Original Article

Revisiting the Black–Litterman model: The case of hedge funds

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ABSTRACT **In this article, we re-examine a new optimization approach introduced by Black and Litterman to overcome the weaknesses of the standard mean-variance optimization model. We also consider the resampling technique to refine our results. Our results show that combining the resampling technique with the Black–Litterman model presents the most robust asset allocation. When we consider the case of hedge funds (HFs), we find that the integration of HFs into traditional investment categories indeed improves the portfolio's risk/return profile, for the period 2002–2007. The importance of HFs is less obvious, however, when using the Black–Litterman model with the resampling technique.**

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INTRODUCTION

Hedge funds (HFs) have grown rapidly over the past 20 years. Current estimates of the industry stand at nearly 8500 funds in existence managing approximately US$1.6 trillion. Nevertheless, the HF industry remains an unknown area of investment. HFs are exempt from certain regulatory requirements applicable to securities investments, and are mostly reserved for sophisticated investors (with a minimum amount of money earned annually and a net worth of more than $1 million, along with a significant amount of investment knowledge). HFs have also managed to distinguish themselves from traditional asset classes, by not being correlated with them. Many studies have examined the added value of HFs to institutional investors. For instance, L'Her *et al*[1] confirm the importance of including HFs in institutional portfolios using Markowitz[2] optimization. Although the modern portfolio theory developed by Markowitz[2] has become a necessary tool for portfolio managers when selecting asset allocations, a few problems have been observed. According to Drobetz,[3] mean-variance optimization creates concentrated portfolios. It is also highly sensitive to small changes in initial variables and does not take parameter uncertainty into account. The purpose of this research is to re-examine HF behaviour in a Canadian institutional portfolio, based on more robust methods such as the Black–Litterman model and the resampling technique, which we will compare to the standard Markowitz[2] model.

The next section presents the results of previous studies on HFs and on the Black–Litterman model. The subsequent section describes the data and models used in our study. The results of our analyses are presented in the penultimate section before concluding in the final section.

BACKGROUND

Many studies have examined HF behaviour. According to Liang,[4] HFs constantly obtain better returns than mutual funds. The author considers 385 HF and 4476 mutual funds from January 1994 to December 1996 and finds that the Sharpe ratio for different HFs is 0.364 versus 0.168 for mutual funds. He also notes that the standard deviation for mutual funds is 2.04 per cent, compared to 2.10 per cent for HFs, thus making them riskier than mutual funds. Amin and Kat[5] study the monthly returns of 77 HFs and 13 HF indexes over the period May 1990 – April 2000. They find that it is better to invest in a portfolio comprised of HFs than in a single fund. Capocci and Hübner[6] analyse HFs performance over the period 1994–2000. They observe that 10 of the 13 HF strategies considered provide superior returns. They note, however, that some managers took enormous risks to obtain additional returns, while also being faced with cases of extreme losses. Brooks and Kat[7] examine whether HFs would be beneficial for investors and conclude that the high Sharpe ratio observed for HFs is usually accompanied by negative skewness and high kurtosis. The authors maintain that investors should be cautious when they invest in HFs giving non-normality of returns, negative skewness accompanied by positive kurtosis, positive correlation between investment categories, powerlessness of the Sharpe ratio in evaluating performance and finally, overvaluation of positive absolute returns. Amin and Kat[8] also confirm that HFs increase the portfolio's performance but at the cost of low skewness and high kurtosis. L'Her *et al*[1] explore the introduction of HFs into a Canadian institutional portfolio for the period January 1990 – December 2002. They use the mean-variance optimization proposed by Markowitz[2] and find that, in general, HFs generate very good returns in a bull market and manage to limit their losses during favourable periods. They conclude that it was attractive to include HFs in a Canadian institutional portfolio composition, but argue that there is still a price to pay: a decrease in skewness combined with an increase in kurtosis. The authors highlight the limitations within the context of a mean-variance analysis. In this article, we supplement their research using a more robust asset allocation model. The modern

