Original Article

The effects of drawdown risk reduction on the US hedge funds

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ABSTRACT We investigate the effects of drawdown risk reduction on the US hedge funds. Despite the existence of numerous evidences on the asymmetric distribution of portfolio returns, the asymmetric risk measures have been extensively applied in risk management during recent years with the considerable applications on the lower partial moment (LPM) methodology. Unlike prior literatures, we use the drawdown risk measure (DRM), which is a special case of LPM, to study the impacts of drawdown risk decrease on management styles of the US hedge funds. The monthly returns are applied for 1720 US hedge funds over the period 2000–2011. The optimization models in the DRM form are run to optimize the risk measures and investigate the effects of drawdown risk reduction. We find that the potential benefits of funds' diversification may weaken decreases in tolerance levels of drawdown risk. Our findings increase the importance of risk (tolerance) perception, in particular drawdown risk, when making many investment decisions. We find that it can negatively affect portfolio returns. Our findings also show that skewness does not impose any significant problem in the DRM model. The DRM optimization model reduces investors' risk more than the conventional models and can be accommodated with risk-averse investors' approach.

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INTRODUCTION

Asymmetric risk studies, in particular the drawdown risk measure (hereafter, DRM), have extensively increased during recent decades. For

instance, Alexander and Baptista (2006), Steiner (2011) and Tavakoli Baghdadabad *et al* (2011) investigate the DRM measures in investment portfolios. The initial academic interest of the

DRM can stem from Markowitz's (1952) seminal paper on portfolio diversification. However, in spite of the success of the mean variance (hereafter, MV) framework and computational costs, his view on the DRM models is widely neglected. On the other hand, the development of asymmetric risk measures and the restrictions of MV increase interests in extending the DRM models. However, previous studies neglect to consider whether the DRM can flexibly describe the changing degrees of risk tolerances. Specifically, the previous studies do not develop the MV framework based on the DRM in analyzing the changing degrees of risk tolerances.

The objective of our article is first to describe the DRM risk measure and its own development, second to propose two new MV models in the DRM form and third to empirically assess the practical indications of portfolio performance from the US investors' perspective. The evaluation of the US investor's perspective provides initial implications about the issue of how the DRM can be used to affect returns' drawdown risk decrease.

The study is organized as follows. The section 'Theoretical basis and literature review' explains utility theory as a basic theory in portfolio optimization, and then reviews the MV literature, below-target-returns variation and the DRM development. The section 'Proposed methodology' proposes the methodology of funds' optimization based on the DRM. The section 'An empirical study' discusses and analyzes the empirical purpose of our article, data and research method. The section 'Discussion of results' discusses the main findings, and the section 'Concluding remarks' summarizes the findings, and discusses their implications.

THEORETICAL BASIS AND LITERATURE REVIEW

Variance and below-target-returns variation

The concept of portfolio optimization is grounded on 'utility theory', which is a theory of ranking the wealth levels. This theory has gradually changed the framework of portfolio theory by focusing on the maximization of expected utility instead of returns maximization on a given risk. However, due to the publication of Markowitz's (1952) seminal study on portfolio diversification, there are a large number of subsequent studies on portfolio performance. The majority of the studies focus exclusively on two initial moments of portfolio return distribution, namely, mean and variance. The downside risk measure is primarily introduced by Markowitz (1959). He assumes that investors equally worry to eliminate two sides of their own portfolio return distribution. However, Markowitz (1959) finds that it does not describe accurately investors' preferences to minimize portfolio losses. Thus, the studies of asymmetric risk measures, which are based on the decreasing losses of below-target mean returns, are able to make a more appropriate model of investors' decision making. By minimizing the portfolio losses of below-target mean returns, it provides portfolio allocations to minimize the likelihood of below-target mean returns.¹

Markowitz (1959) and then Nawrocki (1999) propose two downside risk measures: a semi-variance (SV) calculated by below-target SV and a SV calculated by below-mean SV. These two downside risk measures calculate variance using the returns lower than a target return or below a mean return. As a subset

of returns' distribution is only used, Markowitz calls them SV or partial as:

$$SV_m = \frac{1}{k} \sum_{i=1}^k \{\text{Max}[0, (E - R_{iT})]\}^2 \text{ Or}$$

$$SV_m = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[0, (R_{iT} - E)]\}^2 \text{ Below - mean}$$

semi - variance (1)

$$SV_t = \frac{1}{k} \sum_{i=1}^k \{\text{Max}[0, (t - R_{iT})]\}^2 \text{ Or}$$

$$SV_t = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[0, (R_{iT} - t)]\}^2 \text{ Below - target}$$

semi - variance (2)

Where R_{iT} , k , t and E are the fund return i over the period T , the number of observations, the target return rate² and the expected mean return, respectively. Max displays that the formulas will square higher than two values, 0, or $(R_T - E)$ and Min displays that the formulas will square lower than two values, 0, or $(R_T - t)$.

Harlow (1991) and then Nawrocki (1999) develop both below-mean returns and below-target SVs and find the importance of focusing on tail of returns' distribution, that is, below-target mean returns. Downside risk measure of this type is called as 'DRM', because it considers the left-hand tail (that is, likelihood of underachieving a bindery return) of returns' dispersion along with the loss that an investor suffers from 0 to t . The DRM assumes the variation of below-target returns along with the losses that an investor suffers as risk measure (Grossman and Zhou, 1993; Dacorogna *et al*, 2001; Gilli and Schumann, 2009; Schuhmacher and Eling, 2011; Tavakoli Baghdadabad *et al*, 2012). The DRM may show the extent of skewness, although it cannot be explained as

'third moment' (skewness) because skewness³ supposes variance as initial risk measure, but the DRM assumes the variation of below-target returns with respect to the losses that an investor suffers over the holding period of a given portfolio.

DRM and the relation to lower partial moment (LPM)

The DRM is widely used in practice, for example, by commodity trading advisors. It is extended by practitioners who do not base their work on theoretical considerations. Most of the prior literature on the DRM are found in journals outside of finance (that is, Petroni and Rotundu, 2008), non-refereed finance journals (that is, Burke, 1994; Kestner, 1996) and finance journals geared to investment community (Pedersen and Alfvén, 2003; Hamelink and Hoesli, 2004; Magdon-Ismail *et al*, 2004; Sharma, 2004; Chekhlov *et al*, 2005; Ammann and Moerth, 2008; Cogneau and Hübner, 2009). However, this measure has been applied to finance literature recently as a new risk measure (that is, Tashman and Frey, 2009; Tavakoli Baghdadabad *et al*, 2011, 2012).

The drawdown is the loss incurred over a given investment period (Tavakoli Baghdadabad and Glabadanidis, forthcoming). It is the loss from the prior local maximum to next local minimum of an investment, whereas a draw-up is the gain from the last local minimum to next local maximum. The DRM is also referred to as asymmetric risk measure, which is decomposed to two measures of maximum DRM (hereafter, M-DRM) and average DRM (hereafter, A-DRM) (that is, Chekhlov *et al*, 2000; Caprin and Lisi, 2009; Gilli and Schumann, 2009; Tavakoli Baghdadabad *et al*, forthcoming).

The M-DRM is also an asymmetric risk measure (Hamelink and Hoesli, 2003; Eling and Schuhmacher, 2007). It is the loss suffered when an asset is bought at a local maximum and sold at the next local minimum' (Hamelink and Hoesli, 2004) and/or the worst loss that a portfolio suffers over the holding period (Alexander and Baptista, 2006).

Alternatively, the A-DRM is the loss average occurred over the holding period (Caprin and Lisi, 2009; Gilli and Schumann, 2009). It is a measure of the loss average occurred over the holding period, which is relevant only if one buys and sells the funds under loss condition.

These measures consider the LPM as below-target risk along with the maximum and average losses that an investor suffers over the holding period. Thus, the M-DRM and A-DRM measures are as:

$$M - DRM_t = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[(D_{t-1} + (R_{iT} - E)), 0]\}^2 \quad (3)$$

$$A - DRM_t = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[(A_{t-1} + (R_{iT} - E)), 0]\}^2 \quad (4)$$

Where D_0 and A_0 are equal to 0. D_t denotes the maximum loss that an investor suffers from 0 to t . A_t denotes the loss average that an investor suffers over the holding period. Equations (3) and (4) are, in fact, a special case of the second-degree LPM.⁴ It is the biggest peak-to-trough decrease in the price of a portfolio. It is the worst return that an investor can experience, that is, the return of an investor who buys the funds at the highest price and sells them at the lowest price. Institutional investors often capture the DRM measures, M-DRM and A-DRM, as an important measure in

choosing a portfolio. In fact, institutional investors capture these measures as an indicator of portfolio risk.

