, the article attempts to aim at the empirical analysis of the deterministic volatility models to determine better alternate minimizing of the price bias between market (Nifty index options) and model price.

In order to testify the usability of deterministic models in most turbulent financial vicissitudes, the hypothesized models have been put into a practical implication of complete short-term cycle of financial movement (up–down–up). This particular phase not only rows as an extreme of phenomenal unpredictability but also ranges the high and low tides of financial flux and thus provides the most apt situation for testifying the comparative competitiveness of models in question. As the models deal with the extreme range of wide highs and lows of index movements, the applicability/usability of deterministic models becomes more dominant.

Furthermore, to search out the apt model after having a rigorous session of comparison and contrast among them and to reach to the

appropriate model could emerge as the most potential framework for predicting and protecting the market option price during the uncertain upheavals of financial force. We specifically look at the relative price errors produced by the model with respect to the market. Though we expect to testify and examine various models to explore the specific model, which can lead us to explain market in the most scientific terms. This research article concentrates on empirical techniques used in testing option pricing models, and also to summarize major conclusions for the empirical findings.

The rest of the article is detailed as follows. The section ‘Data description’ describes the data and its selection procedure. The section ‘Option pricing models’ discusses relevant deterministic volatility models. The section ‘Calibration of models’ provides estimation procedure to estimate the deterministic volatility models. The section ‘Result’ compares the relative error of model and market price and compares the out-of-sample pricing accuracy of the models. The section ‘Conclusion’ finally concludes the research.

DATA DESCRIPTION

Data do relate that the period of year 2007–2009 was the fastest ascending–descending movement of capital market, giving rise to the roars of financial experts to estimate the underlying reasons of such event. We have also chosen this specific period to focus on our study and to test our conceptual models. Because the range of the specific period gives the extreme limits of the canvass of the whole range of Indian capital market as during this period, only the market touched the zenith of its highest ascending and

soon after touching it dashing down to make to dike of crisis. The said period provides the best possible laboratory conditions to experiment the suitability and the implacability of apt models measuring the various virtues and vices of options trading. In order to testify the models, we banked on data of Nifty index options, one of the most heavily traded derivative instrument on the bourse of NSE of India. Figure 2 clearly shows that the trading popularity of Index options has gone up exponentially since its inception, and nowadays it accounts three-fourth (75 per cent) of the total turnover of the Future and Option segment of NSE.

This research work required the collection of historical data of the S&P CNX Nifty 50 Index option contract, and the Indian 'risk-free' interest rate data equal to yield of 91 Day T-Bill. The required data were obtained from the official Websites of NSE and Reserve Bank of India. S&P CNX Nifty 50 Index daily closing price records from 1 January 2007 to 31 December 2009 collected manually. The following option parameters such as index price, strike price, time to maturity, interest rate and option have been obtained to testify the various models to

determine the best model and its applicability for pricing Nifty 50 index options. Figures 3 and 4 exhibit display S&P CNX Nifty Index movement and its return pattern and leverage effect for the specific period, whereas Figure 5 displays the normal distribution histogram of Nifty index return. Figures 5 and 6 exhibit that the distribution of frequency of Nifty index return does not follow a normal distribution pattern.

The return curve is slightly skewed to the left (leptokurtic) and peaked to top (platokurtic). Not only that, the left tail is also thicker than the right tail, which indicates that the occurrence of probability of extreme returns, especially of negative returns, is more likely as compared with positive return during the specific period. In addition, QQ plot (Figure 6) of Nifty residuals and its significant departure from the normal distribution pattern supports the view that Nifty index series have thicker left tail and thinner right tail distribution.

Figure 4 exhibits that the frequency of asset return formed clusters, that is, frequency can be grouped in terms of large and small frequency. Besides, it also shows that the returns of Nifty

Figure 2: Business growth of index options.

Figure 3: Plot of Nifty over time.

Figure 4: Time-series characteristics of Nifty index returns.

Figure 5: Plot of frequency distribution of Nifty index return.

Figure 6: Q-Q plot of Nifty index return residuals.

index tend to be followed by change of either sign positive or negative, which means frequency of large/small change tend to be followed by large/small small changes. This confirms the time-series dependence of tails (Mandelbrot, 1963). The above characteristic is also popularly known as volatility clustering. Volatility clustering not only contradicts the fundamental assumption of BS but also provides an additional support (other than Figures 5 and 6) to the fact that the asset return does not follow a normal distribution pattern, as hypothesized by BS.

The phase of rash volatility spanning over the period from January 2007 to December 2009 (Figure 3) has been taken into consideration to pronounce the whole circular motion of equity market. Estimation and observation of the same altogether provide a strong canvass to study and examine the effectiveness of option pricing models and acting forces responsible for creating sudden burst and resulting into unpredictable upward and downward movement of equity market (Nifty Index). During the 2007–2009 period, the large swing and reversals in spot volatilities caused index options volatility surface to display acute volatility term-structure (either upward or downward sloping). The above

quoted phase can be evoked as a period of high unrest and this specific reason makes it an important phase to understand the suitability of implying various option pricing models and to carry out a research study under the acute shadow of cause and effect.

Data screening procedure

We transformed the collected data through various channels to filter it appropriately to obtain the best raw material for structuring the concept and to develop a framework on it. With this objective, all the call option prices for a non-dividend paying stock/index have been plucked from the market to check the following conditions on a daily basis: Call option with less than 50 traded contracts, European call options not satisfying the lower boundary condition

$$\text{Max} \left(0, S_t - Ke^{-r(T-t)} \right) \leq C_{\text{Market}}(S_t, t)$$

an invalid observation and hence overlooked, (where S_t is the current asset price, K is the strike price and r is the risk-free interest rate, and $C(S_t, t)$ is the call price at time t and t is the time to maturity in year, 738 trading days. Very deep out-of-the-money and very-deep in-the-money

options are also considered as an invalid observation as they are not traded actively on NSE and their price quotes may not reflect the true option value, and hence they are ruled out (data of moneyness $(S/K-1)$ greater than ± 15 per cent). Option strike prices having maturity greater than 90 days are also not considered as they are very less actively traded on NSE. Option strikes with less than 5 days of maturity are also excluded, resulting in price volatility-related biases. After rigorous filtration, the compiled data set finally figured into 15 053 call options out of 205 244.

Tables 1 and 2 depict the pre- and post-screening descriptive statistics of Nifty index options for the specified sample period. Rigorous filtering ensures that only relevant data will be used for comparing and contrasting the performance of models, should they be free from anomaly such as price and maturity bias.

Option categories

Furthermore, to substantiate this empirical research work depending on the time to expiration and ratio of the asset price to the strike price, each market option strike left after applying the rigorous screening procedure is placed in 1 of the 15 categories of moneyness–maturity table.