Markowitz[2] theory on portfolios is indeed the mainstay of portfolio management. Investors can reduce risks in their portfolio simply by holding assets that are not positively or only slightly correlated, thus diversifying their investments. This allows them to obtain the same return potential by reducing their portfolio volatility. The mean-variance optimization model has thus become a key financial instrument for choosing asset allocations, but several difficulties arise. According to Mankert,[9] problems incurred with mean-variance optimization include creation of concentrated (or non-diversified) portfolios, an unstable model causing significant changes in portfolios during small variations in initial data, and the use of historical values that maximize errors in estimating expected return. As a result of this optimization, we will overweight those asset classes for which the estimation error is the most significant. Mankert[9] suggests the Black–Litterman method as an alternative to Markowitz optimization. Black and Litterman[10] introduce an intuitive optimization method to resolve the mean-variance model difficulties. This method makes it possible to combine allocations resulting from market equilibrium according to the Capital Asset Pricing Model (CAPM) with portfolio managers' views. Further, Idzoreck[11] develops robust asset allocation with different optimization approaches such as the standard mean-variance model and the Black–Litterman approach, using a resampling technique. His results show that the Black–Litterman model is more capable of diversifying portfolios than the standard Markowitz[2] model. However, combining resampling and the Black–Litterman model provides the most robust optimization when it comes to properly diversifying risky investment categories. Few researchers have identified the Black–Litterman model as a means of optimizing HF portfolios. Martellini and Ziemann[12] improve the Black–Litterman model with the goal of optimizing a portfolio incorporating various HF strategies. They observe few problems with the non-normality of returns and parameter uncertainty, owing to the lack of historical data and low data frequency. They conclude that there is a distinct advantage to adding alternative investments to a portfolio that already includes traditional investment categories. This study stemmed from earlier research by Martellini *et al*,[13] arguing that the success of HFs among institutional managers results from the fact that they provide new diversification opportunities, compared to traditional asset classes. Given the several challenges faced by the Markowitz[2] optimization model, even in the presence of HFs, the authors introduce an extension to the Black–Litterman model in order to demonstrate that institutional investors could take full advantage of the benefits offered by alternative investments.
In our research, we re-examine the various portfolio optimization models proposed by Idzoreck.[14] We also analyse the place held by HFs in institutional portfolios using various optimization models.

DATA AND METHODS

For our reference portfolio, we consider the following indexes: the S&P/TSX (for the Canadian market), the S&P500 (for the US market), the MSCI EM (for emerging markets), the MSCI EAFE (for Europe, Australasia and the Far East), the long-term governmental and corporate bond return rates and the returns from for the 3-month Canadian Treasury Bills. Figure 3 presents the benchmark portfolio asset allocation. To simulate the HFs universe, we consider the Fund Weighted Composite Index from Hedge Fund Research (HFR), comprising more than 2000 funds. This index is renewed three times a month and contains no hedge funds of funds. A weight of 5 per cent is allocated to HFs. Our period of study is January 2002 – January 2007.

The Black–Litterman method challenges one of the principal weaknesses of mean-variance optimization, which is its high sensitivity to future returns. The Black–Litterman model is especially useful owing to the implicit returns resulting from the following relationship:

$$\Pi = \lambda \Sigma \omega_{mkt} \qquad (1)$$

where $\prod$ is the implied excess equilibrium returns ($N \times 1$ column vector); ω_{mkt} is the market capitalization weight of the assets ($N \times 1$ column vector) in our portfolio; λ is the risk aversion coefficient (1×1), corresponding to the market risk premium on the variance; and $\mathbf{\Sigma}$ is the covariance matrix of returns ($N \times N$ matrix). This relationship is obtained by reversing the general mean-variance optimization equation. Our purpose is to find the expected returns using the Black–Litterman model, integrating both market equilibrium and investor views. Managers and analysts have mixed opinions on stock market movement, but until the Black–Litterman method appeared, no model would allow them to be taken into consideration. This method allows investors to express their view, whether absolute or relative. An absolute view is defined as a manager's forecast regarding the future performance of certain assets in his/her portfolio. For example, one analyst anticipated that the S&P/TSX index would obtain an absolute return of 5 per cent. By contrast, a view is relative when simply comparing differences between various asset classes. For example the S&P/TSX will obtain a performance 3 per cent greater than that of the S&P500. These two factors allow investors to incorporate their views on the stock market when optimizing their portfolio. We should also note that our view choices are not limited. Managers can suggest as many views as they want, yet their level of confidence must be taken into account. This latter factor reflects the degree of managers' certainty with respect to their forecasts. In general, if the manager's views suggest that an asset could obtain a higher level of implicit return, then the model will allocate this asset a significant weight in the portfolio. Moreover, while this model allows managers to express their views, they are not forced to reveal or express them for each asset class. In the latter case, the model uses implicit returns, or those from the market balance according to the CAPM.

To take these views into account in the model, we assume the following relationship:

$$\mathbf{Q} + \varepsilon = \begin{bmatrix} \mathbf{Q_1} \\ \cdot \\ \mathbf{Q_K} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \varepsilon_K \end{bmatrix} \qquad (2)$$

where $\mathbf{Q}$ is the view vector ($K \times 1$ column vector), K is the number of views and $\boldsymbol{\varepsilon}$ is the error term vector indicating the uncertainty associated with these opinions with a mean of 0 and covariance matrix $\mathbf{\Omega}$.

