Attitudes Towards Income Risk in the Presence of Quantity Constraints*

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Considering a consumer with standard preferences, I trace out how quantity constraints on markets impact on relative risk aversion and prudence. I first show how this impact decomposes into a local curvature effect and an endogenously changing risk aversion/prudence effect. Next, I calibrate both effects on relative risk aversion and prudence, using estimates on household demand for durables and labour supply. The calibrations show that commitments to durable goods have large effects on attitudes towards risk. And while small wedges between realised and desired levels of labour supply have only moderate effects, becoming full time unemployed on a 60 per cent unemployment benefit significantly raises risk aversion and prudence.

The Geneva Risk and Insurance Review (2013) **38**, 183–209. doi:10.1057/grir.2013.3

Keywords: household demand; income risk aversion; prudence; quantity constraints; labour supply; durables

Article submitted 19 February 2013; accepted 23 September 2013; published online November 2013

Introduction

How are a consumer's attitudes towards income risk affected when her trading opportunities get restricted because of quantity constraints, such as having to work full-time while wishing a part-time job (or vice versa), or being stuck with a small car, when in need for a large one? I consider two types of attitude towards risk: (i) risk aversion and (ii) prudence or downside risk aversion. The Arrow-Pratt coefficient of risk aversion measures a consumer's willingness to pay for disposing of

* This paper is a substantial revision of an earlier paper titled "Income risk aversion with quantity constraints". I am especially grateful to Jacques H Drèze for very stimulating discussions and useful suggestions, and to the Editor (Achim Wambach) and two anonymous referees for their detailed comments. Also thanks to Louis Eeckhoudt for fruitful discussions. The comments by Kåre Petter Hagen, Julien Hardelin, Karl Rolf Pedersen, Agnar Sandmo, and Dirk Schindler on an earlier version are gratefully acknowledged. Part of the paper was written during a very hospitable research stay at CORE (Louvain-la-Neuve, Belgium).

a zero mean risk. Likewise, Kimball's coefficient of prudence measures the certain reduction in income required to bring the *marginal* utility of consumption in line with the expected marginal utility of consumption when a zero mean risk is added.¹

Intuitively, one would expect that quantity constraints make a consumer both more risk averse and more prudent, since they reduce the opportunity set and thus allow for smaller adjustments of the consumption bundle after the income risk has realised. For example, this is true when the consumer's utility function over consumption (c) and leisure (l) is $u(c, l) = v(c) + l$, with v' , $v'' > 0$ and $v''' < 0$. Because *preferences* are quasi-linear in leisure, all exogenous income risk is absorbed by leisure. Since also the *utility function* is linear in leisure, the consumer is risk neutral with respect to this income risk and exhibits zero prudence. But if she faces a binding quantity constraint on her labour supply, the exogenous income risk is absorbed by the consumption of other goods, whose marginal utility is strictly falling and convex. Hence, the constraint turns the consumer into a strictly risk-averse and prudent person with respect to income risk.

This example points to the central role of the income elasticity of the constrained good in determining whether risk attitudes are affected by constraints. At the same time, it shuts down a number of channels that are important to determine the size and sign of the effects. In this paper, I characterise these effects making use of the virtual price and income approach introduced by Neary and Roberts.² They show how constrained consumer behaviour under certainty may be analysed using standard tools by redefining the price vector and income as those that support the optimal bundle under quantity constraints. I show that under a quantity constraint, an income risk translates into a virtual price and a virtual income risk, both of which are positively correlated (under normality). I also show that the positive net evaluation of the price risk (due to substitution possibilities) is more than offset by the negative net evaluation of the correlation between both risks (due to the income effect). The result is a mark-up on the risk premium compared with an unconstrained situation. At least for weakly binding constraints, this mark-up is positive. A similar mark-up applies for the prudence premium, though this time its sign is *a priori* undetermined.

¹ Since the solution to the consumer's decision problem under uncertainty will satisfy a condition on the expected marginal utility of consumption, the optimal response to changes in the zero mean risk will depend on the sign and size of the prudence coefficient. Hence, this coefficient measures "the propensity to prepare or forearm oneself in the face of uncertainty" (Kimball, 1990, p. 54). The notion of *prudence* was originally defined in a temporal context (savings decision). It is predicated by the concept of *downside risk aversion*, defined by Menezes *et al.* (1980), which is often used in the atemporal context. A downside risk-averse decision maker dislikes a mean-preserving spread that is followed by a mean-preserving contraction on the right of the spread that restores the variance to its original value. Menezes *et al.* show that for an EU-decision maker, this is equivalent to $u''' > 0$. For brevity, I use *prudence* throughout in the paper.

² Neary and Roberts (1980).

The size of these mark-ups depends on the behavioural elasticities and the extent to which the quantity constraint is binding. To assess their empirical importance, I calibrate them for two settings: commitments to durable goods (for the U.S.) and hour constraints on the labour market (for the U.S., the U.K. and Sweden). For durable goods, I find that small frictions in the adjustment to their desired level have positive effects on relative risk aversion and negative effects on relative prudence. I also find that these effects are very non-linear, both in the level of the constraint and in income. The effects of restrictions on labour supply are more modest, although “large” restrictions such as being full-time unemployed do boost both risk attitudes.

I believe there are several reasons that call for a better understanding of the effects of constraints on attitudes towards risk. First, the level of relative risk aversion and prudence governs the response of an expected utility (EU) decision maker when adjusting the fraction of initial wealth to be invested in a risky asset, or the level of savings when the rate of return gets more risky.³ In times where underemployment equilibria prevail, the investment responses of consumer/workers to increased uncertainty on financial markets cannot be expected to be identical when labour markets clear at full employment levels. Moreover, if the anticipation of future quantity constraints impacts on the degrees of risk aversion and prudence, this will trigger changes in the optimal amount of savings of which the macroeconomic effects may confirm the expectation. A better understanding of these impacts is therefore relevant for studying the nature of underemployment equilibria.

Second, in recent years several studies have measured the empirical variation in attitudes towards risk and explained this variation in terms of socioeconomic characteristics of the decision maker. See, for example, Barsky *et al.*,⁴ Dohmen *et al.*,⁵ Guiso and Paiella,⁶ and Aarbu and Schroyen⁷ for risk aversion, and Deck and Schlesinger⁸ for prudence. A salient feature of these studies is that observable characteristics leave a large part of the variation in risk attitudes unexplained. I will argue that one source of unobserved heterogeneity may stem from frictions that people experience in adjusting themselves to desired levels of quantities of goods and services. For example, in the calibration part of the paper, I show that small deviations from the optimal level of durable goods may produce large variation in relative risk aversion around its sample mean. Another example are labour market frictions. Dohmen *et al.*⁹ analyse the responses of 22,000 respondents in the 2004 wave of the German Socioeconomic Panel to the hypothetical question how much of

³ See, for example, Choi *et al.* (2001), Paulsson *et al.* (2005), Bauer and Buchholz (2008).

⁴ Barsky *et al.* (1997).

⁵ Dohmen *et al.* (2005, 2011).

⁶ Guiso and Paiella (2008).

⁷ Aarbu and Schroyen (2013).

⁸ Deck and Schlesinger (2010).

⁹ Dohmen *et al.* (2005).

a €100,000 windfall gain one chooses to invest in a hypothetical asset which in two years' time either doubles the invested amount or returns only half of it, both with 50 per cent probability. The answer (on average about €15,000) is regressed on gender, age, height, parental education and a large set of controls, including household income, household wealth and employment status. The unemployment dummy has a coefficient of €-4,938.2 (st error €2,222.5, $p < 5$ per cent—cf. their Table A.1 (column 9)). Since income is controlled for, this result indicates that being unemployed in itself significantly decreases the willingness to take financial risk and begs the question through which mechanism such restriction on labour supply affects risk attitudes. In the calibration part, I show that despite the low income elasticity of labour supply, the prospect of full-time unemployment may more than double both relative risk aversion and relative prudence—even under constant relative risk aversion (CRRA).

The paper is related to a recent literature that investigates how commitments affect risk taking behaviour or the willingness to take risk and the normative implications this may have for contract design. For example, Chetty and Szeidl¹⁰ show that when an EU decision maker finds it optimal to follow a sluggish adjustment policy w.r.t. housing or some other lumpy good, his indirect utility function becomes more concave in some income regions, but convex in others (compared with full flexibility). They explore how this may help to reconcile a number of empirical puzzles, such as the simultaneous purchasing of insurance and lottery tickets, or the presence of substantial aversion towards moderate gambles without implying unrealistically high aversion towards large gambles.¹¹

Finally, the paper is related to the literature on timeless vs temporal risky prospects. This literature asks whether an agent who can choose a lottery and take some action after observing the outcome of the lottery (a timeless prospect) has a larger willingness to bear risks than an agent who has to commit to an action before observing the lottery outcome (a temporal prospect). Machina¹² has shown through an example that this is not always the case. Gollier¹³ has recently generalised this result and identified a set of sufficient conditions for the flexible context to lead to a higher risk tolerance. He then examines how rigidities may induce a household to more risk-prone behaviour in portfolio allocation and/or savings decisions. In the

¹⁰ Chetty and Szeidl (2007).

