Original Article

Time-varying asset allocation across hedge fund indices

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ABSTRACT **This paper looks at the risk-adjusted performance of dynamic asset allocation strategies across hedge fund indices using conditional volatility forecasting methods for constructing optimal portfolios for funds of funds. Monthly out-of-sample comparisons for nine Credit Suisse First Boston/Tremont hedge fund indices, as well as weekly and daily rebalanced dynamic portfolios are examined for the three main sub-indices of Standard & Poor's (S&P) Hedge Fund Index. A multivariate asymmetric Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is also considered for portfolio construction using daily S&P Hedge Fund sub-indices data. Most hedge fund indices exhibit time-varying volatility and volatility clustering. Accounting for forecasted next-period volatility generates portfolios with the best risk-return profile among all portfolios under consideration. After accounting for transaction costs, out-of-sample results indicate that all dynamic hedge fund index portfolios largely outperform the S&P 500 Index, both on an expected return and risk-adjusted return basis.**

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INTRODUCTION

Since the mid-1990s, hedge funds have emerged as a popular investment vehicle for high net-worth individuals and institutional investors. They have also attracted considerable interest from academics (see, for example, Ackermann *et al*,[1] Brown *et al*,[2] Chen and Liang,[3] Fung and Hsieh,[4–7] Liang[8] and Getmansky *et al*,[9] among others). The tremendous popularity of this new investment vehicle can be explained by the highly diverse investment strategies employed by hedge fund managers and their alleged heterogeneous return profiles.

The investable hedge fund indices that have recently appeared, such as the Credit Suisse First Boston/Tremont (CSFB/T) Sector Indices, provide an opportunity to easily exploit tactical asset allocation strategies in the alternative assets space. Funds of funds (FOFs), pension funds, endowments, family funds and other institutional investors have taken substantial positions in investable hedge funds.[10] The work herein proposes dynamic asset allocation strategies to hedge fund indices based on the minimum variance and the maximum Sharpe ratio approaches (Sharpe 1975). Such strategies should be of great interest to FOFs looking to optimise their portfolios through time. Amenc and Martellini[11] demonstrate the benefits of considering a minimum variance portfolio along the efficient frontier when it comes to tactical hedge fund indices asset allocation. Their results suggest the possibility of achieving a reduction in volatility with no detrimental effect on the returns. To implement this approach, however, one requires a reliable estimate of the volatility of the assets under consideration. Cvitanic *et al*[12] adapt the static mean–variance framework for determining a static optimal allocation to hedge funds with uncertain abnormal returns. Our paper extends the static mean–variance asset allocation framework to allow for time-varying volatility of returns. The result is a dynamic optimisation framework, which we apply to the FOF problem of asset allocation across hedge funds.

Numerous statistical models have been proposed for forecasting financial asset volatility. These include rolling variance estimates, autoregressive conditional heteroscedasticity (ARCH) models and non-parametric models. Engle and Patton[13] reveal a distinct advantage for Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models for a wide range of data-generating processes. Frances and Van Dijk[14] find that GARCH models successfully capture excess kurtosis, which is especially relevant to hedge fund indices. Asset allocation within a multivariate GARCH specification uses time-varying volatilities and cross-correlations between the assets to determine their optimal weights within the portfolio.

Our paper is the first that we are aware of to explicitly account for time-varying volatility in the construction of dynamic optimal portfolios of hedge funds, including, where appropriate, a multivariate asymmetric GARCH model for conditional volatility forecasting for optimal hedge fund indices asset allocation.

Several studies undertaken to examine the returns predictability of hedge fund indices find significant results. Agarwal and Naik[15] use the set of excess returns on standard assets and options on these assets as factors to forecast hedge fund returns. Nonlinear factors are proxied for by positions in derivatives. Schneeweis and Spurgin[16] employ passive option strategies, whereas Lhabitant[17] captures nonlinearity by including hedge fund indices as factors. Amenc *et al*[18] examine lagged multi-factor models on hedge fund indices. Given the difficulty of forecasting expected returns, and the expected return-volatility linkages as dictated by finance theory, further work that explores the effects of volatility measurement for dynamic asset allocation is clearly warranted.

This paper is organised as follows: the next section provides a review of the relevant literature on hedge funds and presents testable hypotheses. The subsequent section gives a description of the hedge fund indices data used in this study. The later section introduces models used to forecast conditional covariance matrices, and presents the methodology for constructing dynamic portfolios. Empirical results follow in the penultimate section. The paper provides a summary and suggested areas for future work in the last section.

LITERATURE REVIEW AND HYPOTHESES

The hedge funds literature focuses primarily on the return characteristics of this alternative asset class. Returns are typically either explained by fund-specific characteristics or are linked to relevant global macro factors. Researchers have addressed various issues, such as identifying drivers of hedge fund performance, whether performance is predictable, and the potential benefits of diversifying into hedge funds from a portfolio of stocks and bonds. Little work has been done on optimal FOFs portfolio construction.

