Original Article

A policy proposal to manage the risk of leveraged MBS and CDO purchases

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ABSTRACT This article presents a Monte Carlo-based mechanism for systematically curbing the risk borne by lenders when extending loans for the purchase of mortgage-backed securities (MBS) or collateralized debt obligations (CDOs) on leverage as investment vehicles. On the basis of the contention that the recent credit crisis of 2008 was chiefly derived from perverse incentives created by the enormous borrowing appetite of the hedge funds and other market participants and the fallacious appearance of credit default swaps, the authors propose a more rigorous implementation of well-known mechanisms for systematically quantifying risky debt interest rates, along with a government policy to require capital reserves and regulate maximum leverage, which would have quelled the real estate bubble that burst in 2008, still unfolding today. A thrust towards the potential systematic implementation of the suggested debt risk curbing mechanisms as a precondition to originating loans by the banking community, thus constituting potentially enforceable policy matter for urgent prudential regulation of the financial markets in the future, hints

at the new light under which banks should consider dynamically managing their capital reserve structures. This could contribute to the safe continued growth of the CDO and MBS derivatives markets, while shifting the focus of value creation for bank stockholders to asset growth without undue systematic risk. The authors also recommend academia to consider emphasizing the teaching of risk management and the pricing of risky debt in the MBA curricula, given the pivotal role MBA graduates have in providing intellectually intensive labor to banks and other financial institutions.

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INTRODUCTION

This article analyzes the behavior of bank-issued loans applied to finance the purchase of pools of mortgages, commonly called MBS (mortgage-backed securities), or *tranches* (portions) of such mortgage pools in the form of CDOs (collateralized debt obligations). We start by generating a methodology to estimate the appropriate cost (interest rate) assignable to bank loans to protect them from sudden systematic mortgage defaults. Subsequently, our analysis estimates the capital reserves the banks would need to back up potential sudden systematic rises in the defaults of such loans.

We posit that the 2008 credit crisis was partly due to the perverse incentives created in the *real estate supply chain* (Figure 1) by the large appetite for CDOs and MBS products initiated by the

hedge funds and other market participants. At one point before the crisis, about 1.5 trillion dollars were invested in hedge funds,^{1–2} which, in competition, sought ever-greater performance and used leverage as a magnifier of returns. Bank lenders, on the other hand, felt confident that by using credit default swaps^{3–4} they would be able to lend below the statistical breakeven rates without having to bear the risk of default on the part of their borrowers (or their investment mechanisms). We believe that what was not taken into account by market participants is the possibility of a sudden, *systematic* default of mortgages caused exogenously across the financial markets.

In our view, *systematic risk* across the financial system calls for *systematic preventive measures*, determined on a loan-by-loan basis in a lender's

Figure 1: The real estate 'supply chain'.

portfolio. In order to quell future credit crises, and to adequately protect individual banks' balance sheets, statistically derived mechanisms must be used to ensure that *each of the bank loans* granted to hedge funds for the purchase of asset-backed securities is secure even when a *sudden, systematic* wave of defaults occurs in the assets backing such securities.

Banks could have used standard simulation techniques (such as the Monte Carlo method described in this article and which we have applied in the Excel spreadsheets we have made available at www.cdosimulation.com) to better price their loans to speculative investors planning to purchase MBS or CDOs on leverage. However, before 2008, banks had access to abnormally cheap capital and faced incentives to lend at lower rates, often below statistical breakeven, to cope with competition. As a result, speculative investors were able to borrow excessively and fueled a credit bubble, which busted, contributing to the current financial crisis. In order to realign incentives, we propose the creation of *dynamic* capital reserves that can be monitored, ideally *loan-by-loan*, by the banking regulators. The necessary reserves are lower when the bank assumes less leverage on a CDO or MBS purchase, resulting in an incentive for banks to price their loans at levels at or above statistical breakeven. This will help the banks avoid unnecessary systematic risks that represent a danger to the entire financial system.