Equations (3) and (4) show that investors are not likely to be risk averse across the returns' distribution and show only risk-averse behavior based on target returns and the maximum and average losses that an investor suffers, because they must determine investors' distinguished gains and risk tolerances. If target return and the maximum and average losses are higher than mean value, as defined in Equation (1), the utility measure in the mean-DRM analysis will display asymmetric pattern, that is, risk averse on downside risk and risk neutral on upside returns, which shows an investors' skewness preference.

DRM and the co-maximum and co-average drawdown risk (CA-DRM2) measures

The studies of asymmetric risk measures, such as LPM, for asset allocation problems are negligible (Nawrocki, 1992; Tee, 2009). However, scope of these studies does not develop on the DRM models in improving asset allocation problems. The Nawrocki's (1992) study provides one of the few examples in this context. He investigates the impacts of downside risk tolerances on the performance of investment portfolios using the LPM theory. Nawrocki (1992) explains a methodology to model Co-Lower Partial Moment (CLPM). CLPM combines the interactions of two securities' LPMs and considers the semi-deviation computations as:

$$SD_{ii} = \frac{1}{m} \sum_{t=1}^m [\text{Max}(0, E - R_{it})]^n \quad (5)$$

Where E , m , n and SD_{ni} are the target return, the number of observations, the LPM degrees and the semi-deviation of asset i over the period n , respectively. SD is used in the CLPM⁵ calculations as:

$$CLPM = (SD_{ni})(SD_{nj})(r_{ij}) \quad (6)$$

Where r_{ij} , SD_{ni} and SD_{nj} are the correlation coefficients between assets i and j , the semi-deviation of asset i over the period n and the semi-deviation of asset j over the period n , respectively.

Tee (2009) investigates the impacts of downside risk decrease on the UK equity-based managed futures funds. He applies the CLPM instead of standard deviation (SD) in asset allocation model to combine the interactions of two securities' LPMs.

The aforementioned studies all apply the CLPM methodology instead of SD in the asset allocation model introduced by Nawrocki (1992) and Moreno *et al* (2005). Similar application of the methodology can also be found in recent studies such as Tee (2009). To date, none of the prior studies use the DRM measures instead of SD in optimizing the investment portfolios. This study thus adopts the methodologies introduced by Nawrocki (1992), Moreno *et al* (2005) and Tee (2009) to propose two new optimized models as second-degree measures of co-maximum drawdown risk (hereafter, CM-DRM2) and co-average drawdown risk (hereafter, CA-DRM2). These two models replace the second-degree M-DRM measure (hereafter, M-DRM2) and second-degree A-DRM measure (hereafter, A-DRM2) with covariance and LPM in two models of MV and CLPM. Equations (3) and (4) are first used to compute M-DRM2

and A-DRM2 and then CM-DRM2 and CA-DRM2 as:

$$CM - DRM_{ij} = E\{[\text{Min}(D_{t-1} + (R_i - E)), 0] \times [\text{Min}(D_{t-1} + (R_j - E)), 0]\} \quad (7)$$

$$CA - DRM_{ij} = E\{[\text{Min}(A_{t-1} + (R_i - E)), 0] \times [\text{Min}(A_{t-1} + (R_j - E)), 0]\} \quad (8)$$

The CM-DRM2 and CA-DRM2 are the optimized form of M-DRM2 and A-DRM2 which are computed by Markowitz's optimization model. The next section provides a more detailed discussion of our suggested methodology.

PROPOSED METHODOLOGY

The Markowitz portfolio optimization model, as noted earlier, uses the conventional SD and covariance as risk measure. The objective of the model is to determine assets' weight, which minimizes the portfolio variance at a level of expected return rate and ensures a return that is equal or bigger than expected return. Specifically, rational portfolio selection is first formulated by Markowitz as a trade-off between return and risk. A rational investor tends to minimize his own risk from expected returns. The MV portfolio analysis is applied as an appropriate benchmark, due to its general application in portfolio theory, and then developed by adding the M-DRM2 and A-DRM2 models to that. Thus, the MV portfolio model is defined as:

$$\begin{aligned} & \text{Min } \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \\ & \text{S.t : } \sum_{j=1}^n r_j x_j \geq \mu \\ & \sum_{j=1}^n x_j = 1 \\ & 0 \leq x_j \leq 1, j = 1, 2, \dots, n \end{aligned} \quad (9)$$

Where μ is the expected rate of funds' return, r_j is the expected return of fund j , σ_{ij} is the expected covariance between returns of indicator i and j , and x_j is the proportion invested in fund j .

Nawrocki (1992) also extends the MV optimization model in terms of CLPM and adds the MV constraints to that:

$$LPM = \sum_{i=1}^n \sum_{j=1}^n x_i x_j CLPM_{ij} \quad (10)$$

We use the M-DRM2 and A-DRM2 algorithms to model the downside risk introduced by Nawrocki (1992), Moreno *et al* (2005) and Tee (2009). We minimize variance as:

$$(CA - DRM)_p^2 = \sum x_i (CA - DRM)_{ij} x_j, \quad (11)$$

$$(CM - DRM)_p^2 = \sum x_i (CM - DRM)_{ij} x_j, \quad (12)$$

Over the weights x_i , given the constraints $\sum_{i=1}^N x_i = 1$ and $\sum_{i=1}^N x_i \mu_i = \mu$, where $(CA - DRM)_p$ and $(CM - DRM)_p$ are the M-DRM2 and the A-DRM2 of a given portfolio, respectively, $(CA - DRM)_{ij}$ and $(CM - DRM)_{ij}$ are the covariance matrix of the A-DRM2 and the M-DRM2, respectively, x_i is the asset i 's weight in the portfolio ($i = 1, \dots, N$), μ is the expected return on a given portfolio and μ_i is the expected return on fund i . It is assumed that there is no absolute constraint on short selling, thus weights x_i can be of either sign and of any absolute value. This is, of course, quite unrealistic, partly for legal, partly for liquidity reasons, but this idealized setup can show the instability of portfolio selection in its purest form. Using the above assumptions, we propose two

quadratic programming models for n asset based on the A-DRM2 and M-DRM2 formulations as:

$$M - DRM = \sum_{i=1}^n \sum_{j=1}^n x_i x_j CM - DRM_{ij} \quad (13)$$

$$A - DRM = \sum_{i=1}^n \sum_{j=1}^n x_i x_j CA - DRM_{ij} \quad (14)$$

Where

$$\begin{aligned} CM - DRM_{ij} &= M - DRM_i && \text{when } i = j \\ CM - DRM_{ij} &= CM - DRM_{ji} && \text{when } i \neq j \\ CA - DRM_{ij} &= A - DRM_i && \text{when } i = j \\ CA - DRM_{ij} &= CA - DRM_{ji} && \text{when } i \neq j \end{aligned}$$

We assume that investors are risk averse, thus the objective functions which use the M-DRM2 and A-DRM2 portfolios are defined as:

$$\begin{aligned} &\text{Minimize } \sum_{i=1}^n \sum_{j=1}^n x_i x_j CM - DRM_{ij} \\ &\text{S.t : } \sum_{j=1}^n r_j x_j \geq \mu \\ &\sum_{j=1}^n x_j = 1 \\ &0 \leq x_j \leq 1, j = 1, 2, \dots, n \end{aligned} \quad (15)$$

$$\begin{aligned} &\text{Minimize } \sum_{i=1}^n \sum_{j=1}^n x_i x_j CA - DRM_{ij} \\ &\text{S.t : } \sum_{j=1}^n r_j x_j \geq \mu \\ &\sum_{j=1}^n x_j = 1 \\ &0 \leq x_j \leq 1, j = 1, 2, \dots, n \end{aligned} \quad (16)$$

Where μ is the expected rate of funds' return, r_j is the expected return of fund j ,⁶ x_j is the proportion invested in fund j . $M - DRM_{ij}$ is the expected M-DRM2 covariance, $(M - DRM_{ij} = E\{\min(D_{t-1} + X_{i,t}, 0)\})$

$[\min(D_{t-1} + X_{j,t}, 0)]\}$, between the returns of indicator i and j where $X_{i,t}$ $t = 0, 1, \dots, T$ is the market risk premium and D_t represents the maximum loss that an investor suffers from 0 to t . $A-DRM_{ij}$ is the expected A-DRM2 covariance, $(A-DRM_{ij} = E\{[\min(A_{t-1} + X_{i,t}, 0)] [\min(A_{t-1} + X_{j,t}, 0)]\})$, between the returns of indicator i and j , where $X_{i,t}$ $t = 0, 1, \dots, T$ is the market risk premium and A_t is the loss average that an investor suffers from 0 to t .