Fung and Hsieh[4] and Schneeweis and Pescatore[19] find that sources of expected returns differ across hedge fund strategies. The constituent strategies of the S&P Hedge Fund Index also demonstrate that some strategies provide return opportunities not typically available through traditional investment vehicles. Schneeweis and Pescatore[19] further state that style-based performance analysis and asset allocation frameworks can be used to determine the optimal allocation to hedge funds. Several studies employ factor analysis to explain hedge fund style returns (for example, Fung and Hsieh,[4] Schneeweis and Spurgin,[20] Schneeweis and Pescatore,[19] and Agarwal and Naik[15]).

Hedge fund strategies aim to exploit inefficiencies or changing opportunity sets in the market. Researchers have attempted to isolate factors that might reflect these drivers of return in macro-factor models (Agarwal and Naik[15]), micro-factor models (Kat and Miffre[21]) and models with nonlinear regressors (Agarwal and Naik[15]).

A growing literature on hedge fund portfolio construction suggests that the nature of hedge fund returns renders the static mean–variance optimisation approach problematic. Lo,[22] Brooks and Kat,[23] and Anson[24] note that certain hedge fund strategies have more downside than upside risk, and thus exhibit negative skewness and excess kurtosis. Krokhmal *et al*[25] and Favre and Signer[26] demonstrate that assuming symmetry in hedge funds portfolio construction leads to riskier portfolios. Duarte[27] presents portfolio optimisation as a general problem, with standard optimisation methods as special cases. His results indicate that mean semi-variance and mean downside risk approaches improve overall portfolio characteristics by lowering the negative skew and excess kurtosis, while preserving the same level of return. Lamm[28] uses a VAR approach to account for skewness and kurtosis.

Cvitanic *et al*[12] incorporate uncertain abnormal returns into the static mean–variance framework for determining the optimal allocation to hedge funds, with normality assumed and a constant volatility process.

Our study focuses on the impact of incorporating time-varying volatility models in the construction of optimal dynamic portfolios of hedge funds. This approach allows for the direct accounting of skewness and kurtosis of returns. Furthermore, the asymmetric GARCH framework allows for leverage effects, whereby negative return shocks could exacerbate conditional volatility. We propose several new hypotheses for testing:

Hypothesis 1: Minimum variance hedge fund indices portfolios based on *Past Volatility* provide a better risk-adjusted return than the *S&P500 Index*.

Hypothesis 2: If not rejected initially, Hypothesis 1 still holds after accounting for transaction costs.

Hypothesis 3: Minimum variance hedge fund indices portfolios with the next-period indices volatilities estimated via *Univariate Glosten, Jagannathan, Runkle GJR-GARCH(1, 1)*[29] provide a better risk-adjusted return than the minimum variance hedge fund indices portfolio with the next-period indices volatilities estimated via *Past Volatility.*

Hypothesis 4: If not rejected initially, Hypothesis 3 still holds after accounting for transaction costs.

Hypothesis 5: Hedge fund indices portfolios with the next-period indices volatilities and cross-correlations estimated via *Multivariate Asymmetric GARCH procedures* provide a better risk-adjusted return than the minimum variance hedge fund indices portfolio with the next-period indices volatilities estimated via *Univariate GJR-GARCH(1, 1)* – for daily data.

Hypothesis 6: A *Maximum Sharpe ratio* portfolio composed of hedge fund indices provides a better risk-adjusted return than the *S&P500 Index*.

Hypothesis 7: If not rejected initially, Hypothesis 6 still holds after accounting for transaction costs.

Hypothesis 8: A minimum variance portfolio with the next-period indices volatilities estimated via *Univariate GJR-GARCH(1, 1)* provides a better risk-adjusted return than the *Maximum Sharpe ratio* portfolio.

Hypothesis 9: If not rejected initially, Hypothesis 8 still holds after accounting for transaction costs.

DATA DESCRIPTION

To represent the style-based investment strategies in an alternative investment universe, two of the most prominent hedge fund index providers are selected: Credit Suisse First Boston/Tremont Hedge Fund Indices (CSFB/T HF Indices) and Standard & Poor's Hedge Fund Indices (S&P HF Indices). Numerous academic studies (Lhabitant,[17] Amenc and Martellini,[11] Agarwal and Naik[15] and others) have used these indices because of several advantages they present with respect to competitors in terms of both calculation ease and transparency.

CSFB/T HF Indices

The CSFB/T HF Indices are the industry's only asset-weighted hedge fund indices. Their calculation begins with the TASS + database, which tracks over 2600 US and offshore hedge funds. Funds are retained only if they have a minimum of $50 million under management, have a minimum track record of 1 year, and provide current audited financial statements. Until recently, however, minimum requirements for assets under management were $10 million and a 1-year track record was not a necessity. About 650 hedge funds pass the criteria and are considered within the CSFB/T Indices. Indices are computed on a monthly basis, using net of fees returns, with the hedge funds reselected every quarter. In order to minimise the survivorship bias, hedge funds are not excluded from the indices until they liquidate their assets or fail to provide audited financial statements.

