Original Article

Commodity price risk management: Valuation of large trading portfolios under adverse and illiquid market settings

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PRACTICAL APPLICATIONS This paper aims to capture the liquidity risk arising because of illiquid trading positions and to obtain an estimate of liquidity-adjusted Value-at-Risk (L-VaR) of large commodity trading portfolios (of both long and short positions) under the notion of different correlation factors and liquidity horizons. In contrast to all existing published literature pertaining to the application of the VaR method to commodity markets, this paper proposes a re-engineered model for assessing a closed-form parametric L-VaR with explicit treatment of liquidity trading risk. Moreover, the commodities selected for this study provide a realistic alternative portfolio, as well as new data, for studying existing techniques of L-VaR estimation. This research makes advances in understanding L-VaR assessment techniques and their performance in the context of commodity price risk management. The results of this study also provide an incentive for further research in the area of L-VaR and commodity price risk management.

ABSTRACT Given the rising need for measuring and controlling commodity price risk exposure, trading risk prediction under illiquid and adverse market conditions plays an increasing role in commodity and financial markets. The aim of this paper is to close the void in commodity trading risk management literature, particularly from the perspective of large trading portfolios, by illustrating how the modified Value-at-Risk (VaR) method can be used by a commodity trading unit in reporting risk exposure, assessing risk reduction alternatives and setting optimised risk limits. In this study we put forward a re-engineered VaR model relevant for commodity trading units that have long- and short-selling trading positions and suggest potential applications of VaR in the context of commodity risk management. To the best of our knowledge, this is the first research paper

that addresses the issue of liquidity trading risk management in commodity markets with direct applications to a larger portfolio of distinctive assets. This paper provides real-world techniques and realistic asset allocation strategies that can be applied to commodity trading portfolios in illiquid markets and under adverse market conditions. The modelling technique is based on the renowned concept of liquidity-adjusted VaR (L-VaR), along with the creation of a software tool utilising matrix-algebra techniques. As such, our comprehensive risk model can simultaneously handle market risk analysis under normal and severe market settings as well as take into account the effects of illiquidity of traded commodities. In order to illustrate the proper use of L-VaR under stressed and illiquid market conditions, real-world examples and feasible reports of liquidity trading risk management are presented for a portfolio of 25 distinct commodities, within a multivariate context and under the notion of several correlation factors along with different liquidity horizons. The example and discussions are widely applicable to any commodity end-user, providing potential applications to practitioners and research ideas to academics.

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SYNOPSIS OF COMMODITY PRICE RISK MANAGEMENT

The instability of prices in commodity markets is mostly originated by supply-and-demand discrepancies that stem from the business cycle (metals, energy products and agricultural commodities), unforeseen weather conditions (agricultural commodities) or political upheavals (energy products). Commodity prices tend to fluctuate in the short term because of day-to-day and cyclical variations in supply and demand, but regress towards a long-term equilibrium. The price risk in commodities is often more complex and volatile than that associated with interest rates and currencies. Moreover, commodity markets may also be less liquid than other financial markets and, as a result, changes in supply and demand can have a more dramatic effect on prices and volatility. Consequently, the

characteristics of these markets can make price transparency and the effective hedging of commodity risk much more difficult.

Recent years have witnessed the increasing role of investment funds in most commodity markets, with a particular emphasis on emerging markets. Commodity assets, in addition to offering high rates of return, offer significant risk management benefits, as commodity assets are customarily negatively correlated with each other and with other assets. Investment funds, traditionally dealing with financial markets, are now diversifying into emerging commodity markets, with the objective of achieving significant risk/return benefits. The focus on emerging markets is motivated by the fact that many of these markets have progressively moved towards market-based systems, liberalised stock and foreign exchange

markets, and relaxed restrictions on foreign investor participation.

In addition to particular apparent risks (illiquidity, insider trading, lack of reliable information, political instability, currency risk and so on), emerging commodity markets tend to be risky and illiquid, and are susceptible to macroeconomic shocks, for other reasons related to their economic and market structure. As a result, the increased tradability of commodities in emerging markets – in addition to the recent rise in crude oil prices – necessitates a re-examination of current commodity risk management techniques; specifically for investment funds with large trading portfolios – of both long- and short-selling trading positions – and within short-to-medium horizons of re-balancing and reporting focuses.

Commodity price risk management is still in its early stages compared to the more developed equity, interest rate and foreign exchange markets. It is important to bear in mind, however, that commodity markets are not anywhere near as unambiguous as financial markets; hence few attempts have been made to measure price risk in commodity markets. Modelling market risk for commodity products thus presents an inherent complexity because of the strong interaction between the trading of the products and the supply-and-demand imbalances that stem from the state of the economy. To address the above deficiencies, in this paper we characterise trading risk for diverse commodity products using a multivariate liquidity-adjusted Value-at-Risk (L-VaR) approach that focuses on the modelling of Value-at-Risk (VaR) under illiquid and adverse market conditions.

The overall objective of this paper is to construct a large commodity portfolio that includes several crude oil/energy spot prices

as well as other common commodities and to evaluate the risk characteristics of such a diversified portfolio. To this end, our aspiration is not to quantify market risk using a specific economic model customised to predict supply and demand in any given market, but to propose a general trading risk model that accounts for the characteristics of the series of price returns – for example, fat tails (leptokurtosis), skewness, correlation factors and illiquidity – and adequately forecasts the market risk within a short-to-medium-term time horizon. As such, our focus on a short-to-medium-term time horizon is coherent with the use of a pure risk management method in which a more fundamental economics model would be of little aid *vis-à-vis* short-to-medium-term risk estimations. Over long-term time horizons, however, the additional value of a pure statistical risk model would perhaps be much less compared to the information given by economics models that tend to forecast supply and demand imbalances.

Furthermore, to the best of our knowledge, no work has yet been published in any international literature on commodity trading risk management that takes a large diverse commodity portfolio as a case study. This paper aims to capture the liquidity risk arising because of illiquid trading positions and to obtain an estimate of L-VaR. In contrast to all existing published literature pertaining to the application of the VaR method to commodity markets, this paper proposes a new model for assessing a closed-form parametric VaR, with explicit treatment of liquidity trading risk. The main contribution of this study is to extend VaR calculation to allow for a steady liquidation of the commodity portfolio over the holding period and to show that liquidity risk can be

straightforwardly and intuitively integrated into the proposed L-VaR framework. Rather than modelling liquidity trading risk as such, the central focus of this paper is to overhaul a wide-ranging and adaptable framework for handling liquidity risk in the overall assessment of commodity trading risk. Its essence relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate market risk assessment during market stress periods when liquidity dries up. The liquidity framework presented in this paper does not incorporate all the aspects of liquidity trading risk. It is, however, effective as a tool for evaluating trading risk when the impact of the illiquidity of specified commodity products is significant.

This study makes the following contributions to the literature in this specific commodity risk management field. Firstly, it represents one of the limited practitioners' papers that empirically examine commodity trading risk management using actual data of different commodity markets. Secondly, unlike most empirical studies in this field, this study employs a comprehensive and real-world trading risk management model that considers risk analysis under normal, severe (crisis) and illiquid market conditions. The principal advantage of employing such a model is the ability to capture a complete picture of possible loss scenarios of actual commodity trading portfolios.

LITERATURE REVIEW AND RATIONALE OF PRESENT WORK

Financial risks are usually associated with possible losses in financial markets arising from changes in the price of equities, currencies and interest rates, but also with risk associated with

negative changes in price of commodities, such as crude oil prices. Measurement of financial risks has greatly evolved in the last two decades, from simple indicator-of-market value, through more complex internal models such as scenario analysis to modern stress-testing and VaR measures.

VaR estimates the downside risk of a portfolio of market-priced assets at a particular confidence level over a specified time horizon. In effect, VaR strives to assess adverse events in the lower tail of a return distribution of a trading portfolio – events likely to cause financial distress to a firm if they arise. Because VaR concentrates solely on downside risk (or bad outcomes) and is typically articulated in dollar terms, it is regarded as an insightful and transparent risk assessment tool for top-level management. The recognition of VaR as a risk-assessment tool has triggered ample interest among risk management practitioners and academics alike.

Despite the obvious uses of VaR for risk disclosure and reporting purposes, VaR has also been recommended for enterprise-wide risk management because of its ability to aggregate risk across asset classes. In essence, VaR could be valuable in making asset allocation and hedging decisions, managing cash flows, setting trading limits, and overall portfolio selection and evaluation. Although VaR has received considerable attention in finance literature, the continued volatility of energy and agricultural prices implies several potential applications of VaR in energy and agricultural economics. Likewise, the management of commodity-based risk is likely to be clustered among specialised entities. This tendency requires a rational metric of measuring risk that is easily grasped at the top corporate level.

To quantify the risks involved in their trading operations, major financial and nonfinancial institutions are increasingly exploiting VaR models. As these institutions differ in their individual characteristics, tailor-made internal risk models are more appropriate. Moreover, the increase in the relative importance of trading risk has obliged regulators to reconsider the system of capital requirements as outlined in previous Basel committee capital accords. Fortunately, and in accordance with the latest Basel II capital accord, trading institutions are permitted to develop their own internal risk models for the purposes of providing for adequate risk measures. Furthermore, internal risk models can be used in the determination of capital that entities must hold to endorse their trading of securities. The benefit of such an approach is that it takes into account the relationship among various asset types and can accurately assess the overall risk for a whole combination of trading assets. The use of more advanced risk management techniques does not have to be limited only to financial institutions, but can also provide valuable information to other entities, such as energy and agriculture firms. As such, major commodities traders would be expected overtime to adopt an internal model approach subject to the safeguards set out in the literatures of the Basel II committee on banking supervision.

Nowadays, in forming capital adequacy requirements, commodity and financial institutions can use either standardised methods or advanced internal models, which enable them to hold lower capital requirements, but on the other hand require human and financial capital investments in competent workers and information-technology infrastructure. When assessing risks associated with holding open

positions in commodities, some financial institutions and commodity traders are still using the standardised approach set by the Basel II committee. Unfortunately, a deprived tracking record of the standardised approach to measuring trading risk has been overwhelmingly verified in the academic literature; and, hence, leaves entities using such approaches in a disadvantageous position compared to other advanced institutions.¹

Despite many criticisms and limitations of the VaR method, it has proven to be a very useful measure of market risk, and is widely used in financial and nonfinancial markets. Evidently, the overwhelming emphasis in VaR techniques has come from the finance literature, mostly as it pertains to the need of entities to satisfy regulatory requirements. Based on studies to date, there is little agreement as to the best method for developing a VaR measure. Literature related to VaR is, however, continually growing, as researchers attempt to reconcile several pending issues.