Specifically, we replace the M-DRM2 and A-DRM2 models instead of SD and CLPM2 to optimize investment portfolios. The minimization functions capture the M-DRM2 and the A-DRM2. This describes the investor's maximum and average losses along with the LPM on a given portfolio, which minimizes the relationship of the LPM of asset returns. Thus, the models' allocation is based on the assets with the lowest interacted M-DRM2 and A-DRM2 values in portfolios.

We assume that short selling is prohibited across our analysis. The analysis includes the optimization of objective functions (9), (10), (15) and (16). For objective functions (15) and (16), the M-DRM2 and the A-DRM2 are first used in the portfolio minimization. The portfolio weights are computed with respect to the constraints of objective function. The data are the returns of the US hedge funds' management styles and the S&P500 and MSCI market indexes. We analyze the performance consequences of market indexes in the US investment portfolio from 2000 to 2011. The estimation interval is 5 years from 2006 to 2011. The out-sample estimation periods are from 2000 to 2005. The holding period is 5 years and the returns of holding period are used to compare the portfolio returns

generated by objective functions (9), (10), (15) and (16).

AN EMPIRICAL STUDY

Data

We do an original and empirical study using the M-DRM2 and A-DRM2 models for security allocation problem based on the methods introduced by Nawrocki (1992), Moreno *et al* (2005) and Tee (2009). However, our sample covers a period from 2000 to 2011.

We use the data of the US hedge funds' management styles to investigate the investment strategies that include fund managers' performance on equity portfolios. Moreover, as hedge funds display positively skewed⁷ distributional attributes, this reinforces and suits the application of the M-DRM2 and the A-DRM2 very well, a usage that is rarely observed in the academic literature.

We use the monthly returns of 11 US hedge funds' management styles including Blend, Contrarian, Emerging Markets (Equity Funds), Equity Income, Geographically Focused (Equity Funds), Growth, Growth and Income, Index Fund (Equity Funds), Long Short, Market Neutral (Equity Funds), and Value in the Morningstar database.⁸ Our research population include all the funds (that is, mutual and hedge) in the database. Our sample includes monthly returns adjusted by dividend from 1720 US hedge funds from the first month of 2000 to the third month of 2011. The monthly return for the 90-day Treasury bills as free risk return and S&P500 as market index are extracted from the DataStream.⁹ The lingo software is used to model and optimize the algorithms, MV, CLPM2, CM-DRM2 and CA-DRM2. Table 1 shows the strategies used by management styles.

Table 1: Hedge funds' management styles and descriptions
Hedge funds

The US hedge funds are the aggressively managed portfolio that use advanced investment strategies such as leveraged, long, short and derivative positions in both domestic and international markets with the goal of generating high returns^a

<i>Management style</i>	<i>Brief descriptions regarding the subclasses of management styles</i>
Blend ^a	A category of equity-based mutual funds with portfolios that are made up of a combination of value and growth equities. This is also called as a 'hybrid fund'
Contrarian ^a	It is against prevailing market trends by purchasing securities that are poorly performing and then selling when being performed well
Emerging markets ^a	They invest the majority of their assets in the financial markets of a single developing country or a group of developing countries
Equity income ^a	Dividend income earned by an investment in equities. It usually invests in high-quality companies with a reliable background of dividend payments and growth in the dividend rate
Geographically focused ^a	They are designed to concentrate on a specific geographic region. Instead of purchasing a single international fund that invests all over the world, it can be mixed and matched by buying funds that specialize in each of the main geographic regions
Growth ^a	A diversified portfolio of stocks that has capital appreciation as its primary goal, with little or no dividend payouts. Portfolio companies would mainly consist of companies with above-average growth in earnings
Growth and income ^a	A mutual fund that has a dual strategy of capital appreciation (growth) and current income generation through dividends or interest payments. A growth and income fund may be only invested in equities or in a combination of securities
Index fund ^a	A type of mutual fund with a portfolio constructed to match or track the components of a market index, such as the Standard & Poor's 500 Index
Long-short ^a	A type of mutual fund that mimics some of the trading strategies typically used by a hedge fund. Unlike most mutual funds, long/short funds use leverage, derivatives and short positions in an attempt to maximize total returns
Market neutral ^a	An aggressive type of mutual fund whose purpose is to deliver superior returns by balancing bullish stock picks with bearish ones. They can also generate income from the interest proceeds of the sales of short securities
Value ^a	A stock mutual fund that primarily holds stocks that are deemed to be undervalued in price and that are likely to pay dividends. Value funds are one of the three main mutual fund types; the other two are growth and blend

^aSource: © 2013 Morningstar. All Rights Reserved.

Panels A, B and C from Table 2 show the statistics for the whole, out-sample and in-sample periods. The M-DRM2 and A-DRM2 values, which evaluate the average monthly below-target variations with respect to the maximum losses that an investor suffers from 0 to t , are calculated by the target return rate of 0.20 per cent. Panel A from Table 2 displays that all the M-DRM2 and A-DRM2 measures have the lowest value rather than the SD and the LPM2, a range from 0.22 per cent to 2.18 per cent for the M-DRM2 and 0.52 per cent to 1.77 per cent for the A-DRM2. Moreover, it ranges from about 5.03 per cent to 5.29 per cent and 1.24 per cent to 1.66 per cent for the M-DRM2 and A-DRM2 benchmark indexes, respectively. Table 2 also shows that all the S&P500 and MSCI indexes are positively skewed against four styles being negatively skewed. Panel B from Table 2 shows that all the M-DRM2 and A-DRM2 measures, similar to the results of Panel A, have lower values than other traditional risk measures, a range from 3.43 per cent to 6.56 per cent for the M-DRM2 and 0.66 per cent to 1.87 per cent for the A-DRM2. Among them, Contrarian, Emerging Markets, Geographically Focused and Growth are significantly and positively skewed. However, the benchmark indexes show higher values for the M-DRM2 and the A-DRM2, a range from 4.14 per cent to 5.32 per cent for the M-DRM2 and 1.22 per cent to 1.82 per cent for the A-DRM2. Similarly, Panel C from Table 2 shows lower DRM values than traditional risk measures, a range from 1.7 per cent to 4.76 per cent for the M-DRM2 and 0.67 per cent to 1.82 per cent for the A-DRM2.

DISCUSSION OF RESULTS

In-sample results

Table 3 Panel A displays the fund values optimized by MV, LPM2, M-DRM2 and A-DRM2, and individual funds' skewness for the In-sample periods. They are the weights of hedge funds (management styles) for portfolio constituents. Table 3 Panel B compares the performance of out-sample funds by comparing the average monthly returns, 5-year holding period returns and skewness for the MV, LPM2 and DRM models.

Table 3 Panel A shows that the covariance models have the highest value among management styles, which interprets the lowest diversification effects. The covariance model generates the highest SD when comparing that with the CLPM2 and DRM algorithms. It has the highest monthly return of 0.72 per cent among other measures, which shows a logical trade-off between risk and return in fund market. In contrast, the M-DRM2 and A-DRM2 models have the lowest values among management styles, which generate the lowest monthly return of 0.66 per cent and 0.65 per cent among other measures, respectively. It describes a logical trade-off between risk and return on fund market, in which lower DRM generates lower return. Specifically, majority of the DRM measures have the lowest risk and the lowest average monthly returns rather than other traditional measures. It implies that the DRMs are compatible with risk-averse investors' approach.

Table 3 also shows skewness increase as a result and reports that the preferences of funds' skewness increase in the DRM algorithms rather than traditional algorithms. Moreover, the Table 3 reports that average monthly returns

