Disappointment and the Optimal Insurance Contract

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This paper studies the optimal insurance contract under disappointment theory. We show that, when the individuals anticipate disappointment, there are two types of optimal insurance contract. The first type contains a deductible and a coinsurance above the deductible. We find that zero marginal cost is just a sufficient but not a necessary condition for a zero deductible. The second type has no deductible and the optimal insurance starts with full coverage for small losses and includes a coinsurance above an upper value of the full coverage.

The Geneva Risk and Insurance Review (2012) 37, 258–284. doi:10.1057/grir.2012.2; published online 31 July 2012

Keywords: disappointment; optimal insurance contract; deductible; coinsurance

Introduction

Since Borch¹, many researchers have studied the optimal risk sharing rules between individuals and insurance companies. A well-established result comes from Arrow's² findings. He showed that full coverage will be optimal when the insurance companies are risk neutral and the premium is actuarially fair. He also found that if the premium includes an insurance loading, then the optimal insurance contract will contain a deductible and a coinsurance above the deductible. Raviv³ adopted the assumption of risk-averse insurers, solved the optimal insurance contract, and found the necessary and sufficient conditions for the existence of a deductible. He showed that the deductible is positive if and only if the marginal cost of insurance is positive.

¹ Borch (1962).

² Arrow (1971).

³ Raviv (1979).

Following Raviv's³ paper, many researchers have studied optimal insurance contracts in alternative situations. For example, Schlesinger⁴ and Gollier and Schlesinger⁵ examined the optimal level of a deductible. Gollier and Breuer⁶ discussed the optimal insurance contract by relaxing the non-negativity constraint. They found the conditions for a contract with negative coverage being optimal. Huberman *et al.* and Garratt and Marshall⁷ respectively found that a contract with an upper-limit of indemnity could be optimal when the individual is able to file for bankruptcy and the insured has the option to convert the insured property. In addition, Gollier and Eeckhoudt *et al.*⁸ indicated that a fixed reimbursement contract will be optimal if the indemnity can only be contingent upon an approximate loss amount, while Huang and Tzeng⁹ found that a fixed reimbursement contract could be optimal if the insured commodities are irreplaceable. Furthermore, Dana and Scarsini¹⁰ discussed optimal risk sharing under the assumption that individuals face a non-insurable background risk in addition to an insurable risk. They found that the correlation between the non-insurable and insurable risk is crucial to the form of the optimal insurance contract.

Although the literature on the optimal insurance contract has provided many findings, most of them share a common assumption in that the insured's psychological emotion does not play any role in the decision under uncertainty. In this paper, we intend to relax this assumption by considering the disappointment effect. Specifically, this paper aims at studying the optimal insurance contract when the insured anticipates disappointment.

Disappointment theory was first introduced by Bell¹¹ and Loomes and Sugden.¹² It is a disutility caused by comparing an alternative outcome with the obtained outcome. Disappointment theory has been documented as an important emotion affecting the individual's decision under uncertainty.¹³ For example, Ang *et al.*¹⁴ found that the low degree of participation in the stock-market could be explained by disappointment aversion. Investors anticipate disappointment from the low return on the investment in risky assets and further

⁴ Schlesinger (1981).

⁵ Gollier and Schlesinger (1996).

⁶ Gollier (1987) and Breuer (2006).

⁷ Huberman *et al.* (1983) and Garratt and Marshall (1996).

⁸ Gollier (1996) and Eeckhoudt *et al.* (2003).

⁹ Huang and Tzeng (2006).

¹⁰ Dana and Scarsini (2007).

¹¹ Bell (1985).

¹² Loomes and Sugden (1986).

¹³ Loomes and Sugden (1987) have provided several experiments to test disappointment. Loomes (1988) further supplies the empirical evidence for disappointment.

¹⁴ Ang *et al.* (2005).

ask for an additional equity risk premium. Chan *et al.*¹⁵ found that an analyst will strategically adjust earnings forecasts to avoid earnings disappointment. Sonsino¹⁶ designed an internet auction for several basic gift certificates and found that disappointment plays a significant role in the bidding decision. Since disappointment is an important emotion when individuals face risk, it is worthwhile examining the optimal insurance contract under disappointment.

Disappointment describes the feeling that can be summed up by the words “I feel sorry that things are not as expected”. It has different features from regret, which was first proposed by Bell¹⁷ and Loomes and Sugden.¹⁸ Regret describes a disutility related to “I should have done something else”. If there is another action, rather than the chosen action, that can generalise a better (Bell¹⁷; Loomes and Sugden¹⁸) or the best (Braun and Muermann¹⁹) outcome from the *ex post* point of view, then the individual will regret not having chosen the foregone alternative. Let us use the demand for insurance as an example to distinguish regret from disappointment. Individuals would be disappointed when a loss is obtained regardless of what level the loss is and how much insurance they purchase. Individuals could be characterised by “I feel sorry that the loss occurred”. Regret, however, is different. If the loss is small, a better foregone alternative could be no insurance. Thus, the individual will regret having purchased some insurance. If a large loss results, the individual will regret not having purchased full coverage.

Disappointment can be set as a function of the difference in utility between the reference outcome and the realised outcome from the chosen lottery. The literature has proposed four different alternatives regarding the reference outcome in the disappointment function: the expected value (Bell¹¹; Loomes and Sugden¹²), the certainty equivalent of the chosen lottery (Gul²⁰), any other outcome (Delquie and Cillo²¹) and the best outcome (Laciana and Weber²²).²³ In this paper, we adopt Laciana and Weber’s setting²⁴ mainly based on

¹⁵ Chan *et al.* (2007).

¹⁶ Sonsino (2008).

¹⁷ Bell (1982).

¹⁸ Loomes and Sugden (1982).

¹⁹ Braun and Muermann (2004).

²⁰ Gul (1991).

²¹ Delquie and Cillo (2006).

²² Laciana and Weber (2008).

²³ In contrast to the other literature, the reference outcome with respect to disappointment in Gollier and Muermann (2010) is optimally chosen by the individual by trading off anticipatory utility against disappointment.