Time ranges are classified as:

Time to Maturity (T)

$$\begin{cases} >5 \text{ Days and } \leq 30 \text{ Days} & \text{Short Maturity} \\ >30 \text{ Days and } \leq 60 \text{ Days} & \text{Medium Maturity} \\ >60 \text{ Days and } \leq 90 \text{ Days} & \text{Long Maturity} \end{cases}$$

whereas moneyness ranges are categorized as:

Moneyness (M)

$$\begin{cases} < -10 \% & \text{DOTM} \\ \geq -10 \% \text{ and } < -5 \% & \text{OTM} \\ \geq -5 \% \text{ and } \leq +5 \% & \text{ATM} \\ > +5 \% \text{ and } \leq +10 \% & \text{ITM} \\ > +10 \% & \text{DITM} \end{cases}$$

Table 1: Filter statistics of Nifty index call options

	Year			Total
	2007	2008	2009	2007–2009
Total call contracts	38 890	72 494	93 860	205 244
<i>Criteria</i>	<i>Number of data rejected</i>			
No trading volume/open interest	29 052	56 835	75 785	161 672
Number of traded contracts ≤ 50	4163	7216	9137	20 516
Moneyness $> +15$ per cent	366	211	1459	2036
Moneyness < -15 per cent	9	1866	738	2613
Maturity > 90 days	2	543	390	935
Maturity < 3 days	490	551	486	1527
No arbitrage relationship	573	123	196	892
Rejected data	34 655	67 345	88 191	190 191
Rejected data (in percentage)	89.11	92.90	93.96	91.99
Remaining data	4235	5149	5669	15 053
Remaining data (in percentage)	10.89	7.10	6.04	8.01

Table 2: Nifty index call option statistics for the year 2007–2009 (after filtration)

Maturity	Moneyness					Total/sub total
	DOTM	OTM	ATM	ITM	DITM	
Short	1189	1692	4358	875	412	8526
	7.9	11.2	29.0	5.8	2.7	56.6
Medium	697	1133	2780	470	187	5267
	4.6	7.5	18.5	3.1	1.2	35.0
Long	138	310	755	42	15	1260
	0.9	2.1	5.0	0.3	0.1	8.4
Total/sub total	2024	3135	7893	1387	614	15 053
	13.4	20.8	52.4	9.2	4.1	100

Methodology

In order to establish an effective outcome, we adopted a common method that encompasses calculation of Error Metrics. Error metrics, namely, Percentage Mean Error (PME) and Mean Absolute Percentage Error (MAPE) (Wiggins, 1987; Naik, 1993; Nandi, 1996; Shu and Zhang, 2004), are used to test the out-of-sample forecasting performance of models. Forecasting accuracy of the models will be judged on the basis of pricing bias between model and market. In order to optimize price effectiveness models, parameters are computed analytically by optimization techniques and used as an input to figure out the effectiveness of the models for pricing S&P CNX Nifty 50 Index Option with market value.

PME

$$PME = \frac{1}{n} \sum_{i=1}^n \left(\frac{C_i^{Model} - C_i^{Market}}{C_i^{Market}} \right)$$

MAPE

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left(\frac{|C_i^{Model} - C_i^{Market}|}{C_i^{Market}} \right)$$

where C_i^{Market} and C_i^{Model} are the market and model prices of i th observation, n is the total

number of observations. As discussed earlier, to find out the relative performance of models, we testify and compared the relative errors of the models categorically (moneyness–maturity wise). Positive relative error means that the model overprices the specific option, whereas negative error means that the model underprices the specific option. However, small relative error implies that the model provides a good approximation of the market value for specific option and vice versa.

OPTION PRICING MODELS

To set the basis of this research approach, we have selected Black-Scholes (1973), PBS model of Dumas *et al* (1998), GC Model of Backus *et al* (1997) and CEV Option Pricing Model of Cox (1996).

The BS option pricing model

Utilizing the concept of Geometric Brownian motion in 1973, Fisher Black, Maryon Scholes and Robert Merton derived the world-famous formula of option pricing. The formula is

$$C_{BS} = SN(d_1) - Ke^{-rt}N(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_1 = \frac{\ln(S/K) + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

C denotes the price of a call option, S denotes the underlying Index price, K denotes the option exercise price, t is the time to expiry in years, r is the risk free rate of return and $N()$ is the standard normal distribution of function d_1 and d_2 . $N(d_1)$ is the factor by which the present value of contingent receipt of the stock exceeds the current stock price and $N(d_2)$ is the risk-adjusted probability that the option will be exercised. σ^2 is the variance of returns on the Index. The model not only revolutionized the world of option trading but also started a new era in the field of financial mathematics. Although the model was invented to price options, these days it is used reciprocally to find out the market volatility widely popular as implied volatility. Instead of using the at-the-money implied volatility, this article utilized the parametric implied volatility, which ensures the incorporation of cross-sectional potential information contained embedded in market option prices, across moneyness and maturities, rather than the information content of at-the-money options only.

Contrary to backward-looking econometric time-series models, implied volatility models are forward looking, provide incremental information and thus are good predictors of future variability of index/stock prices (Beckers, 1981; Mayhew, 1995; Blair *et al*, 2001). Therefore, to figure out the price of the BS model, we also banked on this and extracted it cross-sectional from the market data of day (i), for

each day of the sample period, utilized as an input in the BS model to predict the value of option of day($i+1$).

The CEV model

Following the footsteps of BS, in 1975, Cox and Ross (1976) published another option pricing model popularly known as CEV model. Their model was based on the assumption that asset price follows the diffusion process defined as

$$dS = \mu dt + \delta S^{\beta/2} dz$$

In addition, they also assumed that the variance of the asset price return σ^2 follows a deterministic relationship, defined as:

$$\sigma^2(S, t) = \delta^2 S^{(\beta-2)}$$

Where β and δ is the elasticity of the local volatility. The process defined above is capable of reproducing the volatility smile observed in most of the financial asset return time-series data. The option pricing formula when the underlying process follows the CEV model is

$$C = S_t Q(2\gamma; 2 + 2/(2 - \beta), 2\gamma) - e^{-rt} K(1 - Q(2x; 2 + 2/(2 - \beta), 2\gamma))$$

When $\beta < 2$,

$$C = S_t Q(2x; 2 + 2/(2 - \beta), 2\gamma) - e^{-rt} K(1 - Q(2\gamma; 2 + 2/(2 - \beta), 2x))$$

When $\beta > 2$,

$Q(z; \nu, k)$ is a complementary non-central χ^2 distribution function, with z , ν and k being the evaluation point of the integral, degree of freedom and non-centrality, respectively, where

$$k = \frac{2r}{\delta^2(2 - \beta)(e^{r(2 - \beta)t} - 1)}$$

$$x = k S_t^{2 - \beta} e^{r(2 - \beta)t}$$

$$\gamma = k K^{2 - \beta}$$

C is the call price, S is the stock price, t is the time to maturity, r is the risk-free rate of interest, K is the strike price, and β and δ are the unknown parameters of the formula. For $\beta=2$, CEV call price converges to BS (Emanuel and MacBeth, 1982).

The PBS model

In order to overcome smile deficiency of the BS model, Dumas and colleagues describe implied volatility as deterministic volatility functions of strike price and maturity with varying adjustments/combinations. The various combinations of the model were also analysed and improved upon by Christoffersen and Jacobs in 2004. As it is empirically proven that not all versions of PBS provide significant improvement in option pricing, thus out of several possible combinations of Deterministic Volatility Functions (DVF) models, we researched only the following four specifications, ranging from linear to quadratic, namely, PBS 1, PBS 2, PBS 3 and PBS 4.

$$\begin{array}{ll}
 \text{PBS 1} & \sigma_{IV} = a_0 + a_1 M + a_2 T \quad (\text{Linear}) \\
 \text{PBS 2} & \sigma_{IV} = a_0 + a_1 M + a_2 M^2 + a_3 T + a_4 T^2 \quad (\text{Quadratic}) \\
 \text{PBS 3} & \sigma_{IV} = a_0 + a_1 M + a_2 M^2 + a_3 T + a_4 MT \quad (\text{Quadratic}) \\
 \text{PBS 4} & \sigma_{IV} = a_0 + a_1 M + a_2 M^2 + a_3 T + a_4 T^2 + a_5 MT \quad (\text{Quadratic})
 \end{array}$$

In all the above equations, M is the moneyness defined as $(S/K)-1$, where S is the strike price and T is the time to maturity, σ_{iv} is the implied volatility (to be extracted from BS), a_0 , a_1 , a_2 , a_3 , a_4 , and a_5 are model parameters. For the special case when a_0 , a_1 , a_2 , a_3 , a_4 and a_5 are all 0, the PBS models converge to the BS model, that is, when $\sigma_{iv} = a_0$ PBS $\rightarrow$ BS. Each PBS specification stipulates a different DVF function. PBS

specifications ranging from 1 to 4 are found consistent with the models considered by Dumas *et al* (1998) and Christoffersen and Jacobs (2004). Intently, we have not considered PBS specifications of higher order, as it has been already proved that the inclusion of higher orders of maturity, strike price or moneyness does not provide any material improvement compared with their quadratic specifications (Berkowitz, 2009).