Table 2: Descriptive statistics of the management styles

Management styles	Mean (in percentage)	Maximum (in percentage)	Minimum (in percentage)	SD (in percentage)	Skewness	LPM2 (in percentage)	A-DRM2 (in percentage)	M-DRM2 (in percentage)	Kurtosis	Jarque-Bera (JB)
<i>(A) Descriptive statistics of the management styles for the full period from 2000 to 2011</i>										
Blend	-0.23	15.12	-11.01	7.44	0.2912	8.14	1.29	5.44	3.1809	2.2914
Contrarian	-0.19	23.34	-12.33	7.29	0.2744	6.21	1.37	5.18	3.2567	2.0013
Emerging markets	-0.35	22.11	-17.4	7.11	0.2953	8.12	1.31	6.22	3.8129	2.2341
Equity income	0.29	21.08	-14.03	7.84	0.2844	7.67	1.77	5.88	2.2432	2.5617
Geographically focused	-0.21	21.22	-15.54	7.67	0.3914	7.56	1.67	4.57	3.6702	2.1355
Growth	0.54	22.11	-21.23	6.14	-0.3801**	6.07	1.23	4.98	4.0022	4.0344
Growth and income	0.69	11.73	-16.08	5.13	-0.4010	6.12	1.21	4.39	3.8767	2.4427
Index fund	0.12	9.22	-14.61	2.55	-0.9290***	5.13	1.01	2.18	3.0130	1.0987
Long-short	-0.34	14	-12.31	4.23	0.2130	7.04	1.32	4.87	2.887	0.8532
Market neutral	-0.42	14.20	-15.48	5.41	0.2214	5.99	0.52	2.43	2.132	0.5772
Value	1.56	20.47	-19.51	5.93	-0.0842	7.83	1.24	5.03	3.6112	1.6329
S&P500	1.77	33.01	-18.12	10.88	-0.69***	7.25	1.38	5.22	3.7029	2.1132
MSCI	3.20	26.13	-10.16	8.98	0.4112	7.27	1.66	5.29	3.8605	2.8111
<i>(B) Descriptive statistics of the management styles for the out-sample period from 2000 to 2006</i>										
Blend	1.74	10.26	-11.29	5.17	-0.3749	8.28	1.04	5.74	1.7110	0.8423
Contrarian	2.55	19.79	-11.22	5.87	0.5126**	10.23	1.24	5.83	3.2987	4.019
Emerging markets	2.31	14.23	-7.06	5.22	0.1644	9.15	1.51	5.77	2.773	0.2984
Equity income	2.35	20.18	-10.02	5.67	0.5536***	8.12	1.26	5.44	4.3452	10.1242
Geographically focused	2.24	14.11	-8.24	5.33	0.0893	8.33	1.46	5.33	3.594	0.0622
Growth	2.77	21.02	-10.26	5.06	0.4401***	10.04	1.42	5.88	4.0148	10.2902
Growth and income	-0.66	21.14	-22.15	7.16	-0.4456%	8.88	1.34	6.56	3.2162	0.9019
Index fund	0.11	15.09	-13.22	5.12	-0.2310	7.57	1.87	4.86	2.3319	2.0978
Long-short	-0.77	7.75	-10.66	4.98	-0.6475	8.15	1.38	4.22	2.7492	2.4421
Market neutral	1.63	17.22	-14.06	6.02	-0.2231	8.01	0.66	3.43	2.9602	0.7715
Value	0.44	11.23	-16.22	6.23	-0.3848	9.30	1.22	4.14	2.9930	1.2919
S&P500	3.15	30.06	-14.33	12.17	-0.2136	9.76	1.76	5.44	3.5124	2.5604
MSCI	2.44	23.54	-11.42	9.15	0.4061	9.85	1.82	5.32	3.1948	2.8219

Table 2 continued

Management styles	Mean (in percentage)	Maximum (in percentage)	Minimum (in percentage)	SD (in percentage)	Skewness	LPM2 (in percentage)	A-DRM2 (in percentage)	M-DRM2 (in percentage)	Kurtosis	Jarque-Bera (JB)
(C) Descriptive statistics of the management styles for the In-sample period from 2006 to 2011										
Blend	-3.22	14.24	-11.17	6.22	0.8753**	5.22	0.79	4.23	3.2589	5.1251
Contrarian	-2.27	10.44	-12.15	6.09	0.6220	5.64	1.22	4.67	2.1906	4.2311
Emerging markets	-3.36	9.23	-13.24	6.59	0.4678	5.73	1.56	4.51	2.3420	2.1914
Equity income	-2.48	11.29	-11.13	6.21	0.5639	5.81	1.33	3.89	2.2387	4.8203
Geographically focused	-3.11	22.15	-10.12	6.19	1.0173	6.24	1.77	4.76	3.0324	5.2123
Growth	-3.06	16.21	-13.22	6.32	0.9868	5.64	1.82	4.59	3.2309	4.2367
Growth and income	1.22	16.17	-10.04	6.12	0.7230**	6.59	1.61	5.39	3.1819	4.2236
Index fund	1.18	10.12	-9.19	3.65	-0.2542	5.63	1.11	3.77	3.0112	2.4135
Long-short	1.29	6.34	-6.32	5.62	-0.2306	5.72	1.33	2.42	3.2176	2.4311
Market neutral	-1.19	12.15	-13.08	7.24	0.5433	5.22	0.67	1.70	3.1952	0.7304
Value	1.22	17.23	-11.15	6.19	0.5731	5.39	0.88	4.35	3.2231	4.1958
S&P500	-0.77	10.51	-10.19	6.51	-0.3015	5.19	1.23	4.18	2.7750	0.5648
MSCI	3.21	15.17	-10.77	9.44	0.3291	5.82	1.52	4.67	2.1940	0.8679

JB is estimated as $JB = N[s^2/6 + (k-3)2/24]$, where s , k and N are the value of skewness, the value of kurtosis and the number of data applied in the test, respectively. The JB uses a χ^2 distribution with two freedom degrees. (***) shows significance level at 1 per cent (critical value (CV) for χ^2 is 5.21) and (**) shows significance level at 5 per cent (CV for χ^2 is 9.01). The LPM2 computes the below-target variance as $LPM(2, t) = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[0, (R_T - t)]\}^2$, M-DRM2 calculates the below-target variance along with the maximum loss happened over the holding period as $M - DRM_t = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[(D_{t-1} + (R_T - t)), 0]\}^2$. The A-DRM2 also calculates the below-target variance along with the average loss happened over the holding period as $A - DRM_t = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[(A_{t-1} + (R_T - t)), 0]\}^2$, where K , t , R_T , D_{t-1} and A_{t-1} are the number of observations, the target return, the fund return over the period T , the maximum loss that an investor suffers from 0 to $t-1$ and the average loss that an investor suffers from 0 to $t-1$, respectively. Min is also a minimization function that selects smaller than two values, 0 and $(R_T - t)$.

Table 3: In- and Out-sample fund results of the MV, CLPM2, CM-DRM2 and CA-DRM2 models

Management styles	In-sample covariance	Out-sample covariance	In-sample CLPM2	Out-sample CLPM2	In-sample CM-DRM2	Out-sample CM-DRM2	In-sample CA-DRM2	Out-sample CA-DRM2
<i>Panel A: Funds allocations</i>								
Blend	0.1462	0.1545	0.0304	0.0514	0.0136	0.0545	0.0122	0.0373
Contrarian	0.1454	0.1492	0.0387	0.0478	0.0169	0.0512	0.0120	0.0113
Emerging markets	0.1461	0.1524	0.0107	0.0321	0.0153	0.0458	0.0153	0.0758
Equity income	0.1455	0.1534	0.0699	0.0534	0.0078	0.0461	0.0786	0.0461
Geographically focused	0.1460	0.1512	0.0881	0.0612	0.0112	0.0597	0.0110	0.0597
Growth	0.1467	0.1158	0.0698	0.0567	0.0108	0.0321	0.0108	0.0521
Growth and income	0.0461	0.0361	0.0744	0.0314	0.0109	0.0424	0.0988	0.0524
Index fund	0.0362	0.0444	0.0823	0.0634	0.0115	0.0434	0.0115	0.0631
Long-short	0.0215	0.0221	0.0638	0.0738	0.0126	0.0712	0.0126	0.0712
Market neutral	0.0104	0.0111	0.0174	0.0889	0.0145	0.0871	0.0145	0.0097
Value	0.009	0.0098	0.0201	0.0311	0.0176	0.0256	0.0116	0.0109
<i>Panel B: Out-sample results</i>								
Average monthly returns	0.72%	0.71%	0.69%	0.70%	0.66%	0.62%	0.65%	0.64%
5-year (from 2006 to 2011) holding period returns	17.24%	23.16%	19.3%	20.14%	15.29%	16.22%	15.74%	17.45%
SD	5.31%	6.6%	5.22%	6.84%	1.37%	5.1%	1.44%	5.17%
Skewness	2.9	3.1	4.7	4.16	5.2	4.99	5.39%	5.44%
SV	0.09%	0.12%	0.07%	0.09%	0.05%	0.06%	0.07%	0.08%
M-DRM	1.11%	1.06%	2.1%	2.05%	0.89%	0.71%	0.93%	0.77%
A-DRM	1.19%	1.09%	2.24%	2.01%	0.81%	0.75%	0.96%	0.80%
'Reward' to 'M-DRM2' (R/MD)	15.53	21.84	9.19	9.82	17.17	22.84	16.92	22.66
'Reward' to 'A-DRM2' (R/AD)	14.48	21.24	8.61	10.02	18.87	21.62	16.39	21.81