²⁴ Under this setting, the elation effect is not considered since elation is generated when the outcome of the chosen action is greater than the reference outcome. Note that Inman *et al.* (1997) empirically showed that disappointment effects strongly dominate elation effects. Our setting is a simplification of the models supported by Inman *et al.* (1997).

tractability. Laciara and Weber's model implies that the best outcome is no loss in the insurance setting. The reference point will be independent of the insurance indemnity function under a given premium. Thus, the analysis of the optimal insurance indemnity functional form under a given premium in this type of setting will involve less of a technical problem and be more tractable than the other reference settings, because in other settings the reference outcome is dependent on the choice of insurance coverage. Furthermore, Hart and Moore²⁵ provided a theoretical model and argued that parties in a trading relationship will compare the realised outcome with the best outcome generated by the contract. Fehr *et al.*²⁶ constructed an experiment based upon Hart and Moore's²⁵ model and found that parties in a trading relationship often feel disappointment for the realised outcome as long as there are better items available in the trading contract. Their experiments partially support the view that the best outcome could be the reference outcome while modelling disappointment.

As assumed in most papers, we assume that the indemnity function, which is expressed in terms of a loss, cannot be negative or greater than the amount of the loss (the principle of indemnity). We further adopt Raviv's³ setting whereby the insurance companies could be risk-averse. To handle the indemnity, the insurance companies will face a cost of insurance, which is an increasing convex function of indemnity. The optimal insurance contract is solved by maximising the utility function of the insured subject to the insurer's participating condition and the principle of indemnity. The major difference between our model and those in the literature is that we introduce the disappointment effect on the insured.

We find that there are two types of optimal insurance contracts. First, similar to the finding in Raviv,³ the optimal contract contains a deductible and coinsurance above the deductible. Furthermore, we discuss the condition for the existence of a deductible. Raviv³ found that a positive marginal cost is the necessary and sufficient condition for a non-zero deductible. We find that a marginal cost of zero is a sufficient but not necessary condition for a zero deductible.

Second, and more importantly, our paper further indicates that a contract with full coverage for small losses and a coinsurance for large losses could be an optimal insurance contract when individuals anticipate disappointment. The literature has demonstrated that deductible insurance or coinsurance would be an optimal insurance under different settings. In addition to these two popular contract forms, we identify a new form of optimal insurance contract. This type of contract can in practice be observed in some of the medical

²⁵ Hart and Moore (2008).

²⁶ Fehr *et al.* (2011).

insurance, for instance in the case where the medical insurance provides full coverage for small losses in a base plan and partial insurance above the base plan.²⁷

Both optimal contracts could involve coinsurance. The coinsurance rate will be generally increased (unaffected, decreased) by disappointment if the marginal disappointment function is increasing (constant, decreasing). Furthermore, regardless of whether the individual is characterised by a disappointment emotion or not, the optimal marginal indemnity equals one if the insurer is risk neutral and the marginal transaction cost is constant.

The remainder of this paper is organised as follows. The next section develops the setting of the model. The subsequent section examines the optimal insurance contract. The penultimate section provides numerical analyses. We show that a contract with full coverage for small losses and a coinsurance for large losses is socially-optimal. We also provide numerical analyses to show the effect of the intensity of disappointment on the optimal insurance contract. The last section concludes the paper.

Model setting

Assume that there exists one representative individual who has an endowment w and faces a random loss $x \in [0, L]$ following a loss distribution with probability density function $f(x)$. Further assume that the individual can purchase an insurance contract $(P, I(x))$, where P denotes the insurance premium and $I(x)$ is the indemnity function with $0 \leq I(x) \leq x$.

The utility function of the individual contains two parts. The first part is the utility of consumption, $u(w - x + I(x) - P)$, where $u(\cdot)$ is a strictly increasing and concave function as characterised by von Neumann-Morgenstern. The second part of the utility function consists of the disutility from disappointment. As suggested by Laciana and Weber²², we assume that the disutility from disappointment in each state arises from the difference between the highest level of wealth among all possible states and the level of wealth in the realised state. In our model, given an insurance contract, the best outcome among states for the individual would be no loss at all. Thus, the disappointment function D can be written as

$$D(u(w - P) - u(w - x + I(x) - P)),$$

²⁷ For example, the National Health Insurance in Taiwan fully reimburses patients for the medicine-related expenses if the medicines cost less than 100 NT dollars (about 3 US dollars). When the expenses are greater than 100 NT dollars, the coinsurance rate paid by the public insurance decreases as the medicine-related expenses increase.

where $D' \geq 0$. Therefore, the utility function of the insured can be written as

$$U = u(w - x + I(x) - P) - kD(u(w - P) - u(w - x + I(x) - P)),$$

where $k \geq 0$ denotes a linear weight of the disappointment function $D(\cdot)$ on the utility and measures the intensity of disappointment.

The value function of an insurer with initial capital s is assumed to be $v(s + P - I(x) - c(I(x)))$, where $v(\cdot)$ is a strictly increasing and concave function. $c(I(x))$ is the cost function with $c(0) = c_0$, $c' \geq 0$ and $c'' \geq 0$. In other words, the insurer's disappointment is not considered in our model.

As in Raviv's³ paper, the Pareto optimal insurance contract can be solved by the following model:

$$\begin{aligned} \text{Max}_{I(x), P} \quad EU &= \int_0^L [u(w - x + I(x) - P) - kD(u(w - P) \\ &\quad - u(w - x + I(x) - P))] f(x) dx \\ \text{s.t.} \quad Ev &= \int_0^L v(s + P - I(x) - c(I(x))) f(x) dx \geq v(s) \\ 0 &\leq I(x) \leq x. \end{aligned} \quad (1)$$

Before we start to solve the above optimisation problem, we need to further make an additional assumption to simplify our analysis. We assume that the individual's utility function is strictly concave in $I(x)$ for all x ,²⁸ that is,

$$\begin{aligned} \frac{\partial^2 U}{\partial P^2} &= u''(w - x + I(x) - P) \\ &\quad + k \left[-D''(u(w - P) - u(w - x + I(x) - P))(u'(w - x + I(x) - P))^2 \right. \\ &\quad \left. + D'(u(w - P) - u(w - x + I(x) - P))u''(w - x + I(x) - P) \right] < 0. \end{aligned} \quad (2)$$

