Original Article

The options-based model of target zone and its application to the French franc

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ABSTRACT This article develops an options-based model of the target zone arrangements. The exchange rate in a target zone is modeled as an exchange rate in a floating regime adjusted by the price of two options. The article provides an accurate description of the options, which can capture the joint effect of the edges of the band. As an application of the model, the French band widening in 1993 is analyzed. It is found that the band widening was anticipated by the market.

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INTRODUCTION

In his seminal paper, Krugman¹ mentioned the possible link between the theory of the target zone system and the theory of option pricing. The idea is that the exchange rate in a target zone is equivalent to the exchange rate of a currency in a freely floating regime adjusted by the price of two options.² This article further develops Krugman's idea by accurately specifying the type of options, and by providing the corresponding option pricing method.

There is only one paper in the literature, aside from Krugman's, which touches the options-based approach. Copeland³ (Chapter 15) models

the effect of one edge of the band by one option. In contrast to his model, the options-based model introduced in this article is able to take into account the joint effect of the edges of the band. To model the simultaneous effects of the edges is highly relevant in case of narrow exchange rate bands, like the ± 2.25 per cent wide band in the European Monetary System (EMS), where the exchange rate is influenced by both of the edges at the same time no matter where it is within the band.

The options-based model developed in this article is closely related to Krugman's target zone model.⁴ First, in Krugman's model, the

exchange rate would be equal to the fundamental if there were no target zone. Given this fact, the fundamental is identical to the underlying floating exchange rate, that is, the exchange rate that would prevail in a floating system.⁵ Second, the target zone exchange rate is an S-shaped function of the underlying floating exchange rate in both models. In the options-based model, the process of the target zone exchange rate is limited by two options. At the expiration date of the options, the target zone exchange rate is a broken linear function of the underlying floating exchange rate (a flipped Z-curve), which is also the starting point of Krugman's model. In the Krugman model, the forward-looking nature of the market leads to an S-curve instead of the flipped Z-curve, whereas in the options-based model the nonlinear feature of the options explains the shape of the curve. The close relationship between the Krugman model and the options-based model is indicated by the fact that the option's price as a function of the underlying asset has a curve shape caused also by the expectations. Therefore, the options-based model can be thought of as a short cut to the process of the exchange rate, which utilizes the results of the option pricing literature.

THE OPTIONS-BASED MODEL

According to the options-based model developed here, a currency in a target zone is nothing but a currency in a floating regime with two options. One is a long put option, with the strike price equal to the weak edge of the band. The other is a short call option, with the strike price equal to the strong edge of the band. The commitment of the central bank to maintain the exchange rate regime is assumed to be

perfectly credible, and therefore neither the edges of the band, nor the strike prices are expected to change.

The existence of the two options can be explained in the following way. If the central bank promises to keep the exchange rate in the predetermined band, then, on one hand, the bank assumes the obligation of repurchasing its currency at the rate equal to the weak edge of the band. This provides a long put option from the viewpoint of the currency holders. On the other hand, the central bank does not let the exchange rate strengthen beyond the strong edge of the band. From the viewpoint of the currency holders, it looks as if the central bank had a purchasing right at the rate equal to the strong edge of the band.

As foreign exchange market participants can exercise their put option by trading with the central bank, the existence of the put option is obvious. The existence of the call option is less trivial, because the central bank cannot force anybody to sell its strong domestic currency at the edge of the band. Instead, the central bank has the obligation to buy an unlimited amount of weak foreign currency at the strong edge, which has the same effect as a short call option.

Therefore, the effect of the strong edge can be modeled indeed as a call option. The options are American-type, because they can be exercised at any time within the existing target zone system.