In our example:

$$\mathbf{Q} + \varepsilon = \begin{bmatrix} 5 \\ 3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \qquad (3)$$

The error term vector ε is not directly employed in the final Black–Litterman equation; instead we use the variance of each error term (ω). These variances form $\mathbf{\Omega}$, a diagonal covariance matrix. Specifically, all off-diagonal elements of $\mathbf{\Omega}$ are equal to zero because we assume that the views are independent of one another. Note that the greater confidence expressed by investors, the more they assign importance to their views, and the variance (ω_K) is thus less.

$$\mathbf{\Omega} = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \omega_K \end{bmatrix} \tag{4}$$

Matrix $\mathbf{P}$ represents the assets considered by investors in their forecasts. The rows in this matrix correspond to the managers' views, while the columns are the number of assets in the portfolio. In general:

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdot & p_{1,N} \\ \cdot & \cdot & \cdot \\ p_{K,1} & \cdot & p_{K,N} \end{bmatrix} \tag{5}$$

In the previous case, we have two views ($K=2$), the first being absolute and the second relative. If we consider the S&P/TSX as being the top asset among the 6 ($N=6$) in the portfolio, then it should be placed in the first column. Similarly, if we consider S&P500 as the second asset, then we obtain the following matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Note that we assign positive signs to those assets that performed better and vice versa. In addition, in the case of a relative view, the sum of the row must be zero. Finally, the views are expressed by:

$$\mathbf{P}.\mathbf{E}[\mathbf{R}] = \mathbf{Q} + \varepsilon \tag{7}$$

E[R] represents the expected returns that cannot be observed. They will be integrated both with the various investors' views and the equilibrium relations $\mathbf{\Pi}$, taken from the initial market capitalization. We assume that **P.E[R]** would have a normal distribution with a mean $\mathbf{Q}$ and variance $\mathbf{\Omega}$ (covariance matrix):

$$\mathbf{P}.\mathbf{E}[\mathbf{R}] \sim N(\mathbf{Q}, \Omega) \tag{8}$$

We then obtain the equation for the general Black–Litterman model:

$$\mathbf{E}[\mathbf{R}] = \left[(\tau\Sigma)^{-1} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\left[(\tau\Sigma)^{-1}\mathbf{\Pi} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{Q}\right] \tag{9}$$

where K is the number of views; N is the number of assets in the portfolio;τ is a scalar, which depends on investors' confidence in the market (the smaller it is, the greater the implied confidence in returns, and less in their views); $\mathbf{\Sigma}$ is the covariance matrix of excess returns ($N \times N$ matrix); $\mathbf{\Omega}$ is a diagonal covariance matrix for error terms, representing investor confidence thresholds ($K \times K$ matrix); $\mathbf{\Pi}$ is the implicit returns vector ($N \times 1$ column vector); $\mathbf{P}$ is a matrix that identifies the assets involved in the views ($K \times N$ matrix or $1 \times N$ row vector in the special case of 1 view); $\mathbf{Q}$ is the view vector ($K \times 1$ column vector); and **E[R]** is the new return vector according to the Black–Litterman approach ($N \times 1$ column vector). Thus, when managers have less confidence in their views, then expectation is closer to implicit returns, and vice versa. Eventually, thanks to this new returns vector, we can finally optimize our portfolio and build an efficient frontier. This latter indicates the new redistribution for asset classes in the portfolio. The Black–Litterman model is mathematically very complex, combining the implied return with the managers' views.

To overcome the problem of using estimates based on historical data, we consider the resampling technique. This technique combines the traditional mean-variance optimization with Monte Carlo simulation. It helps explain the uncertainty of assumptions made about our parameters: expected returns, standard deviation and correlation. Whether we use the assumptions related to our parameters (parametric approach) or historical data on portfolio asset classes (non parametric approach), Monte Carlo simulation provides us with a simulated set of assumptions, which are used to optimize our portfolio. This results in a simulated efficient frontier, indicating the new asset allocation. The process is repeated several times in order to obtain an average for all asset allocations combinations. We then retain a final resampled efficient frontier.

Note that our objective is to examine whether or not the inclusion of HFs in a portfolio would be beneficial for institutional investors. Finding the optimal HFs weight is beyond the scope of this research.

In this article, we consider the following views as an example:

View no. 1 S&P/TSX will obtain a monthly return of 1 per cent (confidence level: 30 per cent).