The GC model

To overcome the non-normality of asset prices, and to further resolve the issue of volatility smile in option pricing in 2004, Backus and colleagues suggested another simple formula. In their formula, they provide a simple way to account for non-lognormal excess skewness and kurtosis observed in most of the financial asset return series data. They use GC expansion in the distribution of returns of the underlying asset and incorporated non-lognormal skewness and kurtosis into model. Their formula for pricing

call option is

$$C_{GC} \approx S_t \phi(d) - K e^{-rt} \phi(d - \sigma t) + S_t \phi(d) \sigma t \left[\frac{\gamma_{1t}}{3!} \phi^{(3)}(2\sigma t - d) - \frac{\gamma_{2t}}{4!} (1 - d^2 + 3d\sigma t) - \sigma_t^2 \right]$$

Where $\phi(x)$ is the standard normal density function, $\phi^{(k)}(x)$ is the k th derivative of $\phi(x)$, $\gamma_{1,t}$ and $\gamma_{2,t}$ denote the t -period excess skewness and kurtosis. K , d and r have their usual meanings as defined above. In case the log asset return

Table 3: Price statistics of Nifty index call options (2007–2009)

		Maturity statistics					
		Time to maturity ($T \leq 30$)					Sub Total
		DOTM	OTM	ATM	ITM	DITM	
Market	Average	10.86	27.38	118.72	350.28	506.24	128.04
	SD	12.81	25.94	76.57	84.92	107.52	142.84
BS	Average	9.82	26.13	115.09	343.18	497.09	124.62
	SD	14.57	26.52	74.11	82.86	104.94	140.10
CEV	Average	9.23	25.08	114.45	348.61	499.29	124.68
	SD	15.12	27.41	75.18	83.46	104.51	140.79
GC	Average	10.86	26.98	119.09	346.08	497.47	126.17
	SD	13.79	26.41	75.56	83.95	105.44	141.58
PBS 1	Average	9.81	27.21	118.92	348.21	501.49	127.52
	SD	14.95	27.56	76.30	84.34	103.95	141.89
PBS 2	Average	11.14	28.32	118.33	345.19	498.32	127.16
	SD	15.86	28.15	75.12	83.19	104.65	140.38
PBS 3	Average	9.38	26.76	118.52	347.93	501.18	127.12
	SD	13.98	26.77	75.99	84.08	103.85	141.78
PBS 4	Average	11.14	28.65	119.07	346.28	499.28	127.77
	SD	15.65	28.24	75.48	83.58	104.54	140.73
Number of observations		1189	1692	4358	875	412	8526
		Time to maturity ($30 < T \leq 60$)					
		DOTM	OTM	ATM	ITM	DITM	Sub Total
Market	Average	42.52	76.37	188.3	379.7	508.1	173.39
	SD	31.06	41.22	78.68	80.06	104.4	130.01
BS	Average	47.05	80.92	188.2	376.4	501.8	174.39
	SD	35.77	43.34	78.6	80.85	106.37	128.02
CEV	Average	42.78	77.43	182.56	374.93	501.31	171.74
	SD	33.82	42.55	78.61	81.13	106.16	128.87
GC	Average	44.87	80.32	187.87	379.07	499.06	174.32
	SD	32.73	42.28	78.95	80.96	105.46	129.66
PBS 1	Average	41.07	76.62	187.81	381.59	509.26	173.18
	SD	31.14	41.88	79.41	81.77	105.76	130.92
PBS 2	Average	46.39	80.08	187.24	375.86	501.09	173.52
	SD	34.55	42.43	77.81	80.78	106.34	127.68
PBS 3	Average	41.14	76.90	188.17	381.63	508.75	173.42
	SD	31.35	42.29	79.57	81.98	105.61	130.94
PBS 4	Average	45.40	80.18	187.36	377.20	502.88	173.66
	SD	33.16	44.36	78.65	81.55	106.80	128.57

Table 3: (Continued)

Number of observations		697	1133	2780	470	187	5267
		Time to maturity ($60 < T \leq 90$)					
		DOTM	OTM	ATM	ITM	DITM	Sub total
Market	Average	65.25	105.1	214.1	369.2	510.3	179.66
	SD	40.45	44.65	71.44	75.62	132	100.60
BS	Average	70.76	110.58	216.57	367.81	481.73	182.72
	SD	42.12	46.71	73.56	78.10	116.32	99.34
CEV	Average	65.44	106.19	214.06	368.77	493.90	179.73
	SD	37.80	43.63	72.13	76.13	118.55	99.34
GC	Average	61.83	104.13	213.09	370.25	494.13	178.30
	SD	39.34	46.19	72.50	76.74	123.26	100.56
PBS 1	Average	59.21	103.21	214.28	372.71	499.50	178.65
	SD	34.38	42.68	72.30	76.13	120.43	101.32
PBS 2	Average	66.54	105.86	211.27	365.36	484.06	177.87
	SD	36.90	41.49	70.54	74.15	118.91	97.37
PBS 3	Average	59.74	103.32	213.73	372.68	498.81	178.39
	SD	35.45	47.20	72.51	76.51	121.47	101.73
PBS 4	Average	65.28	104.56	211.45	364.54	482.50	177.47
	SD	34.45	40.85	70.89	74.88	117.08	97.67
Number of observations		138	310	755	42	15	1260

distribution follows a normal distribution pattern, except the first term, all other terms of the above equation will become 0, and thus the formula of GC converges to BS.

CALIBRATION OF MODELS

To imply option-related structural parameters (which govern the underlying asset return distribution data), we have used the most trusted linear and non-linear least square (NLLS) loss function (Rubinstein, 1985; Chiarella *et al*, 2000; Christoffersen and Jacobs, 2004; Rouah and Vainberg, 2007). Further, the optimal set of parameters obtained is then used to compute the models' price. Applying the above procedure, we

have obtained the parameter estimates of all the models. Specifically, the volatility and parameters' estimate obtained for day (i) is utilized to estimate the value of next day options, that is, of day ($i+1$, out of sample). Thereafter, model (theoretical) and market option prices are compared analytically on a daily basis by continuously updating the parameters of models. To find the parameters numerically, the NLLS objective loss function $f(\Omega)$ is defined as

$$f(\Omega) = \min_{\Omega} \sum_{i=1}^n (C_i^{Model}(\Omega) - C_i^{Market}(M, T))^2$$

where Ω is a set of vectors (parameters), which will be determined implicitly, M is the moneyness and T is the time of maturity.