Rearranging the above equation yields

$$\begin{aligned} R_u(w - x + I(x) - P) \\ + \frac{kD''(u(w - P) - u(w - x + I(x) - P))u'(w - x + I(x) - P)}{1 + kD'(u(w - P) - u(w - x + I(x) - P))} > 0, \end{aligned} \quad (3)$$

²⁸ The concavity in utility is a necessary condition for explaining the common consequence effect in the paper.

where R_u denotes the degree of Arrow-Pratt absolute risk aversion. Note that the above assumption places a restriction on the shape of the disappointment function. If the disappointment is linear as assumed in most of the papers in the literature or is convex, then the above condition will hold under concave utility and positively sloping disappointment function assumptions. If the disappointment function is concave as proposed by Laciana and Weber,²² then the above condition will restrict the concavity of the disappointment function.

Optimal insurance contract

To find the optimal insurance contract, we adopt the steps proposed by Raviv³ and followed by Gollier²⁹ and Zhou and Wu.³⁰ We first seek the possible optimal form of indemnity under a fixed premium, and then find the condition for the socially-optimal contract.

Optimal insurance indemnity under a fixed premium

The following proposition demonstrates the possible optimal insurance contract under a fixed premium.

Proposition 1 When P is fixed, the optimal insurance coverage could take one of the following two forms:

$$\begin{aligned} \text{Form 1 } I^*(x) &= 0 \text{ if } x \leq x_1 \\ &\in (0, x - x_1) \text{ otherwise,} \end{aligned}$$

$$\begin{aligned} \text{Form 2 } I^*(x) &= x \text{ if } x \leq x_2 \\ &\in (x_2, x) \text{ otherwise.} \end{aligned}$$

In both forms, in the range of partial insurance, $I^*(x)$ satisfies

$$I^{*'}(x) = \frac{R_u(\alpha) + \frac{kD''(\gamma)u'(\alpha)}{1+kD'(\gamma)}}{R_u(\alpha) + \frac{kD''(\gamma)u'(\alpha)}{1+kD'(\gamma)} + (1+c')R_v(\beta) + \frac{c''}{1+c'}}, \quad (4)$$

²⁹ Gollier (1987).

³⁰ Zhou and Wu (2008).

where $\alpha = w - x + I^*(x) - P$, $\beta = s + P - I^*(x) - c(I^*(x))$, $\gamma = u(w - P) - u(w - x + I^*(x) - P)$, $R_u(\alpha) = -u''(\alpha)/u'(\alpha)$ and $R_v(\beta) = -v''(\beta)/v'(\beta)$.

Proof Please see Appendix A. $\square$

By comparing our Proposition 1 with Raviv's (IPDP)³ Theorem 1, we can find that introducing disappointment will not change the possible form of the optimal indemnity function under a fixed premium. The possible form of the optimal indemnity function may either contain a deductible x_1 and coinsurance above the deductible, or else contain a full coverage with an upper amount x_2 and coinsurance above x_2 as shown in Figure 1. Note that when $x_1 = x_2 = 0$, the optimal insurance contract only contains coinsurance. If $x_1 = L$, then there is no insurance. If $x_2 = L$, then the optimal insurance contract is full coverage.

The optimal marginal indemnity in Eq. (4) is positive, that is, $I^{*'}(x) > 0$, from the assumption of Eq. (3). On the other hand, under the assumption that $R_v \geq 0$ and $c'' \geq 0$, we have $I^{*'}(x) \leq 1$. Thus, for an additional one dollar of loss, the individual will not obtain an indemnity of more than one dollar from the insurance company in the optimum. Regardless of whether the individual's preference includes disappointment or not, $I^{*'}(x) = 1$ if the insurer is risk neutral and c' is a constant.

Furthermore, the optimal marginal indemnity is affected by the disappointment function. Eq. (4) shows that whether or not disappointment causes an individual to transfer more risk to the insurer for an additional loss depends on the shape of the disappointment function. If the disappointment function

Figure 1. Under a fixed premium, the optimal indemnity function could be either Form 1 or Form 2.

is linear, that is, $D''=0$, then the marginal indemnity is the same as that in Raviv (IPDP).³ Transferring more risk to the insurance company will not reduce the marginal disutility from disappointment. Thus, there is no need for the individuals to modify the marginal indemnity under disappointment. In other words, although the individual anticipates disappointment, the marginal indemnity will not be affected as long as the marginal disappointment is a constant. If $D'' > 0$, the coinsurance rate is generally increased by disappointment. This is because an increase in the loss amount will increase the marginal disappointment. Transferring more risk to the insurance company will reduce the increase in disutility from disappointment. Thus, $I^*(x)$ will be greater in our model than in Raviv (IPDP).³ On the other hand, if $D'' < 0$, then the individual will transfer less risk to the insurance company in terms of marginal indemnity since the marginal disutility from disappointment is decreasing. Therefore, in this case, $I^*(x)$ will be smaller than in Raviv (IPDP).³

Socially-optimal insurance contract

In the previous section, we find that under a fixed insurance premium, the optimal coverage is either with Form 1 or Form 2. In this section, we would like to find out the optimal premium and thus complete the solution of the socially-optimal insurance contract.

As discussed in Raviv (IPDP),³ the insurer's participation constraint can help to determine whether the optimal contract contains x_1 or x_2 for a given P . Let $I_0^*(x)$ denote the solution of Eq. (4) with the boundary condition $I^*(0)=0$ under a given P . First, if the insurer's expected value function evaluated at that premium and the $I_0^*(x)$ is equal to $v(s)$, that is, $Ev(P, I_0^*(x))=v(s)$, then the optimal contract will be $x_1=x_2=0$. Let $(P_0, I_0^*(x))$ denote the corresponding premium and insurance indemnity. Second, if $Ev(P, I_0^*(x)) < v(s)$, then the optimal indemnity should be decreased to increase the insurer's value. Owing to the constraint that $I^*(x) \in [0, x]$, the optimal indemnity in this case will be with Form 1. Let $(P_1, I_1^*(x))$ denote the corresponding contract. Third, if $Ev(P, I_0^*(x)) > v(s)$, then the optimal indemnity should be increased. Thus the optimal indemnity should be with Form 2. We use $(P_2, I_2^*(x))$ to denote the corresponding contract. Note that in the second and third cases above, $I_i^*(x)$ is equal to $I_0^*(x)$ and P_i is equal to P_0 when $x_i=0$, $i=1, 2$.