METHODOLOGY OF THE MONTE CARLO EXPERIMENTS

We framed the problem as one of a bank lender providing financing to a speculative investor (whom we call equity holder and is typically a

hedge fund) to acquire a pool of *plain vanilla* fixed rate mortgages constituting a pool (emulating an MBS or a CDO). To simplify our analysis, these pools were defined to have only 500 mortgages (real-life pools can be substantially larger), each of them with a US\$1 (or \$ 1000) present value at origination and a term of 180 months to become fully amortized (selected simulations were run using 360 months demonstrating that the variation in results was not significant). The cash flow behavior of these mortgage pools was then simulated using standard Monte Carlo techniques⁵ in which a prepayment pattern following the standard PSA model^{1,6} was applied. Furthermore, we simulated different default patterns and cumulative default rates to determine the ensuing cash flows, which were then partitioned into a cash flow to service debt and, to the extent a residual cash flow remained, cash flow to the equity holder investing in the pool.

Default and prepayment assumptions

In a 'normal' economy, mortgage pools normally show a gradual appearance of defaults through the entire term of the pool's mortgages (Figure 2).

The simulations followed one of four defaulting patterns:

Gradual default: Simulates the smooth accumulation of nonperforming mortgages along time within the pool until the total expected percentage of defaults is reached. This pattern is shown in Figure 2.

1. *Step change 1:* Simulates all defaults occurring at origination (month 1),

Figure 2: Graph of a typical gradual appearance of defaults in a pool of mortgages. Mortgage pool defaulting GRADUAL pattern: 12 per cent, Mortgages pool: 500 mortgages total

2. *Step change 12:* Simulates gradual defaults for the first 11 months with all the remaining defaults occurring in the 12th month.
3. *Step change 30:* Simulates gradual defaults for the first 29 months with all the remaining defaults occurring in the 30th month.

In terms of total default percentage rates, we simulated various scenarios: from a low 4 per cent of total mortgages in the pool, through a 'typical' BBB mortgage pool⁷⁻⁸ defaulting level of about 8 per cent, to 10 per cent and 12 per cent, which in our view are the most likely ranges in which pools of mortgages are likely to default in a systemic way (the current average default rate for all outstanding mortgages is about 12 per cent).⁹ Extreme levels of between 16 and 20 per cent were also simulated to understand the sensitivity of lender and equity holder returns under such critical conditions should they ever materialize in the economy.

For prepayments, we assumed three interrelated rules that should normally be observed by the borrower/equity holder:

1. When prepayment of a particular mortgage occurs, the proceeds are used entirely to repay the bank lender.
2. All available cash flow from all performing mortgages remaining in the pool must be

available to make up for the shortage in the cash flow, which should be going to the lender from all defaulted mortgages residing in the pool up to that given period. This is, of course, in addition to the cash flow corresponding to performing loans for the period.

3. The residual cash flow, if any, is then the cash flow available to the equity holder.

Terminology and definitions

Through our analysis, we use the following terminology:

1. *Quoted lender rate* is the lender's stated rate to collect from an equity holder buying a pool of mortgages on leverage.
2. *Achieved Lender IRR* is the internal rate of return (IRR) calculated using the cash flows received by the bank in each simulation.
3. *Required lender rate* is the minimum achieved lender IRR that enables the lender to meet all its obligations to its depositors and all other sources of funds. We set it at 4.4 per cent p.a., equal to our assumed *risk-free rate*, but we varied it for certain analyses.
4. *Leverage threshold* is the amount of leverage over which the lender will not be ever able to achieve its *required lender rate* (regardless of any

quoted rate) for varying default rates and default patterns.

5. *Real equity IRR* is the IRR calculated using residual cash flows received by the equity holder (typically, a hedge fund).
6. *Required Capital Reserves (RCR)* is the minimum level of reserves that would enable the bank to cope with any cash flow shortages derived from a sudden jump in defaults in the mortgage pool in such a way that the *Achieved Lender IRR* becomes *equal* to the *required lender rate*, for any given *quoted lender rate*. The RCR can be invested by the bank at the risk-free rate through the life of the loan.