The critical values for estimating the differences of fund returns on the LPM2, M-DRM2 and A-DRM2 allocation algorithms are 2.37 for significant level at 5 per cent and 3.41 for significant level at 1 per cent in χ^2 distribution. As the P -value is equal to 0, the null hypothesis of no difference between the monthly returns made by the allocation algorithms can thus be rejected. These fund returns are significantly different from each other. The findings are similar to those in Nawrocki (1992) and Tee (2009). (***), (**), (*) shows significance level at 1 per cent (CV for χ^2 is 9.21), 5 per cent (CV for χ^2 is 5.991) and 10 per cent (CV for χ^2 is 3.11). The test of skewness is associated with JB. We define 'Reward' to 'M-DRM2' ratio, R/MD, as: $(R_i - R_f)/(M-DRM2)$, and 'Reward' to 'A-DRM2' ratio, R/AD as: $(R_i - R_f)/(A-DRM2)$ where R_i denotes the average monthly fund returns for $i=1-6$, for LPM2, M-DRM2, and A-DRM2 and R_f is the risk free rate which uses the US 3-month treasury bills.

and 5-year holding period returns of the CM-DRM2 and CA-DRM2 algorithms have the lowest values among two algorithms of the MV and the CLPM2. However, reduction in the funds (management styles) allocated, as reported in Table 3, does not increase the SD and the DRM as much as expected. The findings show when switching from MV to LPM2 and then M-DRM2 none of the algorithms have SD compatible with return levels, except M-DRM2. This creates a higher 'reward'-to-'M-DRM2' ratio,¹⁰ as reported in Table 3, where the ratio for In-sample funds reduces from 15.53 (MV) to 9.19 (LPM2), and then boosts to 17.17 (M-DRM2). Similarly, when switching from MV to LPM2 and then A-DRM2, none of the algorithms have SD compatible with return levels, except A-DRM2. This creates a higher 'reward'-to-'A-DRM2' ratio,¹¹ as reported in Table 3, where the ratio for In-sample funds reduces from 14.48 (MV) to 8.61 (LPM2), and then boosts to 16.39 (A-DRM2).

Table 4, which reports the CM-DRM2 and CA-DRM2 values of style pairs, provides the possibility to evaluate the DRM algorithms' interactions of management styles' pairs. These values are the weights of hedge funds for individual portfolio constituents. Table 4 shows many negligible values of In-sample covariance, such as Blend/Emerging Markets, Emerging Markets/Long-Short and Geographically Focused/Merging Markets. These provide a lower SD, but also less returns on the styles. The allocation of hedge funds in the MV is related to funds' variance and also to the covariance of returns' distribution. The allocation of hedge funds can be sub-optimal, in particular when three allocated funds have significant positive skewness in sample, as reported in Table 2.

Table 4 shows that the Growth style is insignificantly correlated with the most funds (between 0 and 0.021), whereas the Growth and Income style has a high correlation with the funds (between 0.0016 and 0.0676). The CM-DRM2 values also show that the Emerging Markets style is insignificantly correlated with the most funds (between 0.005 and 0.0164), whereas the Long-Short style has a high correlation with the funds (between 0.0059 and 0.0595). Table 4 also reports the majority of the styles, in MV and CLPM2 have lower values than the S&P500 and MSCI indexes. In contrast, most funds in CM-DRM2 and CA-DRM2 have higher values than the benchmark indexes. This indicates better performance of the CM-DRM2 and CA-DRM2 algorithms than MV and CLPM2. The CM-DRM2 and CA-DRM2 in-sample, remarkably, have lower values than CLPM2 and Covariance in all styles, which indicate better diversifiers of below-target variations than other two algorithms. It is remarkable that, as a result of switching from MV to CM-DRM, an increase in skewness occurs (from 2.9 to 5.2). Similarly, an increase in skewness occurs when switching from MV to CA-DRM2 (a range from 2.9 to 5.39).

The DRM models' specification, unlike the MV models, fits returns' distribution of the changing attributes. Despite their distributional attributes to minimize funds' drawdown risk, funds' return series are transformed to consider below-target variations. Moreover, the funds' return series in the DRM models, unlike the LPM2 models, considers both below-target variations and the maximum and average losses happened over the holding period. The existence of skewness implies that returns' distribution does

Table 4: Analysis of Covariance, CLPM2, CM-DRM2 and CA-DRM2 of the management styles in the models

Styles' pairs	MV		LPM2		M-DRM2		A-DRM2	
	In-sample covariance	Out-sample covariance	In-sample CLPM2	Out-sample CLPM2	In-sample CM-DRM2	Out-sample CM-DRM2	In-sample CA-DRM2	Out-sample CA-DRM2
Blend/contrarian	0.0204	0.0287	0.0311	0.0894	0.0012	0.0082	0.0061	0.0087
Blend/emerging markets	0.0109	0.0162	0.0124	0.0501	0.0043	0.0077	0.0024	0.0036
Blend/equity income	0.0115	0.0108	0.0148	0.0302	0.0043	0.0215	0.0021	0.0376
Blend/geographically focused	0.0156	0.0153	0.0116	0.0105	0.0047	0.0017	0.0021	0.0041
Blend/growth	0.0159	0.0194	0.0216	0.0566	0.0012	0.0015	0.0020	0.0049
Blend/growth and income	0.0749	0.0101	0.0532	0.0154	0.0479	0.0020	0.0422	0.0040
Blend/index fund	0.0181	0.0163	0.0210	0.0707	0.0194	0.0018	0.0161	0.0034
Blend/long-short	0.0196	0.0124	0.0126	0.0621	0.0043	0.0012	0.0148	0.0031
Blend/market neutral	0.0276	0.0280	0.0111	0.0154	0.0077	0.0032	0.0123	0.0033
Blend/value	0.0166	0.0163	0.0120	0.0112	0.0029	0.0044	0.0031	0.0047
Blend/S&P500	0.0288	0.0296	0.0326	0.0511	0.0027	0.0012	0.0021	0.0029
Blend/MSCI	0.0279	0.0287	0.0314	0.0609	0.0029	0.0011	0.0020	0.0027
Contrarian/blend	0.0142	0.0184	0.0406	0.0264	0.0048	0.0064	0.0041	0.0061
Contrarian/emerging markets	0.0136	0.0172	0.0573	0.0814	0.0031	0.0066	0.0044	0.0053
Contrarian/equity income	0.0116	0.0124	0.0360	0.0201	0.0035	0.0085	0.0043	0.0050
Contrarian/geographically focused	0.0146	0.0230	0.0241	0.0150	0.0037	0.0087	0.0041	0.0057
Contrarian/growth	0.0143	0.0178	0.0106	0.0311	0.0043	0.0011	0.0050	0.0075
Contrarian/growth and income	0.0830	0.0100	0.0748	0.0229	0.0031	0.0093	0.0044	0.0055
Contrarian/index fund	0.0213	0.0233	0.0165	0.0228	0.0044	0.0104	0.0046	0.0058
Contrarian/long-short	0.0960	0.0108	0.0895	0.0212	0.0343	0.0087	0.0461	0.0058
Contrarian/market neutral	0.0681	0.0392	0.0144	0.0234	0.0119	0.0081	0.0138	0.0373
Contrarian/value	0.0342	0.0301	0.0165	0.0129	0.0056	0.0062	0.0059	0.0062
Contrarian/S&P500	0.0554	0.0334	0.0421	0.0278	0.0030	0.0014	0.0041	0.0054
Contrarian/MSCI	0.0551	0.0320	0.0414	0.0271	0.0029	0.0012	0.0040	0.0052
Emerging markets/blend	0.0125	0.0186	0.0124	0.0131	0.0050	0.0088	0.0092	0.0075
Emerging markets/contrarian	0.0224	0.0277	0.0129	0.0129	0.0051	0.0051	0.0073	0.0088
Emerging markets/equity income	0.0390	0.0108	0.0168	0.0178	0.0164	0.0039	0.0125	0.0070
Emerging markets/geographically focused	0.0121	0.0147	0.0111	0.0161	0.0050	0.0108	0.0032	0.0067
Emerging markets/growth	0.0123	0.0199	0.0268	0.0372	0.0050	0.0096	0.0025	0.0090
Emerging markets/growth and income	0.0581	0.0961	0.0377	0.0341	0.0051	0.0129	0.0025	0.0650
Emerging markets/index fund	0.0166	0.0169	0.0224	0.0287	0.0051	0.0115	0.0023	0.0068
Emerging markets/long-short	0.0103	0.0104	0.0398	0.0424	0.0050	0.0079	0.0014	0.0068
Emerging markets/market neutral	0.0314	0.0279	0.0816	0.0707	0.0051	0.0020	0.0157	0.0044