The CSFB/T Indices cover nine distinct investment strategies: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event-driven, fixed-income arbitrage, global macro, long/short equity and managed futures.[30] Descriptive statistics of these indices, relative to the S&P 500 benchmark, are provided in Table 1.

The CSFB/T Indices were launched in 1999, with the data extending back to 1994. This study uses data from January 1994 to June 2006 for a total of 150 monthly return observations.

As shown in panel a, eight of the nine CSFB/T HF Indices outperform the S&P 500 benchmark on a risk-adjusted basis (Sharpe ratio). The best risk-adjusted return is achieved by the equity market neutral hedge fund index, with an annualised mean return of 10.07 per cent and a Sharpe ratio of 3.43. Event-driven and convertible arbitrage indices are next in rank, with Sharpe ratios of 2.08 and 1.88, respectively. The worst-performing, and the only hedge fund index that underperforms the S&P 500 benchmark, is the dedicated short bias (Sharpe ratio of −0.06).

In panel b, cross-correlations of the hedge fund indices are reported. As shown therein, the equity market neutral fund is highly correlated with most of the other indices, with the exception of the global macro and managed futures series.

Standard & Poor's Hedge Fund Indices

S&P Hedge Fund Index was launched in October 2002. The index is equally weighed across various alternative investment strategies and is rebalanced annually. The distinctive characteristic of this index is the availability of daily returns data. The main S&P Hedge Fund Index consists of three (style) indices that are deemed to broadly represent the hedge fund investing universe: arbitrage, event-driven and directional/tactical.[31] Each strategy in turn consists of three underlying strategy components. The arbitrage index includes equity market neutral, fixed income arbitrage and convertible arbitrage. The event-driven index includes merger arbitrage, distressed situations and special situations. The directional/tactical index incorporates equity long/short, managed futures and global macro.

The main S&P Hedge Fund Index is an index suitable for dynamic asset allocation. Constituent strategies, however, cannot be invested in on a stand-alone basis. Thus, the results of the analysis

Table 1: CSFB/Tremont Hedge Fund Indices: (a) descriptive statistics versus S&P 500 benchmark; (b) monthly returns cross-correlations (January 1994–June 2006)

	Convertible arbitrage	*Dedicated short bias*	*Emerging markets*	*Equity market neutral*	*Event-driven*	*Fixed-income arbitrage*	*Global macro*	*Long/short equity*	*Managed futures*	*S&P 500 index*
(*a*)										
2006 (until June)	7.48%	3.58%	7.23%	6.80%	7.35%	5.65%	8.60%	5.20%	2.13%	1.76%
2005	−2.55%	17.00%	17.39%	6.14%	8.95%	0.63%	9.25%	9.68%	−0.11%	3.00%
2004	1.98%	−7.72%	12.49%	6.48%	14.47%	6.86%	8.49%	11.56%	5.97%	8.99%
2003	12.90%	−32.59%	28.75%	7.07%	20.02%	7.97%	17.99%	17.27%	14.13%	26.38%
2002	4.05%	18.14%	7.36%	7.42%	0.16%	5.75%	14.66%	−1.60%	18.33%	−23.37%
2001	14.58%	−3.58%	5.84%	9.31%	11.50%	8.04%	18.38%	−3.65%	1.90%	−13.04%
2000	25.64%	15.76%	−5.52%	14.99%	7.26%	6.29%	11.67%	2.08%	4.24%	−10.14%
1999	16.04%	−14.22%	44.82%	15.33%	22.26%	12.11%	5.81%	47.23%	−4.69%	19.53%
1998	−4.41%	−6.00%	−37.66%	13.31%	−4.87%	−8.16%	−3.64%	17.18%	20.64%	26.67%
1997	14.48%	0.42%	26.59%	14.83%	19.96%	9.34%	37.11%	21.46%	3.12%	31.01%
1996	17.87%	−5.48%	34.50%	16.60%	23.06%	15.93%	25.58%	17.12%	11.97%	20.26%
1995	16.57%	−7.35%	−16.91%	11.04%	18.34%	12.50%	30.67%	23.03%	−7.10%	34.11%
1994	−8.07%	14.91%	12.51%	−2.00%	0.75%	0.31%	−5.72%	−8.10%	11.95%	−1.54%
Annualised mean	8.97%	−1.05%	10.41%	10.07%	11.80%	6.40%	14.44%	12.83%	7.34%	9.51%
Annualised St. dev	4.77%	17.20%	16.33%	2.93%	5.67%	3.73%	11.04%	10.24%	12.02%	18.40%
Sharpe ratio	1.88	−0.06	0.64	3.43	2.08	1.72	1.31	1.25	0.61	0.52
Skewness	−1.2959	0.8508	−0.6711	0.3157	−3.3827	−3.0746	0.0175	0.2082	0.0268	−0.2749
Kurtosis	5.8260	5.0472	7.5070	3.3178	26.3733	19.0043	5.7335	6.7270	3.3272	1.8644
Jarque–Bera	90.0571	43.4069	135.4489	3.0604	3626.4930	1800.4450	45.7724	86.1438	0.6733	0.8623