The literature on measuring financial risks and volatility using VaR models is extensive, yet Jorion² and Dowd *et al*³ should be pointed out for their integrated approach to the topic. Similarly, the *RiskMetrics*TM model,⁴ developed and popularised by *J.P. Morgan*, has provided a tremendous impetus to the growth in the use of the VaR concept and other modern risk management techniques and procedures. Since then, the VaR concept has been well known and scores of specific applications have been adapted to credit risk management and mutual funds investments. The general recognition and use of large-scale VaR models has resulted in considerable growth in literature, including statistical descriptions of VaR and assessments of different modelling techniques. For a

comprehensive survey, and the different VaR analysis and techniques, one can refer to Jorion.²

Literature on L-VaR techniques

Methods for measuring market (or trading) risk have been well developed and standardised in the academic world as well as the banking world. Liquidity trading risk, on the other hand, has received less attention, perhaps because it is less significant in the developed countries where most of the market risk methodologies were originated. In all but the most simple of circumstances, comprehensive metrics of liquidity trading risk management do not exist. Nonetheless, the combination of the recent rapid expansion of emerging markets' trading activities and the recurring turbulence in those markets has propelled liquidity trading risk to the forefront of market risk management research and development.

Liquidity trading risk is an all-embracing apprehension for anyone holding a portfolio of any type of trading asset, and liquidity crises proved to be imperative in the failure of many financial entities. More specifically, liquidity trading risk arises from situations in which a party interested in trading an asset cannot carry this out because no one in the market wants to trade that asset. Liquidity risk becomes, for the most part, important to financial market participants who are about to hold or currently hold an asset, as it affects their aptitude to trade or unwind the trading position. Insolvencies often occur because financial entities cannot get out or unwind their holdings effectively, and hence the liquidation value of assets may differ significantly from their current mark-to-market values.

The conventional VaR approach to computing the market (or trading) risk of a

portfolio does not explicitly consider liquidity risk. Typical VaR models assess the worst change in mark-to-market portfolio value over a given time horizon but do not account for the actual trading risk of liquidation. Customary fine-tunings are made on an *ad hoc* basis. At most, the holding period (or liquidation horizon) over which the VaR number is calculated is adjusted to ensure the inclusion of liquidity risk. As a result, liquidity trading risk can be imprecisely factored into VaR assessments by assuring that the liquidation horizon is at a minimum larger than an orderly liquidation interval. Moreover, the same liquidation horizon is employed to all trading asset classes, although some assets may be more liquid than others. Neglecting liquidity risk can lead to an underestimation of the overall market risk and misapplication of capital cushion for the safety and soundness of financial institutions. In emerging financial markets, which are relatively illiquid, ignoring the liquidity risk can result in significant underestimation of the VaR estimate, especially under severe market conditions.

Within the VaR framework, Jarrow and Subramaniam⁵ provide a market impact model of liquidity by considering the optimal liquidation of an investment portfolio over a fixed horizon. They derive the optimal execution strategy by determining the sales schedule that will maximise the expected total sales values, assuming that the period until liquidation is given as an exogenous factor. The correction to the log-normal VaR they derive depends on the mean and standard deviation of both: an execution lag function and of a liquidation discount. Although the model is simple and intuitively appealing, it suffers from practical difficulties for its implementation. It requires the estimation of additional parameters

such as the mean and the standard deviation of the discount factor and the period of execution – for which data are not readily available, may be easy to estimate, and may require subjective estimates such as a trader's intuition.

Bangia *et al*⁶ approach the liquidity risk from another angle and provide a model of VaR adjusted for what they call exogenous liquidity – defined as common to all market players and unaffected by the actions of any one participant. It comprises such execution costs as order processing costs and adverse selection costs, resulting in a given bid – ask spread faced by investors in the market. On the contrary, endogenous liquidity is specific to one's position in the market and depends on one's actions and varies across market participants. It is mainly driven by the size of the position: the larger the size, the greater the endogenous illiquidity. Bangia *et al* propose splitting the uncertainty in market value of an asset into two parts: a pure market risk component arises from asset returns and uncertainty because of liquidity risk. Their model consists of measuring exogenous liquidity risk, computed using the distribution of observed bid – ask spreads and then integrating it into a standard VaR framework.

Le Saout⁷ applies the model developed by Bangia *et al*⁶ to the French stock market. In an attempt to consider the effect of liquidating large positions, Le Saout⁷ incorporates the weighted average spread into the L-VaR measure. Le Saout's results indicate that exogenous liquidity risk, for illiquid stocks, can represent more than a half of total market risk. Furthermore, the author extends the model to incorporate endogenous liquidity risk and shows that it represents an important component of the overall liquidity risk. Roy⁸ relates the model provided by Bangia *et al*⁶ to the Indian debt

market. First, the author presents a comprehensive survey of L-VaR models and then adopts a modified version of the exogenous liquidity approach suggested by Bangia *et al*.⁶ In that paper, a measure of L-VaR, based on bid – ask spread, is presented, and the liquidity risk is found to be an important component of the aggregate risk absorbed by financial institutions.

Almgren and Chriss⁹ present a concrete framework for deriving the optimal execution strategy using a mean-variance approach, and show a specific calculation method. Their approach has high potential for practical application. They assume that price changes are caused by three factors: drift, volatility and market impact. Their analysis leads to general insights into optimal portfolio trading, relating risk aversion to optimal trading strategy and to several practical implications including the definition of L-VaR. Unlike Almgren and Chriss,⁹ Hisata and Yamai¹⁰ turn the sales period into an endogenous variable. Their model incorporates the mechanism of the market impact caused by the investor's own dealings through adjusting VaR according to the level of market liquidity and the scale of the investor's position.

Berkowitz¹¹ argues that unless the likely loss arising from liquidity risk is quantified, the models of VaR will lack the power to explicate the embedded risk. In practice, operational definitions vary from volume-related measures to bid – ask spreads and to the elasticity of demand. Berkowitz asserts that elasticity-based measures are the most relevant because they incorporate the impact of the seller actions on prices. Moreover, he asserts that, under certain conditions, the additional variance arising from seller impact can easily be quantified given observations on portfolio prices and net flows;

and that it is possible to estimate the entire distribution of portfolio risk through standard numerical methods.

Shamroukh¹² contends that scaling the holding period to account for orderly liquidation can only be justified if we allow the portfolio to be liquidated throughout the holding period. The author extends the *RiskMetrics*TM approach⁴ by explicitly modelling the liquidation of the portfolio over time and by showing that L-VaR can be easily obtained by an appropriate scaling of the variance – covariance matrix. Furthermore, market liquidity risk can be modelled by expressing the liquidation price as a function of trade sizes, thus imposing a penalty on instantaneous unwinding of large position. Following this approach, L-VaR can be viewed as a solution to a minimisation problem arising from the trade-off between higher variance associated with slow liquidation and higher endogenous liquidity risk associated with fast liquidation. As a result, the holding period is an output of this model – that is, the solution to the minimisation problem. In another relevant study, Dowd *et al*³ tackle the problem of estimating VaR for long-term horizon. In their paper, they offer a different but rather straightforward approach that avoids the inherited problems associated with the square-root-of-time rule, as well as those associated with attempting to extrapolate day-to-day volatility forecasts over long horizons.

Literature on commodity price risk management with VaR approach

Despite obvious empirical applications, the use of VaR as a risk-reporting and measurement tool has several practical applications for agricultural (and other commodities) risk management

issues. First and foremost, publicly traded agribusiness firms must comply with Securities and Exchange Commission (SEC) regulations concerning the reporting of positions in highly market sensitive assets, including spot commodity, futures and options positions. Thus, VaR was proposed as a potential risk-reporting measure to be used by commodity firms to disclose their market risk exposure.

In spite of the increasing importance of commodity markets, there are very few published studies in this respect, and particularly within the commodity trading risk management context. Given the general interest and acceptance of VaR within the financial risk management community, VaR has the aptitude of being an effective risk measure for energy and agricultural risk management as well. Therefore, it is imperative to gain an understanding of how standard VaR estimation techniques operate in the context of commodity prices.

In commodity markets, the measurement of volatility and the management of related risks is a fairly new field of research and few research papers deal with this particular topic. For instance, applying VaR models to measuring crude oil price volatility and associated risks is a field that is only starting to draw the attention of researchers. Nonetheless, commodity price volatility and dynamics have been studied by Giot and Laurent¹³ and Manfredo and Leuthold,^{14,15} among others.

Several agricultural risk management problems have been investigated within a multiproduct context. These settings offer logical portfolios for probing the performance of alternative VaR estimation techniques under credible portfolio situations. To date, the performance of VaR techniques has not been rigorously tested on portfolios exposed to

agricultural commodity price risk, although Manfredo and Leuthold^{14,15} are an exception. The dynamic nature of agriculture as well as the reduction of government programmes creates a new risky environment in agriculture.

Agribusinesses are exposed to multifaceted market risks. With the growing use of and interest in VaR, there appear to be several practical applications to agriculture, and there is also a greater need to assess and evaluate current methods of estimating volatility. The recent interest in VaR has created a new and additional motivation for accurate and meaningful measures of volatility and correlations.¹⁴

Considering the growing interest in VaR and the variability of the market risk factors of the cattle-feeding margin, Manfredo and Leuthold¹⁵ examine the application of VaR measures in the context of agricultural prices, using several alternative procedures (both parametric and full-valuation) in predicting large losses in the cattle-feeding margin. In another relevant study, Manfredo and Leuthold¹⁴ review the various VaR estimation techniques and empirical findings and suggest potential extensions and applications of VaR in the context of agricultural risk management. For commodity traders with both long- and short-selling trading positions, Giot and Laurent¹³ put forward VaR models relevant for commodity markets, particularly for short-term horizon. In a 5-year out-of-sample study on aluminum, copper, nickel, Brent crude oil and WTI crude oil daily cash prices and cocoa nearby futures contracts, they assess the performance of the *RiskMetrics*TM method⁴ and the skewed Student APARCH and skewed Student ARCH models. Although the skewed Student APARCH model performed best in all cases, the skewed Student ARCH model nevertheless delivered good results. An

important advantage of the skewed Student ARCH model is that its estimation does not require numerical optimisation procedures and could be routinely programmed in a simple spreadsheet environment.