A gap of about 80–160 basis points is generally considered to be ‘normal’ between the rates of the treasury bonds and corresponding mortgages.^{10–11} In light of this rule, we assumed a risk-free rate of 4.4 per cent p.a. and a mortgage rate of 6.0 per cent p.a., in line with historical levels.

RESULTS OBTAINED FROM THE SIMULATIONS

Dynamic bank reserves as policy tool

As mentioned above, there may be a natural incentive for banks to lend at the rates below their statistical breakeven. From a public policy perspective, if banks are to alter their lending practices to cope with the risk of systematic mortgage defaults, a regulation dictating minimum capital reserve requirements (RCR) to be enforced when lenders extend credit may be more easily monitored and could be more easily implemented.

This capital reserve would then have to be invested at the risk-free rate in an ‘escrow’

account, such that it is always available to offset cash flow shortages from the mortgage pool as defaults occur, to cover the monthly obligation towards the lender. The reserve would have to be recorded and identified against the origination of each loan and the remainder of the capital reserve (normally the reserve itself plus accrued interest) would be returned to the bank’s stockholders, or applied to another use, as soon as the loan is fully paid off. This would give the banks the ability to dynamically manage their capital structure, much as has been touted by the Basel II accord¹² to be the proper way to cope with future systemic risks in the financial sector.

We used our simulations to estimate the appropriate reserves necessary for various cases. Our analysis shows, as would be expected, that the greater the gap of the required lender’s rate over the risk-free rate (5.00 per cent p.a. minus 4.4 per cent risk-free rate p.a. in our simulations), the greater the amount of reserves needed to be held in the escrow account to cope with a potential unexpected event of a step default wave across the pool of mortgages in month 12 after the pool was originated, for any given level of leverage.

The total default rate (used in ‘Step 12’ fashion) also had a significant impact in RCR, when seen from a capital reserves perspective. At 10 per cent default levels, the reserves must begin to be accumulated when leverage for the purchase of the mortgage pool is at 90 per cent, whereas at 15 per cent default levels the lender must begin to accumulate reserves at a leverage level of 70 per cent. Reinforcing the previous statement with lender participation at 99 per cent leverage levels, not unusual in the heady days of the CDOs, there is a stark contrast between the level of reserves as a function of

actual mortgage step (sudden) default rate incurred in the economy. To illustrate, as a percentage of the loan amount, when the default rate is 10 per cent the adequate capital reserve required is about 7.19 per cent, whereas with a default rate of 15 per cent the bank would need to reserve a 14.45 per cent of the loan amount. A simple rule of thumb to establish policy stipulations (assuming a 12 per cent default rate on a Step 12 default fashion as sudden systemic shock to the global economy, as seen these days) is: 'From about 70 per cent to 99 per cent of leverage, increase the bank equity reserves about one half of a percent for each 1 per cent increase in the leverage ratio of the loan. At 99 per cent of leverage for a particular loan, the bank should have as dedicated capital reserves about 15 per cent of the loan amount assigned to such individual loan'.

An important feature of this approach should be highlighted: The temporary hoarding of equity reserves represents an opportunity cost to the bank's equity holders. This is because the capital reserves only earn the risk-free rate (about 4.4 per cent). The weighted average return to bank capital is thus continuously diminishing as the loans become riskier

