Original Article

Tail risk hedging strategies for corporate pension plans

Received (in revised form): 27th May 2011

Josh Davis

is a senior vice president and portfolio manager in the global Quantitative Portfolio Group at PIMCO. He focuses on Portfolio Solutions and Quantitative Strategy, including Asset Allocation, Tail Risk Hedging, Foreign Exchange and Variable Annuities. He holds a PhD in Economics with an emphasis on macroeconomics and Finance from Northwestern University, where he also earned his Master's degree. Mr Davis holds undergraduate degrees in pure Mathematics and Management Science from the University of California, San Diego.

Jim Moore

is a managing director at PIMCO. He leads the liability driven investments team and is co-head PIMCO's Solutions group. He is also PIMCO's pension strategist. Before joining PIMCO in 2003, he was in the corporate derivative and asset-liability strategy groups at Morgan Stanley and responsible for asset-liability, strategic risk management and capital structure advisory work. Mr Moore holds a PhD in Finance from the Wharton School of University of Pennsylvania. He earned undergraduate degrees from Brown University.

Niels K. Pedersen

is a vice president in the Client Analytics Group at PIMCO. He focuses on Optimal Asset Allocation Strategies, Quantitative Risk Management, FX Strategies, Dynamic Derivatives Overlay strategies, Tail Risk Hedging Strategies, Liquidity Risk Models and Hedge Fund Risk Factor modeling. Mr Pedersen holds a PhD in Economics and a Master's degree in Economics from Northwestern University. He received an undergraduate degree in Economics from the University of Aarhus in Denmark.

Correspondence: Niels K. Pedersen, 949-720-7538, PIMCO, 840 Newport Center Drive, Suite 100, Newport Beach, CA 92660
E-mail: Niels.Pedersen@pimco.com

ABSTRACT Pension plan sponsors tend to think of their asset risks and liability risks separately. Indeed, most risk management strategies employed by pension plans focus primarily on the main sources of liability risk, especially managing the sensitivity of the plan's funded status to interest rates. A common and popular strategy is to use interest rate swap overlays to partially immunize the plan's funding status against future changes in interest rates. The simple liability-driven investment (LDI) strategy is very transparent and its impact is to reduce the plan's overall funding volatility without moving large fractions of a plan's assets to bonds and thereby reducing the plan's expected return. In this article, we provide an option-theoretic analysis of the typical LDI approach to pension plan risk management, and we show that the LDI strategies really consist of two separate component strategies: (i) a tail risk hedging strategy that limits downside risk to declining interest rates, and (ii) a financing strategy that limits the plan's net upside in rising interest rate environments. We then explicitly quantify both the associated implicit costs paid for tail risk insurance and the implicit premiums received to finance the latter. This framework allows us to explore

alternative cost and premium equivalent solutions that offer quite different risk return profiles to the pension plan. We demonstrate that an approach that recognizes both the interest rate risk on the liability side and the equity risk on the asset side of the pension plan's 'balance sheet', and integrates these risk factors into a common framework, is more effective than a typical liability-centric LDI approach. Specifically, we show that the optimal tail risk hedging solution reduces the tail risk hedging costs when compared with simple LDI approaches that are aimed at interest rate risk and liability risk only.

Journal of Derivatives & Hedge Funds (2011) 17, 237–252. doi:10.1057/jdhf.2011.18;
published online 28 July 2011

Keywords: portfolio choice; tail risk hedging; corporate pension plans; risk management; derivatives

INTRODUCTION

The sensitivity of pension plan liabilities to changes in interest rates is significant and is a dominant source of risk to the funded status of most corporate defined benefit plans – especially when a plan is in deficit. Plan sponsors can mitigate some of this risk by investing a portion of their assets in long-duration fixed income funds so as to closer match the interest rate duration of the liabilities. However, few plan sponsors wish to commit all or nearly all of a plan's assets to fixed income securities.

A common approach to managing rate risk in pension plan portfolios is therefore to add an overlay of interest rate swaps to the plan's asset. By contracting to receive a fixed rate in an interest rate swap, the plan sponsor can add to the duration of their assets, and achieve the same risk-mitigating objectives as they would by investing in long-duration fixed income securities, but retain the ability to invest in equities or other securities presumed to garner high expected returns over a long horizon. We will refer this approach as the 'simple' LDI hedging solution throughout the article.

