Original Article

Higher moment diversification benefits of hedge fund strategy allocation

Received (in revised form): 9th February 2009

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ABSTRACT Hedge funds are often used by institutional investors as a risk reduction tool in order to decrease portfolio volatility and create more stable return patterns. Normally, the portfolio construction process utilises a mean-variance approach and does not account for non-normal return distributions. In this article, we use higher moment betas to examine the effects on portfolio volatility, skewness and kurtosis when hedge funds are added to an equity portfolio. The results show that hedge funds, in general, can lower the volatility, skewness and kurtosis of the portfolio but large variations are seen between different hedge fund strategies. Convertible Arbitrage, Equity Market Neutral, Fixed Income Arbitrage, Merger Arbitrage and Macro are identified as the most attractive strategies to include in an equity portfolio for investors who care about higher moment risks and want to limit downside risk. Positive diversification effects still exist when serial correlation is accounted for but are then less pronounced.

Journal of Derivatives & Hedge Funds (2010) **16**, 53–69. doi:10.1057/jdhf.2010.2

Keywords: hedge funds; higher co-moments; tail-risk; serial correlation; diversification

INTRODUCTION

Many institutional investors use hedge funds as a diversification tool in order to improve the overall risk profile of their existing portfolios. The analysis is often focused on a reduction in correlation against major indexes and a reduction in the volatility of the portfolio. Normally the Capital Assets Pricing Model (CAPM) is utilised in the portfolio construction

process. The CAPM was introduced by Sharpe,¹ and builds upon the Markowitz² mean-variance model, where the optimal portfolio is obtained by minimising the standard deviation for each level of return. The result of the CAPM is that the return of a specific security should equal the risk-free rate plus a risk premium relative to the market portfolio. The level of the security's risk premium is reflected in its beta value.

Numerous investors use the CAPM in their asset allocation process without even reflecting upon whether it is an appropriate approach or not.

Several studies, for example Fung and Hsieh,^{3,4} Lo⁵ and Agarwal and Naik⁶ have shown that the return distributions of hedge funds do not follow a normal distribution. This fact has important implications for the portfolio construction process as demonstrated by, for example, Amin and Kat.⁷ Using the two-moment CAPM can lead to an over- or under-allocation to certain hedge fund strategies because of return patterns that deviate from a normal distribution. A negative skewness together with a high value of kurtosis indicates a high probability of extreme negative returns and this can lead to a portfolio where the downside risks are significantly underestimated when analysed according to a mean-variance approach. McFall Lamm,⁸ for example, studies hedge fund strategy allocation in a mean-variance setting compared to when non-normal return distributions are accounted for. He finds that utilising mean-variance techniques can lead to an over-allocation of up to 30 per cent to Distressed Debt hedge funds because of a negative skewness combined with a high kurtosis.

In this article, we examine the diversification benefits of including hedge funds in a pure equity portfolio consisting of S&P500. To account for the often non-normal return distributions among hedge funds, we study the diversification effects not only in terms of a reduction in volatility but also in terms of the higher moment effects on the portfolio in form of skewness and kurtosis, that is, the co-variance beta, co-skewness beta and co-kurtosis beta. The higher moment betas are studied for the January 1991 – December 2006 period as well

as with a 60-month rolling window analysis in order to examine the change in diversification benefits over time and in different market conditions. We also conduct the same analysis for return series corrected for serial correlation.

The rest of this article is structured as follows. In the first section, we define the higher order co-moments and the higher order co-moment betas. The following section describes the motivation for using higher moment betas as a tool in the portfolio construction process. The subsequent section describes the benchmark portfolio and the hedge fund indexes used in the study. In the following two sections, we analyse the diversification properties of various hedge fund strategies without and with an adjustment for serial correlation. In the last two sections, we discuss the implications our findings have on portfolio construction and provide some concluding remarks.

FROM HIGHER ORDER CENTRED MOMENTS TO HIGHER ORDER CO-MOMENT BETAS

As in Favre and Ronaldo⁹ and Jondeau and Rockinger,¹⁰ we first define the moments of the distribution as shown in equations (1)–(3) respectively.

$$\text{Variance} = E\left((R_i - \bar{R}_i)^2\right) \quad (1)$$

$$\text{Skewness} = E\left((R_i - \bar{R}_i)^3\right) \quad (2)$$

$$\text{Kurtosis} = E\left((R_i - \bar{R}_i)^4\right) \quad (3)$$

where R_i is the return on asset i . Note here the difference in definition from the normal calculation of skewness and kurtosis, in which we relate to the volatility of the return.