(applied in higher leverage situations to mortgage pool purchases). As a result of this, the bank's return on assets for loans placed to buy mortgage pools on leverage will diminish constantly as the lenders become willing to extend credit in higher leverage situations (see Figure 3). This opportunity cost upon the bank's equity holders will naturally incentivize them to direct their management towards lesser leverage loans related to mortgage pool purchases, certainly below the 70 to 80 per cent loan-to-value level. This has a further quelling effect on future real estate bubbles by diminishing the availability of easy credit for the origination of excessively leveraged MBS and/or CDO investments, a beneficial byproduct of this policy. Under this policy, if implemented by the lenders and blessed by the regulators, the banks may grow assets (loans) more slowly, but the overall loan quality is likely to improve across the sector. Furthermore, the volatility of the banks' cash flows should greatly diminish, thus reducing the sector's Beta. In this sense, this policy may eventually enhance the valuation of banks' stocks even though banks' asset growth may be lower. Further, potential reductions of the escrow equity capital reserves

Figure 3: Effect of capital reserve to returns to bank equity. Return to bank's equity holders when lending to an investor buying an MBS or CDO on leverage, fully performing loan. Sudden step in month 12, def. rate 12per cent, risk free 4.4per cent, required lender rate = 5.0per cent & capital portion of the bank's assets: 6.25per cent

whenever the risk profile of the bank's assets is lowered result in effectively yielding cash back to stockholders, with the ensuing favorable effect upon the bank's valuation.

Incorporating tranches into the analysis

In order to study the effect of 'tranching' upon lender's achieved returns and equity holders' returns from holding each tranche, we simulated a single pool of 500 mortgages, which was subjected to various defaulting patterns. This pool was split into two simple sub-pools, the 'AAA' tranche and the 'toxic' tranche. Both were of the same size. The rule we followed was to simply substitute each defaulted mortgage in the AAA tranche by a new functional one from the toxic tranche as each default occurred. Needless to say, the AAA tranche yielded the required rate to the lender and yielded effective returns to the investors following, strictly, the Modigliani-Miller predicted rates on equity. These returns on equity were so high, and the risk to the equity holders so low because of the contractual structure of the payoffs (any bad mortgage gets discarded and gets replaced) that, in orthodox terms, the pricing of these AAA tranches should have risen in the secondary markets so as to approximate the resulting returns to a risk-free rate, purely on no-arbitrage

condition grounds. This issue may have been another perverse incentive driving the packaging of mortgage pools and their subsequent tranching in the real estate supply chain, but that is now merely speculation. The toxic tranche, on the other hand, effectively essentially *doubled* whatever the rate of defaults was for a given whole pool of mortgages. Needless to say, no returns were achievable by the equity holder even for close to nil leverage levels, while the lender's ability to recoup its principal and interest was fatally impaired, generally at leverage levels of 80 per cent and higher (see Figure 4). This analysis begets the question of how and why investors would ever be attracted to lower quality tranche in the CDO markets. Our speculation is that buyers did not know what they were acquiring in the lower-tranche CDO spectrum, given the lack of regulation and specificity about the financial product and cash flow behavior of these tranches.^{2,5,8}

Other effects identified in the simulation

The reader is referred to the graphs and tables shown in the ftp site of this article for further illustrative reference of the results from our research. We present here the following rather general conclusions regarding the behavior of

Figure 4: Effects of tranches to equity returns. Equity returns for a purchase of an ABS, default rate 12 per cent risk free 4.4 per cent, required lender rate: 4.4 per cent

returns to both lenders and the investors (equity holders) acquiring pools of mortgages on leverage:

1. *Effects of leverage:* Over the 80 per cent range, the pool's cash flow available to meet debt service is very sensitive to the rate of default impacting the pool. Over the 90 per cent leverage range, there frequently are situations where if a pool of default suddenly undergoes even reasonably low (8 per cent) default rates by month 12, no quoted lender rate will enable the lender to earn its required lender rate from its loan.
2. *Effects of total default rate:* With default rates under 8 per cent, the lenders fare well in a scenario of adverse economic shock affecting the pool of mortgages. The situation worsens rapidly, and at 12 per cent default rates and higher the capacity of a mortgage pool to yield sufficient cash flow even for the lender providing leverage for a purchase of the pool is gravely imperiled.
3. *Effects of systematic shocks:* A gradual increase in defaults has very slight adverse impacts on the lender providing the financing, even at leverage levels as high as 95 per cent. Step (shock) changes in defaults, in contrast, can