As a simple illustration of the LDI strategy, assume that a pension plan is short 15 years of interest rate duration in its net 'surplus' portfolio, which is calculated as the value of plan assets minus the value of plan liabilities. The plan is short duration, because of a duration mismatch between the plan's assets and liabilities – the sensitivity of the plan's assets to changes in interest rates is smaller than the rate sensitivity of its liabilities. The plan sponsor can mitigate this duration mismatch by adding an interest rate swap position as an overlay to its asset mix. In a typical case, the pension plan would, for instance, match 50 per cent of the duration of its liabilities with long swap transactions and thereby reduce the duration gap between assets and liabilities from minus 15 years to minus 7.5 years. The surplus portfolio would then be partially 'immunized' against changes in interest rates, which could be considered a good thing all around.

However, in reality, it also creates an issue for the pension plan manager. The swap overlay strategy will improve the funding position of the plan as long as interest rates decline, but the strategy will also reduce the benefits associated

with interest rate increases. In general, a plan that wants to protect its funding level against a fall in interest rates might not, at the same time, view it as desirable to give up the potential for an improved funding ratio associated with increasing interest rates, especially if the pension plan manager believes that interest rates might be more likely to increase in the future. Effectively, the swap overlay amounts to buying downside protection, by selling some of the plan's upside and the simple LDI hedging programs, *imposes this decision on the investor*.

In this article, we make this exact point explicit in a simple option theoretic framework. Using the key concept of put-call parity, we show that the LDI swap overlay strategy really consists of two embedded component swaptions strategies. One of the component strategies is a tail risk hedging strategy, which protects the plan against falling interest rates. The other component strategy provides the financing for the tail risk hedge, but limits the plan's upside in increasing interest rate environments.

Our options interpretation of the simple LDI strategy is crucial to our analysis, because it allows us to quantify the tail risk hedging cost that the pension plan implicitly is paying already as part of the simple LDI strategy. This makes it natural to explore other tail risk hedging strategies and zero-cost solutions that may provide a more attractive risk return trade-off or be more cost efficient than any version of the standard approach in a given set of market conditions. We first consider some of the closest alternatives to the LDI swap overlay strategy, within the set of interest rate options strategies. These alternative swaptions strategies change the conditional payoff profile of the hedged portfolio as a function of interest rate movements, and each of the alternatives addresses

some of the concerns that pension plans may currently have with the LDI swap overlays.

However, these strategies, which manage only the liability risk to the plan's funded status, are suboptimal when it comes to managing the pension plan's tail risk exposure. The problem is that they do not address the risk coming from the equity allocation in the pension plan's assets and the missing ingredient in the strategies is equity puts.

To more effectively manage the pension plan's tail risk exposure, we then, instead, solve the tail risk hedging problem in a unified framework, which acknowledges *both* the interest rate duration exposure stemming from the pension plan's liabilities *and* the equity risk exposures from the pension plan's assets. We show how to structure an efficient hedging strategy that is targeted at specific risk constraints at the overall (surplus) portfolio level. The optimal tail risk hedging solution is directly based on the joint distribution of interest rates and equities, and we conclude that the optimal solution generally includes both interest rate derivatives (swaptions) and equity puts.

The remainder of the article is organized as follows. In the first section, we use the concept of put-call parity from options theory to bridge the gap between the swap overlays, which are a generic part of the simple LDI strategies, and other alternative risk management strategies based on options. In the second section, we introduce the pension plan example that we use to illustrate our approach throughout the article and state our market assumptions. In the third section, we discuss alternative interest rate strategies that are related to the standard swap overlay approach, but offer very different interest rate payoff profiles to the pension plan. The fourth and perhaps most important section of the article provides an in-depth discussion of the importance of recognizing equity risks in the

portfolio and provides a framework that we use to derive an optimal and efficient tail risk hedging solution with both equity puts and swaptions derivatives. The final section of the article contains our concluding remarks.

OPTION THEORETIC ANALYSIS OF SIMPLE LDI STRATEGIES

A swaption is an option granting the buyer the right (but not requiring the buyer to accept the obligation) to enter into an underlying interest rate swap at a future date at a pre-specified contract rate of interest. There are two types of swaption contracts. A *payer* swaption gives the owner the right to enter into a swap and pay the fixed leg and receive the floating leg. This is analogous to a put option in the equity market. A *receiver* swaption gives the owner the right to enter into a swap at a future date and receive the fixed leg and pay the floating leg, which is analogous to an equity call option.