Table 4: Price statistics of nifty index call options (2007–2009)

		<i>Moneyness statistics</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
Market	Average	25.47	52.77	152.36	360.83	506.90	148.23
	SD	29.54	44.65	85.73	84.14	107.04	137.30
BS	Average	26.79	54.28	150.56	355.18	498.15	146.90
	SD	33.58	47.66	85.65	83.49	105.53	135.39
CEV	Average	24.17	51.64	149.51	358.35	502.16	145.49
	SD	30.50	45.44	86.39	84.69	106.32	134.34
GC	Average	23.67	51.68	151.87	354.10	498.25	146.43
	SD	29.54	44.49	84.41	81.68	106.32	136.36
PBS 1	Average	23.94	52.58	152.31	360.26	503.81	147.78
	SD	29.12	45.09	85.80	84.68	104.80	137.10
PBS 2	Average	27.05	54.69	151.49	356.19	498.82	147.63
	SD	32.17	45.99	84.39	83.34	105.38	134.93
PBS 3	Average	23.75	52.45	152.16	360.10	503.43	147.62
	SD	29.19	45.61	85.82	84.62	104.71	137.11
PBS 4	Average	23.75	52.45	152.16	360.10	503.43	147.62
	SD	29.19	45.61	85.82	84.62	104.71	137.11
Number of observations		2024	3135	7893	1387	614	15053

RESULT

In order to analyse the comparative behaviour of deterministic models, we consolidated the study with their inter-relations leading towards multilayered correlation. For this, we thoroughly examined and evaluated the various combinations of moneyness and maturity. Tables 3–10 explicitly state the same. The outcome of cross-sectional, comparative and analytical study of the given tables will decide the comparative competitiveness of the models. This section reports the empirical comparison of model performances based on out-of-sample forecasting performance.

Tables 3 and 4 exhibit the moneyness–maturity statistics of S&P CNX Nifty Index Call options for the period ranging from 1 January

2007 to 31 December 2009. Prices exhibited in Tables 8 and 9 validate the theoretical fundamentals of call options pricing.

Theoretically, intrinsic value of call and put option should follow the order $DOTM < OTM < ATM < ITM < DITM$. In our case too, the moneyness–maturity sequences of call options follow the ascending order understood by the two series: $DOTM < OTM < ATM < ITM < DITM$ and short term < medium term < long term.

Tables 5 and 6 enlist the co-movement of implied volatility with time to maturity and strike price (implied volatility have been extracted through the collected sample data). Tables 5 and 6 clearly exemplify the empirical research work of Dumas *et al* (1998), Heynen (1993) and

Table 5: Implied volatility statistics of deterministic models (2007–2009)

		<i>Maturity Statistics</i>					
<i>Models</i>		<i>Time to maturity ($T \leq 30$)</i>					<i>Sub Total</i>
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	
BS	Average	0.30	0.28	0.20	0.27	0.29	0.26
	SD	0.09	0.08	0.08	0.08	0.09	0.09
CEV	Average	0.29	0.27	0.16	0.27	0.30	0.25
	SD	0.11	0.11	0.12	0.13	0.16	0.13
GC	Average	0.27	0.25	0.17	0.25	0.31	0.24
	SD	0.15	0.11	0.13	0.12	0.14	0.13
PBS 1	Average	0.30	0.28	0.25	0.29	0.32	0.27
	SD	0.09	0.09	0.10	0.09	0.11	0.10
PBS 2	Average	0.27	0.25	0.19	0.25	0.27	0.24
	SD	0.09	0.09	0.10	0.09	0.11	0.10
PBS 3	Average	0.26	0.25	0.22	0.27	0.27	0.25
	SD	0.09	0.10	0.10	0.10	0.12	0.11
PBS 4	Average	0.27	0.24	0.20	0.26	0.26	0.24
	SD	0.10	0.10	0.09	0.10	0.12	0.11
Number of observations		1189	1692	4358	875	412	8526
		<i>Time to maturity ($30 < T \leq 60$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>
BS	Average	0.30	0.27	0.24	0.27	0.29	0.26
	SD	0.09	0.08	0.08	0.09	0.07	0.09
CEV	Average	0.25	0.25	0.23	0.27	0.33	0.25
	SD	0.07	0.06	0.09	0.11	0.11	0.09
GC	Average	0.26	0.25	0.24	0.23	0.28	0.24
	SD	0.09	0.06	0.06	0.09	0.16	0.10
PBS 1	Average	0.29	0.26	0.24	0.28	0.32	0.26
	SD	0.08	0.08	0.09	0.09	0.08	0.09
PBS 2	Average	0.27	0.26	0.23	0.23	0.28	0.24
	SD	0.09	0.07	0.09	0.11	0.09	0.10
PBS 3	Average	0.24	0.23	0.27	0.25	0.28	0.24
	SD	0.08	0.05	0.05	0.09	0.07	0.07
PBS 4	Average	0.25	0.24	0.26	0.25	0.28	0.24
	SD	0.09	0.09	0.07	0.10	0.09	0.09
Number of observations		697	1133	2780	470	187	5267

Table 5: (Continued)

		<i>Time to maturity ($60 < T \leq 90$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>
BS	Average	0.26	0.24	0.21	0.26	0.27	0.23
	SD	0.09	0.07	0.07	0.07	0.06	0.08
CEV	Average	0.21	0.21	0.21	0.23	0.28	0.21
	SD	0.15	0.10	0.13	0.16	0.17	0.15
GC	Average	0.22	0.22	0.21	0.23	0.27	0.21
	SD	0.10	0.08	0.10	0.11	0.15	0.11
PBS 1	Average	0.25	0.24	0.21	0.27	0.30	0.22
	SD	0.07	0.07	0.07	0.09	0.06	0.07
PBS 2	Average	0.24	0.25	0.20	0.24	0.30	0.23
	SD	0.10	0.09	0.09	0.10	0.10	0.10
PBS 3	Average	0.20	0.20	0.22	0.25	0.25	0.20
	SD	0.09	0.08	0.07	0.06	0.06	0.08
PBS 4	Average	0.21	0.21	0.23	0.25	0.27	0.21
	SD	0.10	0.11	0.09	0.09	0.09	0.10
Number of observations		138	310	755	42	15	1260

Table 6: Implied volatility statistics of deterministic models (2007–2009)

		<i>Moneyness statistics</i>					
<i>Models</i>		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
BS	Average	0.30	0.27	0.22	0.27	0.29	0.26
	SD	0.09	0.08	0.08	0.08	0.08	0.09
CEV	Average	0.26	0.25	0.22	0.26	0.31	0.25
	SD	0.11	0.10	0.12	0.13	0.15	0.13
GC	Average	0.26	0.25	0.23	0.24	0.29	0.24
	SD	0.11	0.09	0.10	0.11	0.16	0.12
PBS 1	Average	0.29	0.27	0.24	0.29	0.32	0.26
	SD	0.09	0.09	0.09	0.09	0.10	0.09
PBS 2	Average	0.27	0.26	0.23	0.24	0.29	0.24
	SD	0.09	0.09	0.10	0.10	0.11	0.10
PBS 3	Average	0.26	0.23	0.23	0.26	0.27	0.24
	SD	0.09	0.08	0.08	0.08	0.09	0.09
PBS 4	Average	0.25	0.24	0.23	0.25	0.28	0.24
	SD	0.10	0.11	0.09	0.10	0.11	0.10
Number of observations		2024	3135	7893	1387	614	15 053

Table 7: PME statistics of deterministic models (2007–2009)