¹¹ Drawing on a similar observation, Postlewaite *et al.* (2008) show that efficient employment contracts should display wage rigidity and allow for layoffs when consumer/workers make consumption commitments. The strength of the argument, and the optimal degree of wage rigidity depends—among other things—on the effect of consumption commitments on risk aversion. It is an empirical question how strong this effect is and the calibration that I perform in Section “Durable goods” shows that it is large.

¹² Machina (1982).

¹³ Gollier (2009).

present paper, the focus is on timeless risks. I examine the effect of one particular set of constraints—quantity constraints on purchased levels of goods and services—on the willingness to accept small income risks, and decompose it in terms of consumer preferences. To the best of my knowledge, this paper is also the first to examine the consequences of constraints for the decision maker's rate of prudence and thus her willingness to change precautionary behaviour when background risk increases.

The next section briefly reminds of the consumer's decision problem and its properties, and formulates the coefficients of risk aversion and prudence with respect to income risk in terms of the direct utility function. In the subsequent section, I introduce quantity constraints and derive their effect on relative risk aversion using the virtual price approach. In the latter section, I derive the effects for relative prudence. Using estimation results for the demand for durables (U.S.) and the supply of labour (U.K., U.S. and Sweden), I calibrate in the penultimate section the effects of constraints on risk attitudes. The final section concludes.

Income risk aversion and prudence without quantity constraints

A consumer cares about n commodities whose quantities are given by the bundle q , belonging to the consumption set $X \subset \mathbb{R}_+^n$. She has well-defined preferences over simple lotteries on X , which can be represented by a vNM utility function $u(\cdot)$, which is smooth, monotone and strongly concave (implying that she prefers the certain bundle $\alpha\bar{q} + (1-\alpha)\underline{q}$ to the uncertain consumption prospect $(\bar{q}, \underline{q}; \alpha, 1-\alpha)$). Let the price vector be certain and given by $p \in \mathbb{R}_+^n$. The consumer's income is random and given by $\tilde{m} = m + \tilde{x}$, with $E\tilde{x} = 0$. Thus it has expectation m and variance $\sigma_m^2 = E\tilde{x}^2$.

The consumer is informed about the income draw *before* she makes her consumption decision. In other words, she faces a *timeless* income prospect: the decision on q is made after the uncertainty is resolved. With an income draw $\tilde{m}$, her problem is then to solve

$$\max_q u(q) \quad \text{s.t. } p'q = \tilde{m}.$$

Let the unique solution be given by the bundle $q(p, \tilde{m})$, yielding a utility level $v(p, \tilde{m}) \stackrel{\text{def}}{=} u(q(p, \tilde{m}))$. The function $v(p, \cdot)$ is the vNM utility function representing the consumer's preferences for timeless income lotteries.¹⁴ Then this solution satisfies the first-order conditions¹⁵

$$u_q(q(p, \tilde{m})) = v_m(p, \tilde{m})p. \quad (1)$$

The local properties of $q(p, \tilde{m})$ are well known but repeated here for future reference. Defining K as the matrix of Slutsky substitution effects and q_m as the

¹⁴ See, for example, Kihlstrom and Mirman (1974, pp. 372–375), Drèze and Rustichini (2004, pp. 874–875) or Kreps (2013, pp. 134–138).

vector of income effects, we have:

$$\begin{aligned} & \text{(i) } p'q_m = 1, \text{ (ii) } \frac{\partial q}{\partial p'} = K - q_m q', \text{ (iii) } K = K', \\ & \text{(iv) } Kp = 0, \text{ and (v) } y'Ky < 0 \\ & \text{for } y \neq \alpha p \text{ (}\alpha \text{ real scalar).} \end{aligned} \quad (2)$$

Expression (2(ii)) is the Slutsky decomposition. A similar decomposition of the price effect on the marginal utility of income, v_m , is

$$\frac{\partial v_m}{\partial p} = -v_{mm}q - v_m q_m. \quad (3)$$

The first *right-hand side* term is a real income effect that can be neutralised by an appropriate change in income. The second *right-hand side* term is a substitution effect: the change in the marginal utility of income when the consumer is compensated so as to remain at the same utility level.

Differentiating both sides of (1) with respect to $\tilde{m}$, and making use of the adding-up property (2(i)) gives $v_{mm} = q'_m u_{qq} q_m$, which is negative by the concavity of $u(\cdot)$. Since $v_m = q'_m u_q$, the Arrow-Pratt coefficient of relative risk aversion is given by

$$RR(p, m) \stackrel{\text{def}}{=} -\frac{v_{mm}}{v_m} m = -\frac{q'_m u_{qq} q_m}{q'_m u_q} m. \quad (4)$$

The *rhs* expression may be added to Hanoch's list of alternative representations of relative risk aversion (Hanoch,¹⁶ Theorem 1). It is twice the relative risk premium per unit of variance for infinitesimal proportional timeless income risk.

When the consumer faces an uninsurable income risk but can take actions to mitigate it, Kimball's¹⁷ coefficient of relative prudence, RP , measures the sensitivity of these actions to the risk.¹⁸ For a consumer who cares about many goods, one expects this coefficient, defined as $RP(p, m) \stackrel{\text{def}}{=} -(v_{mmm}/v_{mm})m$, to depend on the set of third (cross) derivatives of the utility function $u(\cdot)$. This is indeed the case. In the

¹⁵ Subscripts with u , v and q denote derivatives. They refer to the arguments of the function, and not necessarily to the values of these arguments. Hence, I omit the tilde for notational convenience.

¹⁶ Hanoch (1977).

¹⁷ Kimball's (1990).

¹⁸ Eeckhoudt and Schlesinger (2006) have shown more generally that a decision maker is prudent if and only if she prefers the additive income lottery $(m-k, m+\tilde{x}; (1/2), (1/2))$ to the lottery $(m, m-k+\tilde{x}; (1/2), (1/2))$, for any loss k and any zero mean risk $\tilde{x}$. Thus she prefers to disaggregate the two "pains" rather than to face them both in the same state of the world. Eeckhoudt *et al.* (2009) have shown that RP exceeds 2 if and only if the decision maker prefers the multiplicative income lottery $(m(1-k), m(1+\tilde{y}); (1/2), (1/2))$ to the lottery $(m, m(1-k)(1+\tilde{y}); (1/2), (1/2))$, for any loss factor $k(<1)$ and any zero mean noise $\tilde{y}(>-1)$.

appendix, I show that

$$RP(p, m) = - \frac{\frac{\partial q'_m u_{qq} q_m}{\partial q'_m} \big|_{q_m} q_m}{q'_m u_{qq} q_m} m, \quad (5)$$

where $(\partial q'_m u_{qq} q_m / \partial q'_m) \big|_{q_m}$ is the effect on the quadratic form $q'_m u_{qq} q_m$ because of a perturbation in the Hessian u_{qq} following dq .¹⁹ Expression (5) thus reveals that the coefficient of relative prudence for income risk can be expressed as the ratio of a cubic form in the (three dimensional) array of third derivatives of $u(\cdot)$ to a quadratic form in the Hessian of $u(\cdot)$.

Proposition 1: When a consumer has a utility function $u(\cdot)$ defined over n commodities, the coefficients of relative risk aversion and prudence for timeless income risks are given by (4) and (5), respectively.

For future reference, it is useful to establish how a compensated increase in the price of good i affects these attitudes towards risk. In the appendix, I prove the following result:

Theorem 1: The m -compensated effects of p_i on RR and RP are

$$\frac{\partial RR(p, m)}{\partial \log p_i} \big|_{dv=0} = \frac{p_i q_i}{m} \left(RR(p, m) \left(1 - \frac{\partial q_i m}{\partial m q_i} \right) + \frac{\partial^2 q_i m^2}{\partial m^2 q_i} \right), \quad (6)$$

$$\begin{aligned} \frac{\partial RP(p, m)}{\partial \log p_i} \big|_{dv=0} = & \frac{p_i q_i}{m} \left(RP(p, m) \left(1 - \frac{\partial q_i m}{\partial m q_i} \right) \right. \\ & \left. + \left(3 - \frac{RP}{RR} \right) \frac{\partial^2 q_i m^2}{\partial m^2 q_i} - \frac{1}{RR} \frac{\partial^3 q_i m^3}{\partial m^3 q_i} \right). \end{aligned} \quad (7)$$

The term $(\partial^2 q_i / \partial m^2)(m^2 / q_i)$ measures the curvature of the Engel curve for good i ; $(\partial^3 q_i / \partial m^3)(m^3 / q_i)$ is the third-order counterpart. If (6) [(7)] is zero for all i , then relative risk aversion [prudence] is constant along the indifference curve (but can vary along an Engel curve).²⁰ Clearly, this will be the case with homothetic preferences (when all Engel curves are straight lines through the origin), but in general, attitudes towards risk will not be constant along the indifference curve. The important implication is that, even when these attitudes are constant w.r.t. income (e.g. because of a CRRA assumption), a change in the relative price structure will affect both of them in a non-trivial way.

¹⁹ With two goods, $\frac{\partial u_{qq}}{\partial q_i} = \begin{pmatrix} u_{11i} & u_{12i} \\ u_{21i} & u_{22i} \end{pmatrix}$ ($i = 1, 2$) and the numerator of (5) is the binary cubic form: $(q'_m (\partial u_{qq} / \partial q_1) q_m, q'_m (\partial u_{qq} / \partial q_2) q_m) q_m = q_{m1}^3 u_{111} + 3 q_{m1}^2 q_{m2} u_{112} + 3 q_{m1} q_{m2}^2 u_{122} + q_{m2}^3 u_{222}$.