The socially-optimal insurance contract is the dominant contract among $(P_0, I_0^*(x))$, $(P_1, I_1^*(x))$ and $(P_2, I_2^*(x))$.³¹ Thus, we can find the socially-optimal

³¹ In other words, if we find that $x_1=x_2=0$ can make the social welfare reach the maximum, then $(P_0, I_0^*(x))$ is the socially-optimal contract. Thus, if we find that $x_i > 0$ and $x_j=0$ can maximize social welfare, then $(P_i, I_i^*(x))$ is the socially-optimal contract, $i, j=1, 2$ and $i \neq j$.

insurance contract in the following. In Proposition 2, we focus on the case where $R_v=0$ and $c''(0)=0$.³² Furthermore, in Proposition 3, we then derive the optimal contract for the cases where $R_v > 0$ or $c''(0) > 0$.

Proposition 2 If $R_v=0$ and $c''(0)=0$, then the socially-optimal insurance contract is as follows:

1. the socially-optimal insurance contract contains a positive deductible, that is, $x_1^* > 0$, and $I^*(x)$ above x_1^* follows Eq. (4) if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx > 0. \quad (5)$$

2. the socially-optimal insurance contract is with $x_1^*=x_2^*=0$ and $I^*(x)$ follows Eq. (4) if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx \leq 0. \quad (6)$$

Note that α_0 and γ_0 denote the values of α and γ evaluated at $x_1^*=x_2^*=0$, respectively.

Proof Please see Appendix B. $\square$

Proposition 2 predicts that, under disappointment, the socially-optimal insurance contract will contain a non-negative deductible and a coinsurance above the deductible when the insurer is risk neutral and $c''(0)=0$. The intuition of the proof of Proposition 2 is as follows. Let us start from $x_1=0$ and $x_2=0$, that is, a contract with coinsurance only.

From the insurer's participation constraint, we know that the insurer's marginal rate of substitution of P for x_1 (or x_2) is $1 + c'$. Taking into

³² Note that c' being constant is a sufficient condition for $c''(0)=0$ but not a necessary one.

consideration the insurer's marginal rate of substitution, an increase in x_i will increase the insured's traditional utility by

$$\begin{aligned} & \int_0^L (-P_i') u'(\alpha_0) \left[1 - \frac{1}{1 + c'(I_0^*(x))} \right] f(x) dx \\ &= \int_0^L (-P_i') u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} f(x) dx, \end{aligned} \quad (7)$$

and will increase the insured's disutility from disappointment by

$$\begin{aligned} & \int_0^L (-P_i') k D'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \left[1 - \frac{1}{1 + c'(I_0^*(x))} \right] \right] f(x) dx \\ &= \int_0^L (-P_i') k D'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] f(x) dx, \end{aligned} \quad (8)$$

where P_i' denotes the derivative of P_i with respect to x_i , $i=1, 2$. Thus, whether the socially-optimal insurance contract contains a positive x_1 or x_2 or not is determined by the combination of Eqs. (7) and (8), that is,

$$\int_0^L (-P_i') \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - k D'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx. \quad (9)$$

As shown in Appendix B, $P_2'=0$ at $x_2=0$ under the assumptions $R_v=0$ and $c''(0)=0$. In other words, an increase in x_2 will not increase the individual's expected utility. It is obvious that the individual will choose $x_2=0$. On the other hand, we have $P_1' < 0$. Thus the individual will have a higher level of expected utility by choosing a positive deductible if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - k D'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx > 0.$$

When the above integration is zero or negative, the individual will choose a zero deductible.

An interesting case is that where the insurer is risk neutral and $c'=\theta$, where θ is a constant. This case will demonstrate that $c'=0$ is only a sufficient but not

a necessary condition for a zero deductible. Under this case, Proposition 2 shows that the socially-optimal insurance contract is

$$I^*(x) = \max\{0, x - x_1^*\}, \quad (10)$$

where $x_1^* \geq 0$. The deductible x_1^* is zero when

$$\int_0^L \left\{ u'(\alpha_0) \frac{\theta}{1+\theta} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{\theta}{1+\theta} \right] \right\} f(x) dx \leq 0. \quad (11)$$

Note that α_0 and γ_0 are the values of α and γ evaluated at $x_1^* = x_2^* = 0$, respectively. From Eq. (10), we know that $I_0^*(x) = x$ in this case. Thus, we have

$$\alpha_0 = w - x + I_0^*(x) - P_0 = w - P_0,$$

and

$$\begin{aligned} \gamma_0 &= u(w - P_0) - u(w - x + I_0^*(x) - P_0) \\ &= u(w - P_0) - u(w - P_0) = 0. \end{aligned}$$

Thus, condition (11) can be rewritten as

$$\int_0^L \left\{ u'(w - P_0) \frac{\theta}{1+\theta} - kD'(0) \left[u'(w - P_0) - u'(w - P_0) \frac{\theta}{1+\theta} \right] \right\} f(x) dx \leq 0. \quad (12)$$

Within the integration, all terms are constant with respect to x . The above condition can be rewritten as

$$u'(w - P_0) \frac{\theta}{1+\theta} - kD'(0) \frac{1}{1+\theta} u'(w - P_0) \leq 0. \quad (13)$$