		<i>Maturity Statistics</i>						
		<i>Time to maturity ($T \leq 30$)</i>						
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>	<i>Sub total</i>
BS	Average	−0.29	−0.08	−0.01	−0.02	−0.02	−0.07	0.03
	SD	0.53	0.49	0.21	0.04	0.02	0.34	0.13
CEV	Average	−0.27	−0.07	−0.05	−0.02	−0.03	−0.06	0.01
	SD	0.55	0.45	0.19	0.04	0.02	0.33	0.13
GC	Average	−0.24	−0.04	−0.02	−0.02	−0.03	−0.04	0.01
	SD	0.56	0.45	0.17	0.06	0.02	0.33	0.13
PBS 1	Average	−0.32	−0.09	0.06	0.02	−0.04	−0.05	−0.01
	SD	0.52	0.43	0.18	0.04	0.02	0.32	0.11
PBS 2	Average	−0.32	−0.08	0.04	−0.02	−0.01	−0.02	0.01
	SD	0.52	0.48	0.20	0.04	0.02	0.35	0.15
PBS 3	Average	−0.18	−0.03	0.01	−0.01	−0.01	−0.01	−0.01
	SSD	0.59	0.43	0.18	0.04	0.02	0.32	0.14
PBS 4	Average	−0.19	−0.04	0.02	−0.01	−0.02	−0.06	0.01
	SD	0.59	0.49	0.20	0.04	0.02	0.35	0.14
Number of observations		1189	1692	4358	875	412	8526	1260
		<i>Time to maturity ($30 < T \leq 60$)</i>						
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>	
BS	Average	0.13	0.09	0.02	−0.03	−0.01	0.04	
	SD	0.32	0.21	0.08	0.04	0.03	0.17	
CEV	Average	0.11	0.09	0.03	−0.02	−0.02	0.05	
	SD	0.28	0.18	0.09	0.04	0.03	0.15	
GC	Average	−0.09	−0.04	0.02	0.01	−0.02	0.03	
	SD	0.29	0.17	0.06	0.04	0.03	0.15	
PBS 1	Average	0.14	0.11	0.05	0.03	−0.04	0.07	
	SD	0.26	0.16	0.08	0.04	0.03	0.14	
PBS 2	Average	0.12	0.09	0.04	−0.03	−0.02	0.03	
	SD	0.31	0.20	0.09	0.04	0.03	0.17	
PBS 3	Average	−0.04	0.02	0.01	0.01	0.02	0.02	
	SSD	0.26	0.16	0.09	0.04	0.03	0.14	
PBS 4	Average	−0.05	0.06	0.03	−0.01	−0.04	0.03	
	SD	0.31	0.23	0.09	0.04	0.03	0.18	
Number of observations		697	1133	2780	470	187	5267	

Table 7: (Continued)

		<i>Time to maturity ($60 < T \leq 90$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
BS	Average	0.11	0.06	0.02	−0.02	−0.05	0.03
	SD	0.21	0.15	0.09	0.07	0.04	0.13
CEV	Average	0.09	0.03	0.02	−0.01	−0.04	0.01
	SD	0.18	0.16	0.09	0.07	0.04	0.13
GC	Average	0.06	0.05	0.03	0.02	−0.02	0.01
	SD	0.21	0.16	0.07	0.07	0.05	0.13
PBS 1	Average	−0.09	−0.04	0.03	0.01	−0.05	−0.01
	SD	0.16	0.13	0.08	0.07	0.04	0.11
PBS 2	Average	0.08	0.04	−0.01	−0.01	−0.02	0.01
	SD	0.27	0.19	0.10	0.07	0.04	0.15
PBS 3	Average	−0.07	0.02	0.00	−0.01	−0.01	−0.01
	SD	0.17	0.21	0.10	0.08	0.04	0.14
PBS 4	Average	0.07	0.03	0.02	−0.01	−0.02	0.01
	SD	0.23	0.19	0.09	0.06	0.04	0.14
		138	310	755	42	15	1260

Table 8: PME statistics of deterministic models (2007–2009)

		<i>Moneyneess statistics</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
BS	Average	−0.12	−0.06	−0.01	−0.01	−0.04	−0.04
	SD	0.50	0.39	0.16	0.04	0.03	0.28
CEV	Average	−0.11	−0.04	−0.04	0.00	−0.03	−0.03
	SD	0.47	0.35	0.15	0.04	0.03	0.27
GC	Average	−0.10	0.03	−0.02	−0.01	−0.03	−0.02
	SD	0.49	0.37	0.16	0.05	0.02	0.27
PBS 1	Average	−0.21	−0.08	0.04	0.03	−0.05	−0.06
	SD	0.45	0.33	0.15	0.04	0.03	0.26
PBS 2	Average	−0.19	0.06	0.03	−0.01	0.04	−0.05
	SD	0.52	0.38	0.16	0.04	0.03	0.28
PBS 3	Average	−0.07	0.04	0.02	−0.01	−0.01	−0.02
	SD	0.51	0.39	0.17	0.04	0.02	0.29
PBS 4	Average	−0.09	−0.04	0.03	−0.02	−0.02	−0.03
	SD	0.44	0.34	0.15	0.04	0.02	0.26
Number of observations		2024	3135	7893	1387	614	15 053

Table 9: Mean absolute percentage statistics of deterministic models (2007–2009)

		<i>Maturity statistics</i>					
		<i>Time to maturity ($T \leq 30$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>
BS	Average	0.50	0.37	0.12	0.03	0.02	0.21
	SD	0.33	0.33	0.17	0.03	0.02	0.28
CEV	Average	0.50	0.34	0.12	0.03	0.02	0.20
	SD	0.36	0.33	0.16	0.03	0.02	0.28
GC	Average	0.49	0.32	0.11	0.03	0.02	0.20
	SD	0.36	0.36	0.14	0.06	0.02	0.28
PBS 1	Average	0.50	0.35	0.11	0.03	0.02	0.19
	SD	0.35	0.31	0.15	0.03	0.02	0.27
PBS 2	Average	0.50	0.34	0.11	0.03	0.02	0.20
	SD	0.37	0.34	0.17	0.03	0.02	0.29
PBS 3	Average	0.47	0.30	0.11	0.03	0.02	0.19
	SD	0.34	0.31	0.15	0.03	0.02	0.27
PBS 4	Average	0.49	0.31	0.11	0.03	0.02	0.20
	SD	0.37	0.34	0.17	0.03	0.02	0.29
Number of observations		1189	1692	4358	875	412	8526
		<i>Time to maturity ($30 < T \leq 60$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub Total</i>
BS	Average	0.27	0.17	0.06	0.03	0.02	0.11
	SD	0.22	0.15	0.06	0.03	0.02	0.14
CEV	Average	0.22	0.15	0.06	0.03	0.02	0.10
	SD	0.20	0.15	0.07	0.03	0.02	0.13
GC	Average	0.19	0.12	0.06	0.03	0.02	0.09
	SD	0.23	0.17	0.08	0.05	0.02	0.14
PBS 1	Average	0.21	0.16	0.06	0.03	0.02	0.08
	SD	0.18	0.11	0.06	0.03	0.02	0.11
PBS 2	Average	0.25	0.16	0.06	0.03	0.02	0.10
	SD	0.22	0.15	0.06	0.03	0.02	0.14
PBS 3	Average	0.17	0.11	0.06	0.03	0.02	0.08
	SD	0.18	0.12	0.06	0.03	0.02	0.11
PBS 4	Average	0.19	0.11	0.06	0.03	0.02	0.10
	SD	0.21	0.19	0.07	0.03	0.02	0.15
Number of observations		697	1133	2780	470	187	5267

Table 9: (Continued)

		<i>Time to maturity ($60 < T \leq 90$)</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
BS	Average	0.18	0.12	0.07	0.05	0.05	0.09
	SD	0.15	0.11	0.06	0.05	0.04	0.10
CEV	Average	0.17	0.12	0.06	0.05	0.05	0.09
	SD	0.15	0.12	0.06	0.05	0.04	0.10
GC	Average	0.17	0.12	0.06	0.05	0.04	0.09
	SD	0.13	0.11	0.06	0.05	0.04	0.10
PBS 1	Average	0.19	0.09	0.06	0.05	0.03	0.08
	SD	0.11	0.09	0.06	0.05	0.03	0.08
PBS 2	Average	0.22	0.14	0.07	0.05	0.05	0.10
	SD	0.19	0.14	0.07	0.05	0.04	0.12
PBS 3	Average	0.13	0.11	0.07	0.05	0.03	0.09
	SD	0.12	0.18	0.07	0.06	0.03	0.11
PBS 4	Average	0.14	0.13	0.06	0.05	0.05	0.09
	SD	0.15	0.14	0.07	0.05	0.04	0.11
Number of observations		138	310	755	42	15	1260