²⁰ The case of constant RR along the indifference curve was first studied by Deschamps (1973, Section 3). Hanoch (1977, Section 3) completed the analysis by deriving the indirect utility function that corresponds to this assumption.

Effects of quantity constraints on relative risk aversion

Virtual income and price risk

Suppose now that $q = \begin{pmatrix} x \\ z \end{pmatrix}$, $p = \begin{pmatrix} p_x \\ p_z \end{pmatrix}$, where z is the quantity of a single good or service which is no longer at the consumer's discretion because it is fixed at $\bar{z}$.²¹ Her problem when observing the income draw $\tilde{m}$ then turns into

$$\max_x u(x, \bar{z}) \quad \text{s.t.} \quad p'_x x + p_z \bar{z} = \tilde{m}.$$

Let the solution be given by $x^r(p, \tilde{m}, \bar{z})$, and the indirect utility function defined as $v^r(p, \tilde{m}, \bar{z}) \stackrel{\text{def}}{=} u(x^r(p, \tilde{m}, \bar{z}), \bar{z})$. Repeating the procedure of the previous section, the coefficient of relative risk aversion for income risk is given by

$$RR(p, m | \bar{z}) \stackrel{\text{def}}{=} - \frac{v_{mm}^r}{v_m^r} m = - \frac{x_m'^r u_{xx} x_m^r}{x_m'^r u_x} m. \quad (8)$$

In order to relate $RR(p, m | \bar{z})$ to $RR(p, m)$, I will use the “virtual price” approach of Neary and Roberts.² This consists in defining a virtual price $\tilde{\pi}_z$, and adjusting the consumer's income to the virtual income level $\tilde{\mu} \stackrel{\text{def}}{=} \tilde{m} + (\tilde{\pi}_z - p_z)\bar{z}$ such that the consumer's notional demand for the z -good coincides with the imposed quantity $\bar{z}$. That is,

$$\bar{z} \equiv z(p_x, \tilde{\pi}_z, \tilde{m} + (\tilde{\pi}_z - p_z)\bar{z}), \quad (9)$$

$$x^r(p_x, p_z, \tilde{m}, \bar{z}) \equiv x(p_x, \tilde{\pi}_z, \tilde{m} + (\tilde{\pi}_z - p_z)\bar{z}), \quad (10)$$

$$v^r(p_x, p_z, \tilde{m}, \bar{z}) \equiv v(p_x, \tilde{\pi}_z, \tilde{m} + (\tilde{\pi}_z - p_z)\bar{z}). \quad (11)$$

This is illustrated by Figure 1 for the case of two normal goods. The Engel curve is given by *EE*. Panel (a) sketches a situation of a consumer who faces the timeless income prospect $(m_L, m_H; 1/2, 1/2)$ with expectation m but cannot consume more than $\bar{z}$ units of the second good. Both under the high-income state (m_H) and the low-income state (m_L), she feels rationed ($\bar{z} < z(p_x, p_z, \tilde{m})$). Her choice of $(x^r(p, m_H, \bar{z}), \bar{z})$ can be rationalised by a large virtual price π_{zH} and a virtual income $\mu_H = m_H + (\pi_{zH} - p_z)\bar{z}$. Likewise, (π_{zL}, μ_L) supports the constrained choice $(x^r(p, m_L, \bar{z}), \bar{z})$. With z being a normal good, the virtual price and income are positively correlated. Panel (b) depicts a situation of forced consumption ($\bar{z} > z(p_x, p_z, \tilde{m})$).

²¹ The assumption that only a single good is constrained in the short run can easily be relaxed by using matrix algebra in the analysis that follows. Cf. Schroyen (2010).

Figure 1. Income risk $\{m_L, m_H\}$ translates into a virtual price risk $\{\pi_{zL}, \pi_{zH}\}$ and a virtual income risk $\{\mu_L, \mu_H\}$: (a) rationing; (b) forced consumption.

Again, the supporting virtual price and income are positively correlated. The figures also show the virtual price (π_z) and income (μ) that support the choice when receiving the mean income m —if unconstrained, z^* is demanded.

Implicitly differentiating (9) and using the Slutsky equation (2(ii)) shows that

$$\frac{\partial \pi_z}{\partial m} = -\frac{z_m}{k_{zz}}, \text{ and } \frac{\partial \mu}{\partial m} = 1 - \frac{z_m}{k_{zz}} \bar{z}, \quad (12)$$

where $z_m \stackrel{\text{def}}{=} \partial z / \partial m$ is the income effect for z and k_{zz} is the own compensated price effect, that is, $k_{zz} \stackrel{\text{def}}{=} (\partial z / \partial p_z) + z_m z < 0$. Intuitively, the consumer would like to respond to a marginal income shock by $d\bar{z} = z_m d\bar{m}$. However, because the quantity constraint prevents this, the virtual price of that good has to rise by $-(1/k_{zz})d\bar{z} = -(z_m/k_{zz})d\bar{m}$, and virtual income by $(1 - (z_m/k_{zz})\bar{z})d\bar{m}$ such that she is exactly compensated for the virtual price rise. *The constraint translates the income risk into a virtual income and price risk.*

Figure 1 clearly shows that with a binding constraint, utility in each state (and therefore EU) is lower than when unconstrained, that is, than the utilities corresponding to the indifference curves going through the intersection of the Engel curve and the budget lines m_L and m_H (not drawn). The fact that the consumer gets worse off is going to affect her relative risk attitudes to the extent that RR and RP depend on real income. For example, under increasing relative risk aversion (IRRA), this income effect of rationing will lower RR . Later in the empirical section, I am going to assume away this “standard” effect by imposing CRRA. As I will show next, what remains are ordinal effects (unrelated to RR and RP) and substitution effects on RR and RP (defined in Theorem 1).

A decomposition result

The marginal utility of income in the constrained situation is given by

$$v_m^r \equiv (v_{\pi_z} + v_m \bar{z}) \frac{\partial \pi_z}{\partial m} + v_m = v_m,$$

where the equality sign follows from Roy's identity. Differentiating one more time with respect to $\bar{m}$ yields

$$\begin{aligned} v_{mm}^r &\equiv v_{m\pi_z} \frac{\partial \pi_z}{\partial m} + v_{mm} \left(1 + \bar{z} \frac{\partial \pi_z}{\partial m} \right) \\ &= -v_{mm} \bar{z} \frac{\partial \pi_z}{\partial m} - v_m z_m \frac{\partial \pi_z}{\partial m} + v_{mm} \left(1 + \bar{z} \frac{\partial \pi_z}{\partial m} \right) \\ &= v_{mm} - v_m z_m \frac{\partial \pi_z}{\partial m}, \end{aligned} \quad (13)$$

where the second line follows from (3).²² Let (π_z, μ) be the virtual price and income combination corresponding to the mean income m . Then use of (12) into (13) and the definition (8) of $RR(p, m | \bar{z})$ leads to:

Proposition 2: When facing the quantity constraint $\bar{z}$, the degree of relative risk aversion can be decomposed into a virtual degree of relative risk aversion and a positive ordinal term:

$$RR(p, m | \bar{z}) = RR(p_x, \pi_z, \mu) \frac{m}{\mu} - \mu \frac{z_m^2}{k_{zz}} \Big|_{(p_x, \pi_z, \mu)} \frac{m}{\mu}. \quad (14)$$

The term $RR(p_x, \pi_z, \mu)$ on the right-hand side of (14) can be coined the virtual coefficient of relative risk aversion. The second *rhs* term is a mark-up that is strictly positive (since $k_{zz} < 0$) and entirely ordinal. Multiplying and dividing the second term by $\pi_z \mu / \bar{z}^2$ and defining the income and compensated price elasticity of z as $\eta_z \stackrel{\text{def}}{=} z_m(\mu / \bar{z})$ and $\varepsilon_{zz} \stackrel{\text{def}}{=} k_{zz}(\pi_z / \bar{z})$, and the virtual budget share as $w_z \stackrel{\text{def}}{=} \pi_z \bar{z} / \mu$, (14) may also be written as

$$RR(p, m | \bar{z}) = \left(RR(p_x, \pi_z, \mu) - w_z \frac{\eta_z^2}{\varepsilon_{zz}} \Big|_{(p_x, \pi_z, \mu)} \right) \frac{m}{\mu}. \quad (15)$$

The mark-up term $w_z(\eta_z^2 / \varepsilon_{zz})$ is ordinal because all of its components are derived from the demand function $z(\cdot)$ which is in principle observable from market behaviour—it is invariant to any positive monotone transformation of the cardinal utility function $u(x, z)$, and thus reflects the preference order (under certainty).

²² Mathematically, this result may be seen as an application of the second-order envelope property of maximum value functions (cf. Dixit, 1990, pp. 113–114).