View no. 2 HF monthly index HFRI will reach higher monthly returns 20 basis points greater than the S&P500 return (confidence level: 65 per cent).

View no. 3 Canadian government bonds will obtain monthly returns 30 basis points less than the S&P/TSX (confidence level: 65 per cent).

Table 1 summarizes the different scenario that we consider in this study.

RESULTS

Table 2 summarizes the main statistics for the different indexes over the period 2002–2007. Several researchers find that HFs provide superior returns, at the cost of negative skewness and high kurtosis. For our period of study, we find that the monthly arithmetic HF mean return is relatively low (0.3 per cent) compared with that of MSCI EM (1.58 per cent). HFs also show positive skewness (0.43) along with negative kurtosis (−0.37). Nevertheless, in addition to fixed-income securities, HFs offer protection against bearish stock market movement. The lowest HF performance for the period under review was −3 per cent, compared to −7.47 per cent for the S&P/TSX, −9.10 per cent for the MSCI EM, −9.36 per cent for the S&P500 and 11.78 per cent for the MSCI EAFE.

Table 1: Optimization scenario

Expected returns	*Views*	*Optimization model*	*With hedge funds*	*Without hedge funds*
Historical (Markowitz)	Without views	Traditional	X	X
Black–Litterman	Without views	Traditional	X	—
	With views	Traditional	X	—
		Resampling	X	—

Table 2: Summary statistics of the different indexes, 2002–2007

	Geometric mean (%)	*Arithmetic mean (%)*	*SD (%)*	*Sharpe ratio*	*Skewness*	*Kurtosis*	*Highest return (%)*	*Lowest return (%)*
S&P/TSX	1.03	1.08	3.20	0.3380	−0.7690	0.1144	6.06	−7.47
S&P500	0.02	0.09	3.65	0.0240	−0.3753	−0.2680	6.79	−9.26
91 Day T-Bill	0.24	0.24	0.06	3.6684	0.5992	−0.2975	0.40	0.13
Scotia LT Corporate Bonds	0.81	0.82	1.75	0.4713	0.0029	−0.8420	4.81	−2.23
Scotia LT Government Bonds	0.73	0.75	1.78	0.4178	−0.1280	−0.1556	5.56	−3.48
MSCI EAFE	0.69	0.76	3.66	0.2078	−0.4348	−0.0156	7.60	−9.10
MSCI Emerging Mkts	1.46	1.58	4.96	0.3182	−0.5089	−0.1727	10.77	−11.78
HFRI Fund Weighted Composite	0.28	0.30	2.07	0.1447	0.4251	−0.3709	5.60	−3.01

Returns are in Canadian dollars. S&P/TSX composite index stands fro S&P/TSX Composite Index. S&P500 stands for S&P500 Index. 91-day T-Bill stands for 3-month Canadian T-Bill rate. Canadian corporate and government bond indexes correspond to the Scotia Long Term (LT) Government Bonds Index and the Scotia Long Term (LT) Corporate Bonds Index, respectively. MSCI EAFE stands for MSCI European, Australasia and Far East markets. MSCI Emerging Mkts stands for MSCI emerging markets. HFRI Fund Weighted Composite is the HFR Hedge Funds Index.

Figure 1 presents the optimal portfolio allocation according to managers' risk positions. Position 0 corresponds to a portfolio with minimal variance, while position 100 indicates asset allocation with maximum return. We find that portfolio asset allocation is slightly diversified when risk is low. However, as risk increases the portfolio is more concentrated, comprising fewer assets. Consequently, traditional mean-variance optimization assigns an allocation of 100 per cent to the asset class having the highest return, which is in most cases the most risky (MSCI Emerging Mkts in our example).

Next, we repeat Markowitz optimization by incorporating HFs into our portfolio. Figure 2 shows Markowitz efficient frontiers including and excluding HFs, while Figure 3 presents the optimal portfolio allocation (with HFs) according to managers' risk positions. We find that the addition of HFs leads to different asset allocation. Specifically, according to Figure 3, only three of the eight assets considered are included in the portfolio, between positions 0 and 78. Thus, the inclusion of HFs in an institutional portfolio makes it more concentrated. We turn now to the Black–Litterman model. We consider two cases: (1) using market equilibrium implied expected returns and (2) combining an investor's views with market equilibrium.