be devastating for the lender, especially when these defaults are generally above 10 per cent. The sooner the step change occurs after the pool has originated, the more pronounced the potential deficiency to meet lender obligations. As a rule, if we consider a step increase in defaults from zero to 12 per cent to occur in month 12 since the moment of the origination of the MBS, leverages higher than 75–80 per cent will imply that even with lender's quoted rates higher than the required lender rate, the pool will only be able to yield the required lender rate up to a point. Once the leverage levels are in the order of 95 per cent or higher, when default rates jump up 8 per cent or more, the lender will not be able to achieve its required lender rate no matter how high the quoted lender rate is (see Figure 5).

4. *Effects of the spread between the required lender rate and the mortgage rate:* Although the spread between mortgages and risk-free rates has been generally stable at about 160 bp across different economic times, the lender usually needs to lend at rates higher than the risk-free rate. Our explorations entailed raising the lender's required rate from 4.4 per cent

Figure 5: Leverage threshold to continue achieving the required lender rate = 4.4 per cent (equal to the risk free rate, too), with different default pattern

(the risk-free rate) to 5.0 per cent p.a. In these explorations, we found that the smaller the gap is between the lender's required lender rate and the pool's mortgage rate (in our simulations, the latter being 6 per cent), the more extreme the conditions will be regarding the adverse effects of defaults upon the maximum allowable leverages and the attainability of the required lender rate in the form of the lender's IRR for any given leverage and default levels.

5. *Effects on the equity holders:* For a step default pattern in month 12, for leverages down to 0 per cent, the equity holder investing in an MBS or a CDO captures no better real (IRR) return from his/her investment than the mere risk-free rate. And from there, as leverage increases, the real return gradually decreases, becoming negative for a leverage of about 97 per cent when the step jump in defaults is 8 per cent or higher. In essence, unless defaults are gradual, and unless mortgages constituting the pool are not subprime (subprime mortgages present a total default rate of about 14 per cent over a cohort's life),¹³ the real return to equity holders buying an MBS or a CDO on leverage is non-existent or negative.

KEY CONCLUSIONS FOR BANKING SECTOR POLICY MAKERS AND ACADEMIA

No matter how the derivative products formed in the financial markets slice and repackage cash flows, if they shrink because of defaults of the mortgages within the pools, their adverse impact on the ensuing returns to each party entitled to receiving those reconfigured cash flows

can be considerable. Given this fundamental tenet, we present the following points for the consideration of the financial community:

1. It should be essential, from now on, for all financial analysts and decision makers involved in the packaging and repackaging of pools of mortgages, to analyze the cash flow patterns likely to be derived from such reconfigured mortgage products at a far more intimate, quantitative, level. Present computational availability mandates that the analyses of cash flow be performed mortgage by mortgage within a given pool, with a clear understanding of each of the mortgage components, principal, interest and total monthly installments. It will also be critical to understand the effect of sudden, unexpected high reductions in the pool's cash flows because of exogenous events.
2. The fundamental rule of paying the bank's loan with all available cash flows from the pool first, including prepayments, shall be observed by investors taking loans for the purchase of mortgage pools. This shall be made explicit in all loan covenants, and shall be enforceable by the banking regulatory authorities at all times. This has not been the case perhaps in the myriad loans extended to hedge funds and other investors buying CDOs and/or MBS on leverage preceding the current credit crisis.^{4,14–15}
3. The better, more easily enforceable, mechanism to deal with the risk of mortgage defaults that lenders face when financing the purchase of mortgage pools on leverage is to have banks build equity capital reserves to meet potentially sudden (systemic) mortgage defaults. Our analysis shows that minimum reserve requirements, as percentage of total

loan, start to increase as soon as the loan-to-value ratio gets greater than 70 per cent.