In order to account for the marginal impact of adding a new asset to an existing portfolio, we define the higher order co-moments.

$$\begin{aligned} \text{Co-Var}(R_i, R_m) \\ = E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)] \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Co-Skew}(R_i, R_m) \\ = E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)^2] \end{aligned} \quad (5)$$

$$\begin{aligned} \text{Co-Kurt}(R_i, R_m) \\ = E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)^3] \end{aligned} \quad (6)$$

where $\text{Co-Var}(R_i, R_m)$ is the co-variance between asset R_i and portfolio R_m , $\text{Co-Skew}(R_i, R_m)$ is the co-skewness and $\text{Co-Kurt}(R_i, R_m)$ is the co-kurtosis.

As we want to study the higher order diversification benefits in relation to the original portfolio, we move from higher order co-moments to higher order co-moment betas.

$$\begin{aligned} \text{Co-Var}(R_i, R_m) \text{ beta} \\ = \frac{E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)]}{E[(R_m - \bar{R}_m)^2]} \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Co-Skew}(R_i, R_m) \text{ beta} \\ = \frac{E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)^2]}{E[(R_m - \bar{R}_m)^3]} \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Co-Kurt}(R_i, R_m) \text{ beta} \\ = \frac{E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)^3]}{E[(R_m - \bar{R}_m)^4]} \end{aligned} \quad (9)$$

where R_i is the return on portfolio i and R_m is the return on the benchmark portfolio. A value below 1 for the co-variance beta and the co-kurtosis beta indicates diversification

benefits, whereas a value below 1 for the co-skewness beta indicates diversification benefits when the benchmark portfolio exhibits a negative skewness and a value above 1 when it exhibits a positive skewness. Positive skewness is a desirable feature as it indicates positive deviations from the mean value, and we are therefore interested in identifying assets that result in an upward adjustment of portfolio skewness when added to the portfolio.

The co-variance beta measures to what extent the volatility of the original portfolio can be reduced when the new asset is added to the portfolio. The same goes for the co-skewness beta and co-kurtosis beta, where the reduction in skewness and kurtosis is measured. When the results from the co-skewness beta and the co-kurtosis beta are combined, we can see to what extent we can reduce the risk of extreme negative returns in the original portfolio.

HIGHER CO-MOMENT EFFECTS IN PORTFOLIO RETURNS

The reasons for the existence of co-skewness and co-kurtosis are the very structure and the way in which hedge funds operate. These investment vehicles are often loosely regulated with generous restrictions in terms of products and markets they can trade. This in combination with trading strategies involving short selling, derivatives, illiquid instruments and the use of leverage can result in co-skewness and co-kurtosis. The result is an often non-linear correlation with various underlying return drivers. A number of studies have shown that hedge funds generate option-like non-linear return profiles and that the factors influencing the return for different hedge fund strategies vary. Mitchell and Pulvino,¹¹ Fung and

Hsieh,^{4,12} and Agarwal and Naik⁶ all show that hedge funds exhibit non-linear relations with different market-related factors. Jaeger and Wagner¹³ present a good summary of where we currently are in terms of explaining the main risk – and return drivers for the most common strategies. They also show that Long/Short Equity, Distressed and Event Driven are the strategies where the models achieve the highest explanatory power with an adjusted R^2 of 88.5, 68.4 and 79.3 per cent respectively. For arbitrage strategies, the explanatory power of the models is generally on a low level.

Besides the various biases present in hedge fund indexes, a high level of serial correlation between current and past return observations can also cause an upwards bias in returns, a downward bias in volatility and have a positive effect on co-moments. The reason for serial correlation can be numerous as reported by Getmansky *et al.*¹⁴ They identify market inefficiencies, time-varying expected returns and time-varying leverage as potential sources of serial correlation but come to the conclusion that illiquidity and smoothed returns are the main sources of serial correlation. The effects of a high serial correlation in hedge fund returns are well documented; see, for example, Brooks and Kat¹⁵ or Hayes.¹⁶ The result can be a lower volatility and correlation with major indexes and distributions with less negative skewness and lower kurtosis.