A forward-starting swap (FSS) is exactly

a contract and binding obligation to enter a specific interest rate swap at a future date. An FSS has zero value when the contract rate equals the current FSS rate. Swaptions of a given expiry and term are consequently considered to be at-the-money (ATM) when the strike rate of the swaption contract equals the FSS rate.

It follows from *put-call parity* that an FSS is equivalent to a long receiver swaption position combined with short payer swaption position, both with strike rates equal to the current forward swap rate. Figure 1 illustrates how the put-call parity relationship works in a specific example and shows the payoff profile of the FSS contract, as well as the payoff profiles of a long ATM receiver swaption and a short ATM payer swaption. It can be seen that the payoff profile of the FSS is precisely replicated by the combined payoff profile of this pair of swaptions. These two positions are therefore identical.

Therefore, by adding an overlay consisting of an FSS to the portfolio, the pension plan pursuing the simple LDI risk management

Figure 1: Equivalence of payoffs between an FSS and two ATM swaptions (long receiver + short payer). Non-linearity in graph reflects the convexity effect.

strategy has *implicitly* bought a receiver swaption to hedge the plan's exposure to a fall in interest rates, and has *implicitly* sold a payer swaption to finance the purchase of the receiver swaption.

As the swap overlay is replicated by a combination of two swaptions positions, we can quantify the implicit cost that the investor is paying to hedge away the downside interest rate risk to the funded status.

Similarly, we can quantify the implicit premium that the manager is receiving for selling some of the upside to the plan's funded status in the event rates rises. But before we do so, we first need to set up the example we will be using in the rest of the article and state our market assumptions.

ALTERNATIVES TO THE STANDARD LDI STRATEGIES

We extend the hypothetical example above to make the pension plan risk management problem more realistic by including more of the assets and risks that populate current pension plan portfolios. The example we build is intended to capture the main risks to the funded status of pension plans. It includes and emphasizes both the equity risk factor that tends to dominate pension plans' asset risks and the duration risk that dominates their liability risk. We view these two risk factors as the most relevant and important sources of risk to the pension plan's funded status, and consequently our results and discussion can be extended to more complicated situations without loss of generality.

The hypothetical pension plan's liabilities are discounted using the swap curve and have interest rate duration of 15 years. On the asset side, we assume that the pension plan is invested in

50 per cent in cash, accruing at the short rate of 5 per cent, and 50 per cent in equities. Consequently, the pension plan's surplus portfolio, which is computed as assets minus liabilities, has an equity beta of 50 per cent, and negative interest rate duration of 15 years. Finally, it is assumed that the pension plan sponsor has implemented a swap overlay strategy that matches 50 per cent of the duration risk factor exposure of the net surplus portfolio. The pension plan's situation is summarized in Table 1.

The forward swap overlay can be viewed as a combination of two ATM swaptions and is equivalent to a long position in a 1-year receiver swaption on a 15-year duration swap, struck at a rate of 5 per cent and a short position in a 1-year payer swaption with the same term and strike rate.

Our market and modeling assumptions are provided in Table 2 and are meant to simplify and condense the example to the greatest extent possible. Specifically, equity returns are assumed to be normally distributed, with a mean of 7.5 per cent, reflecting an equity risk premium, and volatility of 16 per cent per year. The swap rate curve is assumed to be flat at a constant rate of 5 per cent, the

Table 1: Overview of the stylized pension plan portfolio used in the examples in the article

<i>Risk factors</i>	<i>Equities</i>	<i>Duration</i>
Assets	50.0% exposure	0.0 years
Liabilities	0.0% exposure	15.0 years
Swap overlay (50% notional, 15 years dur)	0.0% exposure	7.5 years
Surplus: Assets—Liabilities + Swap overlay	50.0% exposure	−7.5 years

volatility or swap rates at 1 per cent (or 100 bps) per year and as a last simplifying assumption the correlation between equity returns and interest rates is set to zero.