Figure 4 presents the optimal portfolio allocation according to managers' risk positions. Figure 4 shows an obvious increase in portfolio

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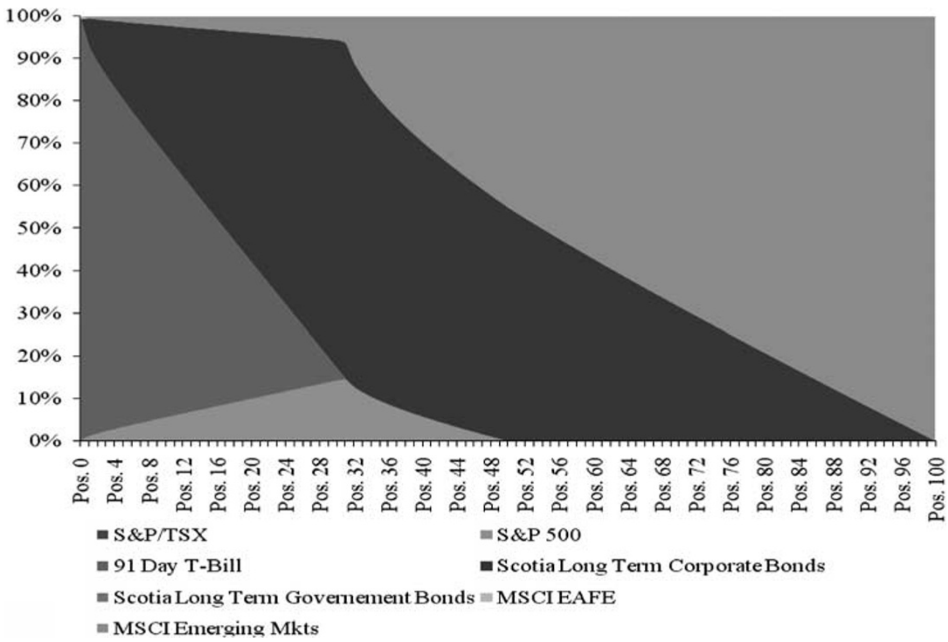

Figure 1: Asset allocation with Markowitz optimization, excluding hedge funds.

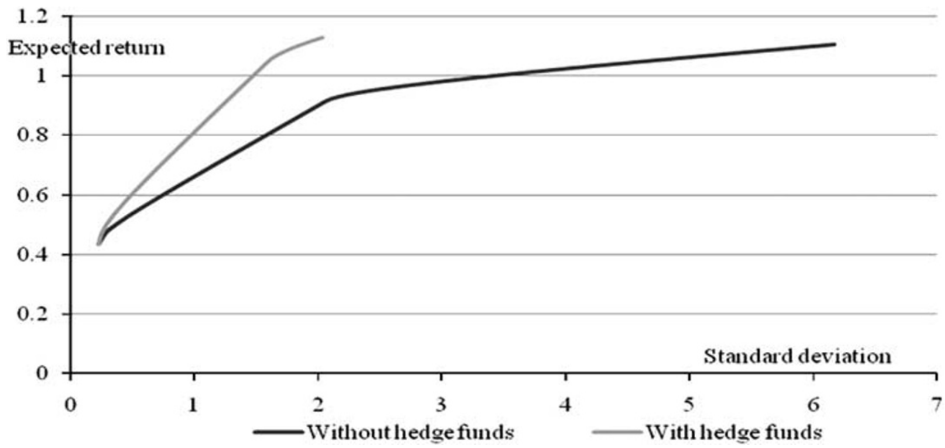

Figure 2: Markowitz efficient frontiers (with and without hedge funds).

diversification; eight assets are included in the first section (positions 0–48). At position 30, the S&P500 and S&P/TSX are assigned a 20 per cent allocation on average. Bonds, HFs and MSCI EM and MSCI EAFE indexes have substantially the same weight (5 per cent on average). By contrast, the Canadian 3-month Treasury bills have a weight of 35 per cent. Unlike traditional optimization with historical returns, we assign significant weight to Canadian and US stocks, while only a small portion to HFs. This difference could be explained by the