Because UK arable farms face substantial price risk, White and Dawson¹⁶ estimate price risk for a representative UK arable farm using the VaR method. To determine the distribution of commodity returns, two multivariate generalised autoregressive conditional heteroscedasticity (GARCH) models, with *t*-distributed and normally distributed errors, and a *RiskMetrics*TM model⁴ are examined. Their analysis of returns shows excess kurtosis and that the GARCH model with *t*-distributed errors fits best. Moreover, estimates of VaR differ between models: both GARCH models perform properly but the *RiskMetrics*TM model⁴ underestimates expected losses. Finally, Wilson and Nganje¹⁷ provide a case study on the application of VaR in bakery procurement. They illustrate the application of VaR to a bread-baking company and demonstrate how it can be used by a commodity processor in reporting risk, evaluating risk reduction alternatives and setting risk limits.

Particular objectives of this study

As indicated above, commodity prices are exposed to a variety of volatile market prices that can be and have been examined in a portfolio context. Despite the rising impact of commodity markets, little attempt has been made to rigorously measure price risk in energy, agriculture and other commodities sectors, and particularly within large trading portfolios under illiquid and adverse market conditions. Thus, large commodity trading portfolios provide a

practical case for testing VaR methodologies in the context of commodity prices, helping to establish the appropriateness of VaR as a viable and important risk management tool for commodity risk managers.

Considering the recent interest in L-VaR and the variability of the market risk factors of different commodities, the overall aim of this paper is to examine VaR measures in the context of large commodity trading portfolios (of both long- and short-selling positions) and under the notion of different correlation factors and liquidity horizons. In particular, this paper develops and tests L-VaR measures using several alternative strategies in predicting large losses, with the aid of different liquidation horizons and under a predetermined confidence level. This research is important and unique because it provides insight into the performance of procedures, often suggested for use in creating VaR estimates for portfolios of financial assets, from the perspective of commodity prices. Given initial evidence of the sensitivity of L-VaR measures to the procedures and data set used, as well as the increasing interest in VaR as a robust tool in risk management, this research makes advances in understanding L-VaR estimation techniques and their performance in the context of commodity price risk management.

This study is the first known attempt at empirically examining the performance of various VaR measures in the framework of a diverse commodity portfolio. To date, all known empirical studies examining the performance of alternative VaR measures have been conducted in the framework of portfolios containing foreign exchange, interest rate or equity data, with portfolios often developed arbitrarily. The commodities selected for this study provide a realistic alternative portfolio, as well as new data,

for studying existing techniques of L-VaR estimation. The results of this study also provide an incentive for further research in the area of L-VaR and commodity price risk management.

The commodity risk management models implemented in this study are applied to 25 different commodities, with particular focus on the energy sector. A database of selected commodities is gathered, filtered and processed in such a manner so as to create meaningful quantitative analysis and a conclusion of commodity market risk measurement. Several case studies are carried out with the objective of assessing L-VaR under adverse illiquid market conditions. The L-VaR estimates have been obtained through a modified closed-form parametric VaR approach. The empirical testing results are then used to draw conclusions about the relative liquidity of the different commodities and the importance of liquidity risk in VaR estimation.

The rest of the paper is organised as follows. The next section lays down the salient features and derives the necessary quantitative infrastructure of VaR, and its limitations. First, we show that L-VaR can be derived for a single-asset portfolio assuming uniform liquidation over the holding period. We then derive a general and broad model that incorporates the effects of multiple illiquid commodity assets in market risk management by simply scaling the multiassets' VaR matrix. Finally, we demonstrate, by applying the L-VaR measures to commodity markets, to what extent the quantified liquidity effects can affect conventional assessment of commodity trading risk. The subsequent section analyses the overall results of the different empirical tests and the further section remarks on conclusions and recommendations. The complete set of relevant

tables of empirical testing and commodity trading risk management reports are included in the appendix.

THEORETICAL FOUNDATION OF L-VAR MODELS FOR COMMODITY RISK MANAGEMENT

Commodity trading risk management with a broad parametric VaR approach

One of the most significant advances in the past two decades in the field of measuring and managing financial risks is the development and ever-growing use of VaR methodology. VaR has become the standard measure that financial analysts use to quantify financial risks, including commodity risk. VaR represents a potential loss in the market value of a portfolio of commodities with a given probability over a certain time horizon. The main advantage of VaR over other risk measures is that it is theoretically simple. As such, VaR can be used to summarise the risk of individual positions in a commodity, such as crude oil cash price, or the risk of large portfolios of commodity assets. Thus, VaR reduces the risk associated with any portfolio of commodities (or other assets) to just one number – the expected loss associated with a given probability over a defined holding period.

VaR is being embraced by corporate risk managers as an important tool in the overall risk management process. In particular, upper-level managers, who may or may not be well versed in statistical analysis, view VaR as an intuitive measure of risk because it concentrates only on adverse outcomes and is usually reported in dollar terms. Initial interest in VaR stemmed,

however, from its potential applications as a regulatory tool. Because of this, VaR is commonly used for internal risk management purposes (for example setting trading limits among traders) and is further being touted for use in risk management decision-making by nonfinancial firms. In the case of a nonfinancial firm, such as an energy enterprise, a risk manager may examine the VaR associated with a portfolio of risky assets (for example cash positions) before and after risk management strategies (for example use of futures, options, forward contracts or a combination of the above) are implemented. This allows the risk manager to assess the potential large losses to the portfolio in the presence of various risk management strategies, thus allowing this manager to more efficiently implement these strategies.

In essence, VaR is intended to measure the largest amount of money a position or trading portfolio could lose, with a given degree of confidence, over a given time horizon and under normal market conditions. Assuming the return of a financial product follows a normal distribution, a linear pay-off profile, and a direct relationship between the underlying product and the income, the VaR is designed to measure the standard deviation of the trading income, which results from the volatility of the different markets, for a certain confidence level. This definition gives latitude in choosing both confidence level and time horizon.

In practice, however, many financial and nonfinancial entities have chosen a confidence interval of 95 per cent (or 97.5 per cent if we only look at the loss side (one-tailed)) for their overall portfolio and a 1-day time horizon to calculate VaR. This means that once every 40 trading days, a loss larger than indicated is expected to occur. Some entities use a

99 per cent (one-tailed) confidence interval, which would theoretically lead to larger loss once every 100 trading days. Because of fat tails of the probability distribution, such a loss will occur more often and can cause problems in calculating VaR at higher confidence levels. Some entities feel that the use of a 99 per cent confidence interval would place too much trust in the statistical model and, hence, some confidence level should be assigned to the 'art-side' of the risk measurement process. Although the method relies on many assumptions and has its drawbacks, it has gained wide acceptance for the quantification and aggregation of trading risks. As a result of the generalisation of this method, capital allocations for trading and active investment activities tend to be calculated and adjusted with the VaR method.

To calculate VaR using the variance – covariance method (also known as the parametric, analytical and delta-neutral method), the volatility of each risk factor is extracted from a predefined historical observation period. The potential effect of each component of the portfolio on the overall portfolio value is then worked out. These effects are then aggregated across the whole portfolio using the correlations between the risk factors (which are, again, extracted from the historical observation period) to give the overall VaR value of the portfolio with a given confidence level.

A simplified calculation process of the estimation of VaR factors (using the variance – covariance method) for single- and multiple-assets' positions is illustrated^{18,19} as follows:

From elementary statistics it is well known that for a normal distribution, 68 per cent of the observations will lie within 1σ (standard deviation) from the expected value, 95 per cent within 2σ and 99 per cent within 3σ from the

expected value. Thus the VaR of a single asset in monetary terms is

$$\text{VaR}_i = \alpha \times \frac{\text{Mark-to-Market Value of Commodity}_i \sigma_i}{\sigma_i} \quad (1)$$

where α is the confidence level (or, in other words, the standard normal variant at confidence level α) and σ_i is the standard deviation (volatility) of the security that constitutes the single position. The mark-to-market value of commodity indicates the amount of investment in commodity i .

Trading risk in the presence of multiple risk factors is determined by the combined effect of individual risks. The extent of the total risk is determined not only by the magnitudes of the individual risks, but also by their correlations. Portfolio effects are crucial in risk management not only for large diversified portfolios, but also for individual instruments that depend on several risk factors. For multiple assets or portfolio of assets, VaR is a function of each individual security's risk and the correlation factor between the returns on the individual securities, as follows:

$$\text{VaR}_p = \sqrt{|\text{VaR}| \times |\rho| \times |\text{VaR}|^T} \quad (2)$$

This formula is a general formula for the calculation of VaR for any portfolio regardless of the number of securities. It should be noted that this formula is presented in terms of matrix-algebra – a useful form to avoid mathematical complexity, as more and more securities are added. This approach can simplify the programming process and permits easy incorporation of short-selling positions in market risk management process. This means, in order to calculate the VaR (of a portfolio of any number of securities), one needs first to create a

matrix $|\text{VaR}|$ of individual VaR positions – explicitly n rows and 1 column ($n \times 1$) matrix – a transpose matrix $|\text{VaR}|^T$ of individual VaR positions – a $(1 \times n)$ matrix, and hence the superscript ‘T’ indicated above on the top of the matrix – and finally a matrix $|\rho|$ of all correlation factors (ρ) – an $(n \times n)$ matrix. Consequently, as one multiplies the three matrices and then takes the square root of the result, one ends up with the VaR_p of any portfolio with any n -number of securities. This simple number summarises the portfolio’s exposure to market risk. Investors and senior managers can then decide whether they feel comfortable with this level of risk. If the answer is no, then the process that led to the estimation of VaR can be used to decide where to reduce redundant risk. For instance, the riskiest securities can be sold, or one can use derivative securities such as futures and options to hedge the undesirable risk.

The VaR method is only one approach to measuring market risk and is mainly concerned with maximum expected losses under normal market conditions. It is not an absolute measure, as the actual amount of loss may be greater than the given VaR amounts under severe circumstances. In extreme situations, VaR models do not function very well. As a result, for prudent risk management, and as an extra management tool, firms should augment VaR analysis with stress-testing and scenario procedures. From a risk management perspective, however, it is desirable to have an estimate for what potential losses could be under severely adverse conditions where statistical tools do not apply.