Since $u' > 0$ and $1 + \theta > 0$, rearranging the above condition yields

$$\theta \leq kD'(0). \quad (14)$$

In other words, the socially-optimal insurance will be full coverage (i.e., with a zero deductible) when the insurer is risk neutral and the marginal cost function is a constant θ in the range $[0, kD'(0)]$. In summary, Proposition 2 predicts that a zero deductible could still be optimal if $c' = \theta > 0$ and is small,

whereas Raviv (IPDP)³³ has indicated that $c'=0$ is a necessary and sufficient condition for the zero deductible.³³

Proposition 3 If either $R_v > 0$ or $c''(0) > 0$, then the socially-optimal insurance contract is as follows:

1. the socially-optimal insurance contract contains a positive deductible, that is, $x_1^* > 0$, and $I^*(x)$ above x_1^* follows Eq. (4) if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx > 0. \quad (15)$$

2. the socially-optimal insurance contract contains full coverage for $x \leq x_2^*$, $x_2^* > 0$, and $I^*(x)$ above x_2^* follows Eq. (4) if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx < 0. \quad (16)$$

3. the socially-optimal insurance contract is with $x_1^* = x_2^* = 0$ and $I^*(x)$ follows Eq. (4) if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx = 0. \quad (17)$$

³³ We thank the referee for providing another interesting example showing that Eq. (4) could suggest that the optimal contract contains full coverage for small losses and partial coverage for large losses if $c''(0)=0$ and $R_v=0$. Assume that the cost function is of the form

$$c(I) = \begin{cases} 0 & \text{for } I \leq \bar{I} \\ c_1(I) & \text{for } I > \bar{I} \end{cases}$$

with $c_1(\bar{I})=0$, $c_1'(I) > 0$, and $\bar{I} > 0$. Furthermore, let us assume that k is large enough such that Eq.(6) holds. Thus, Proposition 2 predicts that the socially-optimal insurance contract will have a zero deductible and follow Eq. (4) since $R_v=0$, $c''(0)=0$ and Eq. (6) holds. In this case, Eq. (4) implies that

$$I(x) = \begin{cases} x & \text{for } x \leq \bar{I} \\ I_1^*(x) & \text{for } x > \bar{I} \end{cases}$$

with $I_1^*(x) < 1$.

Proof. Please see Appendix C. $\square$

Proposition 3 shows that in addition to deductible and coinsurance contracts, Form 2 can be optimal if $R_v > 0$ or $c''(0) > 0$. In Raviv's (IPDP)³ setting, Form 2 is dominated by a contract with a deductible. In our setting, the disappointment effect makes it an optimal contract in certain cases. The intuition of Proposition 3 is similar to that of Proposition 2. The main difference is that, under $R_v > 0$ or $c''(0) > 0$, an increase in x_2 will affect the insurance premium and the individual's expected utility will be increased by increasing x_2 when condition (16) holds.³⁴

Propositions 2 and 3 indicate that our findings include Raviv (IPDP)³ as a special case. When $k=0$, our setting is the same as in Raviv's (IPDP)³ model. In that case, the condition in Propositions 2 and 3 becomes

$$\int_0^L \left\{ u(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right\} f(x) dx \geq 0.$$

A contract with a deductible will dominate other forms of contract as concluded by Raviv (IPDP).³ Note that Form 2 could be reduced to full coverage when $x_2^* > L$. In the following section, we will provide numerical analyses for Form 2 with $x_2^* < L$ as the optimal insurance contract.

Numerical analyses

Since Form 2 is the new optimal contract identified by our paper, we will provide examples for it. We will further demonstrate the effect of k on the socially-optimal insurance contract. Furthermore, because $R_v=0$ is commonly assumed in the literature, all of our numerical analyses in this section are under the risk-neutral insurer assumption.

First of all, let us illustrate that Form 2 is optimal and $x_2^* < L$. From the above analyses, we know that the Form 2 contract might be optimal when

³⁴ The reason why Form 2 can be optimal is that disappointment might lead individuals to demand more indemnity for all levels of loss. Since the optimal marginal indemnity function is less than one, individuals might demand over-insurance for small losses. However, the upper limit constraint of the indemnity function limits the possibility of over-insurance. Thus, we would observe a contract containing full coverage for small losses and partial coverage for large losses as being optimal.

either $R_v > 0$ or $c''(0) > 0$. Since we have assumed that the insurer is risk neutral, that is, $R_v = 0$, we assume that the cost function is quadratic, that is, $c(I) = \theta_1 I + \theta_2 I^2$, $\theta_i > 0$. In this case, $c''(0) = 2\theta_2 > 0$. To simplify the calibration, we assume that $R_u(x) = 1$, $D(\gamma) = 0.5\gamma$ and $k = 1$. Furthermore, we assume that the loss x follows a beta distribution with parameters 1 and 3.³⁵ Thus, x is within the range of 0 and 1, and follows the probability density function

$$f(x) = \frac{(1-x)^2}{\int_0^1 (1-t)^2 dt}.$$

By setting $\theta_1 = 0.1$ and $\theta_2 = 0.5$, we find that Form 2 is socially-optimal. As shown in Panel A of Figure 2, we have $x_2^* = 0.363$, which is smaller than the maximum loss amount, 1. When θ_1 increases from 0.1 to 0.2, Form 2 is still socially-optimal, and the increase in θ_1 causes x_2^* to decrease to 0.215 as shown in Panel B of Figure 2. When $\theta_1 = 0.1$ and θ_2 decreases from 0.5 to 0.3, Panel C of Figure 2 demonstrates that the socially-optimal contract is with Form 2, and the decrease in θ_2 causes x_2^* to increase to 0.665.³⁶

Now, let us focus on the effect of k on the optimal contract. Assume $c(I) = 0.1I + 0.5I^2$, $R_u(x) = 1$, $D(\gamma) = 0.5\gamma$ and x follows a beta distribution with parameters 1 and 3. Let k vary from zero to 1.2. Panel A of Figure 3 shows the relationship between k and the conditions in Proposition 3. Let

$$\Delta = \int_0^L \left\{ u'(x_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(x_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx. \quad (18)$$

In Panel A of Figure 3, we find that Δ decreases in k . The optimal contract is with Form 1 when $k < 0.548$, and it is with Form 2 when $k > 0.548$. The optimal insurance contract only contains coinsurance when $k = 0.548$. This numerical example illustrates that an increase in k could shift the optimal contract from Form 1 to Form 2. Panel B in Figure 3 further shows that $x_2^* = 0$ when $k = 0.548$ and $x_2^* \in (0, 1)$ when $k > 0.548$.³⁷

³⁵ The reason why we choose the beta distribution is that it is a family of continuous probability distributions defined on the interval (0, 1). A beta distribution with parameters 1 and 3 means that the probability of having small losses is large and the density function is strictly convex.