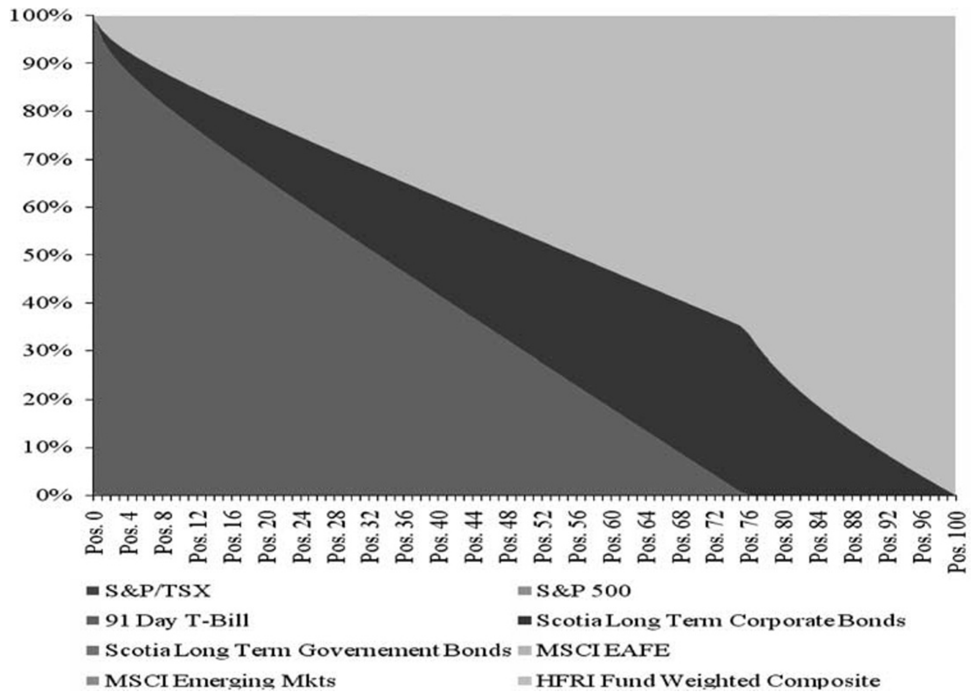

Figure 3: Asset allocation with Markowitz optimization, including hedge funds.

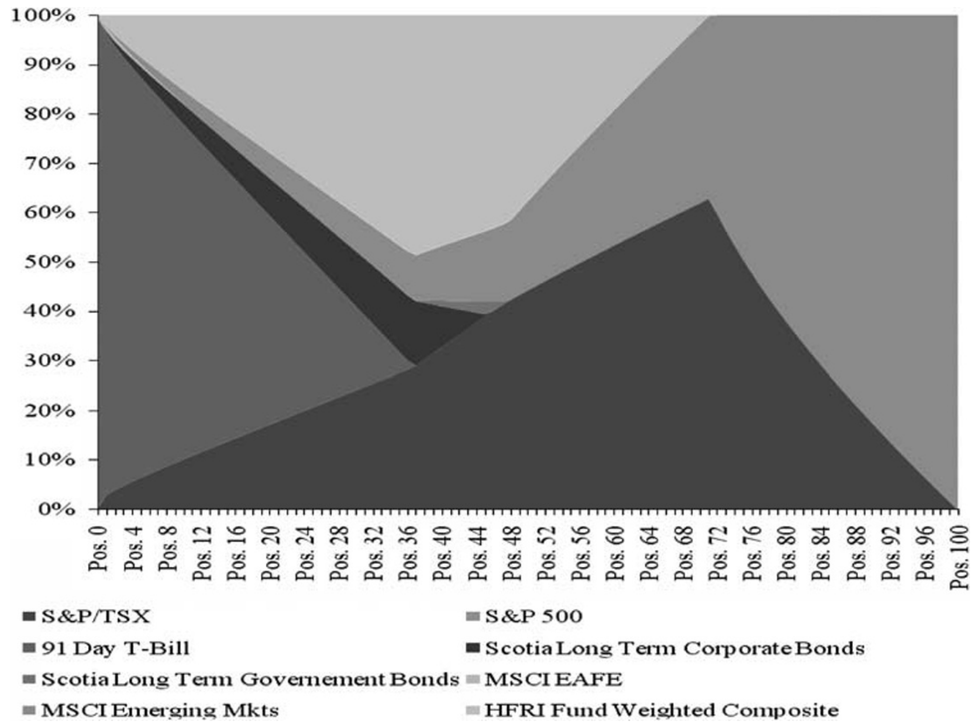

Figure 4: Asset allocation with market equilibrium implied expected returns, including hedge funds.

Black–Litterman return characteristics, which are pre-set.

Figure 5 presents asset allocation combining an investor's views (discussed previously) with market equilibrium, including HFs. We notice that six out of eight asset classes are considered, whereas all the classes are present in Figure 4. Note that a prominent position is given to those asset classes that we overestimate. In this case, we assume a monthly performance of 1 per cent for the S&P/TSX and a higher return than that of government bonds. According to Figure 5, an allocation of approximately 30 per cent is granted to S&P/TSX at position 40, while the government bond index is attributed a weight of 1 per cent. The second view suggests that the S&P500 obtains a performance lower than that of the HFRI HF composite index. The latter obtains a weight of approximately 50 per cent at position 40, in contrast to 3 per cent in the absence of views. By contrast, we notice that the S&P500 is absent from the institutional portfolio's composition. Indeed, a substantial allocation is given to HFs in exchange for a significant reduction in the S&P500 weight.