Given these assumptions, we can compute the premiums associated with the 15-year duration receiver and payer swaptions as a function of their strikes. These premiums are shown in Figure 2. The percentage premiums for the two swaptions that are implicit parts of the swap overlay is 6 per cent of the notional amount for

both the receiver option and the payer option. With the assumed 50 per cent notional exposure, this translates into an implicit total cost or premium of 3 per cent of the value of pension plan assets for both.

Therefore, in this particular example, the put-call parity interpretation of the simple LDI risk management strategy makes explicit that the pension plan implicitly has purchased the ATM receiver swaption for 3 per cent of their asset value to tail risk hedge their funded status against falling interest rates, and that the pension plan implicitly has sold the offsetting ATM payer swaption to finance it.

In the following examples, we use this cost/premium as a benchmark to size hedges with different strikes and notional exposures that offer very different payoff profiles to the pension plan. In each case, we use 150 000 Monte Carlo simulations of the realized 1-year pension plan surplus (assets minus liabilities) returns to assess the impact on the proposed strategies.

Figure 3 shows that the simple LDI strategy significantly reduces the volatility of the plan's surplus by limiting the range of variation in the plan's future funded status. Relative to a situation where the pension plan does not hedge their

Table 2: Overview of the market assumptions used in the examples

<i>Market assumptions</i>	
Swap curve	5% for all maturities
Swaption volatility	100 bps across all strikes, maturities and expiries
Swaption model	Black-normal
Equity risk premium	2.50%
Equity volatility	16% across all strikes and maturities
Equity option model	Black-Scholes
Equity-interest rate correlation	Assumed to be zero

Figure 2: The graph shows swaptions premiums as a function of strike rates for both receiver swaptions and payer swaptions. Premiums computed using market assumptions stated in text and using Black's normal pricing model. ATM swaptions are struck at 5.00 per cent.

Figure 3: Impact of the simple LDI strategy on the pension plan's surplus return distribution.

liability risk at all, the simple LDI strategy does reduce the tail risk exposure of the fund quite dramatically. The question that we pose in this article is whether there are other pairs of tail risk hedging strategies and financing strategies that pension plans should consider, and, if so, what the optimal strategy looks like. In particular, does it resemble the LDI strategy or not?

ALTERNATIVE INTEREST RATE OPTION OVERLAYS

In this section, we explore interest rate strategies that are closely related to the simple LDI swap overlay strategy, but present very different interest rate payoff profiles to the investor. We do this by changing the strikes and the notional amounts of the receiver and payer swaptions that comprise the simple LDI strategy. In general, the pension plan's risk preferences, views, or both, will determine whether these alternatives are appropriate or whether they better match their goals in a particular case than the standard LDI approach.

Case 1: Pension plan has strong aversion to large, interest rate-driven losses, but is less concerned about small losses and wants

to preserve the upside to funding status associated with an increase in interest rates.

The pension plan might in this case choose to replace the ATM receiver swaption that is implicit in the simple LDI hedge with an out-of-the-money receiver option that has a lower strike rate, but a higher notional, and finance it from the same 3 per cent premium obtained by selling the same ATM payer swaption as before. With this change, the effectiveness of the tail risk hedge in the worst interest rate scenarios can be improved, without increasing the implicit hedging costs or changing the payoff profile of the strategy in increasing rate environments. It is clear from Figure 2 that, in our example, a receiver swaption struck 0.5 per cent below the 5 per cent current forward swap, at a rate of 4.5 per cent, will have a cost premium of 3 per cent for a 100 per cent notional. The pension plan can therefore scale up the notional protection by a factor of two, from 50 per cent to 100 per cent, and achieve a full duration hedge below this strike rate of 4.5 per cent for the same cost as the current ATM receiver swaption. This alternative portfolio's net

Figure 4: Conditional expected surplus (asset return minus liability return) of pension plan as function of the change in interest rate for three alternative interest derivative rate strategies.

short-duration exposure is higher for moderate changes in rates than the simple LDI-hedged portfolio, because the strike is out of the money, but it has a lower total duration exposure in an interest rate tail event, because the notional on the receiver swaption is higher.

The associated expected portfolio payoffs conditional on a given realized swap rate for the original hedge and the alternative hedge is shown in Figure 4. The alternative hedging strategy underperforms when interest rates fall by a relatively small amount, but outperforms and fully immunizes the portfolio against incremental interest-driven losses, if interest rates fall a lot. In this way, this customized

payoff profile lines up more closely to the pension plan's risk preferences than the standard LDI hedge.