The underlying assets of the options are far from being obvious. What helps to determine these assets is to view the options as exchange options,⁶ where the holder of the option has the right to exchange the underlying asset for money amounting to the strike price. Whatever is exchanged for the strike price, it is the underlying asset. Now, let us see what is exchanged for what when these options are

exercised. If a currency holder exercises the put option, then she is free not only from the floating currency, but also from the obligation incorporated in the call option. In addition, if the central bank exercises its call option, then the central bank buys the floating currency and withdraws the currency holder's right incorporated in the put option. Therefore, the underlying product of the call component of the target zone exchange rate is the floating currency along with the put option, and the underlying product of the put component of the target zone exchange rate is the floating currency along with the call option.⁷

The options-based model can be formalized as

$$S_t = F_t + P_{t,Kp}(F - C_{Kc}) - C_{t,Kc}(F + P_{Kp}) \quad (1)$$

where S_t and F_t are the exchange rates at time t in the target zone and the floating regime, respectively. The value of the put with the strike price Kp is denoted by $P_{t,Kp}(F - C_{Kc})$. Its underlying product, presented in parentheses, is the floating currency along with the short call. Kp equals the weak edge of the band. $C_{t,Kc}(F + P_{Kp})$ is the value of the call option with the strike price Kc . Its underlying asset is the floating currency along with the long put option. Kc equals the strong edge of the band.

By taking Krugman's idea,² I can formalize a similar but simpler options-based model

$$S_t = F_t + P_{t,Kp}(F) - C_{t,Kc}(F) \quad (2)$$

This model differs from the options-based model of equation (1), as the underlying product of the options is simply the floating currency. The difference is not important if the band is wide enough, because in this case only one of the options is significant, while the value of the other is marginal. Consequently, the accurate and complex determination of the underlying

product does not change the option prices substantially for wide bands, only for narrow bands. Even if the model works well in practice for wide bands, it might lead to a theoretical problem, such as having an exchange rate outside the band. To show that possibility, suppose that the simple put option $P_{t,Kp}(F)$ is exercised. Then this put option along with its underlying product is worth as much as the strike price: $F_t + P_{t,Kp}(F) = Kp$. By plugging this expression into equation (2), we get:

$S_t = Kp - C_{t,Kc}(F)$. Having a positively valued call option, the exchange rate gets outside its band, being less than the weak edge of the band.

OPTION PRICING METHOD

The option pricing method is not straightforward for two reasons. First, these are American-type options, which cannot be priced by a closed form formula. Second, one part of the underlying assets of each of these options is the other option itself. Although the option pricing literature discusses the pricing of compound options,⁸ these results are not applicable here, because one part of the underlying assets of each of these options is the other option itself. Therefore, I develop a suitable pricing method within the basic binomial framework.

By applying the method, I make a sequence of call and put options. The sequence of the put and call options are monotonously increasing, but they never exceed the put and call components of the target zone exchange rate (see Proposition 1 in the Appendix). These two characteristics, the monotony and the boundedness, are sufficient conditions for the convergence of the sequences of put and call options (see Corollary 1 in the Appendix).

The sequence of the put options converges to the put component of the target zone exchange rate and the sequence of the call options converges to the call component of the target zone exchange rate (see Proposition 2 in the Appendix).

The steps of the method are the following. In the first step, we determine the price of the options as if their underlying products were simply the floating part F . We then get a P^1 and a C^1 option. Continuing the method: in the i -th step, the underlying product of P^i is $F - C^{i-1}$ and the underlying product of C^i is $F + P^{i-1}$. The P^1 and the C^1 options, as well as every further options in the sequences, are priced in the usual way in the binomial framework.⁹ First, one should determine the option prices at the final nodes in the binomial tree, and then the current price of the option is calculated by proceeding backwards. The price of an American-type option at a given non-final node is the maximum of its intrinsic value and its discounted expected future value.