Table 1: *Continued*

	Convertible arbitrage	*Dedicated short bias*	*Emerging markets*	*Equity market neutral*	*Event-driven*	*Fixed-income arbitrage*	*Global macro*	*Long/short equity*	*Managed futures*	*S&P 500 index*
Probability	0.0000	0.0000	0.0000	0.2165	0.0000	0.0000	0.0000	0.0000	0.7142	0.6498
Positive months	77.87%	45.58%	63.27%	84.35%	81.63%	80.27%	73.47%	68.03%	55.78%	62.67%

This table presents summary statistics for Credit Suisse First Boston/Tremont Hedge Fund Indices monthly returns data extending from January 1994 until the end of June 2006 (for a total of 150 monthly returns), as well as their benchmark – the S&P 500 Index. CSFB/T Hedge Fund Indices consist of nine distinct strategies: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event-driven, fixed-income arbitrage, global macro, long/short equity and managed futures.

	Convertible arbitrage	*Dedicated short bias*	*Emerging markets*	*Equity market neutral*	*Event-driven*	*Fixed-income arbitrage*	*Global macro*	*Long/short equity*	*Managed futures*
(b)									
Convertible arbitrage	1	—	—	—	—	—	—	—	—
Dedicated short bias	0.3084*	1	—	—	—	—	—	—	—
Emerging markets	0.5746*	0.6770*	1	—	—	—	—	—	—
Equity market neutral	0.5339*	0.2795*	0.3929*	1	—	—	—	—	—
Event-driven	0.2965*	0.4167*	0.3825*	0.4500*	1	—	—	—	—
Fixed-income arbitrage	0.2870*	0.5982*	0.6673*	0.2093*	0.4322*	1	—	—	—
Global macro	−0.1310**	−0.0749	−0.1408**	−0.0617	0.2504*	0.0142	1	—	—
Long/short equity	0.3419*	0.2220*	0.3627*	0.1219	0.2167*	0.3496*	0.1297	1	—
Managed futures	−0.2493*	−0.5550*	−0.6323*	−0.0756	−0.1375**	−0.7210*	0.1177	−0.3273	1

*significant at 0.01 level; **significant at 0.05 level.

conducted on weekly and daily data using three constituent strategies of the S&P Hedge Fund Index are not replicable using a single tradable investment portfolio. Nevertheless, examination of dynamic/tactical asset allocation strategies with weekly and daily rebalancing horizons serves to complement the results obtained using the monthly rebalancing strategies with CSFB/T Indices. The constituent strategies of the S&P Hedge Fund Index also serve as a proxy for the expected characteristics of strategy returns for weekly and daily hedge fund indices soon to enter the marketplace.

Descriptive statistics of the three main S&P HF Indices, compared to the S&P 500 benchmark are provided in Table 2.

For the period analysed, all three hedge fund indices strongly outperformed the S&P 500 Index, on a risk-adjusted basis (Sharpe ratio). The event-driven, directional/tactical and arbitrage indices generated Sharpe ratios of 4.40, 1.44, and 0.94, respectively. This compares to the S&P 500 Index Sharpe ratio of 0.89. It is noteworthy that the S&P 500 Index had the highest annualised return of 12.75 per cent over the period. It also exhibited the highest risk, however (annualised standard deviation of 14.26 per cent). As shown in panel b, only the directional and event-driven series are mildly correlated.

Possible data biases

The CSFB/T HF Indices and S&P HF Indices may be subject to certain biases inherent to indices, including survivorship bias, selection bias, stale price bias and the instant history bias (also referred to as the backfill bias).

Survivorship bias occurs when the database contains only information on funds that survive. According to Fung and Hsieh[6] and Brown *et al*,[2] the difference in the performance of the ‘observable’ portfolio and the portfolio of surviving funds is about 3 per cent per year. The TASS database accounts for this bias by keeping returns of defunct funds in its database from 1994, the same time CSFB began its index returns calculations.

Selection bias is caused by the preponderance of firms with successful past results being added to indices, with poorly performed firms being dropped at the same time. This bias is however, be offset by the non-inclusion of successful managers, who have reached their assets under management objectives. Most of those managers are assumed to have stopped accepting new capital in their funds in order to protect the success of a given investment strategy. According to Fung and Hsieh,[6] the two effects cancel each other out, so that overall selection bias is negligible in these indices.

The stale price bias refers to prices that may not reflect true market conditions. By using the last trade price available in a given security, as is often done in practice, true hedge fund returns may be distorted.