It remains to show that (14) amounts to twice the risk premium (as a fraction of m) per unit of variance in case the consumer is facing a small proportional risk around the expected income m . In the appendix, I show that (12) allows me to approximate the normalised variances of $\tilde{\pi}_z$ and $\tilde{\mu}$, and their covariance, as

$$E\left(\frac{\tilde{\pi}_z - \pi_z}{\pi_z}\right)^2 \simeq \frac{\eta_z^2}{\varepsilon_{zz}^2} \sigma_m^2, \quad (16)$$

$$E\left(\frac{\tilde{\mu} - \mu}{\mu}\right)^2 \simeq \left(1 - \frac{w_z \eta_z}{\varepsilon_{zz}}\right)^2 \left(\frac{\sigma_m}{\mu}\right)^2, \quad (17)$$

$$E\left(\frac{\tilde{\pi}_z - \pi_z}{\pi_z}\right)\left(\frac{\tilde{\mu} - \mu}{\mu}\right) \simeq \left(-\frac{\eta_z}{\varepsilon_{zz}}\right)\left(1 - \frac{w_z \eta_z}{\varepsilon_{zz}}\right)\left(\frac{\sigma_m}{\mu}\right)^2. \quad (18)$$

Taking expectations of a second-order Taylor expansion for $v(p_x, \tilde{\pi}_z, \tilde{\mu})$ around (p_x, π_z, μ) gives

$$\begin{aligned} Ev(p_x, \tilde{\pi}_z, \tilde{\mu}) &\simeq v(p_x, \pi_z, \mu) + \frac{1}{2} \left\{ v_{p_z p_z} \pi_z^2 \text{var}\left(\frac{\tilde{\pi}_z}{\pi_z}\right) \right. \\ &\quad \left. + 2v_{p_z m} \pi_z \mu \text{cov}\left(\frac{\tilde{\pi}_z}{\pi_z}, \frac{\tilde{\mu}}{\mu}\right) + v_{mm} \text{var}\left(\frac{\tilde{\mu}}{\mu}\right) \mu^2 \right\}. \end{aligned}$$

I show in the appendix that the valuations of the variance of the price risk and of the covariance are

$$v_{p_z p_z} \pi_z^2 = v_m \mu w_z^2 \left(-RR(p_x, \pi_z, \mu) + 2\eta_z - \frac{\varepsilon_{zz}}{w_z} \right), \quad (19)$$

$$v_{p_z m} \pi_z \mu = v_m \mu w_z (RR(p_x, \pi_z, \mu) - \eta_z). \quad (20)$$

Thus the consumer is correlation loving (averse) to the extent that RR exceeds η_z . Income risk aversion itself makes the consumer price risk averse, but then a positive income elasticity and a negative Hicksian price elasticity makes her price risk loving. The last result is intuitive: the possibility to substitute in response to price shocks is desirable. Using (16)–(18) and rearranging then results in

$$Ev(p_x, \tilde{\pi}_z, \tilde{\mu}) \simeq v(p_x, \pi_z, \mu) + \frac{1}{2} v_m \mu \left\{ w_z \frac{\eta_z^2}{\varepsilon_{zz}} - RR(p_x, \pi_z, \mu) \right\} \left(\frac{\sigma_m}{\mu}\right)^2. \quad (21)$$

To complete the argument, I mimic the utility of the certainty equivalent income, $v^r(p_x, p_z, m^{CE}, \bar{z})$ as $v(p_x, \pi_z^{CE}, \mu^{CE})$ (cf. (11)) and take a first-order Taylor expansion around (p_x, π_z, μ) :

$$\begin{aligned} v(p_x, \pi_z^{CE}, \mu^{CE}) &\simeq v(p_x, \pi_z, \mu) + v_{p_z} (\pi_z^{CE} - \pi_z) + v_m (\mu^{CE} - \mu) \\ &= v(p_x, \pi_z, \mu) - v_m \bar{z} (\pi_z^{CE} - \pi_z) + v_m (m^{CE} - m + (\pi_z^{CE} - \pi_z) \bar{z}) \\ &= v(p_x, \pi_z, \mu) + v_m (m^{CE} - m), \end{aligned} \quad (22)$$

where the second line follows from the definition of the virtual certainty equivalent income: $\mu^{CE} \stackrel{\text{def}}{=} m^{CE} + (\pi_z^{CE} - \pi_z)\bar{z}$.

Equating the *rhs* of (21) and (22) and rearranging then gives

$$\frac{m - m^{CE}}{m} \simeq \frac{1}{2} \left(\underline{RR(p_x, \pi_z, \mu)} + w_z \frac{\eta_z^2}{(-\varepsilon_{zz})} \Big|_{(p_x, \pi_z, \mu)} \right) \frac{m}{\mu} \frac{\sigma_m^2}{m^2}. \quad (23)$$

The underlined term is exactly $RR(p, m | \bar{z})$ as derived in (15), and hence may be interpreted as the coefficient of relative risk aversion when facing the constraint $\bar{z}$.

Discussion

There are two things worth noticing about $RR(p, m | \bar{z})$. First, all terms in the round bracket expressions (15) or (23) are evaluated at the virtual price and income that support the consumer's decision when facing the mean income m and the constraint $\bar{z}$: (p_x, π_z, μ) . The relative virtual price will exceed its nominal counterpart in case of rationing ($\bar{z} < z(p_x, p_z, m)$)—the case shown in Figure 1(a)) and fall short of it in case of forced consumption ($\bar{z} > z(p_x, p_z, m)$)—Figure 1(b)). For the same reason will virtual income exceed or fall short of nominal income. The extent to which the quantity constraint is binding will thus be reflected by the discrepancy between (p_x, π_z, μ) and (p_x, p_z, m) , and in this way influence the degree of relative risk aversion. Second, the entire round bracket term is multiplied with a factor m/μ . Again, this factor is larger than 1 in case of forced consumption and below one in case of rationing. The reason for this scale effect is our interest in the risk premium as a fraction of nominal income, not of virtual income.

A special case is a weakly binding constraint: when the quantity $\bar{z}$ exactly coincides with the quantity that the consumer wishes to consume at (p_x, p_z, m) . I denote this level by z^* , that is, $z^* = z(p_x, p_z, m)$. In this case, $\pi_z = p_z$ and $\mu = m$, and

$$RR(p, m | z^*) = \underline{RR(p_x, p_z, m)} - w_z \frac{\eta_z^2}{\varepsilon_{zz}} \Big|_{(p_x, p_z, m)}.$$

This expression illustrates the graphical effect that the constraint makes the maximum value function more concave in the neighbourhood of income for which the desired quantity z^* coincides with the imposed level $\bar{z}$. I therefore call the second *rhs* term the *local curvature effect*.²³

²³ It is reminiscent of a result obtained by Drèze and Modigliani (1972). They compared the aversion towards future income risk under two settings: (i) a timeless one where the consumer is informed about her income draw before making her savings decision, and (ii) a temporal one where the savings decision is made before the income draw is known. They showed that the risk aversion for the temporal risk exceeds that for the timeless risk by an ordinal term being the ratio of the squared income effect on current consumption to the substitution effect between current and future

Once the constraint becomes strictly binding, virtual price and income start to deviate from their nominal counterparts. The difference $[RR(p, m | \bar{z}) - RR(p, m)]$ can be decomposed as

$$\begin{aligned} & [RR(p, m | z^*) - RR(p, m)] + [RR(p, m | \bar{z}) - RR(p, m | z^*)] \\ &= -w_z \frac{\eta_z^2}{\varepsilon_{zz}} \Big|_{(p_x, p_z, m)} + \int_{z^*}^{\bar{z}} \frac{\partial RR(p, m | \zeta)}{\partial \zeta} d\zeta. \end{aligned}$$

Thus the difference consists of the local curvature effect and a second effect that I will term the *endogenously changing risk aversion effect*. Since $RR(p, m | \zeta) = (RR(p_x, \pi_z(\zeta), \mu(\zeta)) - w_z(\eta_z^2/\varepsilon_{zz})|_{(p_x, \pi_z(\zeta), \mu(\zeta))})(m/\mu(\zeta))$, all three *rhs* terms depend on π_z and μ which will be affected by ζ . Consider the effect on the first term:

$$\frac{\partial RR(p_x, \pi_z(\zeta), \mu(\zeta))}{\partial \zeta} = \left(\frac{\partial RR(\cdot)}{\partial \pi_z} + \frac{\partial RR(\cdot)}{\partial m} \bar{z} \right) \frac{\partial \pi_z(\zeta)}{\partial \zeta} + \frac{\partial RR(\cdot)}{\partial \mu} (\pi_z(\zeta) - p_z).$$

Thus $\bar{z} \neq z^*$ implies a change in the relative price structure that has a substitution effect on RR (1st *rhs* term). At the end of the previous section, I argued that in general, this substitution effect will not vanish. Second, except for CRRA preferences when $\partial RR(\cdot)/\partial \mu \equiv 0$, there is a real income effect on RR (2nd *rhs* term): a rationed consumer ($\bar{z} < z^*$, $\pi_z > p_z$) will get better off when $\bar{z}$ is increased, and under IRRRA turn more risk averse. In addition, there is the effect on the mark-up $-w_z(\eta_z^2/\varepsilon_{zz})|_{(p_x, \pi_z(\zeta), \mu(\zeta))}$ (clearly depending on higher order behavioural effects), and finally that on the scaling factor $m/\mu(\zeta)$. It is only the last effect that may be signed unambiguously (positive). Thus I will conclude that both the sign and size of the endogenously changing risk aversion effect is an empirical issue, and I will document it in the section on calibration. Before doing so, I briefly discuss the effect on relative prudence.