Figure 6 presents the portfolio asset allocation with Black–Litterman returns using the resampling technique with views. Overall, we find that a slightly higher allocation is attributed to HFs in the presence of views, in exchange for a reduction in the S&P500 weight. The addition of views with the resampling technique has little influence on asset distribution. An interesting feature of the Black–Litterman model is that the distribution of the five asset classes not part of our views remains substantially unchanged. We also notice that the resampling technique reduces the allocation of assets having the largest

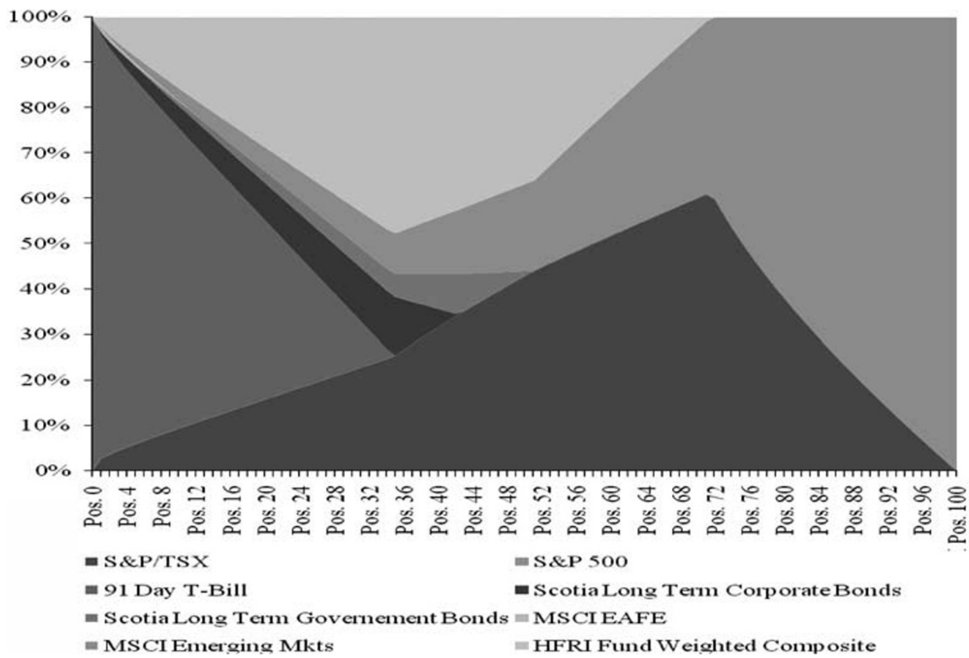

Figure 5: Asset allocation combining an investor's views with market equilibrium, including hedge funds.

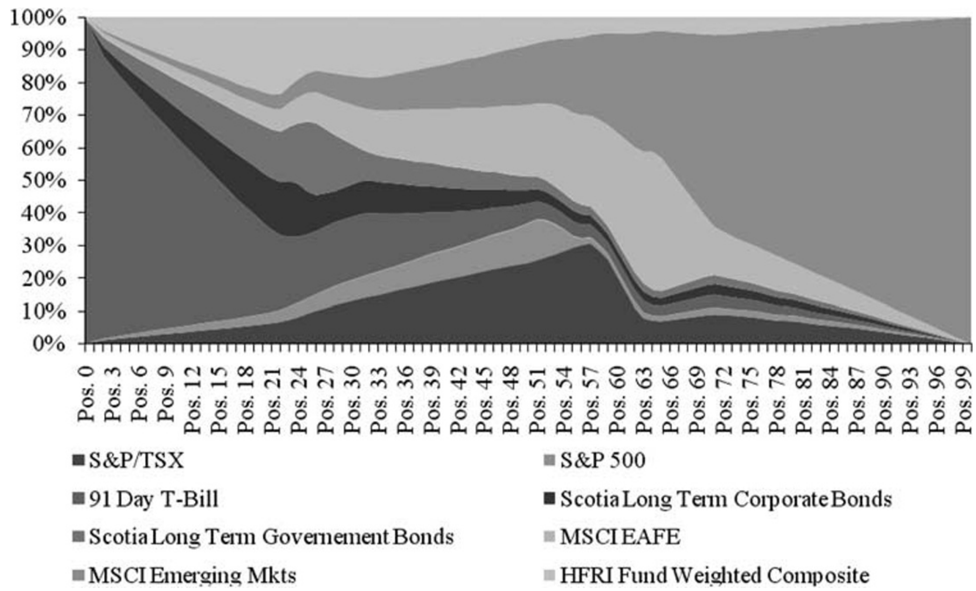

Figure 6: Assets allocation with Black–Litterman optimization using resampling technique, including hedge funds.