The instant history bias occurs when only good returns are backfilled (to the inception date of the fund) for hedge funds added to the index. In other words, bad track records are not backfilled. The bias is therefore the difference between the return of an adjusted observable portfolio and the return of a non-adjusted observable portfolio. Fung and Hsieh[6] estimate the instant history bias to be equal to 1.4 per cent per year for the TASS database using data from 1994 to 1998. Caglayan and Edwards[32] eliminate this bias by dropping the first 12 months of fund returns. CSFB/T HF Indices have recently added a 1-year track record requirement that effectively eliminates the instant history bias.

Table 2: (a) Descriptive statistics: annualised Standard & Poor's Hedge Fund Indices versus S&P 500 benchmark (October 2002–June 2006); (b) Standard & Poor's Hedge Fund Indices cross-correlations (October2002–June 2006)

	Even- driven	*Directional/Tactical*	*Arbitrage*	*S&P 500 Index*
(a)				
2006 (until June)	7.70%	5.67%	6.59%	1.76%
2005	4.61%	2.54%	−0.32%	3.00%
2004	5.66%	3.62%	2.36%	8.99%
2003	16.40%	15.29%	1.60%	26.38%
2002 (starting October)	2.85%	0.53%	1.46%	7.92%
Annualised mean	9.38%	7.08%	3.07%	12.75%
Annualised St. dev	2.13%	4.92%	3.28%	14.26%
Sharpe ratio[a]	4.40	1.44	0.94	0.89
Skewness	0.1751	−0.2588	0.1325	0.3856
Kurtosis	5.6171	4.1309	3.8129	5.2839
Jarque–Bera	275.1025	61.0353	28.8470	228.8040
Probability	0.0000	0.0000	0.0000	0.0000
Positive days	64.52%	55.86%	50.69%	54.39%
Positive weeks	76.06%	64.36%	55.32%	58.16%
Positive months	82.22%	68.89%	62.22%	66.67%

	Arbitrage	*Directional*	*Event-driven*
(b)			
Weekly			
Arbitrage	1	—	—
Directional	0.0250	1	—
Event-driven	−0.0047	0.3139⋆	1
Daily			
Arbitrage	1	—	—
Directional	−0.0499	1	—
Event-driven	−0.1572⋆	0.2397⋆	1

[a]The annualised returns divided by the annualised standard deviation.

⋆significant at 0.01 level; ⋆⋆significant at 0.05 level.

METHODOLOGY

Portfolio construction based on maximum Sharpe ratios

The simplest dynamic hedge fund indices portfolios considered in this study are based on standard mean–variance Markowitz[33] optimisation. Past returns, volatilities, and cross-correlations serve as inputs in the estimation of the next-period efficient frontier. Maximum Sharpe ratio portfolios are constructed for monthly and weekly hedge fund indices data.

Portfolio construction based on past volatility

In order to construct portfolios based on historical volatility, the weights of each hedge fund index within the next period portfolios need to be computed. A global minimum variance (GMV) asset allocation approach is used in this study. Thus, the optimal weights ω_i depend on the predicted covariance matrix H_{t+1}.

Assuming a diagonal covariance matrix for nine univariate CSFB/T HF Indices, the weights of the univariate diagonal portfolio are given by

$$\omega_{t,i} = \frac{\hat{\sigma}^{-2}_{t+1,i}}{\sum_{j=1}^{9} \hat{\sigma}^{-2}_{t+1,j}} \tag{1}$$

where for CSFB/T Indices $i = 1, 2, 3, \ldots, 9$ and for S&P indices $i = 1, 2, 3$. $\hat{\sigma}^2_{t+1,i}$ is the past variance of the monthly returns of the ith CSFB/T Hedge Fund index or is the past variance of weekly or daily returns of the ith S&P Hedge Fund Index. The dynamic variance is either forecasted by the asymmetric univariate GJR-GARCH(1, 1) model or estimated based on past volatility. The same approach is used for finding the optimal weights of S&P Hedge Fund Indices for weekly and daily rebalanced portfolios.

In addition to the univariate GJR-GARCH(1, 1) past volatility risk estimates, multivariate asymmetric GJR-GARCH(1, 1) estimates are used for calculation of weights for daily rebalanced portfolios. The multivariate GJR-GARCH(1, 1) portfolio forecast covariance matrix based on the three S&P HF Indices is then used to find optimal next-period index weights. Portfolio optimisation based on the Markowitz approach requires inputs of expected returns, variances and cross-correlations to generate an efficient investment frontier. The performance of such a portfolio critically depends on the quality of forecasts of the expected returns vector and the covariance matrix. In this paper, next-day variances and cross-correlations are forecasted by the multivariate GJR-GARCH(1, 1) model, whereas the expected returns are equal to the average returns over the in-sample period.