The effects of drawdown risk reduction on the US hedge funds

Table 4 continued

Styles' pairs	MV		LPM2		M-DRM2		A-DRM2	
	In-sample covariance	Out-sample covariance	In-sample CLPM2	Out-sample CLPM2	In-sample CM-DRM2	Out-sample CM-DRM2	In-sample CA-DRM2	Out-sample CA-DRM2
Emerging markets/value	0.0125	0.0144	0.0222	0.0222	0.0069	0.0036	0.0046	0.0060
Emerging markets/S&P500	0.0365	0.0276	0.0401	0.0347	0.0050	0.0033	0.0022	0.0043
Emerging markets/MSCI	0.0348	0.0243	0.0361	0.0322	0.0051	0.0031	0.0021	0.0036
Equity income/blend	0.0963	0.0896	0.0752	0.0655	0.0556	0.0179	0.0491	0.0512
Equity income/contrarian	0.0140	0.0133	0.0123	0.0131	0.0061	0.0091	0.0019	0
Equity income/emerging markets	0.0651	0.0789	0.0214	0.0129	0.0025	0.0098	0.0179	0.0423
Equity income/geographically focused	0.0937	0.0807	0.0309	0.0271	0.0055	0.0258	0.0182	0.0298
Equity income/growth	0.0950	0.0971	0.0441	0.0508	0.0056	0.0196	0.0145	0.0362
Equity income/growth and income	0.0447	0.0463	0.0425	0.0450	0.0125	0.0263	0.0144	0.0291
Equity income/index fund	0.0993	0.0826	0.0222	0.0182	0.0055	0.0236	0.0132	0.0280
Equity income/long-short	0.0797	0.0501	0.0189	0.0160	0.0025	0.0161	0.0108	0.0282
Equity income/market neutral	0.0150	0.0136	0.0397	0.0397	0.0055	0.0318	0.0021	0.0244
Equity income/value	0.0247	0.0288	0.0184	0.0131	0.0110	0.0114	0.0081	0.0093
Equity income/S&P500	0.0842	0.0825	0.0301	0.0470	0.0025	0.0098	0.0011	0.0057
Equity income/MSCI	0.0859	0.0829	0.0332	0.0444	0.0024	0.0090	0.0016	0.0066
Geographically focused/blend	0.0152	0.0145	0.0126	0.0102	0.0110	0.0016	0.0049	0.0039
Geographically focused/contrarian	0.0204	0.0325	0.0729	0.0932	0.0120	0.0099	0.0190	0.0077
Geographically focused/emerging markets	0.0103	0.0128	0.0511	0.0524	0.0047	0.0022	0.0031	0.0034
Geographically focused/equity income	0.0109	0.0940	0.0374	0.0316	0.0059	0.0258	0.0024	0.0347
Geographically focused/growth	0.0151	0.0172	0.0421	0.0591	0.0011	0.0184	0.0020	0.0461
Geographically focused/growth and income	0.0708	0.0886	0.0391	0.0182	0.0378	0.0000	0.0192	0.0371
Geographically focused/index fund	0.0164	0.0146	0.0514	0.0738	0.0011	0.0022	0.0017	0.0035
Geographically focused/long-short	0.0990	0.0152	0.0511	0.0649	0.0478	0.0015	0.0110	0.0036
Geographically focused/market neutral	0.0292	0.0243	0.0202	0.0161	0.0120	0.0039	0.0229	0.0031
Geographically focused/value	0.0910	0.0172	0.0111	0.0134	0.0122	0.0117	0.0413	0.0082
Geographically focused/S&P500	0.0314	0.0298	0.0547	0.0561	0.0053	0.0022	0.0025	0.0032
Geographically focused/MSCI	0.0302	0.0286	0.0516	0.0570	0.0056	0.0018	0.0023	0.0030
Growth/blend	0.0178	0.0219	0.0412	0.0636	0.0109	0.0171	0.0077	0.0056
Growth/contrarian	0.0232	0.0304	0.0234	0.0580	0.0114	0.0101	0.0031	0.0113
Growth/emerging markets	0.0120	0.0195	0.0301	0.0326	0.0045	0.0094	0.0024	0.0047
Growth/equity income	0.0128	0.0130	0.0166	0.0197	0.0052	0.0026	0.0028	0.0048
Growth/geographically focused	0.0174	0.0200	0.0548	0.0684	0.0042	0.0021	0.0021	0.0053
Growth/growth and income	0.0830	0.0122	0.0115	0.0114	0.0052	0.0025	0.0210	0.0052

Growth/index fund	0.0215	0.0204	0.0316	0.0460	0.0110	0.0022	0.0190	0.0050
Growth/long-short	0.0131	0.0144	0.0427	0.0404	0.0045	0.0015	0.0015	0.0051
Growth/market neutral	0.0364	0.0336	0.0315	0.0304	0.0051	0.0040	0	0.0043
Growth/value	0.0282	0.0192	0.0344	0.0365	0.0114	0.0096	0.0074	0.0072
Growth/S&P500	0.0301	0.0297	0.0388	0.0511	0.0042	0.0023	0.0031	0.0046
Growth/MSCI	0.0322	0.0302	0.0382	0.0520	0.0049	0.0022	0.0030	0.0043
Growth and Income/blend	0.0910	0.0122	0.0265	0.0213	0.0220	0.0025	0.0226	0.0050
Growth and income/contrarian	0.0148	0.0159	0.0175	0.0194	0.0055	0.0144	0.0027	0.0132
Growth and income/emerging markets	0.0610	0.0101	0.0149	0.0109	0.0056	0.0033	0.0211	0.0041
Growth and income/equity income	0.0652	0.0675	0.0622	0.0657	0.0556	0.0384	0.0242	0.0425
Growth and income/geographically focused	0.0880	0.0111	0.0245	0.0229	0.0222	0.0130	0.0229	0.0146
Growth and income/growth	0.0890	0.0132	0.0199	0.0123	0.0159	0.0127	0.0181	0.0156
Growth and income/index fund	0.0120	0.0107	0.0136	0.0154	0.0156	0.0132	0.0016	0.0143
Growth and income/long-short	0.0725	0.0598	0.0144	0.0135	0.0113	0.0225	0.0138	0.0440
Growth and income/market neutral	0.0207	0.0186	0.0347	0.0335	0.0156	0.0158	0.0124	0.0138
Growth and income/value	0.0740	0.0112	0.0370	0.0112	0.0101	0.0097	0.0161	0.0042
Growth and income/S&P500	0.1001	0.0198	0.0382	0.0359	0.0055	0.0025	0.0024	0.0044
Growth and income/MSCI	0.0988	0.0193	0.0376	0.0350	0.0061	0.0032	0.0023	0.0046
Index fund/blend	0.0278	0.0248	0.0136	0.0109	0.0029	0.0028	0.0084	0.0060
Index fund/contrarian	0.0479	0.0498	0.0299	0.0990	0.0115	0.0167	0.0340	0.0120
Index fund/emerging markets	0.0221	0.0226	0.0364	0.0556	0.0059	0.0154	0.0026	0.0050
Index fund/equity income	0.0182	0.0152	0.0276	0.0335	0.0132	0.0123	0.0030	0.0051
Index fund/geographically focused	0.0259	0.0231	0.0214	0.0117	0.0060	0.0035	0.0028	0.0056
Index fund/growth	0.0293	0.0224	0.0436	0.0628	0.0096	0.0030	0.0023	0.0068
Index fund/growth and income	0.0152	0.0136	0.0139	0.0194	0.0059	0.0041	0.0028	0.0055
Index fund/long-short	0.0193	0.0208	0.0465	0.0689	0.0040	0.0025	0.0021	0.0053
Index fund/market neutral	0.0553	0.0391	0.0355	0.0171	0.0115	0.0065	0.0173	0.0046
Index fund/value	0.0884	0.0139	0.0194	0.0133	0.0066	0.0072	0.0193	0.0068
Index fund/S&P500	0.0566	0.0390	0.0394	0.0344	0.0056	0.0043	0.0022	0.0045
Index fund/MSCI	0.0557	0.0382	0.0379	0.0339	0.0059	0.0041	0.0020	0.0041
Long-short/blend	0.0721	0.0880	0.0461	0.0438	0.0395	0.0089	0.0104	0.0274
Long-short/contrarian	0.0135	0.0998	0.0571	0.0399	0.0059	0.0526	0.0035	0.0556
Long-short/emerging markets	0.0652	0.0639	0.0511	0.0225	0.0595	0.0287	0.0143	0.0233
Long-short/equity income	0.0680	0.0424	0.0421	0.0135	0.0252	0.0136	0.0111	0.0238
Long-short/geographically focused	0.0827	0.0110	0.0322	0.0471	0.0495	0.0110	0.0128	0.0026
Long-short/growth	0.0821	0.0914	0.0216	0.0253	0.0219	0.0313	0.0121	0.0316
Long-short/growth and income	0.0421	0.0346	0.0341	0.0782	0.0195	0.0130	0.0113	0.0255
Long-short/index fund	0.0915	0.0955	0.0211	0.0317	0.0195	0.0117	0.090	0.0245
Long-short/market neutral	0.0218	0.0302	0.0441	0.0690	0.0119	0.0207	0.0073	0.0213
Long-short/value	0.0192	0.0172	0.0234	0.0536	0.0184	0.0109	0.0091	0.0029