Relative prudence under a quantity constraint

In the appendix, it is shown that

$$v_{mmm}^r = v_{mmm} - 3v_{mm}z_m \frac{\partial \pi_z}{\partial m} - v_m \frac{\tau}{\bar{m}^2}, \quad (24)$$

consumption. The constraint arises in the temporal context because savings decision cannot respond to income shocks. Still, it is weakly binding because the decision has been made optimally.

where τ is a dimensionless term collecting all “second-order” ordinal responses:

$$\tau \stackrel{\text{def}}{=} \left(2z_{mm} \frac{\partial \pi_z}{\partial m} - (z_m^2 - z_{mp_z} - z_{mm}\bar{z}) \left(\frac{\partial \pi_z}{\partial m} \right)^2 + z_m \frac{\partial^2 \pi_z}{\partial m^2} \right) \tilde{m}^2. \quad (25)$$

Then taking the ratio $((24)/(13))(\tilde{\mu}^2/\tilde{\mu})(\tilde{m}/\tilde{\mu})$ and evaluating at mean income m (and therefore at (π_z, μ)), gives:

Proposition 3: When facing a quantity constraint, the coefficient of relative prudence is given by

$$RP(p, m | \bar{z}) \stackrel{\text{def}}{=} - \frac{v_{mmmm}^r}{v_{mm}^r} m = \left(\frac{RR \cdot \left(RP - 3w_z \frac{\eta_z^2}{\epsilon_{zz}} \right) - \tau}{RR - w_z \frac{\eta_z^2}{\epsilon_{zz}}} \right) \frac{m}{\mu}, \quad (26)$$

where all terms in the large round bracket are evaluated at (p_x, π_z, μ) , the terms of trade that mimic the decision when facing mean income m and being constrained at $\bar{z}$.

Thus a positive sign of (26) indicates that a constrained consumer disproves of the income shock $\tilde{x}$ being replaced by a shock $\tilde{y}$ that is the result of a mean preserving spread on the left followed by a mean preserving contraction on the right to keep the variance fixed.²⁴

As with relative risk aversion, I decompose the effect of a constraint as

$$RP(p, m | \bar{z}) - RP(p, m) = [RP(p, m | z^*) - RP(p, m)] + \int_{z^*}^{\bar{z}} \frac{\partial RP(p, m | \zeta)}{\partial \zeta} d\zeta.$$

The square bracket term is a *local curvature effect* (but now of the marginal utility of income function), while the next term is an *endogenously changing prudence effect*. The former effect is no longer unambiguously positive. This can be seen by evaluating the *rhs* of (26) at (p_x, p_z, m) and subtracting $RP^* = RP(p, m)$:

$$RP(p, m | z^*) - RP^* = \frac{w_z^* \frac{\eta_z^{*2}}{\epsilon_{zz}^*} (RP^* - 3RR^*) - \tau^*}{RR^* - w_z^* \frac{\eta_z^{*2}}{\epsilon_{zz}^*}}. \quad (27)$$

The important thing to notice is that this local curvature effect for prudence, as that for relative risk aversion, vanishes when the income effect is zero (since then $\eta_z = \tau = 0$).

²⁴ I could have proceeded as at the end of Section “A decomposition result” to show that (26) measures twice the relative prudence premium per unit of variance of a small proportional risk around m (cf. Kimball, 1990, Eq. 1).

Again, there are four channels that make up the endogenously changing prudence effect: compensated price effects on RR and RP , real income effects on RR and RP , effects on the ordinal mark-up terms $w_z(\eta_z^2/\varepsilon_{zz})$ and τ , and an effect on the scaling factor m/μ . I now turn to the calibration exercises.

Calibrations of the effects on RR and RP .

I will look at two settings. The first is one where the consumer cares about non-durables and durables. The second is a model of labour supply. I am then interested in calibrating the effects of constraints on durable goods or on labour supply on the coefficients of relative risk aversion and prudence.²⁵

Durable goods

In a recent paper, Pakoš²⁶ estimates the Euler equation and the intraperiod first-order condition corresponding to an intertemporal utility maximisation problem where the period utility index is a generalised CES function defined over the consumption of non-durables (x) and the service flow of durables, assumed to be proportional to the stock of durables (z). In my notation, the period-CES index is given by

$$\tilde{u} = \left[x^{1-\frac{1}{\sigma}} + \alpha z^{1-\frac{\theta}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (28)$$

with $\alpha, \sigma, \theta > 0$. It can be shown that the elasticity of intraperiod substitution is given by $\sigma + \varepsilon_{zz}(\theta - 1)$. If $\theta = 1$, preferences reduce to standard homothetic CES preferences and both income elasticities are equal to 1. If $\theta < 1$, durables have a (non-constant) income elasticity larger than 1.

Using aggregate quarterly data for the U.S. (1955:1-2001:4) compiled by Yogo,^{27,28} Pakoš obtains superconsistent estimates for σ and θ by estimating the intratemporal first-order condition as a co-integrating regression.²⁹ This regression picks up long-run co-movements in durables and non-durables, and thus reflect

²⁵ All Maple files used for these calibrations are available upon request.

²⁶ Pakoš (2011).

²⁷ Yogo (2006).

²⁸ Web-appendix to Yogo (2006) (www.sites.google.com/site/motohiroyogo/home/publications/#Asset-Pricing). The data contain quarterly time series for aggregate non-durable household consumption and stock of household durables, and a price index for durables on the basis of which Pakoš (2011) computed an implicit rental cost. Durables include: vehicles, household appliances, books and leisure equipment, jewellery and watches.

²⁹ The intratemporal FOC can be written as $\log x_t \propto \theta \log z_t + \sigma \log(p_z/p_x)_t$. Then σ and θ are estimated as the co-integrating vector for a vector error correction model.

Table 1 Summary of the durable goods model and effects on RR and RP of weakly binding quantity constraints on durable flows

(a)	σ	θ	α	z^*	x^*	
	0.083	0.604	0.012	4.836	3.104	
(b)	p_z^*	m^*	w_z^*	η_z^*	ε_{zz}^*	τ^*
	0.06015	3.395	0.086	1.568	-0.119	14.40
(c)	RR^*	$RR(\cdot z^*)$	LCE_{RR}	RP^*	$RP(\cdot z^*)$	LCE_{RP}
	1.974	3.744	1.77	2.982	0.525	-2.457

behaviour that is unconstrained by adjustment restrictions and costs. The remaining parameters are estimated from the Euler equation, conditional upon the consistent estimates for σ and θ .³⁰ Since $\tilde{u}(\cdot)$ is just a utility index representing the preference order over bundles under certainty, it does not necessarily represent preferences over uncertain consumption prospects. Thus I take a concave iso-elastic transformation by defining the vNM index as $u(\cdot) \stackrel{\text{def}}{=} \tilde{u}(\cdot)^{1-\gamma}/1-\gamma$, and set $\gamma=2$. This choice of γ implies a value of RR^* of about 2, a value frequently found in the literature (the results of a sensitivity analysis w.r.t. γ is discussed below).³¹

Table 1, panel (a) gives the parameter estimates and sample means (x^* , z^*) for non-durables and the stock of durables. Panel (b) gives the relative price (p_z^*) and real income (m^*) that support (x^* , z^*) as an optimal solution, as well as the corresponding budget share, the income and own Hicksian price elasticity for durables and the second-order behavioural elasticity τ^* .³² Panel (c) gives the attitudes towards risk when unconstrained, and with a weakly binding quantity constraint on durables (i.e. $\bar{z} = z^*$). It also gives the local curvature effects.

The reason RR^* is slightly lower than 2 is the non-homotheticity of the CES index ($\theta < 1$). For the same reason RP^* is not exactly 3. The local curvature effects on risk attitudes (panel (c)) are very strong: the mere impossibility to adjust durables in response to small income shocks almost doubles relative risk aversion, and almost reduces relative prudence to a sixth. This reduction is due to the significant size of τ^* . To gauge the endogenously changing risk aversion and prudence effects,

³⁰ The remaining parameters are the weight on durables (α), the discount factor and the intertemporal substitution elasticity.

³¹ I do not use the Pakoš (2011) estimate of the intertemporal elasticity of substitution (0.038) because it would mean an unrealistically large CRRA coefficient.

³² x^* and z^* refer to the mean values of the indices for non-durables and durables in the Yogo (2006) data set; p_z^* is the $MRS(-(dx/dz)|_{du=0})$ at (x^*, z^*) , using the estimated utility function; m^* is the income level supporting this mean bundle: $m^* = x^* + p_z^* z^*$; $w_z^* = (p_z^* z^* / m^*)$. Again using the estimated utility function, the income and compensated price elasticities for z evaluated at (x^*, z^*) can be calculated (η_z^* and ε_{zz}^* in Table 1). Thus the unconstrained "long run" consumer facing the price p_z^* and disposing m^* would freely choose (x^*, z^*) .

Figure 2. $RR^* = 2$ and $RP^* = 3$: (a) $RR(p, m^* | \phi z^*)$ and $RP(p, m^* | \phi z^*)$; (b) $RR(p, \phi m^* | z^*)$ and $RP(p, \phi m^* | z^*)$.