³⁶ For these three cases, the optimal indemnity functions for $x > x_2^*$ are increasing and convex although they look like straight lines in Figure 2.

³⁷ Note that it is not necessary to obtain a Form 2 contract as optimal when k increases. In our simulations, we find some cases where Δ is always positive and increasing in k .

Figure 2. The socially-optimal contract under $R_v=0$, $R_u=1$, $k=1$ and $D'=0.5$. In Panel A, the cost function is $c(I)=0.1 \cdot I + 0.5 \cdot I^2$ and $x_2^*=0.363$. In Panel B, the cost function is $c(I)=0.2 \cdot I + 0.5 \cdot I^2$ and $x_2^*=0.215$. In Panel C, the cost function is $c(I)=0.1 \cdot I + 0.3 \cdot I^2$ and $x_2^*=0.665$.

Figure 3. The effect of k on the socially-optimal contract under the assumptions that $R_v=0$, $c(I)=0.1I+0.5I^2$, $R_u(\alpha)=1$, $D(\gamma)=0.5\gamma$ and x follows a beta distribution with parameters 1 and 3. Panel A shows the relationship between k and the conditions in Proposition 3, Δ . $\Delta=0$ when $k=0.548$. Panel B shows the relationship between k and x_2^* .

Conclusions

This paper studies the optimal insurance contract when the individual anticipates disappointment. We adopt all the assumptions in Raviv (IPDP)³ except that we consider the disappointment effect in the individual's preference. The disappointment function is modelled as a disutility caused by comparing the best outcome with the obtained outcome.

This study is related to two important fields in the literature. The first one shares our main goal of examining the optimal insurance contract. We have found that the form of the optimal insurance contract under disappointment could be quite different from that found by Raviv (IPDP).³ The optimal insurance contract may be a contract with a deductible and a coinsurance above the deductible or with full coverage for small losses and partial coverage for large losses. Second, our paper is related to the recent literature that investigates the

impact of the individual's anticipatory emotion on the decision under risk. We have shown that disappointment plays an important role when we are seeking the optimal risk sharing rules between individuals and insurance companies.

Note that, mainly due to tractability, we choose the best outcome as the reference outcome in our model to evaluate the disutility of disappointment. Therefore, a future study employing other reference outcomes to measure disappointment could be fruitful and deserves further attention.

Acknowledgements

The authors express their deep gratitude to Keith Crocker and the referee for detailed and valuable comments.

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Appendix A

Proof of Proposition 1

Our proof follows the proofs in Raviv (IPDP).³ The Hamiltonian function of problem (1) is

$$H = [u(w - x + I(x) - P) - kD(u(w - P) - u(w - x + I(x) - P))]f(x) + \lambda[v(s + P - I(x) - c(I(x))) - v(s)]f(x),$$

where λ is a constant with respect to x . The necessary conditions are

1. $0 < I^*(x) < x$, if

$$[1 + kD'(u(w - P) - u(w - x + I^*(x) - P))]u'(w - x + I^*(x) - P) - \lambda v'(s + P - I^*(x) - c(I^*(x)))(1 + c'(I^*(x))) = 0. \quad (\text{A.1})$$

2. $I^*(x)=0$, if

$$\mathbf{J} = [1 + kD'(u(w - P) - u(w - x - P))]u'(w - x - P) - \lambda v'(s + P - c_0)(1 + c'(0)) \leq 0. \quad (\text{A.2})$$

3. $I^*(x)=x$, if

$$\mathbf{K} = [1 + kD'(0)]u'(w - P) - \lambda v'(s + P - x - c(x))(1 + c'(x)) \geq 0. \quad (\text{A.3})$$

Rearranging Eq. (A.1) yields

$$\lambda = \frac{[1 + kD'(u(w - P) - u(w - x + I^*(x) - P))]u'(w - x + I^*(x) - P)}{v'(s + P - I^*(x) - c(I^*(x)))(1 + c'(I^*(x)))}. \quad (\text{A.4})$$

Let $\alpha = w - x + I^*(x) - P$, $\beta = s + P - I^*(x) - c(I^*(x))$, and $\gamma = u(w - P) - u(w - x + I^*(x) - P)$. Differentiating Eq. (A.4) with respect to x and then rearranging the result yields

$$I^{*'}(x) = \frac{R_u(\alpha) + \frac{kD''(\gamma)u'(\alpha)}{1 + kD'(\gamma)}}{R_u(\alpha) + \frac{kD''(\gamma)u'(\alpha)}{1 + kD'(\gamma)} + (1 + c')R_v(\beta) + \frac{c''}{1 + c'}}, \quad (\text{A.5})$$

where $R_u(\alpha) = -u''(\alpha)/u'(\alpha)$ and $R_v(\beta) = -v''(\beta)/v'(\beta)$.

Note that under the assumption of Eq. (2), we have

$$\begin{aligned} \frac{d\mathbf{J}}{dx} &= kD''(u(w - P) - u(w - x - P))[u'(w - x - P)]^2 \\ &\quad - [1 + kD'(u(w - P) - u(w - x - P))]u''(w - x - P) > 0, \end{aligned}$$

and

$$\frac{d\mathbf{K}}{dx} = \lambda v''(s + P - x - c(x))(1 + c'(x))^2 - \lambda v'(s + P - x - c(x))c''(x) < 0.$$

Let

$$\mathbf{L} = [1 + kD'(0)]u'(w - P) - \lambda v'(s + P - c_0)(1 + c'(0)).$$

If $\mathbf{L} \geq 0$, then $\mathbf{J} > 0$, which means that Eq. (A.2) cannot be obtained for $x > 0$. On the other hand, if $\mathbf{L} < 0$, then $\mathbf{K} < 0$, that is, Eq. (A.3) cannot be

obtained for $x > 0$. Thus, conditions (A.2) and (A.3) cannot be satisfied simultaneously.