AN APPLICATION OF THE MODEL

There were 56 realignments in the period 1979–1997 in the EMS, implemented in 17 discrete adjustments.¹⁰ Many of these also affected the French franc, as shown in Figure 1. Among these realignments, I analyze the band widening in August 1993, which took place approximately 5 years before the euro was born. In 1993, the exchange rate band has been widened from its original width ± 2.25 per cent to ± 15 per cent, whereas the central parity remained unchanged. The 15 days average of the daily exchange rates just before the widening was 3.416 FRF/DEM and the annualized volatility was 2.88 per cent, whereas the 15 days average of the exchange rate and the volatility characterizing the period after the realignment were 3.503 FRF/DEM and 6.78 per cent, respectively.

I choose the process of the floating exchange rate so as to fit an exchange rate with future locking. I assume that the underlying floating exchange rate will be fixed at the same time and

Figure 1: The exchange rate of the French franc and its band in the EMS.

at the same rate as the target zone exchange rate. As the option pricing method developed here is based on the discrete binomial model, I have a great flexibility at choosing the process of the floating exchange rate. The assumed process of the floating exchange rate is a discretized Brownian Bridge process that can be represented by a binomial-tree. The locking rate and date are set to their values historically developed later. The volatility of the floating exchange rate is calibrated to match the volatility of the target zone exchange rate both before and after the realignment.

The model is used to decompose the effect of band widening into the direct effect of realignment and changing uncertainty. The direct effect captures the changing strike prices, while the other captures the changing volatility. I apply a comparative static analysis. Figure 2 shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its Line 0 demonstrates the relationship before the realignment. As the exchange rate was approximately 3.416 FRF/DEM before the band widening, the floating exchange rate should have been 3.92 FRF/DEM. Line 1 shows the same relationship, but with the post-realignment strike prices. If the floating rate

remained unchanged, then the target zone exchange rate should have weakened to 3.616 FRF/DEM as a result of the changing strike prices. According to Line 2, the exchange rate should have weakened further to 3.77 FRF/DEM because of the changing volatility.

To sum up, the model attributes almost 6 per cent weakening to the direct effect and further 4 per cent to the growing volatility, while the observed weakening was about 2.5 per cent. The model fails to take into account the limited credibility of the regime, and therefore the deviation of the observed depreciation from the one implied by the model can be interpreted as evidence for the anticipation of a realignment.

CONCLUSION

In this article, I developed an options-based model of the target zone arrangements. The exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options. The advantage of showing the analogy between the problems of option pricing and determining the effect of the target zone is that the option pricing literature offers solution for a wide range of processes of

Figure 2: The decomposition of the depreciation of the franc after its band widening.

the underlying asset. I determined the underlying assets of the options accurately and provided an option pricing method applicable for these options.

By applying the options-based approach, I analyzed the band widening of the French franc in 1993. I found that the model overestimates the actual depreciation. One can expect a model to overestimate the change in the exchange rate if it assumes perfect credibility that is violated in practice. Along these lines, the analyses can be thought of as a test of credibility.

One potential generalization of the options-based model is to allow the strike prices to change stochastically. By this modification, one could model not only the exchange rates in target zones under imperfect credibility, but also the exchange rates in countries that engage in manage floats.

REFERENCES AND NOTES

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- 2 'Now the actual exchange rate may be viewed as the price of a compound asset. This asset consists of the imaginary asset (...), plus the right to sell the asset at a price, plus the obligation to sell at the price $\bar{s}$ on demand.' Krugman,¹ p. 677.
- 3 Copeland, L.S. (2000) *Exchange Rates and International Finance*. London: Pearson Education.
- 4 Krugman's target zone model is not identical to his idea of modeling the effect of the target zone by options. Understanding the former assumes no knowledge on derivatives.
- 5 Rangvid, J. and Sørensen, C. (2001) Determinants of the implied shadow exchange rates from a target zone. *European Economic Review* 45: 1665–1696, refer to the same term as the shadow exchange rate.
- 6 Margrabe, W. (1978) The value of an option to exchange one asset for another. *Journal of Finance* 33: 177–186.
- 7 One can also think of these compound options as exotic knockout options. The knockout condition for both options is that the other option is exercised.
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APPENDIX

Proposition 1: The sequence of the created put and call binomial trees are monotonously increasing on each node, but they never exceed the value of the corresponding node of the put and call components of the target zone exchange rate.