4. Policies should also aim at inducing the lenders financing leveraged purchases of mortgage pool products to lend only to loan-to-value ratios up to about 80 per cent.
5. Given the relative simplicity with which the current tools enable researchers to dissect periodic cash flow components at the individual level and exert numerical perturbations upon each of them, the professors of finance in the academic community, especially at the MBA level from which the armies of intellectually intensive laborers equip banks, regulators and credit rating agencies alike, should be well advised to review their teaching curriculum in order to implement coursework geared towards an intimate, deeper, quantitatively based understanding of the simulation methods available at present to manage credit risk in the banking industry.

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APPENDIX

Analytical basis for the Monte Carlo simulations of the MBS/CDO cash flows

Cash flow of a mortgage pool without defaults and without prepayments (ideal)

$$CF_{MBS} = PMT_{CF_{MBS}} (PV_{CF_{MBS}}, r_M\%, T = 180) \\ = PV_{MBS} / \left[\frac{1}{r_M} - \frac{1}{r_M(1 + r_M)^T} \right]$$

where r_M = mortgage monthly effective rate;
 $T = 180$ months (for purposes of most simulations performed); PV_{MBS} = Present value of the mortgage-backed security (mortgage pool), \$500, for a hypothetical pool of 500 mortgages, each of them with a present value of \$1 today ($N = 500$, $PV_N = 1\$$ each)

also:

$$PV_{MBS} = \sum_{n=1}^N PV_{n,0} = \sum_{n=1}^N BAL_{n,0}$$

Alternatively:

$$PV_{MBS} = \sum_{n=1}^{N=500} \sum_{t=1}^{T=180} \frac{CF_{n,t}}{(1+r_m)^t}$$

where $CF_{n,t}$ = PMT (principal = \$1, interest = r_M , $T = 180$); for each of the N mortgages in the pool, originated with a present value of \$1 each at a fixed mortgage rate $r_M = 6.00$ per cent per year*, with a term of 180 months (in the case of most simulations); *effective rate: $(1 + r_M \text{ per cent})^{12} - 1 = 6.00$ per cent p.a., which makes the monthly rate = 0.49 per cent p.m.

Monthly amortization pattern of an individual mortgage in the mortgage pool in any given month:

$$CF_{n,t} = PR_{n,t} + INT_{n,t}$$

for each t , each mortgage n in the pool

where $CF_{n,t}$ is calculated as described above, and:

$$\text{Interest portion } INT_{n,t} = (BAL_{n,t-1}) \times r_{M(\text{monthly})}$$

$$\text{Principal reduction } PR_{n,t} = CF_{n,t} - INT_{n,t}$$

$$\text{Balance outstanding } BAL_{n,t} = BAL_{n,t-1} - PR_{n,t}$$

Cash flow of an individual mortgage in the mortgage pool in a prepayment situation:

$$(CF_{n,t})_{pp} = (BAL_{n,t-1}) \times (1 + r_M)$$

where any n th mortgage in the pool is chosen stochastically for prepayment in any given month t , according to the well-defined PSA prepayment model documented elsewhere.^{2,6} The next parameters were used on the model: PSA multiplier = 1.5, CPR annual prepayment rate = 6.0 per cent and PSA month barrier = 30

Note that:

$$CF_{n,t} = 0 \text{ if:}$$

1. n th mortgage has been prepaid at any month $x, x < t$
or
2. if the n th mortgage has already defaulted at any month $x, x < t$

Here, then, $(CF_{n,t})_{pp} = \emptyset$ (zero) if mortgage n defaulted at any month $x, x < t$

Mortgage default modeling patterns:

1. γ per cent step default event in month x for a particular mortgage f :
 $(CF_{n,t})_{real} = CF_{n,t}$ for all n , other than f , all periods t , except for first γ per cent xN of the mortgages in the pool which have not been already prepaid:
 $(CF_{n,t})_{real} = 0$ for first γ per cent xN of the mortgages in the pool which have not been already prepaid, for $t \geq x$ (that is, if $n < (\gamma \text{ per cent } xN)$, for all $t > x$)
2. γ per cent cumulative default rate uniformly distributed over T months

$$(CF_{n,t})_{real} = \begin{bmatrix} 0xCF_{n,t} \\ 1xCF_{n,t} \end{bmatrix} \text{ as a function of } U[\gamma\%T]$$

where $U[\gamma\%T]$ is a zero/one output, discrete period, and uniform simulated distribution with γ per cent zero outputs uniformly distributed over the discrete range span of T periods. Thus,

the only allowable (discrete) outputs of U over the integer periods $0-T$ are 0 and 1, with γ per cent of U 's outputs being zero, and $(1-\gamma)$ per cent of U 's stochastic outputs (uniformly distributed) are ones.

Keep in mind that $(CF_{n,t})_{real} = CF_{n,t} \times 1$ if prepayment has not occurred for mortgage n , in period t or earlier.

Debt amount taken as a function of leverage

$$D_0 = (L/V) \bullet PV_{MBS} = \sum_{n=1}^N \sum_{t=1}^T (CF_{n,t})(L/V)$$

where $L/V \in [0, 1]$ and stands for loan to (asset, in this case, the MBS or mortgage pool) value ratio and, by definition:

Balance outstanding on loan,
 $BALD_{n,0} = PV_{n,0}^*(L/V)$

Equity amount infused by speculative investor (hedge fund, hypothetically) in the purchase of a mortgage pool in the form of a CDO (collateralized debt obligation) or MBS (mortgage-backed security) with leverage

$$E_0 = PV_{MBS}(1 - L/V)$$

In sum, the total amount paid to both a lender providing a loan to an investor buying an MBS or a CDO on leverage and to the investor or equity holder in the investment, is:

$$(CF_{n,t})_{real} = (CFD_{n,t})_{real} + (CFE_{n,t})_{real}$$

However, in pooled mortgage situations, the following considerations apply:

As the present value of an individual mortgage at origination (when it gets into the mortgage pool of the MBS or CDO) is equal to \$1:

$PV_{MTG} = \$1$ (and the corresponding ideal monthly cash flow from an individual mortgage in the pool is $CF_{n,t}$, determined from the

sinking fund formula) $PMT(PV_{MTG}, r_D, T=180)$, where r_D is the mortgage effective monthly rate, in this case, 0.49 per cent, equivalent to 6.0 per cent effective per year.

Then, in ideal conditions for each individual mortgage in the pool, an individual accounting of the loan balance is kept such that the Cash flow to lender from each mortgage in the pool is defined (ideal case) as:

$$CFD_{n,t} = PRD_{n,t} + INT D_{n,t} \text{ for all } t$$

Interest portion of loan $INT D_{n,t} = (BalD_{n,t-1}) \times r_D$
 Principal reduction $PRD_{n,t} = CFD_{n,t} - INTD_{n,t}$
 Balance of loan outstanding $BalD_{n,t} = BalD_{n,t-1} - PRD_{n,t}$ where r_D is the *requested* monthly rate the lender applies to the loan

When a prepayment of an individual mortgage occurs, the cash flow to the lender is.

$$(CFD_{n,t})_{pp} = BalD_{n,t-1} \times (1 + r_D)$$

When a mortgage in the pool defaults in month x , the cash flow to the lender from that particular mortgage is zero: (for all $n=f$)

$$(CFD_{n,t})_{real} = 0 \forall t \geq x$$

However, given the beneficial pooling nature of an MBS or a CDO, the total cash flow to lender, at any period t follows the conditions described below:

1. If $\sum_{n=1}^N (CF_{n,t})_{real} \geq \sum_{n=1}^N (CFD_{n,t}) \Rightarrow$ Pay to lender: $(CFD_t)_{real} = \sum_{n=1}^N (CFD_{n,t})_{real}$
2. else, pay to lender $(CFD_t)_{real} = \sum_{n=1}^N (CF_{n,t})_{real}$ (that is, all cash flow from the pool preferentially goes to lender)