As such, stress-testing estimates the impact of unusual and severe events on the entity’s value and should be reported as part of the

risk-reporting process. For certain commodities with extreme volatility, the use of stress-testing should be highly emphasised, and a thorough description of the process is included in any trading risk policy manual. Stress-testing usually takes the form of subjectively specifying scenarios of interest to assess changes in the value of the portfolio and it can involve examining the effect of past large market moves on today’s portfolio. In this paper, risk management procedure is developed to assess potential exposure because of an event risk (severe crisis) that is associated with large movements of certain commodities, under the assumption that certain commodities, such as crude oil prices, have typical leaps during periods of turmoil. The task here is to measure the potential trading risk exposure that is associated with a predefined leap, under the notion of several correlation factors.

Modelling commodity liquidity trading risk in VaR framework

Illiquid securities such as foreign exchange rates and equities are very common in emerging markets. Customarily, these securities are traded infrequently (at very low volume). Their quoted prices should not be regarded as representative of the traders’ consensus *vis-à-vis* their real value, but rather as the transaction price arrived at by two counterparties under special market conditions. This of course represents a real dilemma to anybody who seeks to measure the market risk of these securities with a methodology based on volatilities and correlation matrices. The main problem arises when the historical price series are not available for some securities or, when they are available, are not completely reliable because of the lack of liquidity.

Given that institutional investors usually have longer time horizons, the liquidity of their positions will be lower. The investment horizon of the investor as well as the liquidity characteristics of the mutual fund need to be integrated into the risk quantification process. For instance, portfolios with long investment horizons and/or low liquidity need more distinct risk measures than those that have shorter horizons and are very liquid. The choice of the time horizon or number of days to liquidate (unwind) a position is very important factor and has significant impact on VaR numbers, and it depends upon the objectives of the portfolio, the liquidity of its positions and the expected holding period. Typically, for a bank's trading portfolio invested in highly liquid currencies, a 1-day horizon may be acceptable. For an investment manager with a monthly rebalancing and reporting focus, a 30-day period may be more appropriate. Ideally, the holding period should correspond to the longest period for orderly portfolio liquidation.

Liquidity is a key risk factor that, until lately, has not been appropriately dealt with by risk models. Illiquid trading positions can add considerably to losses and can give negative signals to trade because of the higher expected returns they entail. The concept of liquidity trading risk is immensely important for using VaR accurately, and recent upheavals in financial markets confirm the need for the laborious treatment and assimilation of liquidity trading risk into VaR models.

The simplest way to account for liquidity trading risk is to extend the holding period of illiquid positions to reflect a suitable liquidation period. An adjustment can be made by adding a multiplier to the VaR measure of each trading asset type, which in the end depends on the

liquidity of each individual security.

Nonetheless, the weakness of this method is that it allows for subjective estimation of the liquidation period. Furthermore, the typical assumption of a 1-day horizon (or any inflexible time horizon) within the VaR framework neglects any calculation of trading risk related to liquidity effect (that is, when and whether a trading position can be sold and at what price). A broad VaR model should incorporate a liquidity premium (or liquidity risk factor). This can be worked out by formulating a method by which one can unwind a position, not at some *ad hoc* rate, but at the rate that market conditions are optimal, so that one can effectively set a risk value for the liquidity effects. In general, this will significantly raise the VaR, or the amount of capital to support the trading position.

In fact, if returns are independent and can have any elliptical multivariate distribution, it is possible to convert the VaR horizon parameter from daily to any t -day horizon. The variance of a t -day return should be t times the variance of a 1-day return or $\sigma^2 = f(t)$. Thus, in terms of standard deviation (or volatility), $\sigma = f(\sqrt{t})$ and the daily VaR number [VaR (1-day)] can be adjusted for any horizon as

$$\text{VaR}(t\text{-day}) = \text{VaR}(1\text{-day}) \times \sqrt{t} \quad (3)$$

The above formula was proposed and used by *J.P. Morgan* in their earlier *RiskMetrics*TM method.⁴ This methodology implicitly assumes that liquidation occurs in one block sale at the end of the holding period and that there is one holding period for all assets, regardless of their inherent trading liquidity structure. Unfortunately, the latter approach does not consider real-life trading situations, where traders can liquidate (or rebalance) small portions of their trading portfolios on a daily basis. The assumption of a given

holding period for orderly liquidation inevitably implies that assets' liquidation occurs during the holding period. Accordingly, scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period.

In this study, we present a different re-engineered approach for calculating a closed-form parametric VaR, with explicit treatment of liquidity trading risk. The essence of the model relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate overall trading risk assessment during market stress periods when liquidity dries up. To this end, a practical framework of a methodology (within a simplified mathematical approach) is proposed below, with the purpose of incorporating and calculating illiquid assets' horizon VaR, detailed along these lines:

The market risk of an illiquid commodity trading position is larger than the risk of an otherwise identical liquid position. This is because unwinding the illiquid position takes longer than unwinding the liquid position, and, as a result, the illiquid position is more exposed to the volatility of the market for a longer period of time. In this approach, a commodity trading position will be deemed illiquid if its size surpasses a certain liquidity threshold. The threshold (which is determined by each trader) is defined as the maximum position that can be unwound, without disrupting market prices, in normal market conditions during 1 trading day. Consequently, the size of the trading position relative to the threshold plays an important role in determining the number of days required to close the entire position. This effect can be translated into a liquidity increment (or an additional liquidity risk factor) that can be incorporated into VaR analysis. If, for instance,

the par value of a commodity position is US\$10 000 and the liquidity threshold is US\$5000, it will take 2 days to sell out the entire trading position. Therefore, the initial position will be exposed to market variation for 1 day, and the rest of the position (that is, US\$5000) is subject to market variation for an additional day. If it is assumed that daily changes of market values follow a stationary stochastic process, the risk exposure because of illiquidity effects is given by the following illustration, detailed as follows.

In order to take into account the full illiquidity of assets (that is, the required unwinding period to liquidate an asset) one can define the following:

- t number of liquidation days (t -days to liquidate the entire asset fully);
- σ_{adj}^2 variance of the illiquid trading position; and
- σ_{adj} liquidity risk factor or standard deviation of the illiquid position.

The additional liquidity risk factor depends only on the number of days needed to sell an illiquid position. In the general case of t -days, the liquidity factor is given by the following mathematical functional expression:

$$\sigma_{\text{adj}}^2 = f \left(\left(\frac{t}{t} \right)^2 + \left(\frac{t-1}{t} \right)^2 + \left(\frac{t-2}{t} \right)^2 + \dots + \left(\frac{3}{t} \right)^2 + \left(\frac{2}{t} \right)^2 + \left(\frac{1}{t} \right)^2 \right) \quad (4)$$

or

$$\sigma_{\text{adj}}^2 = f \left(\frac{1}{t^2} \{ (t)^2 + (t-1)^2 + (t-2)^2 + \dots + (3)^2 + (2)^2 + (1)^2 \} \right) \quad (5)$$

To calculate the sum of the squares, it is convenient to use a short-cut approach. From mathematical series, the following relationship can be obtained:

$$\begin{aligned} & (t)^2 + (t-1)^2 + (t-2)^2 + \dots \\ & + (3)^2 + (2)^2 + (1)^2 \\ & = \frac{t(t+1)(2t+1)}{6} \end{aligned} \quad (6)$$

Hence, after substituting equation (6) for equation (5), the following can be achieved:

$$\begin{aligned} \sigma_{\text{adj}}^2 &= f \left[\frac{1}{t^2} \{ (t)^2 + (t-1)^2 + (t-2)^2 + \dots \right. \\ & \quad \left. + (3)^2 + (2)^2 + (1)^2 \} \right] \\ \text{or } \sigma_{\text{adj}}^2 &= f \left(\frac{(2t+1)(t+1)}{6t} \right) \end{aligned} \quad (7)$$

Accordingly, from equation (7), the liquidity risk factor (in mathematical functional form) is obtained as:

$$\begin{aligned} \sigma_{\text{adj}} &= f \left\{ \sqrt{\frac{1}{t^2} [(t)^2 + (t-1)^2 + (t-2)^2 + \dots + (3)^2 + (2)^2 + (1)^2]} \right\} \\ \text{or } \sigma_{\text{adj}} &= f \left\{ \sqrt{\frac{(2t+1)(t+1)}{6t}} \right\} \end{aligned} \quad (8)$$

The final result of equation (8) is of course a function of time and not the square root of time, as employed by some financial market's participants based on the *RiskMetrics*TM methodologies.⁴ The above approach can also be used to calculate L-VaR for any time horizon. Likewise, in order to perform the calculation of L-VaR under illiquid market conditions, it is possible to use the liquidity factor of equation (8) and to define the following:

$$\text{L-VaR}_{\text{adj}} = \text{VaR} \sqrt{\frac{(2t+1)(t+1)}{6t}} \quad (9)$$

VaR is the Value-at-Risk under liquid market conditions; L-VaR_{adj} is the Value-at-Risk under illiquid market conditions.

The latter equation indicates that L-VaR_{adj} ≫ VaR, and for the special case when the number of days to liquidate the entire assets is 1 trading day, L-VaR_{adj} = VaR. Consequently, the difference between L-VaR_{adj} - VaR should be equal to the residual market risk because of the illiquidity of any asset under illiquid markets conditions.

The above mathematical formulas can be applied for the calculation of L-VaR for each trading position and for the entire portfolio. In order to calculate the L-VaR for the full trading portfolio under illiquid market conditions (L-VaR_{Padj}), the above mathematical formulations can be extended, with the aid of equation (2), into a matrix-algebra form

to yield the following:

$$\text{L-VaR}_{P_{\text{adj}}} = \sqrt{|\text{L-VaR}_{\text{adj}}| \times |\rho| \times |\text{L-VaR}_{\text{adj}}|^T} \quad (10)$$

The above mathematical structure (in the form of three matrices, |L-VaR_{adj}|, |L-VaR_{adj}|^T and |ρ|) can facilitate the mathematical algorithm and programming process so that the trading risk manager can specify different liquidation horizons for the whole portfolio and/or for each individual trading commodity according to the necessary number of days to liquidate the entire asset completely. The latter can be achieved by specifying an overall benchmark liquidation

period to fully liquidate the entire constituents of the portfolio. The number of days required to liquidate a position (depending on the type of commodity, of course) can be obtained from the various publications of commodity markets and can be compared with the assessments of individual traders of each trading unit. As a result, it is possible to create simple statistics of the commodity volume that can be liquidated and the necessary time horizon to unwind the whole volume.