Table 10: Mean absolute percentage statistics of deterministic models (2007–2009)

		<i>Moneyness statistics</i>					
		<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>	<i>Sub total</i>
BS	Average	0.40	0.27	0.07	0.03	0.02	0.16
	SD	0.32	0.28	0.13	0.03	0.02	0.23
CEV	Average	0.38	0.24	0.08	0.03	0.02	0.15
	SD	0.34	0.28	0.13	0.03	0.02	0.23
GC	Average	0.36	0.23	0.08	0.03	0.02	0.16
	SD	0.31	0.29	0.11	0.04	0.03	0.23
PBS 1	Average	0.40	0.26	0.09	0.03	0.02	0.14
	SD	0.33	0.26	0.12	0.03	0.02	0.22
PBS 2	Average	0.39	0.26	0.08	0.03	0.02	0.16
	SD	0.34	0.29	0.13	0.03	0.02	0.24
PBS 3	Average	0.35	0.21	0.07	0.03	0.02	0.14
	SD	0.32	0.26	0.12	0.03	0.02	0.22
PBS 4	Average	0.37	0.22	0.07	0.03	0.02	0.16
	SD	0.34	0.30	0.14	0.03	0.02	0.24
Number of observations		2024	3135	7893	1387	614	15 053

Rubinstein (1994) that the implied volatilities tend to vary systematically with respect to maturity and expiry. Tables 5 and 6 exhibit that implied volatility tends to vary from DOTM to DITM options and make a systematic upward trend when it moves either from ATM to DITM or towards DOTM call options. The variation in implied volatility ranging from DOTM to DITM is highest in the case of BS followed by CEV, GC

overvaluing Nifty index call options prices. However, the primary focus remains on finding out the model strongly reacting, resulting and causing undervaluing or overvaluing Nifty index call options. In case of all deterministic models, we noticed a systematic decrease in mean percentage price error going from DOTM to DITM. The sequence based on behaviour pattern can be put like

$$\text{Moneyiness} \left\{ \begin{array}{l} \text{BSM : } \text{DOTM}(-12\%) > \text{OTM}(-6\%) > \text{ATM}(-1\%) \geq \text{ITM}(-1\%) \\ \text{CEV : } \text{DOTM}(-11\%) > \text{OTM}(-4\%) \geq \text{ATM}(-4\%) > \text{DITM}(-3\%) > \text{ITM}(0\%) \\ \text{GC : } \text{DOTM}(-10\%) > \text{OTM}(+3\%) > \text{DITM}(-3\%) > \text{ATM}(-2\%) > \text{ITM}(-1\%) \\ \text{PBS1 : } \text{DOTM}(-21\%) > \text{OTM}(-8\%) > \text{DITM}(-5\%) > \text{ATM}(+4\%) > \text{ITM}(+3\%) \\ \text{PBS2 : } \text{DOTM}(-19\%) > \text{OTM}(+6\%) > \text{ATM}(+3\%) > \text{ITM}(-1\%) \\ \text{PBS3 : } \text{DOTM}(-7\%) > \text{OTM}(+4\%) > \text{ATM}(+2\%) > \text{ITM}(-1\%) \geq \text{DITM}(-1\%) \\ \text{PBS4 : } \text{DOTM}(-9\%) > \text{OTM}(-4\%) > \text{ATM}(+3\%) > \text{ITM}(-2\%) \geq \text{DITM}(-2\%) \end{array} \right.$$

and PBS, which further dictates the volatility smile-capturing capacity of models. PBS 3 and 4 depict least variation in implied volatility. Evidentially, *prima facie*, it can be claimed that the PBS model (specifically PBS 3 and PBS 4) prices S&P CNX Nifty index call options are better than the rest others, including the traditional BS model.

Tables 7 and 8 demonstrate the Mean Percentage Error statistics of S&P CNX Nifty Index Call options moneyness and moneyness–maturity wise.

Hence, having viewed the data presented in Tables 7 and 8, it can confidently be said that the BS, CEV, GC and PBS models underprice short-, medium- and long-term DOTM, OTM, ITM and DITM Nifty index call options, whereas the models slightly overprice prices ATM call options in almost all maturity groups. As far as the findings are concerned, it is seen that the models in question are either undervaluing or

Tables 7 and 8 reveal that the pricing error of PBS 3 and 4 is lowest, whereas the pricing error of BS and PBS 1 (linear version) is highest, estimating DOTM, OTM, ITM and DITM category options. When put together, and examining the outcome of Tables 5, 6, 7 and 8, we jointly deduced that the several distinguished deterministic models' capacity of PBS 3 and 4 model in explaining/capturing the smile is higher than the others. We also noticed that, for the period specified, all models underpriced DITM options; however, the degree of pricing error was found to be low, varying in the range of –1 per cent to –5 per cent. GC is also a close competitor of PBS 3 and PBS 4, pricing all moneyness–maturity call options with pricing error close to PBS 3 and 4. We find that PBS 3 tends to underprice all category DOTM and DITM options, except OTM and ATM call options. However, PBS 3 proves potential enough to price short-term DOTM, OTM, ITM and

DITM options, whereas in similar categories the pricing performance of GC and PBS 4 is somewhat comparable. The short-term pricing sequence is patterned as:

$$\text{ShortTerm} \left\{ \begin{array}{l} \text{BSM : } \text{DOTM}(-29\%) > \text{OTM}(-8\%) \geq \text{ITM}(-2\%) > \text{DITM}(-2\%) > \text{ATM}(-1\%) \\ \text{CEV : } \text{DOTM}(-27\%) > \text{OTM}(-7\%) > \text{ATM}(-5\%) > \text{DITM}(-3\%) > \text{ITM}(-2\%) \\ \text{GC : } \text{DOTM}(-24\%) > \text{OTM}(-4\%) > \text{DITM}(-3\%) > \text{ATM}(-22\%) \geq \text{ITM}(-2\%) \\ \text{PBS1 : } \text{DOTM}(-32\%) > \text{OTM}(-9\%) > \text{ATM}(+6\%) > \text{DITM}(-4\%) > \text{ITM}(+2\%) \\ \text{PBS2 : } \text{DOTM}(-32\%) > \text{OTM}(-8\%) > \text{ATM}(+4\%) > \text{ITM}(-2\%) > \text{DITM}(-1\%) \\ \text{PBS3 : } \text{DOTM}(-18\%) > \text{OTM}(-3\%) > \text{ATM}(+1\%) > \text{ITM}(-1\%) \geq \text{DITM}(-1\%) \\ \text{PBS4 : } \text{DOTM}(-19\%) > \text{OTM}(-4\%) > \text{ATM}(+2\%) > \text{DITM}(-2\%) > \text{ITM}(-1\%) \end{array} \right.$$

All deterministic models tend to underprice short-term DOTM, OTM, ITM and DITM options, whereas, except PBS variants, others such as BS, CEV and GC underprice short-term ATM options. As the pattern resulted in short-term maturity options, we had the same models that were causing underpricing of DOTM, OTM, ITM options and overpricing of short-term ATM call options. The same price bias sequence and the effect seem to be repeated in medium- and long-term DOTM, OTM, ITM and DITM options. The price bias sequence can be ordered as

From Tables 9 and 10 we also observed that there is a systematic reduction in the price performance of models, which can be understood by the series: DOTM < OTM

< ATM < ITM < DITM and long term < medium term < short term, implies that lower (higher) the time to maturity is, higher (lower) the pricing bias is amid liquidity. This also in conjunction with the evidence (Tables 5 and 6) that volatility of short term is higher than the long term for Nifty index options. We also found that the degree of error of DITM (DOTM) options is the highest for long (short)-term options amid time sensitivity of options.