CHARACTERISTICS OF THE BENCHMARK PORTFOLIO AND THE HEDGE FUND INDEXES

Benchmark portfolio

We have chosen the S&P500 in US dollars as the equity benchmark, and the studied period ranges from January 1991 – December 2006 and

therefore includes not only several periods of equity market turbulence and bear markets, but also periods with a very strong market. As displayed in the upper panel in Figure 1, the S&P500 has a negative skewness and a positive kurtosis for the studied period, and, as a result, the return pattern does not follow a normal distribution according to the Bera-Jarque test. We also include two VaR measures to see the impact of the non-normal return distribution on downside risk. The VaR assumes a normal distribution, whereas the Modified VaR accounts for skewness and kurtosis in form of a Cornish-Fisher expansion.¹⁷

As shown, the downside risk is underestimated when only the normal VaR is utilised. We also display the 60-month rolling volatility, skewness and kurtosis of S&P500 in Figure 1. The skewness for the S&P500 is negative except for a short period in the beginning of the studied period, and the highest negative values are seen from August 1998 to January 2000, when we also see the highest values of kurtosis.

Hedge fund indexes

As a proxy for the return on various hedge fund strategies, we have chosen the HFR hedge fund indexes. These indexes are equally weighted dollar denominated without any limitations in terms of minimum assets under management or required length of active period for a fund to be included. The return of the underlying funds in the index is net of all fees. Furthermore, funds that close down or are liquidated will still be part of the index with their historic return series until the last reported performance figure. Backfilling bias has a limited impact on the indexes, as the final historic performance does not change when new funds are added to the indexes.

	Annualised return	Volatility	Skewness	Kurtosis	Bera-Jarque statistic	95% VaR	95% Modified VaR
S&P500	9.54%	13.62%	-0.508	1.079	17.59	-6.87%	-7.96%

Figure 1: Risk and return statistics for S&P500 (upper panel) and 60-month rolling volatility (upper graph), skewness and kurtosis (lower graph) January 1991 – December 2006.

ANALYSIS OF DIVERSIFICATION EFFECTS

We now turn to analyse the higher moment diversification effects of including different hedge fund strategies in an equity portfolio. First, we calculate some risk- and return statistics

for the indexes and as displayed in Table 1, Convertible Arbitrage, Distressed, Fixed Income Arbitrage, Merger Arbitrage, Emerging Markets and Event Driven all demonstrate a negative skewness and a large value of kurtosis. Only the return series of Equity Market Neutral can

Table 1: Risk and return statistics for the different sub-indexes in HFR hedge fund index January 1991 – December 2006

	<i>Annualised return (%)</i>	<i>Volatility (%)</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Bera-Jarque statistic</i>	<i>95% VaR (%)</i>	<i>95% Mod VaR (%)</i>
Convertible Arbitrage	10.61	3.38	−1.212	2.816	110.46	−1.06	−1.60
Distressed	15.73	5.69	−0.613	7.137	419.53	−1.98	−3.18
Equity Hedge	16.98	8.56	0.201	1.721	24.98	−3.50	−3.54
Equity Market Neutral	8.65	3.09	0.278	0.540	4.81	−1.05	−0.95
Fixed Income Arbitrage	8.06	4.14	−1.787	11.525	1164.86	−1.69	−3.09
Merger Arbitrage	11.03	3.56	−1.937	8.822	742.75	−1.13	−2.14
Macro	15.38	8.10	0.436	0.772	10.86	−3.36	−2.93
Emerging Markets Total	17.53	14.03	−0.829	4.738	201.56	−6.50	−9.00
Event Driven	15.57	6.01	−1.253	5.800	319.34	−2.17	−3.50

be assumed to follow a normal distribution according to the Bera-Jarque test. The effects of skewness and kurtosis can be seen when the two VaR measures are studied; the downside risk is greatly underestimated for certain arbitrage-focused strategies and this is especially the case for Merger Arbitrage and Fixed Income Arbitrage, with an 89 and 83 per cent increase in VaR respectively.