EMPIRICAL APPLICATION – ANALYSIS OF LARGE PORTFOLIOS OF DISTINCTIVE COMMODITIES

For the purpose of this study, and in order to examine various L-VaR estimation alternatives for commodity price risk management, price return series are needed. Returns are constructed from the cash (or spot) prices of 25 different commodities, in addition to an 'All Commodities Index'. The sample portfolio of commodities is included in Table A1 and the classification of the commodities is along these lines: energy sector (seven commodities with main focus on crude oil), metal sector (nine commodities), agriculture sector (seven commodities) and miscellaneous sectors (two commodities).

Moreover, in this study, price returns are defined as $R_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$ where $R_{i,t}$ is the monthly return of commodity i , $\ln$ is the natural logarithm, $P_{i,t}$ is the current month price of commodity i , and $P_{i,t-1}$ is the previous month price. Furthermore, for this particular study we have chosen a confidence interval of 95 per cent (or 97.5 per cent with 'one-tailed' loss side) and several liquidation time horizons to compute L-VaR. The commodities database

series span from 1987 through 2007, providing 20 years of relevant price returns for estimation and out-of-sample testing. The historical database of monthly prices is obtained from the International Financial Statistics Browser (<http://imfStatistics.org>) of the International Monetary Fund.²⁰

In the process of analysing the data, as well as the empirical testing, monthly returns of the various commodities have been calculated. These returns are essential factors for the estimation of standard deviations, correlation matrices, sensitivity factors (or beta coefficients), skewness, kurtosis and to perform the Jarque – Bera (JB) test for all commodities and their relationship *vis-à-vis* the All Commodities Index. A software package is created for the purpose of creating a realistic commodity trading portfolio and consequently for carrying out L-VaR, optimisation of trading limits and scenario analysis under extreme illiquid market conditions. The analysis of data and discussions of relevant empirical findings are organised and explained in the following sections.

Descriptive statistical analysis and testing for normality

In this section, analysis of the particular risk of each commodity (monthly and annual volatilities) of the commodity relationships with respect to the benchmark index (that is, All Commodities Index) and a test of normality are performed on the large portfolio of commodities. Table A1 illustrates the monthly volatility of each commodity under normal and severe (crisis) market conditions. Crisis market volatilities are calculated as the maximum negative returns (losses) that are witnessed in the historical time series, for all commodities.

From the table it is clear that the commodity with the highest volatility is gasoline (under normal market conditions), whereas coffee has demonstrated the highest volatility under severe market conditions. Table A1 also depicts annualised volatilities: these are calculated by adjusting (multiplying) the monthly volatilities by the square root of 12. An interesting outcome of the study of sensitivity factors (beta factors for systematic risk) is the manner in which the results are varied across the sample commodities, as indicated in Table A1. Gasoline appears to have the highest sensitivity factor (1.97) *vis-à-vis* the All Commodities Index (that is, the highest systematic risk), and both tobacco and phosphate rock seem to have the lowest beta factor (0.01).

In another study, tests of normality (symmetry) are performed on the sample commodities. In the first study, the measurements of skewness and kurtosis are achieved on the sample commodities. The results are depicted in Table A2. It can be seen, in general, that almost all commodities have shown asymmetric behaviour (between both positive and negative values). Moreover, kurtosis studies have shown similar patterns of abnormality (that is peaked/flat distributions). At the furthest extent, iron ore has shown the greatest positive skewness (8.29), which is combined with a high kurtosis – peakedness of (96.83). Some commodities, such as lead, have shown a close relationship to normality (skewness of 0.20 and kurtosis of 3.11). As evidenced in Table A2, the above results of general departure from normality are also confirmed with the JB test. The JB statistics are calculated in the following manner:

$$JB = \frac{n}{6} [S^2 + (K - 3)^2/4] \approx \chi^2(2) \quad (11)$$

where S is the skewness, K is the kurtosis and n is the number of observations. The JB statistics approximately reassemble a χ^2 distribution [$\chi^2(2)$] with 2 degrees of freedom. The 95 and 99 per cent percentage points of the χ^2 distribution with 2 degrees of freedom are 5.99 and 9.21, respectively; thus, the lower the JB statistics, the more likely it is that a distribution is normal. Nonetheless, the JB test shows an obvious general deviation from normality and thus rejects the null hypothesis that the actual commodity portfolio's time series returns are normally distributed. The interesting outcome of this study suggests the necessity of combining L-VaR calculations – which assumes normal distributions of returns – with other methods such as stress-testing and scenario analysis to obtain a detailed picture of other remaining risks (fat tails in the probability distribution) that cannot be captured with the simple assumption of normality.

Finally, Table A5 depicts a matrix of empirical correlation factors that are used next for the estimation of L-VaR. The objective here is to establish the necessary quantitative infrastructures for the advanced risk management analysis that will follow shortly. The empirical correlation matrix is an essential aspect, along with volatilities matrices for the creation of L-VaR and stress-testing calculations for commodity market risk management processes and procedures. Contrary to general belief, our analysis indicates that in the long-run period, there are very small and/or negative correlations between the various commodity markets. The result of Table A5 confirms several well-known facts in the commodity markets, including the strong relationships between energy sector commodities, with the exception of natural gas and coal. In general, it seems that

the energy sector, excluding natural gas and coal, dominates the panorama of actions and therefore has the greatest effect (and correlation relationship) on the All Commodities Index.

Commodity trading risk management and control reports

In order to illustrate the linkage between the theoretical constructs of L-VaR and its practical application and value as a tool for commodity risk management, the following hypothetical examples with full case studies are presented. These case studies parallel descriptions of L-VaR often found in the finance literature, where portfolios of interest rate, equities or currencies are the norm. These case studies also help in understanding the methods used in determining the performance of alternative L-VaR estimation procedures in the context of commodity trading risk management.

Using the definition of L-VaR in section 'Empirical application—analysis of large portfolios of distinctive commodities' and assuming that a given commodity portfolio has both long- and short-selling trading positions, Table A6 illustrates L-VaR at the one-tailed 97.5 per cent level of confidence over the upcoming 1-month period. In this first full case study, the total portfolio value is US\$10 000 000, with different asset-allocation percentages and 1-month liquidity horizon – that is, 1 month to unwind all commodity trading positions. Furthermore, Table A6 illustrates the effects of stress-testing (that is, L-VaR under severe market conditions) and different correlation factors on monthly L-VaR estimations. The L-VaR report also depicts monthly volatilities, in addition to their respective sensitivity factors *vis-à-vis* the All Commodities Index. Crisis (or severe) market

monthly volatilities are calculated and illustrated in the report as well. These monthly severe volatilities represent the maximum negative returns (losses) that perceived in the historical time series, for all commodities.

The L-VaR results are calculated under normal and severe (or crisis) market conditions by considering four different correlation factors (+1, 0, −1 and empirical correlations between the various risk factors). Under correlation +1, the assumption is for 100 per cent positive relationships among all risk factors (risk positions) all the time, whereas for the 0 correlation case, there is no relationship between any of the positions. Although −1 correlation case assumes 100 per cent negative relationships, the empirical correlation case considers the actual empirical correlation factors between all positions and is calculated via a variance – covariance matrix. Therefore, with 97.5 per cent confidence, the actual commodity portfolio should be expected to realise no more than a US\$2 228 671 decrease in the value over a 1-month time frame. In other words, the loss of US\$2 228 671 is one that a commodity portfolio should realise only 2.5 per cent of the time. If the actual loss exceeds the L-VaR estimate, this would be considered a violation of the estimate. From a risk management perspective, the L-VaR estimate of US\$2 228 671 is a valuable piece of information. As every commodity trading business has different characteristics and tolerances toward risk, the commodity risk manager must examine the L-VaR estimate relative to the overall position of the entire business. Simply put, can the firm tolerate or even survive such a rare event – a loss of US\$2 228 671 (or 22.3 per cent of total portfolio value)? This question is not only important to the commodity trading unit, but also to financial

institutions that lend money to these firms. The inability of a commodity trading unit to absorb large losses may jeopardise its ability to make principal and interest payments. Therefore, various risk management strategies could be examined in the context of how they might affect the L-VaR estimate. Presumably, risk management strategies, such as the use of futures and options contracts in hedging possible fluctuation in commodity prices, should reduce the L-VaR appraisal. In other words, those extreme losses in commodity trading, that would normally occur only 2.5 per cent of the time should be smaller with the incorporation of some type of risk management strategy.

Finally since the variations in L-VaR are mainly related to the ways in which the assets are allocated, in addition to the liquidation horizon, it is instructive to examine the way in which L-VaR figures are influenced by changes in such parameters. All else equal, and under the assumption of normal and severe market conditions, Table A7 illustrates the nonlinear alterations to L-VaR figures when the liquidation periods is increased in line with the liquidity-holding horizons and commodity-specific liquidity-adjusted risk factors of Tables A3 and A4, respectively.

Optimisation of maximum L-VaR limits for a commodity trading risk management unit

The optimisation of maximum risk limits (or risk budgeting) is an important element of any commodity trading risk management unit and it should be defined clearly and used wisely to ensure control of the trading/investment unit's exposure to risk. All limit-setting and control, monitoring and reporting should be performed

by the risk management unit, independent of the front office's commodity traders. In most professional trading and asset management units (such as commercial banks, foreign exchange dealers, commodity traders, institutional investors, and so on), VaR limit-setting is based on the concept of risk appetite. The risk appetite is defined as the maximum loss potential that management is willing to accept when an adverse move in commodity prices occurs within a specified time horizon. In general, risk appetite will be dependent upon the following factors:

- the performance track record in trading commodities;
- the strategic importance of commodity trading by the trading unit in question;
- the quality of the trading unit's infrastructure in handling the traded products;
- the overall exposure that the trading unit wants to have to proprietary trading and/or active investment risks in general and commodity risks in particular;
- the volatilities of the commodities (as determined by the risk manager) and the correlation factors among the different commodities;
- local and/or global regulatory constraints for the operations of commodity securities.

How should we set risk limits to safeguard against maximum loss amounts? These are some of the central questions risk managers must envisage. In this paper, a simplified but practical approach is presented for the setting of optimised maximum L-VaR limits. To this end, an optimisation technique with different L-VaR calculations has been examined in order to set up procedures for the establishment of

maximum L-VaR limits. These maximum limits serve as a strict policy for handling situations in which commodity trading unit are above the authorised L-VaR limits. The L-VaR limits methodology and procedure must be analysed and approved by the board of directors of the commodity trading entity. All trading/investment units need to have such limits of L-VaR as a guideline and also as a strict policy for their risk taking behaviour. Any excess of L-VaR beyond the ratified limits must be reported to top management by the risk management unit. Moreover, traders/asset managers need to give full and justified explanations of why their L-VaRs are beyond the approved limits.