The effects of drawdown risk reduction on the US hedge funds

Table 4 *continued*

<i>Styles' pairs</i>	<i>MV</i>		<i>LPM2</i>		<i>M-DRM2</i>		<i>A-DRM2</i>	
	<i>In-sample covariance</i>	<i>Out-sample covariance</i>	<i>In-sample CLPM2</i>	<i>Out-sample CLPM2</i>	<i>In-sample CM-DRM2</i>	<i>Out-sample CM-DRM2</i>	<i>In-sample CA-DRM2</i>	<i>Out-sample CA-DRM2</i>
Long–short/S&P500	0.0814	0.0727	0.0512	0.0517	0.0062	0.0071	0.0067	0.0057
Long–short/MSCI	0.0812	0.0732	0.0472	0.0519	0.0066	0.0080	0.0060	0.0050
Market neutral/blend	0.0184	0.0187	0.0144	0.0103	0.0125	0.0021	0.0082	0.0022
Market neutral/contrarian	0.0651	0.0375	0.0491	0.0191	0.0435	0.0115	0.0173	0.0351
Market neutral/emerging markets	0.0182	0.0162	0.0314	0.0526	0.0175	0.0019	0.0081	0.0019
Market neutral/equity income	0.0120	0.0108	0.0219	0.0317	0.0110	0.0033	0.0086	0.0020
Market neutral/geographically focused	0.0200	0.0166	0.0192	0.0110	0.0194	0.0106	0.0076	0.0021
Market neutral/growth	0.0215	0.0199	0.0252	0.0594	0.0175	0.0023	0.0090	0.0025
Market neutral/growth and income	0.0113	0.0102	0.0119	0.0183	0.0113	0.0031	0.0075	0.0020
Market neutral/index fund	0.0151	0.0169	0.0690	0.0742	0.0116	0.0028	0.0075	0.0020
Market neutral/long–short	0.0206	0.0293	0.0311	0.0652	0.0175	0.0196	0.0070	0.0201
Market neutral/value	0.0992	0.0142	0.0431	0.0133	0.0401	0.0114	0.0191	0.0066
Market neutral/S&P500	0.0267	0.0209	0.426	0.0521	0.0110	0.0076	0.0069	0.0020
Market neutral/MSCI	0.0245	0.0204	0.0486	0.0494	0.0121	0.0093	0.0068	0.0019
Value/blend	0.0110	0.0101	0.0129	0.0144	0.0015	0.0032	0.0016	0.0033
Value/contrarian	0.0425	0.0498	0.0303	0.0125	0.0144	0.0124	0.0128	0.0114
Value/emerging markets	0.0217	0.0176	0.0212	0.0325	0.0041	0.0036	0.0034	0.0028
Value/equity income	0.0840	0.0102	0.0224	0.0122	0.0123	0.0112	0.0151	0.0032
Value/geographically focused	0.0268	0.0258	0.0216	0.0119	0.0066	0.0069	0.0074	0.0061
Value/growth	0.0422	0.0429	0.0509	0.0791	0.0311	0.0186	0.0446	0.0225
Value/growth and income	0.0218	0.0223	0.0348	0.0536	0.0044	0.0047	0.0036	0.0042
Value/index fund	0.0182	0.0152	0.0245	0.0335	0.0038	0.0059	0.0030	0.0051
Value/long–short	0.0577	0.0502	0.0654	0.0792	0.0344	0.0316	0.0313	0.0345
Value/market neutral	0.0662	0.0654	0.0436	0.0324	0.0302	0.0400	0.0402	0.0416
Value/S&P500	0.0677	0.0634	0.0591	0.0626	0.0047	0.0039	0.0039	0.0038
Value/MSCI	0.0611	0.0622	0.0576	0.0603	0.0042	0.0037	0.0037	0.0035

not influence allocation process as much as the MV portfolio model.

Out-sample results

Table 3 Panel A shows the values of MV, LPM2 and DRM, and individual funds' skewness for the out-sample periods. Table 3 Panel B also compares and analyzes the out-sample performances, comparison of average monthly returns, 5-year holding period returns and funds' skewness on the MV, LPM2 and DRM models.

Table 3 panel A shows that the covariance models have the highest value among management styles. This shows that the covariance has the lowest diversification, rather than all the other models. The covariance generates the highest SD when comparing that with the CLPM2 and DRM models. Similar to the In-sample results, it has the highest monthly out-sample return of 0.71 per cent among other risk algorithms. Table 3 also shows increase in skewness and finds that the preference of funds' skewness increases in the DRM algorithms relative to other two algorithms. It reports that average monthly returns and 5-year holding period returns of the DRM algorithms have the lowest values among two algorithms of MV and CLPM2. Similar to the In-sample results, a decrease in the funds (management styles) allocated, as reported in Table 3, does not increase SD as much as expected. The evidence shows when switching from MV to LPM2 and then DRM, none of the measures have SD compatible with the return levels, except M-DRM2 and A-DRM2. This provides higher 'reward'-to-'M-DRM2' and 'reward'-to-'A-DRM2' ratios, as reported in Table 3. The 'reward'-to-'M-DRM2' ratio

reduces from 21.84 (MV) to 9.82 (LPM2), and then boosts to 22.84 (M-DRM2). The 'reward'-to-'A-DRM2' ratio also reduces from 21.24 (MV) to 10.02 (LPM2), and then boosts to 21.81 (A-DRM2).

Table 4 shows the covariance, CLPM2 and DRM models of the funds allocated in the MV, CLPM2, CM-DRM2 and CA-DRM2 algorithms. The results reported in the out-sample styles well support the In-sample results. Table 4 reports the values optimized in the MV, LPM2, M-DRM2 and A-DRM2 algorithms. The allocation of the funds in the MV model is based on funds' variance and covariance of total return distribution. The outcome of funds' allocation can be sub-optimal, especially when three allocated styles have significant positive skewness for the out-sample periods, as reported in Table 2. Table 4 also shows many relatively lower CA-DRM2s, where it reports low values of the out-sample CM-DRM2 such as those of Equity Income/Contrarian, Market Neutral/Equity Income, and Market Neutral/Growth and Income. Similar to In-sample styles, these low CA-DRM2s implicate better diversifiers generated in funds' below-target variations. The out-sample funds generate a monthly return of 0.62 per cent. It is notable that, as a result of moving from MV to CA-DRM2, an increase in skewness occurs (from 3.1 to 5.44). Table 4 also displays some lower CM-DRM2 where, it shows the out-sample CM-DRM2 values such as those of Blend/MSCI and Contrarian/Growth. The CM-DRM2s, in fact, have lower values than the traditional measures, which indicate their better diversifiers based on below-target variations. The out-sample CM-DRM2 generates a monthly return of 0.62 per cent. It is notable that, as a result of

moving from MV to CM-DRM2, an increase in skewness happens (from 3.1 to 4.99).

Table 4 shows the CLPM values where the Growth style is insignificantly correlated with the most styles (between 0.0015 and 0.0171), while the Long–Short style has the highest correlation with the styles (between 0.0071 and 0.0526). Table 4 also shows that majority of the styles in two models of MV and CLPM2 are underperformed by the S&P500 and MSCI indexes, whereas the most funds of CM-DRM2 and CA-DRM2 are underperformed by the benchmark indexes. It indicates a better performance of CM-DRM2 and CA-DRM2 than MV and CLPM2. Similar to the In-sample results, the CM-DRM2 and CA-DRM2 values In-sample are lower than the CLPM2 and MV algorithms in all styles, which indicates that they are better diversifiers based on below-target variations than other two algorithms.

CONCLUDING REMARKS

The applications and concepts of the DRM reinforce the methodology of risk management under asymmetric risk condition. The effects of asymmetric risk reduction in recent studies, especially the studies of risk management such as Hyung and Vries (2005), Danielsson *et al* (2006) and Tee (2009), show much greater applicability of the LPM and its relevant models on funds' allocation problems when investors show an extensive range of risk-averse behaviors in below-target returns. Accordingly, using the DRM models, it is analytically indicated that the M-DRM2 and the A-DRM2 are simply a special case of the LPM when $n = 2$. It shows that the M-DRM2 and A-DRM2 measures include much greater restrictions compared with

the LPM2 model in describing the risk tolerance levels. However, despite some problems on the DRM, such as reduction of information content of a data set and effects of common factors of the styles on the measures, drawdowns are suitable as performance evaluation measure.