In order to account for the problems described earlier with the two-moment CAPM model, we use an approach that accounts for non-normal return distributions and investors' often asymmetric risk preferences. This framework builds upon the extension of Sharpe's original two-moment CAPM model into a three-moment CAPM, as for example in Jurczenko and Maillet,¹⁸ and a four-moment CAPM as in Favre and Ronaldo.⁹

We study here the diversification benefits of hedge funds with the help of higher moment betas, here named co-variance beta, co-skewness

beta and co-kurtosis beta. Géhin and Vaissé¹⁹ and Martellini and Ziemann²⁰ use the higher moment betas to show that significant diversification effects can be achieved when hedge funds are added to an equity or bond portfolio. They identify certain hedge fund strategies to be especially suitable and show that adding hedge fund strategies to a bond- or equity portfolio can result in a decrease in portfolio volatility, an increase in skewness and a decrease in kurtosis. Other approaches have also been used in the context of studying the higher moment diversification benefits of hedge funds. Popova *et al*²¹ use Monte Carlo simulations incorporating higher moments and show that a significant allocation to hedge funds can lower the risk of a traditional portfolio with a 60/40 split between bonds and stocks.

In Table 2, we display the results of the higher moment beta analysis spanning over the period from January 1991 to December 2006. As the S&P500 has a skewness of −0.508 for the

Table 2: Higher moment betas for HFR hedge fund indexes against S&P500 from January 1991 to December 2006

	<i>Co-variance beta</i>	<i>Co-skewness beta</i>	<i>Co-kurtosis beta</i>
Convertible Arbitrage	0.07	0.11	0.10
Distressed	0.16	0.65	0.27
Equity Hedge	0.42	0.66	0.43
Equity Market Neutral	0.04	0.07	0.06
Fixed Income Arbitrage	-0.01	0.20	0.02
Merger Arbitrage	0.11	0.39	0.19
Macro	0.21	0.30	0.20
Emerging Markets Total	0.57	1.21	0.81
Event Driven	0.28	0.71	0.35

studied period, as shown earlier in Figure 1, we are looking for values below 1 for all higher moment betas in order to provide for positive diversification effects. Adding any of the indexes except Emerging markets will lower the variance, skewness and kurtosis of a portfolio invested in S&P500. Convertible Arbitrage, Equity Market Neutral and Fixed Income Arbitrage, Merger Arbitrage and Macro are the indexes with the best diversification effects. Equity Hedge, Emerging Markets and Event Driven are less attractive in this sense. In the case of Equity Hedge and Event Driven, it can be the result of the underlying risk exposure of these strategies with both strategies being exposed to an equity market factor and the spread between small and large capitalisation stocks, as shown by Fung and Hsieh²² and Jaeger and Wagner.¹³ It is therefore natural that these two strategies display less positive diversification benefits when added to an equity portfolio.

Rolling higher moment beta analysis

The next step in the analysis is to conduct a rolling window analysis of the higher moment

betas in order to find any possible time variations in the level of diversification benefits of including the different hedge fund indexes in an equity portfolio. We therefore calculate the co-variance beta, co-skewness beta and co-kurtosis beta using a 60-month rolling window with a jump of one month at a time. The results are presented in Figure 2.

Figure 2 reveals some interesting findings regarding the diversification benefits of the nine hedge fund strategies. First, the co-variance beta and the co-kurtosis beta are generally stable over time with values below 1 for all indexes except Emerging Markets. Second, the co-skewness beta varies over time and large spikes are seen for the various sub-strategies with a start in August 2003. We now turn to analyse the co-skewness beta in more detail.

Analysis of time-varying properties of the co-skewness beta

With our original rolling time window of 60 months, the co-skewness beta started to increase (decrease for Convertible Arbitrage and Equity

Figure 2: Co-variance beta, co-skewness beta and co-kurtosis beta for HFR hedge fund indexes against S&P500 January 1991 – December 2006, 60-month rolling window.