In this study, the optimisation process is based on the definition of L-VaR as the maximum possible loss over a specified time horizon within a given confidence level. The optimisation technique solves the problem by finding the market positions that maximise the loss, subject to the fact that all constraints are satisfied within their boundary values. Further, in all cases the liquidation horizons as indicated in Table A3 are assumed constant throughout the optimisation process. For the sake of simplifying the optimisation routine and thereafter its analysis, a volume trading position limit of US\$10 million is assumed as a constraint – that is, the commodity trading entity must keep a maximum overall market value of different commodities of no more than US\$10 million (between long- and short-selling positions). Furthermore, for optimisation purposes, and in order to set up a more realistic risk management case, other constraints are imposed, detailed as follows:

- $\sum$ (Energy Commodities volume) = 15 million US\$ between long- and short-selling trading positions.

- Trading volume in any energy commodity should be between $[-7, +7]$ million US\$.
- Trading volume in any metal commodity should be between $[-2, +2]$ million US\$.
- Trading volume in any agriculture commodity should be between $[-1, +1]$ million US\$.
- Trading volume in any miscellaneous commodity should be between $[-0.5, +0.5]$ million US\$.

The complete results of the optimisation process are given in Table A8. Based on the above optimisation budget constraints and the results of Table A8, senior management of the commodity trading entity can set maximum monthly L-VaR limits for a large commodity portfolio as follows:

- Maximum approved L-VaR limit under normal market conditions, with the assumption of empirical correlations = 5 231 353 US\$.
- Maximum approved L-VaR limit under normal market conditions and with the assumption of positive 100 per cent correlations = 3 942 736 US\$.
- Maximum approved L-VaR limit under normal market conditions and with the assumption of 0 correlations = 2 897 432 US\$.
- Maximum approved L-VaR limit under normal market conditions and with the assumption of negative –100 per cent correlations = 1 115 821 US\$.
- Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of empirical correlations = 15 961 144 US\$.
- Maximum approved L-VaR limit under severe or crisis market conditions and with

- the assumption of positive 100 per cent correlations = 10 242 513 US\$.
- Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of 0 correlations = 9 016 181 US\$.
 - Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of negative -100 per cent correlations = 7 594 338 US\$.
 - Maximum approved trading volume limit for all commodities between long- and short-selling positions = 10 000 000 US\$.
 - Maximum approved total asset allocation limit for energy sector commodities between long- and short-selling positions = 15 000 000 US\$.
 - Trading volume in any energy commodity (VE) should be $-7\,000\,000 \leq VE \leq 7\,000\,000$ US\$, with the possibility of short-selling.
 - Trading volume in any metal commodity (VM) should be $-2\,000\,000 \leq VM \leq 2\,000\,000$ US\$ with the possibility of short-selling.
 - Trading volume in any agriculture commodity (VA) should be $-1\,000\,000 \leq VA \leq 1\,000\,000$ US\$, with the possibility of short-selling.
 - Trading volume in any miscellaneous commodity (VMS) should be $-500\,000 \leq VMS \leq 500\,000$ US\$, with the possibility of short-selling.

SUMMARY AND CONCLUDING REMARKS

Despite the recent attention and popularity of L-VaR, the risk measure should only be viewed as another measure in the risk manager's toolkit

and not as a substitute for prudent risk management practices. As suggested several times, the performance of L-VaR estimating techniques are likely to be sensitive to the data set used in developing estimates, the length of the forecast horizon, confidence level and departures from assumed distributional forms. Other less quantifiable issues important to L-VaR are the ease of use, flexibility, cost of implementation and degree to which management and investors find the risk measure useful.

There are many methods and ways to identify, to measure and to control trading risk, and trading risk managers have the task to ascertain the identity of the one that suits their needs. In fact, there is no right or wrong way to measure/ manage commodity trading risk; it all depends on each entity's objectives, lines of business, risk appetite and the availability of funds for investment in trading risk management projects. Regardless of the methodology chosen, the most important factors to consider are the establishment of sound risk practices, policies and standards and the consistency in the implementation process across all lines of businesses and risks. L-VaR is becoming a standard risk management tool that provides a quantification of the primary risk exposures that a commodity entity faces, and offers key information of the overall risk profile of the firm to senior management, traders and regulators. It is important to point out that L-VaR is just a tool to measure and manage risk, and not a goal in itself. It is the process that leads to the estimation of the L-VaR figures which really adds value to the risk management role, not just the L-VaR estimates. Once the exposures to several risk factors have been identified and quantified, it is possible to analyse how those risk

exposures interact with each other, which ones are acting as a natural hedge to the portfolio, and which ones represent the largest sources of risk for the firm. With L-VaR it is possible to minimise the variability of the entity's trading earnings, decide which risks are worth taking, and hedge those which may cause further variability to those earnings. One of the main advantages of L-VaR is that it comprises a user-friendly way to present concise reports to top management and comply with regulatory requirements.

This study is the first known attempt at empirically examining the performance of L-VaR measures in the framework of a diverse commodity portfolio. To date, all known empirical studies examining the performance of L-VaR measures have been conducted in the context of portfolios containing currency, interest rate or equity data with portfolios often developed randomly. The use of commodity prices would bring new data to the empirical evaluation of L-VaR. However, the performance of L-VaR techniques when applied to commodity prices might be different than those found with financial asset prices. Because of this, risk managers that build L-VaR models in which the portfolio of interest contains positions in energy commodities or other seasonal commodities should be more cautious of their L-VaR estimates during these known times of increased volatility. Moreover, a large commodity cluster provides a realistic alternative portfolio, as well as new data, for studying new measurement techniques of L-VaR estimation. The results of this study also provide an impetus for further research in the area of L-VaR. For instance, research is needed that focuses on the applicability and performance of L-VaR in the prospect of other commodity prices and

portfolios as well as the implementation of alternative parametric procedures when options positions, which have a nonlinear pay-off structure, are included in a portfolio. Therefore, as interest and use of L-VaR increases among risk managers, research should focus on models that are robust for a variety of prices and portfolios.

This paper has presented a framework for calculating L-VaR incorporating the liquidity of trading assets. In this work we proposed an enhanced L-VaR model which, unlike the standard version that assumes that all trading positions can be sold instantaneously with no friction at the end of the holding period, takes into account different liquidation horizons with which the securities of a given commodity portfolio could be liquidated. Our framework facilitates the relatively simple L-VaR under certain assumptions and recognises liquidity trading risk as a significant risk factor that should be integrated within the framework of VaR. Furthermore, the model is theoretically simple with moderate demands on additional computing power while capturing the essential aspects of liquidity. Liquidity trading risk is found to be an important component of the aggregate risks absorbed by commodity trading entities. Our results suggest that ignoring liquidity trading risk can produce substantial underestimates of overall trading risk. Finally liquidity trading risk should be included in any performance evaluation as part of returns adjusted for risk compensation. Likewise, it should be induced in the process of setting of maximum trading limits.

Our empirical testing results suggest that in almost all tests, there are clear asymmetric behaviours in the distribution of returns of the sample commodities. The appealing outcome of

this study suggests the inevitability of combining L-VaR calculations with other methods such as stress-testing and scenario analysis to grasp a thorough picture of other remaining risks (such as, fat tails in the probability distribution) that cannot be revealed with the plain assumption of normality. In conclusion, the implications of the findings of this study on a large commodity portfolio suggest that although there is a clear departure from normality in the distribution of price returns, this issue can be tackled without the need of complex mathematical and analytical procedures. In fact, it is possible to handle these issues for cash commodities with the simple use of variance – covariance method (in its matrix-algebra form); along with the incorporation of a credible stress-testing approach (under adverse market conditions); as well as by supplementing the risk analysis with a realistic liquidity risk factor that takes into account real-world trading circumstances. In this research, a reasonable model for the measurement of illiquidity of both short-selling and long trading positions is incorporated. In contrast to other commonly used liquidity models, the liquidity approach that is applied in this work is more appropriate for real-world trading practices as it considers selling small fractions of the long/short trading commodity on a specific liquidation horizon. This liquidity factor can be implemented for the entire portfolio or for each commodity in the trading portfolio.

However, albeit our liquidity model allows a better understanding of the role played by those liquidity trading risk components and a more precise quantification of their impact, several outstanding issues still remain opened. We hope that additional research will be conducted on trading liquidity risk and that a broad methodology of assessing commodity market

liquidity risk will be developed based on the framework presented in this paper. It is suggested that future research can centre on the following areas and/or lines of interest:

- Although our proposed liquidity risk factor model reflects the time-volatility aspects of liquidity trading risk, which represents just one aspect of trading liquidity risk, we did not examine the dynamic interaction between commodity trading activities and price/bid – ask spread fluctuations. A feasible next step would be to tackle the dynamics of market impact by utilising time-series analysis.
- Implementation of matrix-algebra approach along with real-world L-VaR and liquidity model techniques to construct realistic commodity portfolios that comply with asset managers' risk aversion while maximising expected return. This can be achieved by considering the present or expected correlations and individual L-VaR matrices of the portfolio's components, under the assumption that the expected portfolio's returns are accurate and provided that correlations along with individual L-VaR matrices will remain constant for the selected time horizon.
- The fact that several commodity prices have associated futures and options contracts allows implied volatility estimates to be employed in L-VaR valuations. The use of implied volatility estimates in developing VaR measures has been suggested by researchers and practitioners as being superior to other methods, as implied volatility is the markets' forecast of volatility. The performance of implied volatility in the context of L-VaR has not, however, been

rigorously examined. Furthermore, relevant risk horizons observed in commodities tend to be longer than the daily horizon commonly used by financial practitioners in calculating VaR. This issue calls into question the performance of different volatility forecasting procedures to calculate L-VaR at longer horizons.