Case 2: Pension plan wants to retain more upside if interest rates rise, but is less concerned with outperforming in the most favorable interest rate environments.

The pension plan can express these preferences on interest rates by moving the strike of the payer swaption further out of the money, but increase the notional. As the payer is struck out of the money, it will only pay off if interest rates rise by a relatively large amount.

The pension plan will therefore retain all of the immediate upside associated with an increase in interest rates and will also receive the premium associated with the sales of the payer swaption. Again, referring to Figure 2, a swaption with a strike 50 bps above the current forward rate (that is, at 5.5 per cent) with a notional amount equal to 100 per cent of the net interest rate exposure will provide enough premium, namely 3 per cent, to finance the existing (LDI) receiver position. The associated payoff profile and surplus return distribution from this new portfolio is shown in Figure 4. With this change in positions, the investor will maintain the hedged portfolio's exposure to a decline in interest rates, but improve the plan's performance for moderate rises in interest rates, creating an asymmetric payoff profile to changes in interest rates.

Case 3: Pension plan believes that interest rates will rise moderately in the future and wants

to retain more upside in those scenarios, but would still like to have an effective tail risk hedge in place.

To better address the views of this pension plan, we can combine the changes to the portfolio that were made in the previous two examples. This would move the strikes of both the receiver and the payer swaptions 50 bps out of the money to 4.5 per cent and 5.5 per cent, respectively, and increase the notional exposures of both to 100 per cent. Figure 4 also shows the simulated payoff profile for this so-called 'swaption collar' risk management strategy. The 'swaption collar' improves the tail risk profile compared with the simple LDI strategy and will also outperform the swap overlay for a moderate range of increases in interest rates. In a favorable interest rate environment, it will only underperform for large increases in rates where the funding ratio of the plan already improves substantially. At the same time, the strategy still hedges

Table 3: Table showing the probability of return falling in particular return bucket for unhedged, simple LDI hedge(50% swap overlay) and the swaption collar

<i>Upside</i>				<i>Downside</i>			
<i>Return (%)</i>	<i>Portfolio (%)</i>	<i>Simple LDI (50%)</i>	<i>Swaption collar (%)</i>	<i>Return (%)</i>	<i>Portfolio (%)</i>	<i>Simple LDI (50%)</i>	<i>Swaption collar (%)</i>
2.50	5.67	8.40%	8.74	−2.50	5.59	8.21%	8.26
5.00	5.50	7.62%	8.02	−5.00	5.29	7.32%	7.43
7.50	5.14	6.70%	6.98	−7.50	4.88	6.28%	6.49
10.00	9.22	10.14%	10.36	−10.00	8.49	8.94%	9.40
15.00	7.27	6.13%	5.93	−15.00	6.52	4.50%	4.46
20.00	9.04	4.72%	4.07	−20.00	7.47	2.38%	1.37
30.00	3.64	0.87%	0.60	−30.00	2.56	0.16%	0.00
40.00	1.48	0.13%	0.07	−40.00	0.77	0.00%	0.00

against drops in interest rates in tail events that the investor perhaps do not fully understand or is able to quantify the probability of, such as a low growth, deflationary scenario. To allow for a more detailed comparison of the different strategies, we have summarized the probabilities of realizing various upside and downside scenarios in Table 3.

We note that the swaption collar strategy has become more popular recently. One reason is that the strategy appears more attractive to pension plans in an upward sloping swap curve environment, where the receiver swaption can be struck just below the current spot rate, and the out-of-the-money payer can be struck quite far above the current spot rate. As pension plans in many cases effectively view changes in the value of their liabilities from a spot rate perspective, the interest rate risk-return profile of the swaption collar strategy then appears very attractive.

In the next section, we consider risk management strategies that also take into account the asset risk to the pension plan's funded status and integrate it with the liability risk.