Market Neutral) when we came to the time window spanning the period September 1998 – August 2003, that is, the first full 60-month window after the sharp fall of –14.58 per cent

for the S&P500 in August 1998. In order to analyse the change in co-skewness beta over time, and to get an insight into the time-varying properties, we alter the length of the rolling

Figure 2 *continued*

window from 60 months to 24, 36 and 48 months respectively. The same pattern is visible when we use these shorter time windows and the same sharp increase (decrease for Convertible Arbitrage and Equity Market Neutral) in co-skewness beta occurs with the first full time window after August 1998. Figure 3a and b illustrates this effect with a 48-month rolling window for the Macro strategy. The first large spike started in August 2002 and it was the first 48-month time window without the inclusion of August 1998. The next spike started in September 2006, the first full 48-month time window, which did not include the negative skewness effects of the -11 per cent move in the S&P500 in September 2002 and the skewness

of the S&P500 during the period October 2002 – September 2006 is -0.000235541 compared to -0.0000108 for the previous window spanning the period September 2002 – August 2006. The skewness is calculated here according to equation (2).

In Figure 3c, we present the same analysis using a 48-month rolling window for the HFR Equity Market Neutral but here with MSCI Europe as the reference portfolio instead of S&P500. The MSCI Europe did not display the same very near zero skewness during the period discussed above when we used S&P500 as the reference portfolio but instead during, for example, February 1995 with a skewness of 0.000013449 and a resulting co-skewness beta

Figure 3: (a) Skewness and kurtosis for S&P500 (upper graph) and co-skewness beta for HFR Macro against S&P500 (lower graph) January 1991 – December 2006, 48-month rolling window. (b) Co-skewness beta for HFR Macro against S&P500 with same value of co-skewness beta in August 2002 as previous month January 1991 – December 2006, 48-month rolling window. (c) Co-skewness beta for HFR Equity Market Neutral against MSCI Europe January 1991 – December 2006, 48-month rolling window. (d) Co-skewness for HFR Equity Market Neutral and MSCI Europe January 1991 – December 2006, 48-month rolling window. (e) Skewness for MSCI Europe January 1991 – December 2006, 48-month rolling window.

of 9.61, according to equations (2), (5) and (8). In Figure 3d and e, we display the co-skewness of HFR Equity Market Neutral and MSCI Europe and the skewness of MSCI Europe calculated with the same 48-month rolling window. Studying Figure 3e in more detail gives us more insight into the properties of the co-skewness beta. The large spikes seen in the co-skewness beta in Figure 3c coincide with skewness values very near zero for MSCI Europe.

After studying the co-skewness beta with different length of the time window used and different reference portfolios, our findings suggest that the large positive or negative spikes are a result of a very close to zero skewness of the reference portfolio and not as first appeared a drastic change in the diversification effects of including hedge funds in the portfolio. The skewness and kurtosis of the reference portfolio vary over time, as seen in Figure 1, a fact also noted by Anson *et al.*,²³ and it is

Figure 3 *continued*

therefore important to use a time period including tail events in order to correctly assess the diversification benefits of including hedge fund in the portfolio.

ADJUSTING FOR SERIAL CORRELATION IN THE RETURNS

As pointed out earlier in this study, serial correlation can have a severe impact on volatility and higher moments for some hedge fund strategies. In order to evaluate the diversification benefits when serial correlation is corrected for, we analyse here the serial correlation in the nine hedge fund indexes at lags of one, two and three months, here named ρ_1 , ρ_2 and ρ_3 respectively. Table 3 displays the results.

The first-order autocorrelation is highest for the strategies trading in instruments where the liquidity is low, that is, Convertible Arbitrage, Distressed, Fixed Income Arbitrage, Emerging Markets and Event Driven. In general, the serial correlation is low at lags longer than one month. To study the higher moment effects after adjusting for the serial correlation, we do an unsmoothing of the original return series. Serial correlation is a well-known problem in real estate data, and several methods of

unsmoothing have been developed by real estate researchers. We here apply a technique used by Geltner²⁴ to unsmooth real estate data. The same process has also been applied by Brooks and Kat¹⁵ to unsmooth hedge fund data.

The following equation is used to create a new return series with a zero first-order autocorrelation and the same mean value as the original series:

$$r_t = \frac{r_t^* - \alpha r_{t-1}^*}{1 - \alpha} \quad (10)$$

where r_t is the true unobserved return and r_t^* the observed return at time t . α is a weight factor ranging from 0 to 1 assigned to past returns and it is here set equal to the autocorrelation coefficient at a one-month lag, ρ_1 , as displayed in Table 3.