Furthermore, define x_1 and x_2 to respectively satisfy the following equations:

$$[1 + kD'(u(w - P) - u(w - x_1 - P))]u'(w - x_1 - P) - \lambda v'(s + P - c_0)(1 + c'(0)) = 0, \quad (\text{A.6})$$

and

$$[1 + kD'(0)]u'(w - P) - \lambda v'(s + P - x_2 - c(x_2))(1 + c'(x_2)) = 0. \quad (\text{A.7})$$

Since $\mathbf{J}(\mathbf{K})$ is a strictly increasing (decreasing) function of x , x_1 and x_2 can be defined uniquely.

Therefore, the optimal insurance contract will take one of the following forms:

$$\begin{aligned} \text{Form 1 } I^*(x) &= 0 \text{ if } x \leq x_1 \\ &\in (0, x - x_1) \text{ otherwise,} \end{aligned}$$

$$\begin{aligned} \text{Form 2 } I^*(x) &= x \text{ if } x \leq x_2 \\ &\in (x_2, x_1) \text{ otherwise.} \end{aligned}$$

In both forms, for the range $0 < I^*(x) < x$, $I^*(x)$ satisfies Eq. (A.5).

Appendix B

Proof of Proposition 2

The socially-optimal insurance contract will maximize the social welfare (SW) function

$$SW = EU + \lambda E_v.$$

Since there are only two possible contract forms and these two forms become identical when $x_1 = x_2 = 0$, we will first follow the proof in Blazenko³⁸ to find out the if and only if conditions for the positive optimal levels of x_1 and x_2 , respectively. Then, we will compare the social welfare levels under the two forms and conclude the proof.

³⁸ Blazenko (1985).

First, rearranging the total differential of Eq. (A.6) yields

$$\begin{aligned} P'_1 &= \frac{dP_1}{dx_1} \\ &= - \frac{\frac{kD''(u(w-P_1)-u(w-x_1-P_1))u'(w-x_1-P_1)}{1+kD'(u(w-P_1)-u(w-x_1-P_1))} + R_u(w-x_1-P_1)}{\frac{kD''(u(w-P_1)-u(w-x_1-P_1))u'(w-x_1-P_1)}{1+kD'(u(w-P_1)-u(w-x_1-P_1))} + R_u(w-x_1-P_1) + R_v(s+P_1+c_0)} \\ &< 0. \end{aligned}$$

Let

$$\underline{x} = w - x + I_1^*(x) - P_1,$$

$$\underline{\beta} = s + P_1 - I_1^*(x) - c(I_1^*(x)),$$

and

$$\underline{\gamma} = u(w - P_1) - u(w - x + I_1^*(x) - P_1).$$

The individual's expected utility with a deductible policy is

$$\begin{aligned} EU(x_1) &= \int_0^{x_1} [u(w - x - P_1) - kD(u(w - P_1) - u(w - x - P_1))]f(x)dx \\ &\quad + \int_{x_1}^L [u(\underline{x}) - kD(\underline{\gamma})]f(x)dx. \end{aligned}$$

The derivative of the above equation with respect to x_1 is

$$\begin{aligned} &- P'_1 \int_0^{x_1} [1 + kD'(u(w - P_1) - u(w - x - P_1))]u'(w - x - P_1)f(x)dx \\ &+ P'_1 \int_0^{x_1} kD'(u(w - P_1) - u(w - x - P_1))u'(w - P_1)f(x)dx \\ &+ \int_{x_1}^L \left[\frac{\partial I_1^*(x)}{\partial x_1} - P'_1 \right] [1 + kD'(\underline{\gamma})] u'(\underline{x})f(x)dx \\ &+ \int_{x_1}^L P'_1 kD'(\underline{\gamma}) u'(w - P_1)f(x)dx. \end{aligned}$$

On the other hand, the insurer's expected value function is

$$Ev = \int_0^{x_1} v(s + P_1 - c_0)f(x)dx + \int_{x_1}^L v(\underline{\beta})f(x)dx.$$

The derivative of the above equation with respect to x_1 is

$$\int_0^{x_1} P'_1 v'(s + P_1 - c_0)f(x)dx - \int_{x_1}^L \left[(1 + c'(I_1^*(x))) \frac{\partial I_1^*(x)}{\partial x_1} - P'_1 \right] v'(\underline{\beta})f(x)dx.$$

Thus, evaluating the first derivative of the maximum of the problem $EU + \lambda Ev$ with respect to x_1 at $x_1=0$ yields

$$\begin{aligned} \left. \frac{\partial SW}{\partial x_1} \right|_{x_1=0} &= \int_0^L \left[\frac{\partial I_1^*(x)}{\partial x_1} - P'_1 \right] [1 + kD'(\gamma_0)]u'(\alpha_0)f(x)dx \\ &\quad + \int_0^L P'_1 kD'(\gamma_0)u'(w - P_0)f(x)dx \\ &\quad - \lambda \int_0^L \left[(1 + c'(I_0^*(x))) \frac{\partial I_1^*(x)}{\partial x_1} - P'_1 \right] v'(\beta_0)f(x)dx \\ &= \int_0^L \frac{\partial I_1^*(x)}{\partial x_1} \{ [1 + kD'(\gamma_0)]u'(\alpha_0) - \lambda v'(\beta_0)(1 + c'(I_0^*(x))) \} f(x)dx \\ &\quad - \int_0^L P'_1 \{ [1 + kD'(\gamma_0)]u'(\alpha_0) - \lambda v'(\beta_0) \} f(x)dx \\ &\quad + \int_0^L P'_1 kD'(\gamma_0)u'(w - P_0)f(x)dx, \end{aligned}$$

where α_0 , β_0 and γ_0 respectively denote the values of $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$ evaluated at $x_1=0$. Eq. (A.1) ensures that the first term in the above equation is zero. Rearranging the above equation yields

$$\left. \frac{\partial SW}{\partial x_1} \right|_{x_1=0} = \int_0^L (-P'_1) \{ u'(\alpha_0) - kD'(\gamma_0)[u'(w - P_0) - u'(\alpha_0)] + \lambda P'_1 v'(\beta_0) \} f(x)dx.$$