STRUCTURING A COMPLETE LDI RISK MANAGEMENT SOLUTION

In practice, pension plans generally maintain a significant equity exposure on the asset side of their portfolios, which is expected to produce superior investment results over the long run. It follows that both lower interest rates and negative equity returns reduce the funded status of the pension plan. In the case of lower interest rates, the plan's liabilities increase in value and in the case of equity market sell-offs the plan's assets take a hit. Indeed, the worst tail risk scenarios in terms of the pension plan's funded status are those where interest rates fall and equity markets drop at the same time, as evidenced by events of 2008 where the average defined benefit pension plan's funded status fell by about 25 per cent.¹

However, as we have discussed above, existing risk management solutions essentially focus only on liability risk, and as a consequence the strategies do not clearly address the equity risk to the plan's funded status, and do not properly take into account the interaction between equity risk and interest rate risk.

To illustrate the importance of equity risk to the plans' funded status, Figure 5 shows a risk

Figure 5: Risk decompositions of surplus for the simple hypothetical pension plan portfolio. Left-hand side shows unhedged portfolio. Right-hand side shows risk decomposition for the simple LDI-hedged portfolio.

decomposition for our hypothetical pension plan's surplus portfolio for both the completely unhedged portfolio and the LDI hedged portfolio. Although the duration risk is the dominant source of risk for the pension plan's unhedged surplus portfolio and contributes about 78 per cent of its 17 per cent annual volatility, the figure also reveals that the equity risk factor exposure accounts for more than half of the LDI hedged portfolio's surplus volatility of 11 per cent. The equity risk that originates from the pension plan's assets cannot and should not be ignored in a tail risk hedging and risk management strategy.²

Next, we show how to structure a more efficient risk management solution that accounts for both liability-driven and asset-driven tail risks to the pension plan's funded status. However, to derive an efficient solution, we first have to define what exactly the tail risk hedging problem is. An intuitive objective for a pension fund that wants to tail risk hedge its

portfolio is to target a specific loss threshold for the hedged portfolio. The optimal tail risk hedging strategy would then seek to protect the pension plan's portfolio against potential losses beyond the threshold and would do so in the most cost-effective manner.

Given the many risk factors that the pension plans are exposed to, it is, however, unrealistic to structure a perfect hedge of the pension plan's surplus that eliminates *all* downside risk beyond a given point in the surplus distribution.

Therefore, the tail risk hedging mandate should instead specify a limit on the probability that the hedged portfolio will lose more than the desired threshold amount. The tail risk hedge should, for instance, reduce the probability that the pension plan's assets will underperform the plan's liabilities by more than 15–2.5 per cent or lower. These ideas can be formalized in terms of two important tail risk hedging concepts that we refer to as the *attachment point* and the *basis risk* of the tail risk

Figure 6: Figure shows probability density of surplus returns for hedged and unhedged portfolio and illustrates the 'attachment point' and 'basis risk' concepts in tail risk hedging analysis.

Figure 7: The figure shows three sets of lines. The ‘cost lines’ are combinations of hedges that have the same total option premium. The ‘attachment point’ lines are combinations of options that produce the same attachment points as defined in the text. The ‘Optimal Tail Risk Hedges’ line is the set of cost-efficient tail risk hedging programs.

hedged portfolio. In this case, the attachment point would be 15 per cent and the basis risk would be 2.5 per cent. An overview of these two concepts is provided in Figure 6.

In the figure, the tail risk hedge has eliminated most of the downside, beyond the attachment point, but has not completely eliminated it, as the surplus return in some cases fall below the attachment point. The specified basis risk, however, dictates that the total percentage of such returns, which fall below the 15 per cent attachment point, must be less than or equal to 2.5 per cent. We can now state the pension plan’s tail risk hedging problem:

The tail risk hedging problem for the pension plan that we aim to solve is to minimize the upfront premium paid for a tail risk hedging strategy that achieves the pension plan’s specified attachment point with a given basis risk.

To solve this problem in an efficient and intuitive manner, we first set the notional

exposures to 50 per cent on the equity put to match the equity beta of the portfolio and a 100 per cent for the receiver swaption.³ Figure 7 then illustrates how the optimal combination of strikes on the equity puts and receiver swaptions are obtained given these notional exposures, by comparing the costs and the attachment points for all the pairs of tail risk hedges that are available.