Although, pricing performance of the BS model is poor as compared with other deterministic models in most of the

$$\text{MediumTerm} \left\{ \begin{array}{l} \text{BSM : } \text{DOTM}(+13\%) > \text{OTM}(+9\%) > \text{ITM}(-3\%) > \text{ATM}(+2\%) > \text{DITM}(-1\%) \\ \text{CEV : } \text{DOTM}(+11\%) > \text{OTM}(+9\%) > \text{ATM}(+3\%) > \text{ITM}(2\%) \geq \text{DITM}(-2\%) \\ \text{GC : } \text{DOTM}(-9\%) > \text{OTM}(-4\%) > \text{ATM}(+2\%) > \text{DITM}(2\%) > \text{ITM}(+1\%) \\ \text{PBS1 : } \text{DOTM}(+14\%) > \text{OTM}(+11\%) > \text{ATM}(+5\%) > \text{DITM}(4\%) > \text{ITM}(3\%) \\ \text{PBS2 : } \text{DOTM}(+12\%) > \text{OTM}(+9\%) > \text{ATM}(+4\%) > \text{ITM}(3\%) > \text{DITM}(-2\%) \\ \text{PBS3 : } \text{DOTM}(-4\%) > \text{OTM}(+2\%) \geq \text{DITM}(+2\%) > \text{ATM}(+1\%) \geq \text{ITM}(+1\%) \\ \text{PBS4 : } \text{OTM}(+6\%) > \text{DOTM}(-5\%) > \text{DITM}(-4\%) > \text{ATM}(+3\%) > \text{ITM}(-1\%) \end{array} \right.$$

$$\text{LongTerm} \left\{ \begin{array}{l} \text{BSM : } \text{DOTM}(+11\%) > \text{OTM}(+6\%) > \text{DITM}(-5\%) > \text{ATM}(+2\%) > \text{ITM}(-2\%) \\ \text{CEV : } \text{DOTM}(+9\%) > \text{DITM}(-4\%) > \text{OTM}(+3\%) > \text{ATM}(+2\%) > \text{ITM}(-1\%) \\ \text{GC : } \text{DOTM}(+6\%) > \text{OTM}(+5\%) > \text{ATM}(+3\%) > \text{DITM}(-3\%) > \text{ITM}(+2\%) \\ \text{PBS1 : } \text{DOTM}(-9\%) > \text{DITM}(-5\%) > \text{OTM}(-4\%) > \text{ATM}(+3\%) > \text{ITM}(+1\%) \\ \text{PBS2 : } \text{DOTM}(+8\%) > \text{OTM}(+4\%) > \text{DITM}(-2\%) > \text{ATM}(-1\%) \geq \text{ITM}(-1\%) \\ \text{PBS3 : } \text{DOTM}(-7\%) > \text{OTM}(+2\%) > \text{ITM}(-1\%) \geq \text{DITM}(-1\%) > \text{ATM}(0\%) \\ \text{PBS4 : } \text{DOTM}(+7\%) > \text{OTM}(+3\%) > \text{ATM}(+2\%) > \text{DITM}(-2\%) > \text{ITM}(-1\%) \end{array} \right.$$

moneyness–maturity categories. However, simultaneously it can be noted that the BS model works well for options that are near the money (at-the-money) and for options with next striking price on either side of the at-the-money options compared with all others. However, contrary to the at-the-money options, it does not yield satisfactory results for OTM options (most heavily traded options in any market) and misprices them severely.

Specifically, CEV model underprices short-term DOTM, OTM and ATM options but overprices same in medium and long maturity categories; however, it continues to underprice ITM and DITM options in all maturities. CEV model prices call options with PME ranging from –27 per cent to +11 per cent and like others degree of pricing bias is the highest for short-term options. However, by introducing only one extra parameter compared with BS, it improves the pricing performance notably (compared with BS) in most of the out-of-sample moneyness–maturity groups. The CEV option pricing model performs better than the BS model in 8, GC model in 3 and PBS model in 2 of the 15 moneyness–maturity options groups (defined in the subsection ‘Option categories’), compositely.

Contrary to CEV, GC model underprices DOTM and OTM medium-term call options in addition to short-term options; however, similar to the CEV model pricing bias, it decreases with the time to maturity and the price bias ranges from –24 per cent to +6 per cent. GC model is strong enough to incorporate the phenomenon of excess skewness and kurtosis observed in Nifty asset return series. GC model outperforms BS in 8, CEV in 9 and PBS in 2 of the 15 moneyness–maturity groups, compositely.

Tables 7 and 8 provide empirical evidence that the BS model generally underprices short-term

options across moneyness, whereas it overprices medium- and long-term DOTM, OTM and ATM call options. However, PME is highest for short-term call option compared with medium- and long-term call option and measures short-term ATM and ITM options more accurately compared with medium- and long-term options, measures with pricing error –1 per cent and –2 per cent, respectively. The BS model outperforms CEV and GC in 3, whereas it outperforms the PBS models in only 1 of the 15 moneyness–maturity categories.

Of the PBS class models, PBS 3 model outperforms the benchmark BS in 13, CEV in 14 and GC in 12 of the 15 moneyness–maturity groups. PME pricing bias of the PBS model is the highest for short- and long-term DOTM call options, whereas it is lowest for short-, medium- and long-term ATM, ITM and DITM call options. Although, the pattern of sequential effect represents that time to maturity is a crucial factor in the pricing performance of the models. The sequential representation of models remains unchanged in all maturity classes and PBS 3 continues to outperform. Most interestingly, we found that the degree of mispricing of deterministic models decreases with the increase in the time to maturity of Nifty index options. This is in line with the practical experience as long maturity options are less actively traded on the browser of NSE thus less volatile, further implies, implied volatility of short maturity options is higher than the long maturity options and supports the figures of Tables 5–10.

Our empirical research strongly suggests that the deterministic volatility functions, especially those having joint combinations of moneyness and maturity, comparatively reduce the pricing bias and give values closer to market.

On the basis of MAPE (depicted in Tables 9 and 10), the sequence based on the behaviour pattern of option pricing models can be jumbled like PBS 3>PBS 4>GC>CEV>BS. Results of Tables 9 and 10 reaffirm that efficiency of the PBS models is superior to others, across moneyness and maturity. Similar to PME, here we also noticed a systematic reduction in price error going from DOTM to DITM, and long maturity to short maturity. The sequence based on the behaviour pattern based on moneyness can be arranged like

$$\text{Moneyness} \left\{ \begin{array}{l} \text{BSM : DOTM (40 \%) > OTM (27 \%) > ATM (7 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{CEV : DOTM (38 \%) > OTM (28 \%) > ATM (8 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{GC : DOTM (36 \%) > OTM (23 \%) > ATM (8 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{PBS1 : DOTM (40 \%) > OTM (26 \%) > ATM (9 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{PBS2 : DOTM (39 \%) > OTM (26 \%) > ATM (8 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{PBS3 : DOTM (35 \%) > OTM (21 \%) > ATM (7 \%) > ITM (3 \%) > DITM (2 \%)} \\ \text{PBS4 : DOTM (37 \%) > OTM (22 \%) > ATM (7 \%) > ITM (3 \%) > DITM (2 \%)} \end{array} \right.$$

Mean absolute price error tends to follow similar pattern across moneyness–maturity as well. From above, we deduced that of all the deterministic models discussed here, when put together and examined, the PBS model (especially PBS 3 and PBS 4) is the only model that stands firmly on the ground of comparative competitiveness. Together, Table 11 and Figure 7 also evidentially prove this (Figure 7 generated randomly).