The *proof* of the boundedness and the monotony rely on the fact that the value of a put option is monotonously decreasing in its underlying asset, whereas a call option is monotonously increasing in its underlying asset.

First, I check the boundedness of the sequences of the binomial trees of the options, by comparing their underlying assets with the underlying assets of the put and call component of the target zone exchange rate. The boundedness of the sequences of the binomial trees is proved by the logic of total induction: by showing that the binomial trees of P^1 and C^1 are bounded and the boundedness of P^{i-1} and C^{i-1} is inherited to P^i and C^i . The boundedness of the binomial trees of P^1 and C^1 is obvious, as the underlying asset of P^1 is not less than the underlying asset of the put component of the target zone exchange rate ($F - C_{K,a} \leq F$) for each node, or in other words, at each state of the world. And the underlying asset of C^1 is not

higher than the underlying asset of the call component of the target zone exchange rate ($F + P_{Kp,a} \geq F$) at each state of the world. What we have to show next is that if the binomial trees of P^{i-1} and C^{i-1} are bounded, then the binomial trees of P^i and C^i are bounded. This comes from the fact that the boundedness of C^{i-1} implies that $F - C^{i-1} \geq F + C_{Kc,a}$ at each state of the world. So the underlying product of the P^i is not less than the underlying product of the put component of the target zone exchange rate, consequently, no node value of the binomial tree of P^i is greater than the corresponding node value of the binomial tree of the put component. Similar argument holds for the C^i . Therefore, I proved by total induction that for every i , P^i and C^i are bounded.

Second, I prove that the put and call sequences are monotonously increasing on each node by comparing the underlying assets again. It is obvious that P^2 and C^2 are not less than P^1 and C^1 , respectively, as $F - C^1 \leq F$ and $F + P^1 \geq F$ at each state of the world. Therefore, all we have to show is that if P^{i-1} and C^{i-1} are not less than P^{i-2} and C^{i-2} , respectively, then P^i and C^i are not less than P^{i-1} and C^{i-1} , respectively. The binomial tree of C^{i-1} is not less than the binomial tree of C^{i-2} is equivalent with $C^{i-2} \leq C^{i-1}$ at each state of the world, which implies that $F - C^{i-2} \geq F - C^{i-1}$. Thus, the underlying product of the P^i is not greater than the underlying product of the P^{i-1} , consequently, no node value of the P^i binomial

tree is less than the corresponding node value of P^{i-1} . Similar argument holds for the C^i .

Corollary 1: According to a convergence theorem (see for example: Pownall,¹¹ p. 162, Theorem 2.6.1 part (iii)) the sequences of the put and call binomial trees are convergent on each node, as they are both monotonous and bounded.

Proposition 2: The sequence of the put binomial trees converges to the binomial tree of the put component of the target zone exchange rate and the sequence of the call binomial trees converges to the binomial tree of the call component of the target zone exchange rate.

I provide an *indirect proof of Proposition 2*: if either the put sequence or the call sequence or both sequences would converge to a lower upper bound, that is, $C^\infty < C_{Kc,a}(F + P_{Kp,a})$ or $P^\infty < P_{Kp,a}(F - C_{Kc,a})$ at least at one state of the world, then we get a contradiction. Let us assume that $C^\infty < C_{Kc,a}(F + P_{Kp,a})$. In this case, the comparison of the underlying product of the limit of the put options with the underlying product of the put component indicates the following: the binomial tree of the limit of the sequence of the put options exceeds at least at one node the binomial tree of the put component. The argument goes similarly if $P^\infty < P_{Kp,a}(F - C_{Kc,a})$.