Specifically, the figure shows two sets of contour lines. The first set of lines are so-called ‘iso-attachment point’ lines, which represent pairs of receiver swaption and equity puts that result in the same total attachment point for the hedged portfolio. It is intuitive that as the strike of the put option or the strike of the receiver swaption is lowered, the attachment point of the hedged portfolio increases, because the chosen hedges provide less downside protection. The second set of lines that we show in Figure 7 are so-called ‘iso-cost’ lines, which represent pairs of equity puts and receiver swaptions that have the same

combined total cost premium. Again as expected, the closer the strikes are to the current spot rate and/or level of the equity market, the higher is the cost of the specific pair of tail risk hedges. As the pension plan's goal is to minimize the total hedging premium for a given portfolio attachment point, we can use these two sets of lines to determine the optimal set of tail risk hedges.

It is straightforward to show that the optimal tail risk hedging strategies are located at the tangency points between the iso-attachment point lines and the iso-cost lines. At these points, there is no way to reduce the cost of the tail risk hedge without simultaneously moving to a higher attachment point for the hedged portfolio.

For instance, if we move along the 15 per cent attachment point line, the tangency point with the iso-cost lines occurs at an equity put option strike of 93.50 per cent, and a receiver swaption strike of 4.25 per cent. As can be seen, the associated cost premium is 3 per cent. The other efficient solutions that are associated with different attachment points, and which we also have highlighted in Figure 6, are similarly generally balanced in the sense that they include both a meaningful equity hedging

component and a meaningful interest rate hedging component. This brings us to the key point of this article:

In no case is the simple LDI or its close relatives an optimal solution to the tail risk hedging problem, when solved and viewed at the total portfolio level.

The trade-off between the optimal hedging strategies, which includes equity puts and simpler LDI strategies that do not do so, becomes very clear when we consider the fact that the efficient hedge's total cost premium of 3 per cent notional exactly matches the payer premium embedded in the simple LDI hedge example. This tail risk hedging strategy could therefore be combined with the same ATM payer swaption position that is part of the standard LDI hedge at a zero total cost, instead of the ATM receiver swaption, and this risk management strategy would be much more efficient in reducing the pension plan's tail risk.

As Figure 8 shows, the joint equity and interest rate hedge clearly cuts off the left tail of the distribution of potential surplus returns, at a point that corresponds precisely to the attachment point of 15 per cent.

Figure 8: Distribution of pension plan surplus for the efficiently hedged portfolio and simple LDI swap overlay.

Almost all of the returns that would have fallen below this threshold are precisely offset by the tail risk hedge in contrast to the typical LDI hedge. Therefore, the optimal tail risk hedging strategy provides more efficient and targeted downside protection. This also explains the dramatic ‘spike’ in the return distribution above the 15 per cent return level. This happens because both of the key risk factor exposures to the funded status are controlled *at the same time* with constant amounts of both swaptions and equity puts purchased.

We note that to obtain the same attachment point for the hedged portfolio using interest rate receiver swaption hedges alone would be much more expensive. This fact is documented in Figure 9, which compares the relative costs between the two approaches as function of the hedged portfolio’s attachment point. As can be seen, if the same attachment point is attainable using receiver swaptions alone, it is significantly more expensive than a tail risk hedging solution that also includes equity puts. And in some cases, the desired tail risk hedging attachment point cannot even be achieved using swaptions alone.

In the example above, instead of the 3 per cent premium that would be spent on the

optimal tail risk hedge for the 15 per cent attachment point, it would require a strike of 4.73 per cent with an associated cost of more than 4.5 per cent to obtain the same attachment point using receiver swaptions only.

Managing the tail risk of pension plans by addressing *both* principal risk factors and both asset and liability risk simultaneously should be the principal organizing theme in a more comprehensive LDI tail risk hedging program for pension plans. This reorientation of the pure interest rate hedging program, which really accounts only for liability risk, to a hedging program that recognizes the equity risk stemming from the pension plans asset mix and which embraces equity hedges is based on a holistic view of the risks to a plan’s funded status. The shift in thinking also invites a close examination of the magnitude and composition of risks in a defined benefit plan portfolio, the sponsor’s financial outlook and risk tolerance levels and the relative richness and cheapness of options in the interest rate and equity markets.

Our analysis has shown that by spending some of the premium embedded in a standard LDI overlay on equity market hedges instead, the plan sponsor can dramatically shrink the

Figure 9: Figure compares the costs associated with different attachment points in our example, between the tail risk hedges that only use interest rate swaptions, and the efficient solution that also includes equity puts.

overall left-tail risks of the overall portfolio. And this example emphasizes the value of understanding the risks to funding status in a holistic, multifactor framework.