This article first develops two portfolio optimization models of MV and CLPM2 and then proposes two new portfolio optimization models based on the M-DRM2 and A-DRM2 risk measures. Then, it empirically concentrates on the M-DRM2 and A-DRM2 models to analyze the effects of drawdown risk reduction on the US hedge fund diversification and the effects of making a target threshold return to adjust the risk tolerance levels in those two models. The MV and CLPM2 models are used along with the CM-DRM2 and CA-DRM2 models in funds' allocation process. In addition, the US hedge funds' management styles and the S&P500 and MSCI benchmark indexes are used in the process of our analysis, due to their general usage in portfolio theory. The DRM models' allocation is based on funds' return, which is transformed by algorithms in the models to consider drawdown risk.

Similar to the LPM2 models and unlike the MV portfolio, skewness does not impose any significant problem on the DRM models. This implication is similar to Tee's (2009) optimization approach on the LPM models. However, it is revealed that the effects of changing the drawdown risk tolerances in the DRM models makes a reduction in funds' return. The CM-DRM2 and CA-DRM2 optimization models reduce investors' risk levels more than other conventional models due to lower values of CM-DRM2 and CA-DRM2 than other two risk algorithms, MV and CLPM2. On the other hand, we conclude that

fund returns generated by the DRM measures get lower value than other conventional measures. Thus, as another importance implication, the M-DRM2 and A-DRM2 measures can be accommodated with risk-averse investors' approach.

An increase in management styles' skewness indicates the existence of a logical trade-off between skewness and returns, a finding that is similarly found by Huang and Yau (2006) and more recently by Tee (2009). Finally, we use the DRM models to investigate the funds' diversification outcomes on the US investors with many tolerances of drawdown risk. We use data on the US hedge funds that are already diversified based on management styles.

The results have implications on the US hedge funds so that the potential advantages of funds' diversification may improve remarkable reductions in drawdown risk-tolerance levels. This enhances the importance of risk tolerances' perception (that is, Veld and Veld-Merkoulova, 2008), especially drawdown risk when making decisions on the US portfolios.

NOTES

1. Due to the computational costs and complexity of asymmetric risk measures (that is, SV, downside risk, LPM and DRM), in particular when such analyses can only be considered repeatedly, Markowitz (1959) selects not to follow this line of investigation. He refuses the SV as a preferred risk measure and focuses instead on his own mean-variance approach. However, Markowitz (1959, p. 194) mentions that the superiority of variance with respect to the computational costs, familiarity and convenience may disappear in the future, which prevents the application of SV and its relevant developed risk measures.
2. The target rate is normally supposed to be 'zero'. It depends on how to be defined, thus mean return or risk free rate can be used as target return.
3. In order to elaborate their differences, we consider a portfolio selection problem along with skewness, which fits Polynomial Goal programming (PGP) framework to optimization; see for more details Chunnachinda *et al* (1997) and Prakash *et al* (2003). To make the optimization, the standard statistical moment of distributions are incorporated where investors show a preference on higher values than skewness and mean returns, and detestation on higher values than kurtosis and variance (Scott and Horvath, 1980). Thus, purposes of three moments are described to minimize variance, maximize the expected return rate and maximize skewness. Unlike the DRM model, the PGP optimization improves the portfolio selection problem with respect to variance. Thus, skewness and other two moments are used to reflect the attitudes toward both higher and lower returns of the distributions. In contrast, the optimization form of DRM improves the portfolio selection problem by minimizing the variation of below return-target levels.
4. Bawa (1975), Fishburn (1977) and then Nawrocki (1999) introduce the LPM as below-target risk $LPM(n, t) = \frac{1}{k} \sum_{i=1}^k \{\text{Min}[0, (R_{iT} - t)]\}^n$, where n values are the 'weights' that are placed on the tolerances of below-target variation. The higher n value describes that investor is risk averse with respect to below-target variation. Equations (1), (3) and (4) imply

that investors are not probably to be risk averse across the total return distribution and reveal only risk-averse behavior in target returns, because the returns must consider investors' preferred gains and their risk tolerances (Fishburn, 1977). If target value is mean return, as defined in Equation (1), then the utility measure in the mean-LPM analysis ($n > 1$) will show asymmetric pattern, that is, risk averse on downside risk and risk neutral on upside returns, implying investors' skewness preference, and a variety of attitudes toward lower risks than a certain target level. The DRM, as a special case of second-degree LPM, is based on the aforementioned descriptions and implies that investors are risk averse to below-target returns.

5. It is also called as the symmetrical CLPM. Elton *et al* (1978), Nawrocki (1992) and recently Tee (2009) provide the motivation to apply symmetrical CLPM. They indicate that a simple framework can make better prediction than a complex optimum framework. More specifically, Nawrocki (1992) formulates the symmetrical CLPM and proposes positive semi-definite matrix, which is an important attribute to solve an optimization problem.
6. The expected fund returns, μ , is associated with the expected returns of individual securities and their respective weightings. The expected returns of individual security are calculated on the In-sample mean returns from 2000 to 2005.
7. This is well reported in the literature; see, for instance, Lamm (2003, 2005) and Tee (2009). In this article, all management styles report significantly positive values.

There is significant positive skewness in some management styles' returns over the Out-sample and In-sample funds (see Table 2).

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9. The source of our data is Thomson Reuters.
10. This ratio is primarily proposed by Nawrocki (1992) and then applied by Tee (2009) to analyze funds' performance in the LPM framework. In this article, we use the concept of this ratio for introducing a new performance measure as 'reward'-to-'M-DRM' ratio.
11. In this article, we use the concept of this ratio for introducing a new performance measure as 'reward'-to-'A-DRM2' ratio. See Footnote 10.

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APPENDIX

The M-DRM

The M-DRM is the loss suffered when an asset is bought at a local maximum, and sold at the next local minimum (Hamelink and Hoesli, 2004). In the mean-drawdown behavior framework, investor's utility is $U = U(\mu_P, \Sigma_P^2)$, where Σ_P^2 denotes returns' drawdown risk on investor's portfolio. In this case, the risk of a certain fund i is individually measured by downside SD of a combination of loss from a local maximum to next local minimum plus risk premium, which is given by

$$\sum_i = \sqrt{E\{\min[(D_{t-1} + (R_{it} - \mu_{it}), 0)]^2\}} \quad (\text{A.1})$$

Where D_0 is equal to 0. D_t denotes, at time t , the maximum loss that an investor suffers from 0 to t . Equation (A.1) is, in fact, a special case of semi-deviation, which can be more generally

expressed with respect to any benchmark return $B(\Sigma_{Bi})$ as

$$\sum_{Bi} = \sqrt{E\{\min[(D_{t-1} + (R_{it} - B)), 0)]^2\}} \quad (\text{A.2})$$

To clearly describe the M-DRM, let us define drawdown in a recursive way as:

$$D_t(X_{i,t}) = \min[(D_{t-1} + (R_{i,t} + R_{f,t})), 0] \quad (\text{A.3})$$

Where D_0 is equal to 0. Given the sample observations for $X_{i,t}$ $t = 0, 1, \dots, T$, maximum drawdown $D_t(X_{i,t})$ or simply D_t represents, at time t , the maximum loss that an investor suffers from 0 to t . The risk measures are defined as ordering drawdown and computing quantities such as maximum drawdown, $D_1(X_{i,t}) = \min_{D_{t=1}^T$, or the second largest drawdown $D_2(X_{i,t}) = \min(D_{t=1}^T - D_1(X_{i,t}))$ and so on.

The A-DRM

The A-DRM is the loss average suffered when an asset is bought under loss condition (Chekhlov *et al*, 2000; Caprin and Lisi, 2009; Gilli and Schumann, 2009). In the mean-drawdown behavior framework, investor's utility is defined as $U = U(\mu_P, \Sigma_P^2)$, where Σ_P^2 denotes the returns' drawdown risk average on investor's portfolio. In this case, the risk of a certain fund i is individually measured by downside SD of a combination of loss average from a local maximum to next loss plus risk premium, which is given by

$$\sum_i = \sqrt{E\{\min[(A_{t-1} + (R_{it} - \mu_{it}), 0)]^2\}} \quad (\text{A.4})$$

Where A_0 is equal to 0. A_t denotes, at time t , the average loss that an investor suffers from 0 to t . Equation (A.4) is a special case of semi-deviation, which may be more generally expressed with respect to any benchmark return $B(\Sigma_{Bi})$ as

$$\sum_{Bi} = \sqrt{E\{\min[(A_{t-1} + (R_{it} - B), 0)]^2\}} \quad (\text{A.5})$$

To clearly describe the average DRM, let us define drawdown in a recursive way as:

$$A_t(X_{i,t}) = \min[(A_{t-1} + (R_{i,t} + R_{f,t})), 0] \quad (\text{A.6})$$

Where A_0 is equal to 0. Given the sample observations for $X_{i,t}$ $t = 0, 1, \dots, T$, average drawdown or simply A_t represents, at time t , the average loss that an investor suffers from 0 to t .