I have plotted in Figure 2(a), $RR(1, p_z^*, m^* | \bar{z})$ where I vary $\bar{z}$ from 80 per cent to 120 per cent of the notional level z^* . Being constrained at 90 per cent of the optimal flow of durable services further increases relative risk aversion to about 5. On the other hand, being rationed at 90 per cent again increases relative prudence, but still to only a third of its unconstrained value. Figure 2(a) also shows that $|dRR/d \log \bar{z}|$ is about 10 $((5-3)/(1.1-0.9))$. The empirical implication is that *a sample of individuals, all identical except for some small heterogeneity in their degree of adjustment to the desired level of durables may display a considerable degree of heterogeneity in relative risk aversion*. The U-shape for $RP(\bar{z})$ in panel (a) also suggests that an EU saver who has recently optimised w.r.t. z (and therefore is stuck at a $\bar{z}$ close to z^*) reacts more fiercely to an increase in risk in the rate of return on savings (by saving less) than another one, stuck at far more suboptimal levels of z .³³

In Figure 2(b), I keep fixed $\bar{z}$ at z^* and change mean disposable income from 80 per cent to 120 per cent of the value that supports z^* (i.e. $m^* = 3.395$). When unconstrained, relative risk aversion is almost constant w.r.t. income (the bold dashed line is slightly falling), but the figure shows that it becomes strongly increasing in income when constrained at z^* . Relative prudence, on the other hand, is non-monotone in income—again, if unconstrained this risk attitude would be almost constant around 3.

³³ An EU consumer relying only on savings for future consumption reacts to an increase in risk about the rate of return by saving more iff $RP > 2$ (cf. Eeckhoudt and Schlesinger, 2008, p. 1335).

In the appendix, I have drawn similar figures for $\gamma = 1$, which corresponds to $RR^* = 1.03$ and $RP^* = 2.02$, and for $\gamma = 4.2$, which corresponds to $RR^* = 4.04$ and $RP^* = 5.06$ (see Figures A1 and A2).³⁴ While the qualitative properties of the curves are preserved, the main difference is that with a low baseline level for RR^* and RP^* , relative prudence turns negative when constrained—the consumer turns imprudent or downside risk loving. With $\gamma = 4.2$, the local curvature effect for relative prudence is still negative, but much smaller. A back of the envelope calculation shows that as γ becomes large, LCE_{RP} approaches twice the value of LCE_{RR} , and thus positive.³⁵

I believe these results warrant the conclusion that risk attitudes are very sensitive to the presence of consumption commitments, and that small heterogeneity in the possibility to adjust may result in large heterogeneity in relative risk aversion, and in market behaviour governed by RR and RP . I next turn to labour supply restrictions.

Labour supply

The second calibration is based on estimates of the labour supply equation

$$L = \beta_0 + \beta_1 \log a + \beta_2 m,$$

where a is the real wage rate and m denotes non-labour income. This equation has been estimated by Blundell *et al.*³⁶ for U.K. married or cohabitating women, by de Linde Leonard³⁷ for U.S. male heads of household, and by Flood and MaCurdy³⁸ for Swedish married men.

Stern³⁹ showed that the above labour supply equation integrates to the following utility index:

$$\begin{aligned} \tilde{u}(c, L) = & \frac{\exp(\beta_2 a(c, L))}{\beta_2} (\beta_2 (c - a(c, L)L) + \beta_0 + \beta_1 \log a(c, L)) \\ & - \frac{\beta_1}{\beta_2} Ei(\beta_2 a(c, L)), \end{aligned}$$

where Ei is the standard exponential integral, and $a(c, L)$ is the reservation wage when consuming c and supplying L hours.⁴⁰ I define the vNM utility function as

³⁴ Chetty (2006) concludes that 1 is “a central estimate of the coefficient of relative risk aversion implied by labor supply studies”. The calculations of de Linde Leonard (2012), also based on labour supply elasticities, confirm this conclusion. Other elicitation methods, for example, using hypothetical income gambles, typically get much higher average values for RR (about 8 in Kimball *et al.* (2008), 3.8 in Aarbu and Schroyen (2013)).

³⁵ With CRRA preferences $RP^* = RR^* + 1$, and $\lim_{RR^* \rightarrow \infty} (27) = -2w_z^*(\eta_z^{*2}/\hat{\epsilon}_{zz}^*) > 0$. With CARA preferences $RP^* = RR^*$, and the same limit result holds.

³⁶ Blundell *et al.* (1998).

³⁷ de Linde Leonard (2012).

³⁸ Flood and MaCurdy (1992).

³⁹ Stern (1986).

Table 2 Sample means, parameters and implied elasticities for labour supply studies

	a^*	L^*	m^*	$(a^*L^*)/m^*$	β_1	β_2	η_L^*	ε_{LL}^*
U.K. ^a	£/hour	hours/week	£/week					
No child	2.97	32	88.63	1.072	4.493	0	0	0.140
Youngest child 0–2 yo	3.36	20	129.69	0.518	4.105	–0.028	–0.185	0.301
Youngest child 3–4 yo	3.10	18	143.64	0.388	6.686	–0.022	–0.173	0.439
Youngest child 5–10 yo	2.86	21	151.13	0.397	2.777	–0.014	–0.102	0.173
Youngest child 11+ yo	2.83	25	147.31	0.480	3.260	–0.011	–0.063	0.160
U.S. ^b	\$/hour	hours/year	\$/year					
	18.05	2223	17462	2.297	1156	–0.038	–0.342	1.206
Sweden ^c	SEK/hour	hours/year	SEK/year					
	22	1932	51076	0.832	446.8	–0.0034	0.090	0.306

^aBlundell *et al.* (1998), weekly data (U.K. Family Expenditure Survey—FES; 1978–1992) on labour supply of married or cohabitating women in age range 20–50 years.

^bde Linde Leonard (2012), yearly data (Michigan Panel Study of Income Dynamics—PSID; 1996) on labour supply of male heads of household in age range 25–61 years.

^cFlood and MaCurdy (1992), yearly data (Swedish Household Market and Non-market Activities Panel Survey—HUS; 1983) on labour supply of married men in the age range 25–65 years).

$u(c, L) \stackrel{\text{def}}{=} [\tilde{u}(c, L) + \nu]^{1-\gamma} / 1 - \gamma$. The indirect utility function is then given by

$$v(a, m) = \frac{1}{1-\gamma} \left[\frac{\exp(\beta_2 a)}{\beta_2} (\beta_2 m + \beta_0 + \beta_1 \log a) - \frac{\beta_1}{\beta_2} \text{Ei}(\beta_2 a) + \nu \right]^{1-\gamma}.$$

Again I assume $\gamma = 2$, and calibrate the constant ν such that the “unconstrained” coefficient of relative risk aversion $RR^* = -(v_{mm}/v_m)m^*$ attains the value 2 at (a^*, m^*) , the sample mean values for m and a . This also ensures that $RP^* = 3$. The sample means for the wage rate, hours worked and non-labour income, as well as the parameter estimates and the income and compensated wage elasticities are given in Table 2.⁴¹

Using the information in Table 2, I have calculated the effects of labour supply constraints on relative risk aversion and prudence. The results are given in Table 3. Unlike for durables, constraints on labour supply hardly affect attitudes towards risk. The local curvature effects are strongest for U.S. male workers: 0.17 for relative risk aversion and 0.24 for relative prudence. The reason these effects are small are the low-income elasticities. Indeed, for U.K. females without children the income effect is absent. In Figure 3, I have plotted $RR(\cdot|\phi L^*)$ (lower curves) and $RP(\cdot|\phi L^*)$ (upper curves) for $\phi \in [0.8, 1.2]$. Both risk attitudes are decreasing in the constraint on labour

⁴⁰ $Ei(x) = \int_{-\infty}^x (\exp(t)/t) dt$ and $a(c, L)$ is the solution to $L = \beta_0 + \beta_1 \log a + \beta_2(c - aL)$.

⁴¹ The Slutsky condition requires that $\beta_1 > \beta_2 \omega L$. Since $\beta_2 \leq 0$ for all the studies listed in Table 2, it is automatically satisfied.

Table 3 Effects of labour supply constraints on relative risk aversion and prudence

	RR^*	$RR(\cdot L^*)$	LCE_{RR}	RP^*	$RP(\cdot L^*)$	LCE_{RP}
U.K.						
No child	2	2	0	3	3	0
Youngest child 0–2 yo	2	2.057	0.057	3	3.072	0.072
Youngest child 3–4 yo	2	2.027	0.027	3	3.036	0.036
Youngest child 5–10 yo	2	2.023	0.023	3	3.030	0.030
Youngest child 11+ yo	2	2.012	0.012	3	3.017	0.017
U.S.	2	2.170	0.170	3	3.240	0.240
Sweden	2	2.030	0.030	3	3.043	0.043

Figure 3. $\gamma = 1(RR^* = 1.03, RP^* = 2.02)$: (a) $RR(p, m^*|\phi z^*)$ and $RP(p, m^*|\phi z^*)$; (b) $RR(p, \phi m^*|z^*)$ and $RP(p, \phi m^*|z^*)$.

supply.⁴² A constraint to supply about 10 per cent more than the optimal amount of hours typically makes the worker as risk averse and prudent as without any constraint. On the

⁴² A figure similar to Figure 2(b), for $RR(\phi m|L^*)$ and $RP(\phi m|L^*)$, $\phi \in [0.8, 1.2]$ (not shown) would reveal that these attitudes towards risk are almost constant in income.

other hand, an involuntary 20 per cent reduction in labour supply has non-negligible positive effect on both risk attitudes, especially for male U.S. workers.