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APPENDIX I

Tables of relevant empirical testing and commodity trading risk management reports

See Tables A1–A8

Table A1: Risk analysis data: Volatility under normal and crisis market conditions and sensitivity factors

<i>Commodity name</i>	<i>Monthly volatility (normal market) (%)</i>	<i>Monthly volatility (crisis market) (%)</i>	<i>Annual volatility (normal market) (%)</i>	<i>Annual volatility (crisis market) (%)</i>	<i>Sensitivity factors</i>
Petroleum: average crude price	8.1	24.6	28.1	85.2	1.72
Petroleum: Dubai	8.5	30.6	29.3	106.1	1.65
Petroleum: West Texas intermediate	7.7	20.6	26.6	71.5	1.63
Petroleum: UK Brent	8.8	25.2	30.4	87.2	1.88
Gasoline	10.4	25.4	36.0	88.0	1.97
Natural gas	5.8	20.6	20.0	71.2	0.14
Coal	4.0	13.5	13.9	46.9	0.26
Gold	3.3	12.5	11.3	43.2	0.18
Silver	5.4	21.6	18.7	75.0	0.18
Copper	6.2	20.0	21.5	69.2	0.48
Zinc	6.1	24.9	21.3	86.4	0.34
Lead	6.3	23.8	21.9	82.3	0.15
Aluminium	5.8	32.6	20.0	113.1	0.31
Nickel	8.9	22.2	30.7	76.9	0.54
Iron ore	4.4	12.9	15.2	44.7	0.18
Phosphate rock	2.3	21.7	8.1	75.2	0.01
Wheat	5.1	15.1	17.7	52.3	0.08
Cotton	4.9	12.6	17.0	43.5	0.14
Sugar	2.1	11.0	7.3	38.2	−0.05
Maize	5.3	25.2	18.4	87.2	−0.08
Tobacco	1.8	4.9	6.2	16.8	0.01
Coffee	8.0	37.1	27.6	128.6	0.04
Tea	7.7	23.6	26.8	81.8	0.11
Rubber	6.0	18.1	20.8	62.7	0.37
Wool	4.7	16.5	16.4	57.3	−0.02
All Commodities Index	3.6	12.3	12.5	42.5	1.00

Table A2: Risk analysis data: Descriptive statistics, skewness, kurtosis and Jarque – Bera (JB) test

<i>Commodity name</i>	<i>Maximum (%)</i>	<i>Minimum (%)</i>	<i>Median (%)</i>	<i>Arithmetic mean (%)</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>JB- Statistics</i>
Petroleum: average crude price	45.7	−24.6	0.8	0.5	0.50	3.73	15**
Petroleum: Dubai	52.1	−30.6	0.9	0.5	0.55	6.08	107**
Petroleum: West Texas intermediate	39.1	−20.6	0.8	0.5	0.36	2.17	12**
Petroleum: UK Brent	46.6	−25.2	0.4	0.5	0.43	2.82	8*
Gasoline	36.4	−25.4	1.1	0.5	0.09	0.40	68**
Natural gas	40.5	−20.6	0.0	0.6	1.77	12.26	978**
Coal	15.9	−13.5	0.0	0.4	0.29	2.69	4
Gold	16.2	−12.5	−0.1	0.2	0.59	3.16	14**
Silver	19.7	−21.6	−0.2	0.2	0.04	2.30	5
Copper	24.8	−20.0	0.7	0.6	0.39	2.02	16**
Zinc	23.4	−24.9	0.4	0.6	−0.11	1.84	14**
Lead	30.7	−23.8	0.0	0.5	0.20	3.11	2
Aluminium	18.0	−32.6	0.1	0.3	−0.67	4.85	52**
Nickel	58.1	−22.2	−0.1	1.0	1.34	6.85	219**
Iron ore	53.9	−12.9	0.0	0.5	8.29	96.83	90 413**
Phosphate rock	15.0	−21.7	0.0	0.2	−1.02	46.43	18 828**
Wheat	15.9	−15.1	0.0	0.2	0.18	0.80	50**
Cotton	16.3	−12.6	−0.2	−0.1	0.31	0.86	50**
Sugar	9.8	−11.0	0.0	0.0	−0.11	5.93	86**
Maize	28.7	−25.2	0.4	0.3	0.01	5.40	57**
Tobacco	5.5	−4.9	0.1	0.0	0.00	0.24	76**
Coffee	29.1	−37.1	−0.5	0.0	−0.17	4.43	21**
Tea	21.5	−23.6	0.1	0.1	−0.14	0.52	62**
Rubber	19.6	−18.1	0.2	0.4	0.31	1.33	31**
Wool	19.9	−16.5	−0.2	0.2	0.49	2.63	11**
All Commodities Index	10.0	−12.3	0.0	0.4	−0.29	1.31	32**

Note: Asterisks, * and **, denote statistical significance at the 0.05 and 0.01 levels, respectively.

Table A3: Risk analysis data: Liquidity-holding horizon and liquidity-adjusted VaR factors (normal market condition)

<i>Commodity name</i>	<i>Liquidity-holding horizon (in months)</i>	<i>Monthly volatility (%)</i>	<i>Liquidity-adjusted monthly volatility (%)</i>	<i>Commodity specific liquidity-adjusted risk factors (%)</i>
Petroleum: average crude price	1.2	8.1	8.3	16.5
Petroleum: Dubai	1.2	8.5	8.6	17.2
Petroleum: West Texas intermediate	1.2	7.7	7.8	15.7
Petroleum: UK Brent	1.2	8.8	8.9	17.9
Gasoline	1.3	10.4	10.7	21.4
Natural gas	1.4	5.8	6.0	12.0
Coal	1.5	4.0	4.2	8.4
Gold	1.2	3.3	3.3	6.6
Silver	1.5	5.4	5.7	11.4
Copper	1.7	6.2	6.7	13.4
Zinc	2.0	6.1	6.9	13.7
Lead	2.5	6.3	7.5	15.0
Aluminium	1.5	5.8	6.1	12.2
Nickel	2.5	8.9	10.5	21.0
Iron ore	1.8	4.4	4.8	9.6
Phosphate rock	3.0	2.3	2.9	5.8
Wheat	1.5	5.1	5.4	10.8
Cotton	1.8	4.9	5.4	10.7
Sugar	1.7	2.1	2.3	4.5
Maize	2.0	5.3	5.9	11.9
Tobacco	3.0	1.8	2.2	4.4
Coffee	1.6	8.0	8.5	17.0
Tea	1.7	7.7	8.3	16.7
Rubber	3.0	6.0	7.5	14.9
Wool	2.0	4.7	5.3	10.6

Table A4: Risk analysis data: Liquidity-holding horizon and liquidity-adjusted VaR factors (severe market condition)

<i>Commodity name</i>	<i>Liquidity-holding horizon (in months)</i>	<i>Monthly volatility (%)</i>	<i>Liquidity-adjusted monthly volatility (%)</i>	<i>Commodity specific liquidity-adjusted risk factors (%)</i>
Petroleum: average crude price	1.2	24.6	25.1	50.1
Petroleum: Dubai	1.2	30.6	31.2	62.4
Petroleum: West Texas intermediate	1.2	20.6	21.0	42.1
Petroleum: UK Brent	1.2	25.2	25.7	51.3
Gasoline	1.3	25.4	26.2	52.3
Natural gas	1.4	20.6	21.4	42.9
Coal	1.5	13.5	14.3	28.6
Gold	1.2	12.5	12.7	25.4
Silver	1.5	21.6	22.8	45.6
Copper	1.7	20.0	21.5	43.1
Zinc	2.0	24.9	27.9	55.8
Lead	2.5	23.8	28.1	56.2
Aluminium	1.5	32.6	34.4	68.8
Nickel	2.5	22.2	26.3	52.5
Iron ore	1.8	12.9	14.1	28.2
Phosphate rock	3.0	21.7	27.1	54.1
Wheat	1.5	15.1	15.9	31.8
Cotton	1.8	12.6	13.7	27.4
Sugar	1.7	11.0	11.9	23.8
Maize	2.0	25.2	28.1	56.3
Tobacco	3.0	4.9	6.1	12.1
Coffee	1.6	37.1	39.6	79.2
Tea	1.7	23.6	25.5	51.0
Rubber	3.0	18.1	22.6	45.1
Wool	2.0	16.5	18.5	37.0

Table A5: Risk analysis data: Correlation matrix for a diverse portfolio of commodities

Commodity name	Petroleum: average crude price (%)	Petroleum: Dubai (%)	Petroleum: WTI (%)	Petroleum: UK Brent (%)	Gasoline (%)	Natural gas (%)	Coal (%)	Gold (%)	Silver (%)	Copper (%)	Zinc (%)	Lead (%)	Aluminium (%)	Nickel (%)	Iron ore (%)	Phosphate rock (%)	Wheat (%)	Cotton (%)	Sugar (%)	Maize (%)	Tobacco (%)	Coffee (%)	Tea (%)	Rubber (%)	Wool (%)	All Commodities Index (%)
Petroleum: average crude price	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Petroleum: Dubai	97	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Petroleum: WTI	98	92	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Petroleum: UK Brent	99	94	95	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Gasoline	73	67	75	72	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Natural gas	-5	-8	-2	-5	-2	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Coal	11	7	14	10	17	1	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Gold	21	19	20	21	6	3	16	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Silver	7	5	8	8	6	-2	11	57	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Copper	18	17	17	18	11	5	22	27	16	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Zinc	4	4	3	5	7	3	12	14	21	45	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Lead	3	3	4	2	-4	-11	14	22	20	36	44	100	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Aluminium	22	21	22	22	13	3	13	20	24	41	37	29	100	—	—	—	—	—	—	—	—	—	—	—	—	—
Nickel	15	14	14	17	11	-2	21	24	21	38	40	34	44	100	—	—	—	—	—	—	—	—	—	—	—	—
Iron ore	10	8	11	10	16	23	11	0	-4	1	7	2	-2	0	100	—	—	—	—	—	—	—	—	—	—	—
Phosphate rock	3	3	3	4	9	-13	1	-7	-1	-13	6	-4	0	1	22	100	—	—	—	—	—	—	—	—	—	—
Wheat	-7	-7	-6	-6	2	-3	9	8	8	9	7	1	5	1	-2	-5	100	—	—	—	—	—	—	—	—	—
Cotton	0	-1	2	0	7	7	2	-5	-1	8	8	11	8	9	6	-6	-4	100	—	—	—	—	—	—	—	—
Sugar	-6	-4	-6	-8	-2	6	10	-5	0	0	-5	8	-9	-7	8	-3	12	17	100	—	—	—	—	—	—	—
Maize	-18	-18	-16	-20	-6	-4	18	8	15	-1	12	5	7	5	0	6	48	15	9	100	—	—	—	—	—	—
Tobacco	7	10	6	6	0	-18	-3	-7	-1	6	8	9	-14	-5	3	12	0	-8	-3	2	100	—	—	—	—	—
Coffee	2	-1	3	2	7	6	8	-2	7	8	0	3	-3	7	0	-1	7	-2	-3	3	0	100	—	—	—	—
Tea	6	3	9	5	1	5	8	2	-8	3	4	1	-1	-5	13	3	8	11	-9	-10	-12	6	100	—	—	—
Rubber	20	21	19	19	16	-3	19	24	10	13	-1	-7	20	18	0	0	11	19	-4	10	-13	6	7	100	—	—
Wool	-3	-1	-3	-4	7	7	13	9	10	5	7	3	15	10	5	1	-4	4	-1	5	-7	7	1	7	100	—
All Commodities Index	77	71	77	78	69	9	24	20	12	28	20	9	20	22	15	1	6	10	-8	-6	2	2	5	23	-2	100