The corresponding return statistics for the new unsmoothed return series are shown in Table 4. When comparing the results in Table 4 with the results of the original return series in Table 1, we can see the effects of the serial correlation identified in Table 3. For indexes with a high serial correlation a significant increase in volatility is notable, with Convertible Arbitrage being the most extreme with a volatility of 6.28 per cent compared to

3.38 per cent for the original return series. The skewness and kurtosis of the unsmoothed series are at the same levels as for the original series with two exceptions: Fixed Income Arbitrage, where a significant reduction in both negative skewness and kurtosis is visible; and Convertible Arbitrage, where a less negative skewness is seen. Indexes where a high serial correlation is present are also subject to much higher VaR

Table 3: First- to third-order autocorrelation in HFR hedge fund indexes

	ρ_1	ρ_2	ρ_3
Convertible Arbitrage	0.55	0.26	0.08
Distressed	0.48	0.16	0.03
Equity Hedge	0.18	0.09	0.01
Equity Market Neutral	0.06	0.09	0.15
Fixed Income Arbitrage	0.40	0.12	0.12
Merger Arbitrage	0.24	0.15	0.15
Macro	0.19	0.01	0.03
Emerging Markets Total	0.34	0.12	0.03
Event Driven	0.29	0.08	0.01

and Modified VaR figures, indicating higher downside risk, when we adjust the return series for smoothening- and illiquidity effects.

We now turn to analyse the differences in higher moment diversification effects of the original series, Table 2, versus the serial correlation corrected returns in Table 5. As can be seen, the co-moments for Equity Market Neutral, Fixed Income Arbitrage, Merger Arbitrage and Macro are at the same levels or increase very slightly. On the other hand, Convertible Arbitrage, Distressed, Equity Hedge, Emerging Markets and Event Driven all display markedly worse values for all co-moments implying less positive diversification effects when serial correlation is accounted for.

Rolling higher moment beta analysis of serial correlation corrected return series

To examine the higher moment diversification effects and the impact of serial correlation over

Table 4: Return statistics for the serial correlation corrected return series in HFR hedge fund indexes January 1991 – December 2006

	<i>Annualised return (%)</i>	<i>Volatility (%)</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Bera-Jarque statistic</i>	<i>95% VaR (%)</i>	<i>95% Mod VaR (%)</i>
Convertible Arbitrage	10.60	6.28	−0.505	3.655	115.02	−2.69	−3.51
Distressed	15.42	9.59	−0.727	8.353	575.14	−4.19	−6.51
Equity Hedge	16.80	10.26	0.215	1.340	15.84	−4.46	−4.41
Equity Market Neutral	8.63	3.28	0.283	0.573	5.20	−1.16	−1.06
Fixed Income Arbitrage	7.94	6.32	−0.893	7.258	446.91	−2.92	−4.39
Merger Arbitrage	10.99	4.55	−1.736	8.040	613.62	−1.69	−2.92
Macro	15.20	9.82	0.354	0.886	10.30	−4.33	−3.97
Emerging Markets	16.42	19.96	−1.013	5.465	271.74	−9.85	−13.91
Event Driven	15.40	8.12	−1.046	5.262	256.53	−3.36	−5.00

Table 5: Higher moment betas for the serial correlation corrected HFR hedge fund indexes against S&P500 January 1991 – December 2006

	<i>Co-variance beta</i>	<i>Co-skewness beta</i>	<i>Co-kurtosis beta</i>
Convertible Arbitrage	0.15	0.33	0.21
Distressed	0.35	1.05	0.49
Equity Hedge	0.51	0.77	0.51
Equity Market Neutral	0.04	0.08	0.06
Fixed Income Arbitrage	0.00	0.31	0.07
Merger Arbitrage	0.14	0.47	0.24
Macro	0.27	0.33	0.24
Emerging Markets Total	0.94	1.72	1.23
Event Driven	0.39	0.93	0.48

time, we run a 60-month rolling window analysis of the unsmoothed return series as well. The corresponding graphs are displayed in Figure 4. In general, the results are similar to the results presented in Figure 2 earlier. When we compare Figure 4 with the results from the 60-month rolling analysis of the original returns series in more detail, some observations can be made. Convertible Arbitrage still results in positive diversification effects when included in an S&P500 portfolio but the extent is somewhat less positive when serial correlation is accounted for. Equity Hedge, Equity Market Neutral, Merger Arbitrage and Macro all display effects very similar to the ones described earlier for the non-serial correlated return. This is due to the generally low level of serial correlation as presented in Table 3. For Distressed, we see less positive effects on portfolio variance, skewness and kurtosis after correcting for serial correlation because of the often illiquid nature of the investments these funds are involved in. Fixed Income Arbitrage and Event Driven display similar effects on volatility and kurtosis, but

somewhat worse effects on portfolio skewness. Emerging Markets stands out as being the index in which a worse picture is seen across the board for all three higher moments.