The investors, as equity holders in the investment in an MBS or CDO using leverage, receive as their cash flow in month t the residual, if any, cash flow once the payment to the lender

has occurred each month from the total mortgage pool cash flow:

$$(CFE_n)_{real} = \text{Max} \left(0, \left[\sum_{n=1}^N (CF_{n,t})_{real} - (CFD_{n,t})_{real} \right] \right)$$

Effective returns from an MBS or a CDO bought by an investor on leverage whenever defaults and prepayments occur:

It can easily be demonstrated that a pool of mortgages without defaults and with any prepayment pattern yield effective returns such that:

Effective total return of the MBS or CDO

$$IRR_{MBS} = r_M$$

Effective return of the loan to buy the MBS or CDO on leverage

$$IRR_D = r_D$$

Effective return of the equity-holders who invest in an MBS or a CDO on leverage without defaults:

$$IRR_E = r_E | E_0 = \sum_{t=1}^T \sum_{n=1}^N \frac{(CF_{n,t})_{real} - (CFD_{n,t})_{real}}{(1 + r_E)^t}$$

Under a scenario where defaults occur, the effective resulting rates of return are:

$IRR_{MBS_{real}}$ such that:

$$PV_{MBS} - \sum_{n=1}^N \sum_{t=1}^T \frac{(CF_{n,t})_{real}}{(1 + IRR_{MBS_{real}})^t} \equiv 0$$

$IRR_{D_{real}}$ such that:

$$D_0 - \sum_{t=1}^T \frac{(CFD_{n,t})_{real}}{(1 + IRR_{D_{real}})^t} \equiv 0$$

$IRR_{E_{real}}$ such that:

$$E_0 - \sum_{t=1}^T \frac{\sum_{n=1}^N (CF_{n,t})_{real} - (CFD_{n,t})_{real}}{(1 + IRR_{E_{real}})^t} \equiv 0$$

In the equation above, the term within the inner summation is zero if the differences between $(CF_{n,t})_{real} - (CFD_{n,t})_{real}$ are negative.

Calculation of Equity Capital Reserves to be kept by the lender

The bank's equity capital reserve that the lender must keep in order to avoid the effects of defaulting pattern in the CDO package and achieve its required lender rate, at any period t , follows the conditions described below:

$$1. \text{ if } \sum_{n=1}^N (CFD_{n,t})_{real} < \sum_{n=1}^N (CF_{n,t})_{real} \Rightarrow$$

$$\text{Cash Reserve Needed } (CRN)_t = \sum_{n=1}^N (CF_{n,t})_{real} - (CFD_{n,t})_{real}$$

$$2. \text{ else } (CRN)_t = 0$$

In sum, the total capital reserve to be kept by the lender for the duration of the loan is:

$$CRN_{total \ t=0} = CRN_{T=0} = \frac{\sum_{t=1}^T CR_t}{(1 + r_{Fmonthly})^t}$$

where $r_{Fmonthly}$ = risk-free rate monthly = 0.36 per cent per month in the simulations

Reference rates:

As a reference, the following rates were used in the simulations:

$r_M = 0.49$ per cent per month, equivalent to 6 per cent p.a., unless as noted in specific tables and graphs

Required rate achieved by the lender on a loan to buy an MBS or a CDO on

leverages has to be $r_{Dreal} = 0.36$ per cent per month, equal to 4.4 per cent p.a., unless as noted in specific tables and graphs

As a basis for the lender to successfully continue its operations long term

r_{rF} = risk-free rate of the economy = 4.4 per cent p.a. (0.36 per cent per month)

Whenever $IRR_{Dreal} < r_{Dreal}$ the lender is in distress as a result of lending to the investor to buy an MBS or CDO on leverage, attaining less than the risk-free rate.