Portfolio construction based on ARCH/GARCH conditional volatility estimation

As a first step in the construction of the portfolios, residuals from OLS estimation of returns are tested for ARCH behaviour. As shown in Tables 1 and 2, all of the hedge fund indices show evidence of skewness and leptokurtosis, consistent with ARCH effects. In most cases, we reject normality, based on the Jarque–Bera test. GARCH residuals are also confirmed using Engle[34] ARCH/GARCH tests based on various lags for daily, weekly and monthly series.[35]

To predict the volatilities of next-period returns, an Asymmetric GARCH model

(GJR-GARCH) with t-distributed errors is used. While a standard ARMA-GARCH model with normality adequately captures time-varying volatility, it is not the most effective approach for capturing the excess kurtosis or fat tails observed in hedge fund indices returns. A student-t distribution[36] is therefore used in place of a normal distribution.

The GJR-GARCH specification is used to account for possible leverage effects, with negative shocks serving to enhance conditional volatility. Krokhmal *et al*[25] and Favre and Signer[26] state that assuming normality in hedge fund returns leads to portfolios that are more risky than those in which asymmetry is accounted for. Conditional variances are parameterised by a GJR-GARCH model of orders p and q.

The GJR-GARCH(p, q) model is thus of the following form:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} (\alpha_i + \gamma_i S_{t-i})\varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{2}$$

where S_t is a dummy variable for negative residuals, defined as

$$S_t = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t > 0 \end{cases} \tag{3}$$

Using the GJR-GARCH model, the next-day conditional volatility for monthly, weekly and daily-rebalanced hedge fund indices is then forecasted by

$$\hat{\sigma}_{t+1}^2 = \hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\gamma}_1 S_t)\varepsilon_t^2 + \hat{\beta}_1 \sigma_t^2 \tag{4}$$

where S_t is a dummy variable for negative residuals, as defined in equation (3).

A univariate GJR-GARCH(1, 1) model with a BHHH[37] algorithm is estimated on the Hedge Fund Indices. For the S&P daily returns, 800 rolling in-sample observations are used to forecast volatilities for 143 out-of-sample days, from December 2005 until the end of June 2006. For the S&P weekly returns data, 157 rolling in-sample weekly observations are used to forecast volatilities for 31 out-of-sample weeks, from the beginning of December 2005 until the end of June 2006.

As expected, ARCH/GARCH terms are generally significant for the daily and weekly S&P Hedge Fund Indexes. For the daily series, the asymmetric volatility term is negative and significant, consistent with leverage effects typically found for equity markets – with negative shocks in returns serving to enhance conditional volatility.

For the monthly CSFB/T HF Indices, 50 rolling windows of 100 observations are used in the estimation. January 1994 through March 2002 serves as an initial calibration period for subsequent volatility forecasts from April 2002 until June 2006. In the pre-tests, ARCH/GARCH effects are only observed for the CFSB/T Market Neutral and Fixed Income series. Consequently, GARCH forecasts of volatility are only applied to these series.

In addition to the univariate GJR-GARCH(1, 1) specification, a multivariate asymmetric GARCH model extending the study by Switzer and El Khoury[38] is applied for the daily S&P Hedge Fund Indices data, where ARCH/GARCH effects are identified. The added benefit of the multivariate GARCH specification in dynamic asset allocation is that the covariance matrix is estimated jointly across assets, as opposed to being inferred from the forecasts of the global minimum variance formula.

The covariance matrix of the multivariate asymmetric bivariate GARCH can be written as

$$H_t = C'C + A'H_{t-1}A + B'\varepsilon_t\varepsilon_t'B + G'\eta_{t-1}\eta_{t-1}'G \tag{5}$$

where G is a matrix of coefficients, and η_t is the additional quadratic form of the vector of negative return shock. H_t is a linear function of its own past values as well as of values of squared shocks. The inclusion of η_t in the above form not only accounts for asymmetry in the conditional variances, but also allows for an asymmetric effect in the conditional covariance. This approach allows for time variation in the correlations across the various series.

Parameter estimates are obtained by maximising the log-likelihood function. Conditional log-likelihood functions are computed as

$$L_t(\theta) = -\log 2\Pi - \tfrac{1}{2}\log|H_t| - \tfrac{1}{2}e_t'(q)H_{t-1}(\theta)e_t(\theta) \tag{6}$$

where θ is the vector of all parameters of the model. To maximise this log-likelihood function, we use the simplex and Berndt *et al*[37] algorithms.

Benchmark portfolio and transactions costs

Four investment portfolios are examined: the maximum Sharpe portfolios, past volatility portfolios, GJR-GARCH(1, 1) portfolios and the benchmark S&P 500 Index. The latter is held as a passive portfolio. For the case of daily rebalancing, the multivariate GJR-GARCH(1, 1) portfolio replaces the maximum Sharpe portfolio in the analysis.

We also incorporate transaction costs in the analysis. The results reported here assume transaction costs of 25 basis points. Comparative and lower levels have been used in prior academic studies that looked into investment strategies for traditional asset classes, and are believed to be appropriate for an alternative investment universe composed of 'investable' hedge fund indices.