To check the stability of our results, we also conduct the same analysis of higher moment diversification effects using the corresponding indexes in CSFB/Tremont hedge fund indexes for the period January 1995 to December 2006. Both the results of the higher moment betas analysis covering the whole period from January 1995 to December 2006 and the results from the 60-month rolling window analysis confirm the findings from the analysis of the sub-indexes of HFR hedge fund indexes. No material changes are seen when the serial correlation-corrected series of the CSFB/Tremont indexes are studied.

IMPLICATIONS FOR PORTFOLIO CONSTRUCTION

The results above have some interesting implications for portfolio construction when hedge fund investments are included.

Figure 4: Co-variance beta, co-skewness beta and co-kurtosis beta for unsmoothed HFR hedge fund indexes against S&P500 January 1991 – December 2006, 60-month rolling window.

As indicated by the low values of the co-variance beta, all indexes except Emerging Markets can be used as a tool to lower the volatility of an equity portfolio even when serial correlation

is corrected for. When it comes to reducing the likelihood of extreme negative returns, the effects of adding Convertible Arbitrage, Equity Market Neutral, Fixed Income Arbitrage,

Figure 4 *continued*

Merger Arbitrage and Macro are positive and these are therefore the most suitable strategies to add to an equity portfolio for investors with a high aversion against downside risk. Strategies with a high serial correlation, for example Distressed, are generally less suitable to be included in the portfolio if the aim is to decrease the likelihood of extreme negative returns.

Investors who are concerned with downside risk and use a mean-variance approach when constructing their portfolios will not account for the hidden risks in terms of co-skewness and co-kurtosis and a better alternative is therefore to utilise an asset allocation framework that accounts for non-normal distributions and

co-moment effects. A more appropriate alternative is to apply a Modified VaR optimisation instead of a standard mean-variance optimisation. This has been shown in earlier studies, for example, by McFall Lamm,⁸ and is confirmed in this study. One other implication for investors is the time varying style-allocation often applied by fund of funds. As shown here, the different hedge fund styles exhibit various degrees of diversification benefits when added to an equity portfolio, with some strategies being directly inappropriate from a diversification point of view when the investor cares about higher moment risks. As a result, the diversification effects of a specific fund of funds will to a large extent depend on the

actual style allocation within the fund and this is normally out of investor control.

CONCLUSION

After studying the diversification effects of including nine different hedge fund strategies in an equity portfolio, we come to the conclusion that Convertible Arbitrage, Equity Market Neutral, Fixed Income Arbitrage, Merger Arbitrage and Macro are the best diversifiers for investors who want to reduce the risk of extreme negative returns in their equity portfolios. Our results from the 48- and 60-month rolling window analysis also indicate that the co-moment betas vary over time and the choice and length of the time period used in the portfolio construction process are of great importance. Furthermore, we conclude that the large spikes seen in the co-skewness beta are a result of the reference portfolio having a skewness very close to zero and do not reflect a significant change in the diversification properties. Finally, we also study the diversification effects of the various hedge fund indexes after correcting for serial correlation. Here, we find that strategies with a high serial correlation will look more attractive as diversification tools when the serial correlation is not corrected for.

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- 17 The Modified VaR is an extension of the VaR subject to a normal distribution in form of a Cornish-Fisher correction for skewness and kurtosis. The Modified VaR is defined as

$$\text{Modified VaR}_i = E(R_i) + \left(z_c + \frac{z_c^2 - 1}{6} S_i + \frac{z_c^3 - 3z_c}{24} K_i - \frac{2z_c^3 - 5z_c}{36} S_i^2 \right) \sigma_i$$

where z_c depends on the level of confidence interval, and S_i and K_i are the skewness and kurtosis of portfolio i and σ_i is the standard deviation of the portfolio. If the distribution is normal, S_i and K_i are equal to zero.

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