Table A6: Commodity trading risk management and control report: Commodity asset allocation and liquidity-adjusted Value-at-Risk (L-VaR) report

<i>Commodity name</i>	<i>Market value (US\$)</i>	<i>Asset allocation (%)</i>	<i>Liquidity- holding horizon</i>	<i>Monthly volatility (normal) (%)</i>	<i>Monthly volatility (severe) (%)</i>	<i>Sensitivity factors</i>
Petroleum: average crude price	3 000 000	30.0	1.0	8.11	24.60	1.72
Petroleum: Dubai	6 000 000	60.0	1.0	8.46	30.62	1.65
Petroleum: West Texas intermediate	2 000 000	20.0	1.0	7.68	20.64	1.63
Petroleum: UK Brent	3 000 000	30.0	1.0	8.78	25.18	1.88
Gasoline	2 000 000	−20.0	1.0	10.40	25.40	1.97
Natural gas	1 000 000	−10.0	1.0	5.76	20.57	0.14
Coal	1 000 000	−10.0	1.0	4.00	13.54	0.26
Gold	3 000 000	−30.0	1.0	3.25	12.48	0.18
Silver	1 000 000	−10.0	1.0	5.40	21.65	0.18
Copper	1 000 000	10.0	1.0	6.21	19.96	0.48
Zinc	1 000 000	10.0	1.0	6.14	24.94	0.34
Lead	1 000 000	10.0	1.0	6.33	23.76	0.15
Aluminium	1 000 000	10.0	1.0	5.78	32.64	0.31
Nickel	1 000 000	10.0	1.0	8.86	22.19	0.54
Iron ore	2 000 000	20.0	1.0	4.37	12.91	0.18
Phosphate rock	1 000 000	10.0	1.0	2.33	21.71	0.01
Wheat	1 000 000	−10.0	1.0	5.12	15.11	0.08
Cotton	1 000 000	−10.0	1.0	4.91	12.56	0.14
Sugar	1 000 000	−10.0	1.0	2.10	11.03	−0.05
Maize	1 000 000	−10.0	1.0	5.32	25.18	−0.08
Tobacco	1 000 000	−10.0	1.0	1.78	4.86	0.01
Coffee	2 000 000	−20.0	1.0	7.96	37.11	0.04
Tea	1 000 000	10.0	1.0	7.73	23.62	0.11
Rubber	1 000 000	10.0	1.0	5.99	18.09	0.37
Wool	1 000 000	10.0	1.0	4.72	16.53	−0.02
All Commodities Index	—	0.0	1.0	3.62	12.26	1.00
Total portfolio value (US\$)	10 000 000	100	—	—	—	—

Table A6: Continued

<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (normal market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
2 228 671	1 975 109	1 479 635	691 075
22.29%	19.75%	14.80%	6.91%
<i>Diversification effects</i>			
US\$253 562	-11.38%		
<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (severe (crisis) market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
7 555 895	6 333 949	5 051 325	3 303 462
75.56%	63.34%	50.51%	33.03%
<i>Diversification effects</i>			
US\$1 060 814	-13.58%		
<i>Overall sensitivity factor: Portfolio of commodities</i>			
2.139			

Table A7: Commodity trading risk management and control report: Commodity asset allocation and liquidity-adjusted Value-at-Risk (L-VaR) report

<i>Commodity name</i>	<i>Market value (US\$)</i>	<i>Asset allocation (%)</i>	<i>Liquidity- holding horizon</i>	<i>Monthly volatility (normal) (%)</i>	<i>Monthly volatility (severe) (%)</i>	<i>Sensitivity factors</i>
Petroleum: average crude price	3 000 000	30.0	1.2	8.11	24.60	1.72
Petroleum: Dubai	6 000 000	60.0	1.2	8.46	30.62	1.65
Petroleum: West Texas intermediate	2 000 000	20.0	1.2	7.68	20.64	1.63
Petroleum: UK Brent	3 000 000	30.0	1.2	8.78	25.18	1.88
Gasoline	2 000 000	−20.0	1.3	10.40	25.40	1.97
Natural gas	1 000 000	−10.0	1.4	5.76	20.57	0.14
Coal	1 000 000	−10.0	1.5	4.00	13.54	0.26
Gold	3 000 000	−30.0	1.2	3.25	12.48	0.18
Silver	1 000 000	−10.0	1.5	5.40	21.65	0.18
Copper	1 000 000	10.0	1.7	6.21	19.96	0.48
Zinc	1 000 000	10.0	2.0	6.14	24.94	0.34
Lead	1 000 000	10.0	2.5	6.33	23.76	0.15
Aluminium	1 000 000	10.0	1.5	5.78	32.64	0.31
Nickel	1 000 000	10.0	2.5	8.86	22.19	0.54
Iron ore	2 000 000	20.0	1.8	4.37	12.91	0.18
Phosphate rock	1 000 000	10.0	3.0	2.33	21.71	0.01
Wheat	1 000 000	−10.0	1.5	5.12	15.11	0.08
Cotton	1 000 000	−10.0	1.8	4.91	12.56	0.14
Sugar	1 000 000	−10.0	1.7	2.10	11.03	−0.05
Maize	1 000 000	−10.0	2.0	5.32	25.18	−0.08
Tobacco	1 000 000	−10.0	3.0	1.78	4.86	0.01
Coffee	2 000 000	−20.0	1.6	7.96	37.11	0.04
Tea	1 000 000	10.0	1.7	7.73	23.62	0.11
Rubber	1 000 000	10.0	3.0	5.99	18.09	0.37
Wool	1 000 000	10.0	2.0	4.72	16.53	−0.02
All Commodities Index	—	0.0	1.0	3.62	12.26	1.00
Total portfolio value (US\$)	10 000 000	100	—	—	—	—

Table A7: Continued

<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (normal market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
2 300 071	2 095 117	1 528 778	533 672
23.00%	20.95%	15.29%	5.34%
<i>Diversification effects</i>			
US\$204 954	-8.91%		
<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (severe (crisis) market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
7 814 212	6 753 398	5 231 602	3 021 742
78.14%	67.53%	52.32%	30.22%
<i>Diversification effects</i>			
US\$1 060 814	-13.58%		
<i>Overall sensitivity factor: Portfolio of commodities</i>			
2.139			

Table A8: Commodity trading risk management and control report (L-VaR limit-setting) (optimisation technique outcomes): Commodity asset allocation and liquidity-adjusted Value-at-Risk (L-VaR) report

<i>Commodity name</i>	<i>Market value (US\$)</i>	<i>Asset allocation (%)</i>	<i>Liquidity-holding horizon</i>	<i>Monthly volatility (normal) (%)</i>	<i>Monthly volatility (severe) (%)</i>	<i>Sensitivity factors</i>
Petroleum: average crude price	7 000 000	70.0	1.2	8.11	24.60	1.72
Petroleum: Dubai	7 000 000	70.0	1.2	8.46	30.62	1.65
Petroleum: West Texas intermediate	1 000 000	10.0	1.2	7.68	20.64	1.63
Petroleum: UK Brent	7 000 000	70.0	1.2	8.78	25.18	1.88
Gasoline	7 000 000	70.0	1.3	10.40	25.40	1.97
Natural gas	7 000 000	−70.0	1.4	5.76	20.57	0.14
Coal	7 000 000	−70.0	1.5	4.00	13.54	0.26
Gold	2 000 000	−20.0	1.2	3.25	12.48	0.18
Silver	2 000 000	−20.0	1.5	5.40	21.65	0.18
Copper	2 000 000	20.0	1.7	6.21	19.96	0.48
Zinc	2 000 000	20.0	2.0	6.14	24.94	0.34
Lead	2 000 000	20.0	2.5	6.33	23.76	0.15
Aluminium	2 000 000	20.0	1.5	5.78	32.64	0.31
Nickel	2 000 000	20.0	2.5	8.86	22.19	0.54
Iron ore	2 000 000	−20.0	1.8	4.37	12.91	0.18
Phosphate rock	2 000 000	−20.0	3.0	2.33	21.71	0.01
Wheat	1 000 000	−10.0	1.5	5.12	15.11	0.08
Cotton	1 000 000	−10.0	1.8	4.91	12.56	0.14
Sugar	1 000 000	−10.0	1.7	2.10	11.03	−0.05
Maize	1 000 000	−10.0	2.0	5.32	25.18	−0.08
Tobacco	1 000 000	−10.0	3.0	1.78	4.86	0.01
Coffee	1 000 000	−10.0	1.6	7.96	37.11	0.04
Tea	1 000 000	−10.0	1.7	7.73	23.62	0.11
Rubber	500 000	5.0	3.0	5.99	18.09	0.37
Wool	500 000	−5.0	2.0	4.72	16.53	−0.02
All Commodities Index	—	0.0	1.0	3.62	12.26	1.00
Total portfolio value (US\$)	10 000 000	100	—	—	—	—

Table A8: Continued

<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (normal market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
5 231 353	3 942 736	2 897 432	1 115 821
52.31%	39.43%	28.97%	11.16%
<i>Diversification effects</i>			
US\$1 288 617	-24.63%		
<i>Monthly liquidity-adjusted Value-at-Risk (L-VaR) in US\$ (severe (crisis) market conditions)</i>			
<i>Correlation=Empirical</i>	<i>Correlation=1</i>	<i>Correlation=0</i>	<i>Correlation=-1</i>
15 961 144	10 242 513	9 016 181	7 594 338
159.61%	102.43%	90.16%	75.94%
<i>Diversification effects</i>			
US\$5 718 632	-35.83%		
<i>Overall sensitivity factor: Portfolio of commodities</i>			
5.191			