Substituting Eq. (A.4) into the above equation yields

$$\begin{aligned} & \left. \frac{\partial SW'}{\partial x_1} \right|_{x_1=0} \\ &= \int_0^L (-P'_1) \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx. \end{aligned}$$

Since $P'_1 < 0$ and the second-order condition holds, it is obvious that $x_1^* \geq 0$ if and only if

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx \geq 0. \quad (\text{B.1})$$

Second, from Eq. (A.7), we have

$$P'_2 = \frac{dP_2}{dx_2} = \frac{(1 + c'(x_2))R_v(s + P_2 - x_2) + \frac{c''(x_2)}{1 + c'(x_2)}}{R_u(w - P_2) + R_v(s + P_2 - x_2)} \geq 0. \quad (\text{B.2})$$

Let

$$\alpha(x_2) = w - x + I_2^*(x) - P_2,$$

$$\beta(x_2) = s + P_2 - I_2^*(x) - c(x_2),$$

and

$$\gamma(x_2) = u(w - P_2) - u(w - x + I_2^*(x) - P_2).$$

The individual's expected utility with the Form 2 policy is

$$EU(x_2) = \int_0^{x_2} u(w - P_2) f(x) dx + \int_{x_2}^L [u(\alpha(x_2)) - kD(\gamma(x_2))] f(x) dx.$$

The derivative of the above equation with respect to x_2 is

$$\begin{aligned} & -P'_2 \int_0^{x_2} u'(w - P_2) f(x) dx \\ & + \int_{x_2}^L \left[\frac{\partial I_2^*(x)}{\partial x_2} - P'_2 \right] [1 + kD'(\gamma(x_2))] u'(\alpha(x_2))] f(x) dx \\ & + \int_{x_2}^L P'_2 kD'(\gamma(x_2)) u'(w - P_2) f(x) dx. \end{aligned}$$

On the other hand, the insurer's expected value function is

$$Ev = \int_0^{x_2} v(s + P_2 - x - c(x))f(x)dx + \int_{x_2}^L v(\beta(x_2))f(x)dx.$$

The derivative of the above equation with respect to x_2 is

$$\begin{aligned} & \int_0^{x_2} P_2' v'(s + P_2 - x - c(x))f(x)dx \\ & - \int_{x_2}^L \left[(1 + c'(I_2^*(x))) \frac{\partial I_2^*(x)}{\partial x_2} - P_2' \right] v'(\beta(x_2))f(x)dx. \end{aligned}$$

Thus, evaluating the first derivative of the maximum problem $EU + \lambda Ev$ with respect to x_2 at $x_2=0$ yields

$$\begin{aligned} \left. \frac{\partial SW}{\partial x_2} \right|_{x_2=0} &= \int_0^L \left[\frac{\partial I_2^*(x)}{\partial x_2} - P_2' \right] [1 + kD'(\gamma_0)]u'(\alpha_0)f(x)dx \\ &+ \int_0^L P_2' kD'(\gamma_0)u'(w - P_0)f(x)dx \\ &- \lambda \int_0^L \left[(1 + c'(I_0^*(x))) \frac{\partial I_2^*(x)}{\partial x_2} - P_2' \right] v'(\beta_0)f(x)dx \\ &= \int_0^L \frac{\partial I_2^*(x)}{\partial x_2} \{ [1 + kD'(\gamma_0)]u'(\alpha_0) - \lambda v'(\beta_0)(1 + c'(I_0^*(x))) \} f(x)dx \\ &- \int_0^L P_2' \{ [1 + kD'(\gamma_0)]u'(\alpha_0) - \lambda v'(\beta_0) \} f(x)dx \\ &+ \int_0^L P_2' kD'(\gamma_0)u'(w - P_0)f(x)dx. \end{aligned}$$

Eq. (A.1) ensures that the first term in the above equation is zero. Substituting Eq. (A.4) into the above equation yields

$$\begin{aligned} & \left. \frac{\partial SW}{\partial x_2} \right|_{x_2=0} \\ &= \int_0^L (-P_2') \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx. \end{aligned}$$

From Eq. (B.2), we know that P_2' evaluated at $x_2=0$ is 0 when $R_v=0$ and $c''(0)=0$. Since the second-order condition holds, we have $x_2^*=0$ if and only if either P_2' evaluated at $x_2=0$ is 0 or

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx = 0. \quad (\text{B.3})$$

Since $P_2' \geq 0$, we have $x_2^* > 0$ if and only if P_2' evaluated at $x_2=0$ is positive and

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx < 0. \quad (\text{B.4})$$

Socially-optimal insurance contract

From the above analyses, we know that $x_2^*=0$ when $R_v=0$ and $c''(0)=0$. In other words, a Form 2 contract will be reduced to a coinsurance contract. Thus, the socially-optimal insurance will contain a non-negative deductible and coinsurance above the deductible. The above analyses have demonstrated that the necessary and sufficient condition for a positive deductible is

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx > 0. \quad (\text{B.5})$$

This concludes Proposition 2.

Appendix C

Proof of Proposition 3

When $R_v > 0$ or $c''(0)=0$, the analyses in Appendix B indicate that x_2^* is positive if and only if condition (B.4) holds. From conditions (B.1) and (B.4), we could obtain that $x_1^* > 0$ when

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx > 0; \quad (C.1)$$

$x_2^* > 0$ when

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx < 0; \quad (C.2)$$

and $x_1^* = x_2^* = 0$ when

$$\int_0^L \left\{ u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} - kD'(\gamma_0) \left[u'(w - P_0) - u'(\alpha_0) \frac{c'(I_0^*(x))}{1 + c'(I_0^*(x))} \right] \right\} f(x) dx = 0. \quad (C.3)$$

This concludes Proposition 3.

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