Invited Editorial

An exceedingly simple method of pricing American options

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ABSTRACT This article offers a very simple way of pricing American (and European) options without the need for volatility estimation. In addition, it relaxes assumptions such as normality and a geometric Brownian motion. Moreover, this method allows the estimation of the probability of a positive/negative payoff.

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INTRODUCTION

The existing methods of the European option pricing (especially the Black–Scholes formula) require a needlessly complicated mathematics and strong assumptions such as a geometric Brownian motion and normality. More importantly, these methods require the arbitrary (inaccurate) estimation of the volatility of the underlying asset. It is well known that there are several (controversial) methods of estimating volatility and therefore the option price changes accordingly (see, for example, Alghalith, 2012). Needless to say, pricing American options is much more cumbersome and controversial (see, for example, Cvitanic and Zapatero (1994) for a detailed discussion).

In response to these limitations, this article offers a very simple way of pricing both American

and European options without the need for volatility estimation. In addition, it relaxes assumptions such as normality and a geometric Brownian motion. Moreover, this method allows the estimation of the probability of a positive/negative payoff based on real-life data. This is very useful in forecasting the price of the underlying asset.

PRICING EUROPEAN OPTIONS

It is well known that the (theoretical) price of the European call option is given by

$$c_t = \frac{E[S_T - K]^+}{R}, R \equiv 1 + r, \quad (1)$$

where c is the price of the option, S_T is the price of the underlying asset at the exercise date T , r is the discount rate, and K is the

strike price. The above formula can be expressed as

$$c_t = \frac{\pi(uS_t - K)}{R},$$

where $0 \leq \pi \leq 1$ is the probability of a positive payoff, S_t is the current price and uS_t is the price that yields a positive payoff. Rearranging yields

$$\pi u S_t = R c_t + \pi K.$$

Alternatively, the above equation can be rewritten as

$$S_t^{-1} = \frac{\pi u}{R c_t + \pi K}.$$

The above equation can be expressed as the following regression equation:

$$\bar{S} = \frac{\beta_0}{\beta_1 R + \beta_2 K} \quad \bar{S} \equiv S_t^{-1}, \beta_0 = \beta_2 u \equiv \beta_2 \beta_3, \\ 0 \leq \beta_2 \leq 1, \beta_3 > 1, \quad (2)$$

where β_i is the parameter to be estimated, while $\bar{S}$, R and K are observed data. Clearly, β_1 is an estimate of the option price, whereas β_2 is an estimate of π . The above equation can be estimated using historical data of $\bar{S}$, R and K .

Using simulated data, we estimated (2) (using non-linear LS) and obtained the following results: the European option price $\hat{\beta}_1 = 3.21$, the probability of a positive payoff $\hat{\beta}_2 = 0.44$, $\hat{\beta}_0 = 0.59$ and thus $\hat{\beta}_3 = 1.34$.

PRICING AMERICAN OPTIONS

The price of the equivalent American option is given by:

$$A_t \geq \frac{E[S_T - K]^+}{R}, \quad (3)$$

where A is the price of the American option. As before,

$$A_t \geq \frac{\pi(uS_t - K)}{R}.$$

The above equation can be expressed as

$$A_t = \frac{\pi(uS_t - K) + V}{R}, \quad V \geq 0. \quad (4)$$

Rearranging yields

$$S_t = \frac{A_t R + \pi K - V}{\pi u}. \quad (5)$$

The above equation can be estimated by a very simple linear regression. We use the estimated values of π , u and β_0 ($\hat{\beta}_0$, $\hat{\beta}_2$ and $\hat{\beta}_3$, respectively) from the previous regression (that was used to estimate the price of the equivalent European option (Equation (1)) in order to estimate (5) as the following

$$S_t = \beta_4 + \frac{\beta_5}{\hat{\beta}_0} R + \frac{1}{\hat{\beta}_3} K \quad \beta_4 \leq 0.$$

Thus, our regression equation is

$$S_t = \beta_4 + \frac{\beta_5}{0.59} R + 2.272 K \quad (6)$$

where β_5 is an estimate of the price of the American option. As before, R and K are the observed (historical) data. Using simulated data, we estimated (6) and obtained $\hat{\beta}_5 = 3.53$.

References

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