WHAT ABOUT MORE EXOTIC OPTION STRATEGIES?

In this article, we have restricted attention to plain-vanilla swaptions and equity puts. These are very deep and liquid, mark-to-market options with low bid-ask spreads, which can be unwound if the investor chooses to do so during a crisis. One could ask whether alternative and more exotic structures that have payoffs that depend on both equity returns and interest rate changes would not be better suited for the type of risk that the pension plan faces. Indeed, as the pension plan's realized surplus and funded status is determined by a combination of two factor returns, exotic options contracts, such as basket options, could appear attractive. Although this may be true in principle, there are very serious drawbacks associated with trading such customized exotics or basket options, which are quoted in terms of implied correlation between the factors. And these problems are in our view so significant that they disqualify them from serious consideration for most audiences. Specifically, we note that such exotic derivative contracts are very illiquid, have very high bid-ask spreads and that, in our view, they are not quoted at premiums anywhere near 'fair value' to compensate dealers for both the costs of dynamic hedging and for uncertainty about the correlations. These contracts are in addition subject to potentially significant counter-party risk and they cannot easily be unwound in a crisis period without significant give up.

For these reasons, we do not view them as viable components of a tail risk hedge for a pension plan. An alternative and perhaps better approach may be some form of dynamic trading strategy between equity puts and swaptions, which takes into account transaction costs. This is a topic for future research.

CONCLUDING REMARKS

In this article, we review pension plan risk management strategies. In the first section, we use the concept of put-call parity from options theory to bridge the gap between the swap overlays, which are a generic part of the simple LDI strategies, and other alternative risk management strategies based on options. This approach leads to a simple, but important, insight, as it shows that the typical LDI strategies really consist of two distinct component strategies, a tail risk hedging strategy and a financing strategy. We quantify the implicit hedging costs associated with the interest rate hedging component and the premium obtained from the financing component. We then use these implicit costs and premiums as a reference point to discuss *all* the alternative cost-equivalent hedging strategies that are available. The first three of the alternative strategies that we consider are close relatives of the simple LDI hedge, because they also only target the plans interest duration risk and only focus on liability risk. They represent different opportunities to express the pension plan's view on interest rates or to tailor the hedge to achieve a given interest rate payoff profile. One of the strategies is the zero-cost swaption collar strategy, which protects against downside interest rate risk but allows for more upside in favorable interest rate environments.

In the second section of the article, we then show how pension plans may redirect part of the implicit costs in an LDI strategy toward protection in the equity market and more broadly toward hedging of the risks that originate from their assets.

We explicitly define the tail risk hedging problem for the pension plan and show how to derive an efficient and optimal tail risk hedging solution that addresses both interest rate risk and equity risk in a consistent, integrated framework. The optimal tail risk hedge includes both equity put options and interest rate swaptions and we conclude that the balanced tail risk hedging strategy is superior to any LDI strategy that uses interest rate derivatives only.

In future research, we plan to provide empirical evidence of the historical performance of the type of multi-factor tail risk hedging strategies for pension plans that we propose in this article. We also plan to explore how robust optimization techniques that do not rely on a specific correlation estimate between the

factors affect the optimal tail risk hedging strategy. Finally, we plan to explore the effect of market conditions, such as options market skews and the relative pricing of derivatives in interest rate and equity markets, on the composition of the optimal tail risk hedging solution.

NOTES

- 1 Source: Credit Suisse tabulation of companies in SP500 with defined benefit plans. This number includes some hedging done previous 2008. A number of funds fared significantly worse than the 25 per cent average.
- 2 Also note that equity volatility varies more over time and it increases significantly in crisis periods. Indeed, implied volatilities hit more than 50 per cent in the crisis of 2008. Thus, the risk decomposition provides a lower bound on the equity contribution to volatility, and the relative significance of each factor in forward-looking assessments of volatility is therefore also time varying.
- 3 The completely general solution would also allow the notional exposures to be optimized. The general solution would however be very similar to the solution we propose in this article, but would not admit the same intuitive graphical representation that we want to provide the reader with in this article.