weighting and increases the allocation of those assets having the lowest weighting. It thus allows managers to adequately diversify their portfolios. Further, we confirm that the Black–Litterman model is a robust diversification tool. Its use along with the resampling technique provides a robust asset allocation. However, even if we confirm the importance of HFs in a Canadian institutional portfolio, we should note that their role is less evident when using the Black–Litterman model and the resampling technique.

CONCLUSION

The mean-variance optimization is an essential tool for portfolio managers. However, it does have some weaknesses. The model's sensitivity to small changes in initial values, the estimation errors and the use of historical data are three elements that lead to a concentrated distribution of assets. In this article, we compared two robust asset allocation models, the Black–Litterman method and the resampling technique, in order to counteract the powerlessness of standard Markowitz[2] optimization. The Black–Litterman model integrates the market equilibrium hypothesis according to the CAPM into the investors' views, creating a new return vector. The resampling method involves sensitivity analysis using Monte Carlo simulation and mean-variance optimization to create perfectly diversified portfolios. Compared to the traditional model, each of these approaches is capable of adequately diversifying the portfolio. By combining the two it is possible to develop a robust asset allocation model.

We applied various optimization models to examine the role of HFs in a Canadian institutional portfolio. Our results show that for the period 2002–2007, the integration of HFs into investment categories improves the portfolio's traditional risk/return profile.

However, the significance of HFs is less obvious when using the Black–Litterman approach. Although in this study we were able to examine the importance of robust asset allocation versus the Markowitz[2] optimization, certain limitations appeared, and these could become additional avenues of research. It would be interesting, for example, to apply the Black–Litterman approach to different HF strategies. The universe of alternative investments is in fact highly heterogeneous, and each fund has its specific characteristics (styles, risk, performance and so on) that could influence portfolio behaviour. It would be worth revisiting the robust model within the framework of factors models.

REFERENCES

1 L'Her, J.-F., Desrosiers, S. and Isabelle, C. (2003) Les hedges funds ont-ils leur place dans un portefeuille institutionnel canadien? *Canadian Journal of Administrative Sciences* 20(3): 193–209.
2 Markowitz, H. (1952) Portfolio selection. *The Journal of Finance* 7(1): 77–91.
3 Drobetz, W. (2001) How to avoid pitfalls in portfolio optimization? Putting the Black-Litterman approach at work. *Financial Markets and Portfolio Management* 15(1): 59–75.
4 Liang, B. (1999) On the performance of hedge funds. *Financial Analysts Journal* 55(4): 11–18.
5 Amin, G. and Kat, H.M. (2001) Hedge Fund Performance 1990–2000: Do the 'Money Machines' Really Add Value? ISMA Center, The University of Reading. Working Paper.
6 Capocci, D. and Hübner, G. (2004) Analysis of hedge fund performance. *Journal of Empirical Finance* 11(1): 55–89.
7 Brooks, C. and Kat, H. (2002) The statistical properties of hedge fund index returns and their implications for investors. *Journal of Alternative Investments* 5(2): 26–44.
8 Amin, G. and Kat, H.M. (2003) Stocks, bonds and hedge funds: Not a free lunch! *The Journal of Portfolio Management* 29(3): 113–120.
9 Mankert, C. (2006) The Black-Litterman model – Mathematical and behavioral finance approaches towards its use in practice. Licentiate Thesis, Stockholm Royal Institute of Technology.
10 Black, F. and Litterman, R. (1992) Global portfolio optimization. *Financial Analysts Journal* 48(5): 28–43.
11 Idzoreck, T.M. (2005) A Step-by-step Guide to the Black-Litterman Model – Incorporating ser-Specified Confidence Level. Ibbotson Associates. Working Paper.
12 Martellini, L. and Ziemann, V. (2007) Extending Black-Litterman analysis beyond the mean-variance framework. *The Journal of Portfolio Management* 33(4): 33–44.
13 Martellini, L., Vaissié, M. and Ziemann, V. (2005) Investing in Hedge Funds: Adding Value through Active Style Allocation Decisions. EDHEC Risk and Asset Management Research Center. Working Paper.
14 Idzoreck, T.M. (2006) Developing Robust Asset Allocations. Ibbotson Associates. Working Paper.