RESULTS

CSFB/Tremont monthly rebalanced portfolios

The performance of the CSFB/Tremont monthly rebalanced dynamic portfolio based on conditional volatility forecasting from GJR-GARCH(1, 1) is compared to the past volatility portfolio and the S&P 500 Index. The risk-adjusted performance of the portfolios under consideration (maximum Sharpe, past volatility, univariate GARCH and S&P500) are compared based on Sharpe ratio, defined as per Sharpe[39] as the ratio of the annualised mean portfolio return to the annualised portfolio standard deviation:

$$SR_P = \mu_P/\sigma_P \tag{7}$$

The out-of-sample testing period for the monthly analysis extends from May 2002 until June 2006, for a total of 50 return observations.

As shown in Table 3 after accounting for transaction costs, based on the Sharpe ratio rankings the past volatility portfolio ($SR_P = 3.46$) performs as well as the Maximum Sharpe ratio portfolio ($SR_P = 3.44$), whereas the GJR-GARCH(1, 1) portfolio dominates ($SR_P = 3.53$).[40] We therefore fail to reject Hypotheses 2, 4, 7 and 9, for monthly data. All three portfolios still largely outperform their benchmark S&P 500 Index ($SR_P = 0.37$).

Figure 1 shows the evolution of wealth after transaction costs of the portfolios.

The S&P weekly rebalanced portfolios

For weekly data, after transactions costs are accounted for, as shown in Table 4, GARCH

Table 3: Out-of-sample (May 2002–June 2006) monthly-rebalanced portfolios composed of nine Credit Suisse First Boston/Tremont Hedge Fund Indices, after accounting for transaction costs

	Max Sharpe portfolio	*Past volatility portfolio*	*Univariate GJR-GARCH portfolio*	*S&P 500 Index*
Annualised mean return	6.52%	7.82%	7.18%	4.72%
Annualised St. dev.	1.89%	2.26%	2.03%	12.83%
Sharpe ratio	3.44	3.46	3.53	0.37
Out-of-sample months	50	50	50	50
Positive months	82.69%	86.00%	80.77%	62.00%
Average decline	−0.19%	−0.36%	−0.26%	−3.07%
Worst month	−0.75%	−0.82%	−0.71%	−11.00%
Largest drawdown	−0.88%	−1.26%	−1.09%	−16.05%

This table shows the maximum Sharpe, past volatility and the univariate GJR-GARCH investment portfolios characteristics versus the S&P 500 Index benchmark, after transaction costs of 25bp are incorporated into the performance calculations. Monthly return data from nine Credit Suisse First Boston/Tremont Hedge Fund Indices are used for the construction of portfolios.

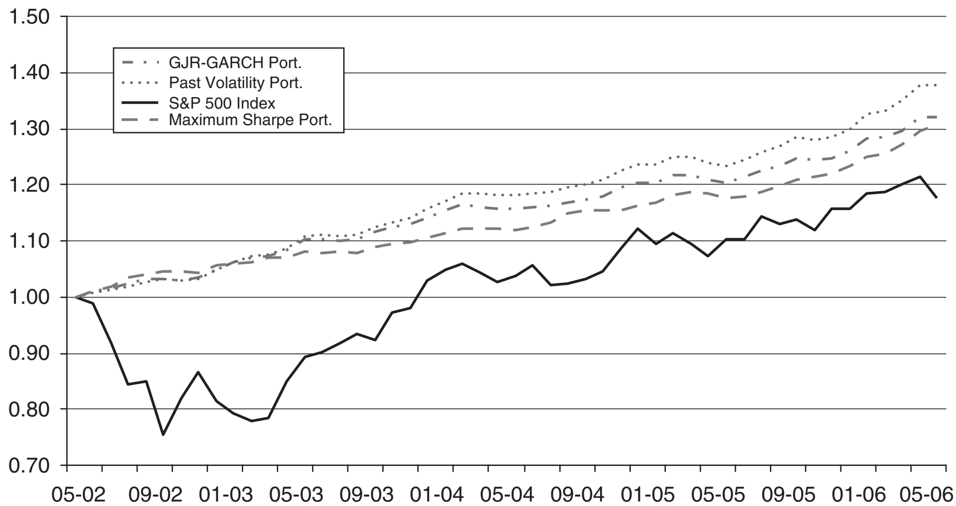

Figure 1: Out-of-sample wealth effects of monthly rebalanced Credit Suisse First Boston hedge fund indices portfolios, after transaction costs are included.

(1, 1) exhibits the best performance ($SR_P = 5.07$), followed by past volatility ($SR_P = 5.06$), the Maximum Sharpe ratio ($SR_P = 4.47$) and the S&P 500 Index ($SR_P = 0.03$), respectively. Thus, we fail to reject Hypotheses 2, 4, 7, and 9.

