Original Article

Intraday *realised* volatility relationships between the S&P 500 spot and futures market

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ABSTRACT In this paper, we provide additional evidence on the intraday lead-lag relationship in the S&P 500 stock index futures market. In particular, we focus on the dynamic interactions of market volatility. In contrast to previous studies, we follow Andersen *et al* by using realised volatility to estimate market volatility. The empirical findings support the existence of a unidirectional causal relationship between futures market volatility and spot market volatility, suggesting that the arrival of new information disseminates faster in the derivative market.

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INTRODUCTION

One of the most important issues in financial economics involves the nature of the dynamic interactions between stock index and stock index futures markets. Given that spot and futures prices are linked by arbitrage operations, market linkages are of interest to traders and regulators. If markets were frictionless, futures prices would reflect the opportunity cost of a long spot position that replicates the underlying stock index. In this case, spot and futures returns would be perfectly and positively correlated and no lead-lag relationship between spot and futures

markets would arise. However, the existence of transaction costs, information asymmetries and capital requirements leads to potential explanatory capacity from one market to another. Indeed, there is extensive empirical evidence in the literature reporting significant cross-correlations between spot and futures market returns (see Kawaller *et al*,¹ Herbst *et al*,² Stoll and Whaley,³ Brooks *et al*,⁴ Turkington and Walsh,⁵ among others).

Although most studies generally find that futures market returns lead to spot market returns (see, for example, Frino and West,⁶

Min and Najand,⁷ Lafuente and Novales,⁸ Gwilym and Buckle,⁹ and Chatrath *et al.*¹⁰ among many others), the empirical evidence regarding market volatility is far from conclusive. For example, Chin *et al.*¹¹ analyse the intraday volatility interactions between the S&P 500 stock index and stock index futures market for the period covering 1985–1989. These authors find that the impact of past cash market shocks and volatility on futures market volatility is stronger than that observed in the predictability among market returns, suggesting that the pattern of new information flows to the market may be more symmetric than that inferred from examining only market returns. On the other hand, Kawaller *et al.*¹² provide empirical evidence supporting the unidirectional predictability from futures to spot volatility.

The main objective of this paper is to provide additional insights into the price discovery process in the S&P 500 stock index futures market by examining the intraday lead-lag relationship among market volatilities. In contrast to previous literature, we use *realised* volatility, as proposed in Andersen *et al.*¹³ As far as we are aware, this volatility measure has not yet been used to test the lead-lag relationship between the stock index and stock index futures market. In addition, Lindley's paradox is taken into account to test Granger causality using sample size adjusted *t* and *F* critical values.

The rest of the paper is organised as follows: the next section describes the data and the construction of the realised volatility. The subsequent section presents the methodological aspects to analyse the nature of the volatility transmission. The penultimate section reports empirical results and the final section gives concluding remarks.

DATA

Intraday data on the S&P 500 spot index and stock index futures market were provided by 'Tick Data Inc.', for the period 17 January, 2000 through 26 November 2002. As the nearest to maturity contract is systematically the most actively traded, only data for the nearby futures contract were used. We selected 15-min prices and then generated the per cent return series for S&P 500 by taking the first difference of the natural logarithm. We excluded overnight returns because they are measured over a longer time period. This procedure finally gave 35 return observations for each trading day, including the overnight (close-to-open) return. Overall, we obtained 20 256 return observations.

To compute daily realised volatility, Andersen *et al.*¹⁴ show that using high frequency sampling leads to daily volatility that is indistinguishable from the true latent volatility. On the other hand, Andersen *et al.*¹⁵ also show that realised volatility results become *biased* if too high a sample frequency is chosen. Taking into account both aspects, the daily volatility of the spot index is measured by the sum of squared 15-minute intraday returns and the squared close-to-open return

$$RV_t = (r_t^{\text{overnight}})^2 + \sum_{j=1}^{34} (r_{t,j}^{\text{intraday}})^2$$

where $r_t^{\text{overnight}}$ denotes the return from the close on day $t-1$ to the open on day t , and $r_{t,j}^{\text{intraday}}$ refers to the intraday 15-min return on day t for intraday interval j . Finally, we obtain 729 daily observations of spot realised volatility for both spot and futures markets. Figure 1 depicts the time dynamics of spot and futures daily realised volatility.

Figure 1: Impulse response functions.

MODELLING AND TESTING THE INTERACTIONS BETWEEN SPOT AND FUTURES VOLATILITY

We use a vector autoregressive (VAR) model to investigate the simultaneous interactions of spot and futures volatility. The VAR technique allows us not only to analyse Granger causality, but also to study the nature of the volatility transmission between the spot and futures market.

$$\begin{pmatrix} RV_{spot_t} \\ RV_{fut_t} \end{pmatrix} = C + \sum_{j=1}^p \Psi_j \begin{pmatrix} RV_{spot_{t-p}} \\ RV_{fut_{t-p}} \end{pmatrix} + U_t$$

where $U_t \sim N(0, \Sigma)$, Ψ_j are 2×2 matrices that capture the impact of past volatility in

both markets and p is the lag length. The lag structure is determined according to the Akaike information criterion.