I therefore conclude that labour supply constraints have small effects on relative risk aversion and prudence, at least when these constraints are not too binding. But this does not imply that risk attitudes are immune to unemployment status. For example, using the estimates of de Linde Leonard,⁴³ I find that for an average worker hit by full-time unemployment and receiving an unemployment benefit at 60 per cent of past earnings during 26 weeks (the maximum benefit period in most U.S. states), relative risk aversion increases to 4.26 and relative prudence to 6.30.⁴⁴

Conclusion

I have traced out how quantity constraints on a good or service affect the consumer's attitude towards timeless income risk. Using the virtual price approach, I have decomposed the effect into a local curvature effect and an endogenously changing risk aversion/prudence effect.

Calibration of the model shows that restrictions on service flow of durables—for example, because of adjustment costs—have large effects on both relative risk aversion and prudence. Moreover, for relative prudence the effect turned out to be non-monotone in the degree that the constraint is binding, and in the income level. A natural next question is how durables are decided upon when consumers realise the consequences of costly adjustment for risk attitudes. Under simplifying assumptions, Shore and Sinai⁴⁵ provide a first answer to this (difficult) question.

Constraints on the supply of labour were shown to be more moderate, the main reason being the small income effect of notional labour supply. Still, assuming a base rate relative risk aversion of 2, this attitude towards risk raises with more than 0.4 when labour market frictions force a U.S. worker to 80 per cent of his notional labour supply, and with almost 2.3 when fully unemployed on a 60 per cent unemployment benefit.

Eeckhoudt and Schlesinger,⁴⁶ p. 1335) show that an increase in the skewness of the interest rate distribution triggers more saving if relative temperance (*RT*) exceeds 3. An obvious extension of the paper would consider the effects of constraints on *RT*, and verify whether, for example, costly adjustment of durables makes *RT* trespass this critical value.

⁴³ de Linde Leonard (2012).

⁴⁴ I.e., $RR(w^*, m^* + (26/52)(6/10)w^*L^* | L = 0) = 4.26$ and $RP(w^*, m^* + (26/52)(6/10)w^*L^* | L = 0) = 6.30$.

⁴⁵ Shore and Sinai (2010).

⁴⁶ Eeckhoudt and Schlesinger (2008).

Quantity constraints may also apply to savings or portfolio decisions, in the form of borrowing constraints or constraints on taking short positions. However, such decisions are made before the uncertainty (about future income and/or rates of return) is resolved. Future research should focus on the effects of such constraints on the attitudes towards *temporal* risks.

References

- Aarbu, K.O. and Schroyen, F. (2013) 'Mapping risk aversion in Norway using hypothetical income gambles', mimeo, Norwegian School of Economics.
- Barsky, R., Juster, T., Kimball, M. and Shapiro, M. (1997) 'Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement survey', *Quarterly Journal of Economics* 112(2): 537–579.
- Bauer, C. and Buchholz, W. (2008) *How changing prudence and risk aversion affect optimal savings*, CESIFO wp 2438, Munich.
- Blundell, R., Duncan, A. and Meghir, C. (1998) 'Estimating labor supply responses using tax reforms', *Econometrica* 66(4): 827–861.
- Chetty, R. (2006) A new method of estimating risk aversion, *American Economic Review* 96(5): 1821–1834.
- Chetty, R. and Szeidl, A. (2007) 'Consumption commitments and risk preferences', *Quarterly Journal of Economics* 122(2): 831–877.
- Choi, G., Kim, I. and Snow, A. (2001) 'Comparative statics predictions for changes in uncertainty in the portfolio and savings problems', *Bulletin of Economic Research* 53(1): 61–72.
- Deck, C. and Schlesinger, H. (2010) 'Exploring higher order risk effects', *Review of Economic Studies* 77(4): 1403–1420.
- de Linde Leonard, M. (2012) 'Risk preferences and expected utility: Evidence from labor supply data', *Economic Inquiry* 50(1): 264–276.
- Deschamps, R. (1973) 'Risk aversion and demand functions', *Econometrica* 41(3): 455–465.
- Dixit, A. (1990) *Optimization in Economic Theory*. 2nd edn, Oxford: Oxford University Press.
- Dohmen, Th., Falk, A., Huffman, D., Sunde, U., Schupp, J. and Wagner, G. (2005) 'Individual risk attitudes: new evidence from a large, representative, experimentally-validated survey', mimeo, IZA discussion paper 1730.
- Dohmen, Th., Falk, A., Huffman, D., Sunde, U., Schupp, J. and Wagner, G. (2011) 'Individual risk attitudes: Measurement, determinants, and behavioral consequences', *Journal of the European Economic Association* 9(3): 522–550.
- Drèze, J.H. and Modigliani, F. (1972) 'Consumption decisions under uncertainty', *Journal of Economic Theory* 5(3): 308–355.
- Drèze, J.H. and Rustichini, A. (2004) 'State-dependent utility and decision theory', in S. Barberà, P.J. Hammond and Ch. Seidl (eds.), Chapter 16, *Handbook of Utility Theory*. Vol. 2, Boston: Kluwer Academic Publishers.
- Eeckhoudt, L., Ether, J. and Schroyen, F. (2009) 'The values of relative risk aversion and prudence: A context-free interpretation', *Mathematical Social Sciences* 58(1): 1–7.
- Eeckhoudt, L. and Schlesinger, H. (2006) 'Putting risk in its proper place', *American Economic Review* 96(1): 280–289.
- Eeckhoudt, L. and Schlesinger, H. (2008) 'Changes in risk and the demand for saving', *Journal of Monetary Economics* 55(7): 1329–1336.
- Flood, L. and MaCurdy, Th. (1992) 'Work disincentives effects of taxes: An empirical analysis of Swedish men', *Carnegie-Rochester Conference Series on Public Policy* 37: 239–278.

- Gollier, C. (2009) 'Does flexibility enhance risk tolerance'? mimeo, University of Toulouse.
- Guiso, L. and Paiella, M. (2008) 'Risk aversion, wealth and background risk', *Journal of the European Economic Association* 6(6): 1109–1150.
- Hanoch, G. (1977) 'Risk aversion and consumer preferences', *Econometrica* 45(2): 413–426.
- Kihlstrom, R. and Mirman, L. (1974) 'Risk aversion with many commodities', *Journal of Economic Theory* 8(3): 361–388.
- Kimball, M. (1990) 'Precautionary saving in the small and in the large', *Econometrica* 58(1): 53–73.
- Kimball, M., Sahm, C. and Shapiro, M. (2008) 'Imputing risk tolerance from survey responses', *Journal of the American Statistical Association* 103(483): 1028–1038.
- Kreps, D. (2013) *Microeconomic Foundations I: Choice and Competitive Analysis*, Princeton: Princeton University Press.
- Machina, M. (1982) 'Flexibility and the demand for risky assets', *Economics Letters* 10(1–2): 71–76.
- Menezes, C., Geiss, C. and Tessler, J. (1980) 'Increasing downside risk', *American Economic Review* 70(5): 921–932.
- Neary, J.P. and Roberts, K.W.S. (1980) 'The theory of household behaviour under rationing', *European Economic Review* 13(1): 25–42.
- Pakoš, M. (2011) 'Estimating intertemporal and intratemporal substitutions when both income and substitution effects are present: The role of durable goods', *Journal of Business and Economics Statistics* 29(3): 439–454.
- Paulsson, Th., Sproule, R. and Wagener, A. (2005) 'The demand for a risky asset: Signing, jointly and severally', the effects of three distributional shifts, *Metroeconomica* 56(2): 221–232.
- Postlewaite, A., Samuelson, L. and Silverman, D. (2008) 'Consumption commitments and employment contracts', *Review of Economic Studies* 75(2): 559–578.
- Schroyen, F. (2010) 'Income risk aversion with quantity constraints', mimeo, Norwegian School of Economics.
- Shore, S. and Sinai, T. (2010) 'Commitment, risk, and consumption: Do birds of a feather have bigger nests'? *Review of Economics and Statistics* 92(2): 408–424.
- Stern, N. (1986) 'On the specification of labour supply functions', in R. Blundell and I. Walker (eds.), *Unemployment, Search and Labour Supply*. Cambridge: Cambridge University Press.
- Yogo, M. (2006) 'A consumption-based explanation of expected stock returns', *Journal of Finance* 61(2): 539–580.