Table 4: Out-of-sample (1 December 2005–30 June 2006) weekly-rebalanced portfolios composed of three Standard and Poor's Hedge Fund Indices, after accounting for transaction costs

	Max Sharpe portfolio	*Past volatility portfolio*	*Univariate GJR-GARCH portfolio*	*S&P 500 Index*
Annualised mean return	10.54%	10.61%	10.95%	0.32%
Annualised St. dev.	2.36%	2.10%	2.16%	10.03%
Sharpe ratio	4.47	5.06	5.07	0.03
Out-of-sample weeks	31	31	31	31
Positive weeks	74.19%	77.42%	74.19%	48.39%
Average weekly decline	−0.19%	−0.18%	−0.22%	−1.01%
Worst week	−0.41%	−0.27%	−0.57%	−2.79%
Largest drawdown	−0.41%	−0.27%	−0.59%	−4.48%

This table shows the maximum Sharpe, past volatility and the univariate GJR-GARCH investment portfolios characteristics and how they compare against each other and to the S&P 500 Index benchmark, after transaction costs of 25bp are incorporated into the performance calculations. Weekly return data from three Standard & Poor's Hedge Fund Indices are used for the construction of portfolios.

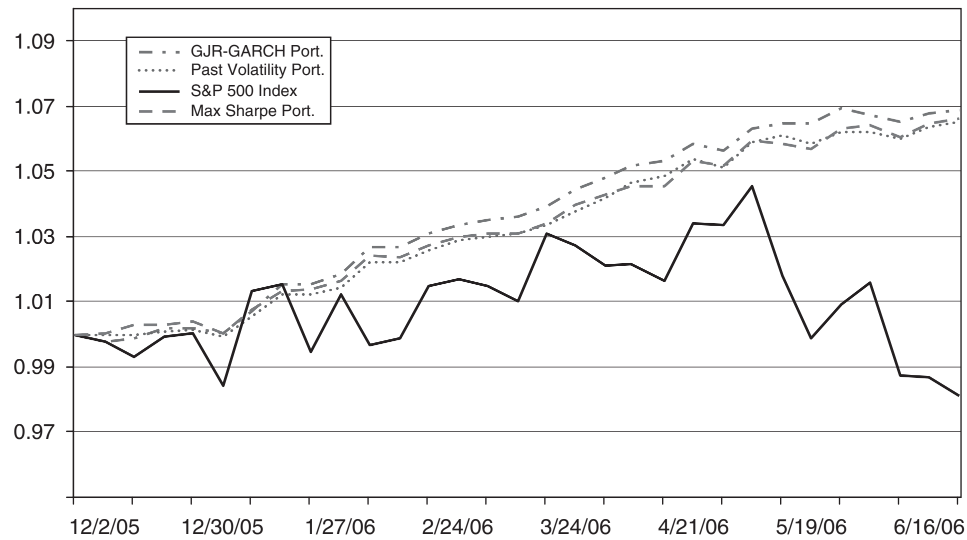

Figure 2: Out-of-sample wealth effects of weekly rebalanced Standard & Poor's Hedge Fund Indices portfolios, after transaction costs are included.

Figure 2 shows the wealth-changes-through-time effects associated with weekly-rebalanced hedge fund indices portfolios versus the S&P 500 Index.

We also conducted the analysis using daily data.[41] Ignoring transactions costs, multivariate GARCH(1, 1) model is shown to dominate based on the Sharpe ratio ($SR_P = 7.45$), followed by the univariate GARCH(1, 1) model ($SR_P = 7.30$), the past volatility model ($SR_P = 6.20$) and the distant S&P 500 Index benchmark ($SR_P = 0.32$). In general, the annualised returns are higher for more actively managed portfolios. Thus, we fail to reject Hypothesis 5. When transactions costs are accounted for, however, the benefits of more frequent rebalancing strategies are diminished by the higher trading costs, in terms of the risk-adjusted returns. Nevertheless, when plausible levels of transactions costs are included, all of the active portfolios dominate the passive benchmarks.

CONCLUSION AND IMPLICATIONS FOR FUTURE RESEARCH

This paper examines the return/risk benefits of portfolios of hedge fund indices with time-varying volatility, and with returns distributions that are skewed and leptokurtotic. The results show that there are distinct benefits in volatility reduction for portfolios constructed based on conditional volatility forecasting relative to static portfolios including the S&P 500 benchmark. These results are robust to transactions costs.

For the S&P hedge funds, portfolios constructed based on conditional volatility models that embody asymmetric volatility outperform on a risk-adjusted basis because of the larger returns, as opposed to a reduction in volatility, versus a portfolio structured based on the past volatility model.

Potential topics for future studies include modelling changes in volatility clustering patterns of hedge fund styles through time, sources of the macro-economic and other shocks that have in the past led to unusually high conditional volatility for a given hedge fund strategy, and common factors that have led to spikes in cross-correlations across hedge fund styles.

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41 These results are available on request.