Given the relatively large sample size used in this paper (697 observations), one additional issue that arises in the context of our empirical analysis is Lindley's¹⁶ paradox. When using large sample sizes, there is a tendency to reject the null hypothesis at conventional significance levels even when posterior odds favour the null (see Zellner and Siow¹⁷ for a discussion of this question in the context of regression analysis). To overcome this problem, Connolly¹⁸ provides formulas for calculating sample

size adjusted critical values for t and F statistics^{19,20}

$$t^* = [(T - k)(T^{1/T} - 1)]^{1/2}$$

$$F^* = [(T - k_1/p)(T^{p/T} - 1)]$$

where T is the sample size, K is the number of parameters estimated, K_1 is the number of parameters to be estimated under the null hypothesis and p is the number of restrictions being tested. The above expressions correspond to prior odds of 1 to 1. According to this Bayesian inference procedure, if a calculated standard statistic exceeds the appropriate critical value from the above expressions, the null hypothesis should be rejected.

EMPIRICAL RESULTS

Panel A of Table 1 presents the results of VAR estimation. From a classical point of view, the parameters associated with the first two lags of futures volatility in the futures equation appear to be statistically different from zero at the 10 per cent significance level, whereas relative to the spot equation all parameters estimated regarding lagged futures volatility are significant at the 1 per cent significance level. As to the potential explanatory power of lagged spot volatility, only the first lag in the spot equation becomes statistically different from zero at the 1 per cent significance level. To forecast futures volatility, the estimated model suggests that the relevant past information includes the futures volatility on the previous two days, whereas only the spot volatility in the previous day appears to be relevant in forecasting the relevant information set to forecast current spot volatility. All parameters estimated regarding volatility spillovers are positive. This is consistent with

spot and futures prices evolving according to a long-run equilibrium relationship. Given that market prices are linked by arbitrage operations, this empirical finding basically reflects the fact that price innovations in both markets most often have the same sign.

Relative to cross effects, we observe that all parameters estimated corresponding to lagged futures volatility are statistically different from zero in the spot equation at the 1 per cent significance level. However, only the first two lags of spot volatility are significant at the 10 per cent significance level in the futures equation. In spite of this individual explanatory power, the test of the joint significance of spot volatility in the futures equation leads to acceptance of the null hypothesis. However, the empirical value of the F -statistic with regard to the joint significance of futures volatility in the spot equation clearly leads to rejection of the null. Overall, the empirical results reveal a unidirectional causal relationship from futures to spot market volatility, suggesting that the arrival of new information to the market tends to be first incorporated in the derivative market. This pattern is expected, as trading in the spot market is more expensive and some components of the index might be infrequently traded. Confronting classical and Bayesian perspectives, the nature of the empirical results remains qualitatively unchanged when sample size adjusted critical values are used.

We carry out an impulse response analysis to further investigate the dynamic relationship between the spot and futures volatility of the S&P 500 stock index. To recover the structural VAR, according to the above-mentioned empirical findings, we use a Choleski decomposition of the variance-covariance matrix of residuals, which assumes that futures shocks are exogenous.

Table 1: Testing lead-lag relationships

Regressors	Dependent variable	
	Spot realized volatility	Futures realized volatility
Panel A. VAR estimation results		
Intercept	0.00279 (0.004) ^a	0.0010 (0.0003) ^a
$RV_{spot_{t-1}}$	−0.0729 (0.035)	−0.0287 (0.0417)
$RV_{spot_{t-2}}$	−0.0356 (0.036)	−0.0299 (0.040)
$RV_{spot_{t-3}}$	−0.0444 (0.035)	−0.0131 (0.039)
$RV_{spot_{t-4}}$	−0.0445 (0.035)	−0.0112 (0.038)
$RV_{fut_{t-1}}$	0.1817 (0.033) ^a	0.0714 (0.037)
$RV_{fut_{t-2}}$	0.2323 (0.034) ^a	0.0619 (0.035)
$RV_{fut_{t-3}}$	0.2319 (0.035) ^a	0.0042 (0.039)
$RV_{fut_{t-4}}$	0.1145 (0.036)	0.0642 (0.040)
Panel B. Causal inference		
H ₀ : Spot volatility does not cause futures volatility	F=0.30	
H ₀ : Futures volatility does not cause spot volatility	F=33.05	
Panel C. Diagnosis tests		
	Spot volatility	Futures volatility
	11.02 (0.75)	20.43 (0.16)

^adenotes that the parameter associated is statistically different from zero. The sample size-adjusted critical t value is 2.54.

Notes: Panel A. Standard errors are given in brackets.

Panel B. The sample size-adjusted critical F value is 6.72.

Panel C. Empirical values of the Ljung-Box Q-statistics for the null hypothesis of absence of autocorrelation allowing for 15 lags. P -values are in parentheses.

Figure 1 depicts the impulse response functions. The responses of the variables can be judged by the strength and the length over time. If we focus on volatility spillovers, we observe that the response of the futures volatility to spot volatility is initially stronger than that corresponding to spot response, and also that futures shocks tend to be more persistent. In sum, these plots in

Figure 1 suggest that futures volatility leads to spot volatility.

CONCLUDING REMARKS

This study used VAR techniques and impulse response function analysis to examine the dynamic inter-relationships between market

volatility in the S&P 500 stock index and stock index futures market from 17 January 2000 to 26 November 2002. We use the realised volatility measure, as proposed in Andersen *et al* (2003) as a proxy of market volatility. In particular, spot and futures realised volatility is generated from intraday 15-minute market returns. The empirical results in this study indicate a unidirectional causal relationship from futures volatility to spot volatility. This is consistent with the idea that the futures market acts as a leader in incorporating the arrival of new information. We also check the robustness of our empirical findings by comparing classical and Bayesian perspectives. The nature of the empirical results remains qualitatively unchanged even when the sample size adjusted t and F critical values are used.

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