Appendix

Derivation of (5) Since $v_{mm} = q'_m u_{qq} q_m$, differentiating both sides with respect to m yields

$$v_{mmm} = 2q'_m u_{qq} q_{mm} + \frac{\partial q'_m u_{qq} q_m}{\partial q'} q_m$$

where $\partial q'_m u_{qq} q_m / \partial q'$ is the effect on the quadratic form $q'_m u_{qq} q_m$ because of a perturbation in the Hessian following dq . Differentiating the first-order condition (1) with respect to m yields

$$v_{mm} p = u_{qq} q_m.$$

Doing the same with the adding-up condition (2(i)) gives $p'q_{mm}=0$. Therefore, $q'_m u_{qq} q_{mm}=0$ and $v_{mmm} = ((\partial q'_m u_{qq} q_m)/(\partial q'))q_m$. The coefficient of relative prudence with respect to income risk is then given by

$$RP(p, m) = -\frac{v_{mmm}}{v_{mm}} = -\frac{\frac{\partial q'_m u_{qq} q_m}{\partial q'} q_m}{q'_m u_{qq} q_m}.$$

Proof of Theorem 1: expression (6)

$$\begin{aligned}\frac{\partial RR}{\partial p_i} &= -\frac{m}{v_m} \left(v_{mmp_i} - \frac{v_{mm}}{v_m} v_{mp_i} \right), \\ \frac{\partial RR}{\partial m} &= -\frac{v_{mm}}{v_m} - \frac{m}{v_m} \left(v_{mmm} - \frac{v_{mm}}{v_m} v_{mm} \right).\end{aligned}$$

To keep utility constant, dp_i should be accompanied by $dm = q_i dp_i$. Therefore

$$\begin{aligned}\frac{\partial RR}{\partial p_i} \Big|_{dv=0} &= \frac{\partial RR}{\partial p_i} + \frac{\partial RR}{\partial m} q_i \\ &= -\frac{m}{v_m} \left(v_{mmp_i} + v_{mmm} q_i - \frac{v_{mm}}{v_m} (v_{mp_i} + v_{mm} q_i) \right) - \frac{v_{mm}}{v_m} q_i.\end{aligned}$$

From (3), it follows that $v_{mp_i} = v_{p_i m} = -v_{mm} q_i - v_m (\partial q_i / \partial m)$. Deriving once more w.r.t. m , gives

$$v_{mmp_i} = v_{p_i mm} = -v_{mmm} q_i - 2v_{mm} \frac{\partial q_i}{\partial m} - v_m \frac{\partial^2 q_i}{\partial m^2},$$

so that

$$\frac{\partial RR}{\partial p_i} \Big|_{dv=0} = \frac{v_{mm} m}{v_m} \frac{\partial q_i}{\partial m} + m \frac{\partial^2 q_i}{\partial m^2} - \frac{v_{mm} m}{v_m} \frac{q_i}{m}.$$

Multiplying through by p_i and making use of the definition of RR and the fact that $\partial p_i q_i / \partial m = (p_i q / m) (\partial q_i / \partial m) (m / q_i)$ then gives (6).

Proof of Theorem 1, expression (7)

$$\begin{aligned}\frac{\partial RP}{\partial p_i} &= -\frac{m}{v_{mm}} \left(v_{mmmp_i} - \frac{v_{mmmm}}{v_{mm}} v_{mmp_i} \right), \\ \frac{\partial RP}{\partial m} &= \frac{v_{mmmm}}{v_{mm}} - \frac{m}{v_{mm}} \left(v_{mmmmm} - \frac{v_{mmmm}}{v_{mm}} v_{mmmm} \right).\end{aligned}$$

Therefore

$$\begin{aligned} \frac{\partial RP}{\partial p_i} \Big|_{dv=0} &= \frac{\partial RP}{\partial p_i} + \frac{\partial RP}{\partial m} q_i, \\ &= -\frac{m}{v_m} \left(v_{mmmp_i} + v_{mmmm} q_i - \frac{v_{mmm}}{v_{mm}} (v_{mmp_i} + v_{mmm} q_i) \right) - \frac{v_{mmm}}{v_{mm}} q_i. \end{aligned}$$

Since

$$v_{mmmp_i} = v_{p,mmmm} = -v_{mmmm} q_i - 3v_{mmmm} \frac{\partial q_i}{\partial m} - 3v_{mm} \frac{\partial^2 q_i}{\partial m^2} - v_m \frac{\partial^3 q_i}{\partial m^3},$$

the previous expression can be written as

$$\frac{\partial RP}{\partial p_i} \Big|_{dv=0} = -\frac{v_{mmmm}}{v_{mm}} \left(\frac{q_i}{m} - \frac{\partial q_i}{\partial m} \right) + \left(3 - \frac{v_{mmmm} m}{v_{mm}} \frac{v_m}{v_{mm} m} \right) \frac{\partial^2 q_i}{\partial m^2} m + \frac{v_m m}{v_{mm}} \frac{\partial^3 q_i}{\partial m^3}.$$

Multiplying through by p_i and making use of the definitions of RR and RP then gives (7).

Derivation of (16)–(18) First, notice that the virtual price for the z -good in case of income realisation $\tilde{m}$, $\tilde{\pi}_z = \pi_z(p_z, \tilde{m}, \bar{z})$, can be approximated as

$$\tilde{\pi}_z \simeq \pi_z + \frac{\partial \pi_z}{\partial m} (\tilde{m} - m) = \pi_z - \frac{z_m}{k_{zz}} (\tilde{m} - m) = \pi_z - \frac{\eta_z}{\varepsilon_{zz}} \frac{\pi_z}{\mu} (\tilde{m} - m),$$

where k_{zz} , z_m , η_z and ε_{zz} are all evaluated at the triple (p_x, π_z, μ) . Virtual income in case of income realisation $\tilde{m}, \tilde{\mu}$, is given by $\tilde{m} + (\tilde{\pi}_z - p_z)\bar{z}$. Since $\mu = m + (\pi_z - p_z)\bar{z}$, we can make use of the previous result to write

$$\tilde{\mu} = \mu + (\tilde{\pi}_z - \pi_z)\bar{z} \simeq \mu + \left(1 - \frac{z_m}{k_{zz}} \right) (\tilde{m} - m) = \mu + \left(1 - \frac{\eta_z}{\varepsilon_{zz}} \frac{\pi_z}{\mu} \right) (\tilde{m} - m).$$

It is then easy to show that the normalised variances of $\tilde{\pi}_z$ and $\tilde{\mu}$, and their covariance, are given by (16)–(18).

Derivation of (19) Since $v_{p_z} = -v_m z$, differentiating through w.r.t. p_z gives

$$\begin{aligned} v_{p_z p_z} &= -v_{mp_z} z - v_m \frac{\partial z}{\partial p_z} = -v_{mp_z} z - v_m k_{zz} + v_m z_m z \\ &= v_{mm} z^2 + 2v_m z_m z - v_m k_{zz}, \end{aligned}$$

where the second equality follows from (2) and the third from (3). Multiplying through by π_z^2 then gives

$$\begin{aligned} v_{p_z p_z} \pi_z &= v_m \left(\frac{v_{mm}}{v_m} \mu^2 \left(\frac{\pi_z z}{\mu} \right)^2 + 2\pi_z z_m \pi_z z - k_{zz} \pi_z \right) \\ &= v_m \left(-RR\mu \left(\frac{\pi_z z}{\mu} \right)^2 + 2\mu \left(\frac{\pi_z z}{\mu} \right)^2 \eta_z - \mu \varepsilon_{zz} \frac{\pi_z z}{\mu} \right), \end{aligned}$$

where the second equality follows from the definitions of RR , η_z and ε_{zz} . Moving μ and the squared budget share of z ($w_z = (\pi_z z)/\mu$), outside brackets gives (19).

Derivation of (20) From (3) we get

$$v_{mp_z} = -v_{mm}z - v_m z_m = v_m \left(-\frac{v_{mm}}{v_m} z - z_m \right).$$

Multiplying through by $\pi_z \mu$ gives

$$v_{mp_z} \pi_z \mu = v_m \left(-\frac{v_{mm}}{v_m} \mu^2 \frac{\pi_z z}{\mu} - z_m \pi_z \mu \right) = v_m (RR\mu w_z - \eta_z w_z \mu),$$

where the second equality follows from the definitions of RR , η_z and ε_{zz} . Moving μ and w_z outside brackets gives (20).

Derivation of (24) Differentiating (13) with respect to m , gives

$$\begin{aligned} v_{mmm}^r &= v_{mm\pi_z} \frac{\partial \pi_z}{\partial m} + v_{mmm} \left(1 + \bar{z} \frac{\partial \pi_z}{\partial m} \right) - \left(v_{mm} - v_m z_m \frac{\partial \pi_z}{\partial m} \right) z_m \frac{\partial \pi_z}{\partial m} \\ &\quad - v_m \left[z_{mp_z} \frac{\partial \pi_z}{\partial m} + z_{mm} \left(1 + \bar{z} \frac{\partial \pi_z}{\partial m} \right) - v_m z_m \frac{\partial^2 \pi_z}{\partial m^2} \right]. \end{aligned} \quad (\text{A.1})$$

Totally differentiating (3) for π_z with respect to m , gives

$$v_{mm\pi_z} = -2v_{mm}z_m - v_m z_{mm} - v_{mmm}\bar{z}.$$

Then (A.1) can be written as (24) in the text.

The effects of constraints on durables on risk attitudes: alternative base levels for relative risk aversion and relative prudence.

Figure A1. $\gamma = 4.2$ ($RR^* = 4.04, RP^* = 5.06$): (a) $RR(p, m^* | \phi z^*)$ and $RP(p, m^* | \phi z^*)$; (b) $RR(p, \phi m^* | z^*)$ and $RP(p, \phi m^* | z^*)$.

Figure A2. RR (lower panel) and RP (upper panel) when labour supply is constrained at ϕL^* .

About the Author

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