The above results strongly suggest that the PBS models are not only easy to implement, but they also yield values closer to market prices, when compared with others. As far as the quintessence of this research drive, comparing and contrasting different models are concerned, we can comfortably estimate that the PBS model outperforms others in most of the moneyness–maturity categories during the specific period of

turbulence. However, none of the models dominate the rest in all categories of moneyness–maturity, which further corresponds to producing relative price errors lower than the other models. Table 12 focusing on Comparative Effectiveness of models (moneyness–maturity wise) suggests deterministic option pricing models to be used for pricing the S&P CNX Nifty 50 Index option of India. Thus, of all the models tested and critically examined, we did not procure a single model that comprises or caters to all the best requirements of the options traders. As a result, we ended up that no

single model supports multifarious moneyness–maturity dimension.

CONCLUSION

The article empirically investigates the out-of-sample performance of various deterministic volatility models for the pricing of S&P CNX Nifty 50 index options. The article focuses on the performance (as measured by pricing PME and pricing MAPE) of three families of models, the PBS, GC and CEV. On the basis of the study period 2007–2009, the quadratic version of the PBS outperforms the other models consistently for both error metrics and for most of the moneyness/time-to-maturity buckets.

As the PBS model outshines during the most disturbed phase of Indian financial economy, it

Table 11: Descriptive price bias statistics of BS and deterministic models, relative to market
(1 October 2008, parameters: index price: 3950.8, expiry: 29 October 2008, time to expiry: 28 days)

	Moneyness (in percentage)	Market price of Nifty index option	Models price					Models price bias (relative to market)				
			BS	GC	CEV	PBS 3	PBS 4	BS	GC	CEV	PBS 3	PBS 4
DOTM	−14.11	13.05	12.52	6.89	6.28	13.68	15.38	−0.04	−0.47	−0.52	0.05	0.18
	−13.17	20	15.98	10.42	8.60	17.24	19.09	−0.20	−0.48	−0.57	−0.14	−0.05
	−12.21	20.1	20.24	14.94	11.64	21.66	23.65	0.01	−0.26	−0.42	0.08	0.18
	−11.22	28.05	25.45	20.62	15.57	27.09	29.22	−0.09	−0.26	−0.44	−0.03	0.04
	−10.21	30.9	31.78	27.61	20.61	33.72	35.97	0.03	−0.11	−0.33	0.09	0.16
OTM	−9.18	38.05	39.40	36.08	26.95	41.74	44.09	0.04	−0.05	−0.29	0.10	0.16
	−8.12	47.7	48.48	46.19	34.85	51.33	53.77	0.02	−0.03	−0.27	0.08	0.13
	−7.04	59.4	59.20	58.06	44.55	62.70	65.19	0.00	−0.02	−0.25	0.06	0.10
	−5.93	70.85	71.76	71.84	56.28	76.03	78.55	0.01	0.01	−0.21	0.07	0.11
ATM	−4.80	86	86.32	87.63	70.31	91.49	94.02	0.00	0.02	−0.18	0.06	0.09
	−3.64	104.35	103.06	105.53	86.84	109.22	111.73	−0.01	0.01	−0.17	0.05	0.07
	−2.45	127.75	122.11	125.59	106.05	129.35	131.81	−0.04	−0.02	−0.17	0.01	0.03
	−1.23	149.6	143.60	147.87	128.10	151.94	154.33	−0.04	−0.01	−0.14	0.02	0.03
	0.02	176	167.60	172.41	153.06	177.02	179.32	−0.05	−0.02	−0.13	0.01	0.02
	1.30	204.15	194.16	199.21	180.96	204.60	206.79	−0.05	−0.02	−0.11	0.00	0.01
	2.62	221.4	223.29	228.26	211.76	234.61	236.69	0.01	0.03	−0.04	0.06	0.07
	3.97	269.15	254.94	259.56	245.36	266.97	268.91	−0.05	−0.04	−0.09	−0.01	0.00
ITM	5.35	310	289.03	293.04	281.58	301.54	303.36	−0.07	−0.05	−0.09	−0.03	−0.02
	6.78	337	325.44	328.67	320.22	338.18	339.86	−0.03	−0.02	−0.05	0.00	0.01
	8.24	365	364.00	366.35	361.03	376.71	378.26	0.00	0.00	−0.01	0.03	0.04

Empirical competitiveness of deterministic option pricing models

Figure 7: Plot of relative price bias of the BS and deterministic models (depict values of Table 11).

Table 12: Effectiveness of deterministic models (2007–2009)

Time to maturity	Moneyness $((S/K)-1)$				
	<i>DOTM</i>	<i>OTM</i>	<i>ATM</i>	<i>ITM</i>	<i>DITM</i>
Short maturity	PBS	PBS	BS, PBS, GC	BS, PBS, CEV	BS, PBS, CEV
Medium maturity	PBS, GC, CEV	PBS, CEV	BS, PBS, GC, CEV	BS, PBS, GC, CEV	BS, PBS, GC, CEV
Long maturity	PBS, GC, CE	PBS	BS, PBS	BS, PBS, GC	PBS

BS: Black-Scholes

PBS: Practitioner Black-Scholes

CEV: Constant Elasticity of Variance

GC: Gram-Charlier

could prove to be a touchstone to testify and justify the quality of a model. This also supports to understand and estimate its outperforming virtue as compared with the other qualifying models for the same purpose. The PBS model travelled successfully across all the turbulent phases, which could be an ordeal to examine and observe the capacity of a model to sustain during such periods. Such hardships prove to be the best arena to test various concepts developed into forms of models, and PBS is not an exception to this rule. We can also interpret the other facet of it that, if the PBS model performs during such

hardships of financial contraction, it is but obvious that it can perform par smoothly and effectively during the normal course of time. Attractiveness of the PBS model lies in the powerful duality of its tractability and robustness relative to other models. Further empirical research can be projected by studying the information content of implied skewness and kurtosis carefully through new sophisticated models. Empirical research work can also be conducted to test the forecasting performance of deterministic volatility option pricing models with stochastic volatility option pricing models

such as SV model, two-factor SV model or SV model with jumps, defining volatility as a stochastic process, for pricing Nifty index options. Work can also be launched to test the study of Christoffersen and Jacobs (2004); according to them, the PBS model not only outperforms deterministic volatility models but also complex stochastic models. Doing so would render the empirical investigation of out-of-sample pricing more interesting.

Time has always been evidential in challenging all speculated concepts. This has compelled and led the researchers to frame out better strategies to beat the probability of losses or to protect well the money already invested. However, as a constant effort to optimizing and maximizing profits of financial assets and its various operative components, the suggested solutions have performed satisfactory to protect the investment tossing and tumbling under high volatility of the market. With best of our effort, the suggested solution to such a problem could have been resolved to a great extent. The proposed concept can be further worked upon to explore various scopes and immense probability in option pricing models. This study brings out a new volume, giving a direction to proceed with and find out still unexplored opportunities. The efforts of young researchers with fresh enthusiasm can definitely design better option pricing model. The futurity of the suggested concept is highly applicable, as it can surely forecast and predict a future course of investments by protecting it on positive corner. There have been none or very thin studies of Indian market testing the applicability of option pricing models incorporating volatility and non-log distributional characteristics of Nifty indices into option pricing. We hope this study will surely set the foundation of applicability, testing and

designing of more advanced option pricing formulas for pricing of Nifty index options, safeguarding the interest of traders/investors, and will lead to a new vista of empirical and theoretical research into the area.

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