Risk Measures and Capital Requirements: A Critique of the Solvency II Approach

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In this paper the Solvency II VaR-based capital requirement is analysed and discussed. The new European risk-based system of prudential regulation for insurers could in fact increase, and not decrease, the fragility of the insurance industry. More specifically, the VaR capital requirement exposes insurance companies to a potentially huge systemic effect, as the bigger/better diversified insurers have high default probabilities in case of market shortfalls. This paper shall suggest and discuss some adjustments to the current Solvency II framework.

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Introduction

The insurance industry has only been marginally impacted by the financial crisis and the few insurers that experienced serious difficulties were brought down by non-insurance activities. The relative strengthening of the insurance sector compared with the systemic crisis of the banking system is due to different factors. Among these, life insurance products often shift investment risks to the policyholder and in non-life insurance products, the insurer assumes diversifiable risk quite exclusively. A common key factor is less exposure to systematic risk that increases the insurance system's resilience to a global systemic crisis.

Furthermore, the Solvency II European Directive (2009/138/EC) is to introduce a risk-based approach to capital requirements as from 2015 or 2016. More specifically, the directive prescribes that the Solvency Capital Requirement "shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5 per cent over a one-year period". Apparently, the new Solvency II framework is a step towards a more resilient insurance system seeing that the principle of more risk/more capital provides the right incentives to better risk management practices while lowering the risk-reward profile.

Unfortunately, as this paper demonstrates, this is not the case. The Solvency II framework could increase the insurance system's probability to be involved in a global crisis, thus contributing to the crisis itself. Even if the principles and the overall Solvency II structure are economically viable, they fail when a wrong risk measure—the Value-at-Risk—is applied to the correct principle of more risk/more capital.

The simple model developed in this paper demonstrates that the Value-at-Risk measure is incorrect, simply because it is a total risk measure. Nonetheless, the financial economics literature reminds us that there are two sharply different types of risk: systematic risk and diversifiable risk. Broadly speaking, two insurance companies with the same total risk but with different compositions between systematic risk and diversifiable risk have, according to the Solvency II directives, the same Solvency Capital Requirement. The more diversified the company is (i.e. the more exposed to systematic risk), the more exposed to market shortfalls it is. Therefore, in the case of a global crisis, it has a higher probability for bankruptcy. Under the Solvency II framework, there are clear incentives for diversification, growth in size (which implies more diversification) and systematic risk assumption. Under the Solvency II regimes, the bigger/better diversified insurance companies could have a higher probability to be involved in the next global crisis.

Related literature and outline of the paper

This paper contributes to the insurance economics literature, exploiting the relationship between risk-reward profile, default probability and systematic vs. diversifiable risk, in a VaR capital requirement framework. Even if it is based on a very simple theoretical model, this paper exposes a large pitfall in the Solvency II Directives—which is also relevant for similar regulatory systems, such as the Swiss Solvency Tests (SST), where a Tail-VaR capital requirement is used—and suggests some means to correcting it.

The financial economics literature has only recently indicated the drawbacks of diversification for financial institutions.¹ Wagner² noted that “even though diversification reduces each institution’s individual probability of failure, it makes systemic crises more likely”. In this paper, results are shown that could also be extended to the insurance sector. In addition, within the Solvency II framework a marked inducement to diversification emerges and this makes the insurance sector more vulnerable to systemic risk.³

The new regulatory framework for insurance companies has been widely discussed and sometimes criticized.⁴ Hereafter, a new specific and original critique of the Solvency II framework emerges. The Solvency II regime uses an inadequate risk measure to compute the Solvency Capital Requirement.

¹ Goldstein and Pauzner (2004); Allen and Gale (2005); Allen and Carletti (2006); Ibragimov *et al.* (2011).

² Wagner (2010).

³ The term “systemic risk” has been widely used with different meanings. Often, only a micro-systemic risk definition is considered, for example, Csiszar (2002) (“the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default, with the chain reaction leading to broader financial difficulties”). However, Baluch *et al.* (2011) also include the macro-systemic risk, that is, the risk that “a single exogenous event leads to simultaneous adverse effects on multiple entities”. Hereinafter, only the macro-systemic risk has been considered in the model, since a single exogenous event, a “market shortfall”, could negatively affect a large part of insurers. However, the model predicts that the bigger/better diversified insurance companies are more exposed to “market shortfall” and, as a consequence, the micro-systemic risk could also be relevant, even if not explicitly addressed in the model.

⁴ Cf. among others Eling *et al.* (2007); Doff (2008); Eling and Schmeiser (2010); Ashby (2011).

The VaR measure is often criticized since only in a normally distributed world it has desirable properties such as subadditivity.⁵ Additionally, it does not adequately consider what happens in the scenarios exceeding the VaR measure.⁶ In this paper, the Value-at-Risk is suboptimal for regulatory purposes in view of the fact that it is a total risk measure and from this point of view, the other coherent total risk measures suffer the same drawbacks. Here the central point of the model is the distinction between systematic risks and diversifiable risks which implies that a capital constraint based on a total risk measure, which does not consider this distinction, leads to some undesired side effects.

Lastly, the effects of a VaR capital constraint have already been analysed in literature for banks.⁷ As of now, only a few theoretical studies exist for insurance companies. Among them, Bernard and Tian⁸ analyse the effects of the introduction of a VaR capital requirement on insurance contract optimal design obtaining mixed results (“insured are better protected”, but “insurer’s insolvency risk might be increased and there are moral hazard issues in the insurance market”). With a very different approach, this paper strengthens the incertitude about the VaR capital requirement, which might increase the macro-systemic risk of the insurance sector.

The layout of this paper is as follows: In the next section the model is introduced and analysed with particular regard to the risk-return threshold induced by the VaR capital requirement. In the subsequent section the insurer default probability in case of market shortfall is computed and related to the insurer diversification degree. The model is relatively simple and analytically tractable since it is based on a normally distributed world with perfect and frictionless financial markets. The normal distribution and other assumptions of the model are discussed in a dedicated section. In the final section, the policy implication of the model is discussed and some suggestions for Solvency II regulation improvement are introduced.

The VaR solvency capital requirement and the insurance company risk-return profile

In this section the model is outlined. Firstly, the insurance company economic balance sheet and net asset value distribution are introduced. Secondly, the shareholder’s equity value risk-return profile is computed. Thirdly, the VaR solvency capital requirement is introduced and the upper limit for the risk-reward profile is computed. Numerical examples and a sensitivity analysis conclude this section.

The insurance company economic balance sheet and the net asset value distribution

Consider an insurer that operates in one single period and in a perfect and normally distributed financial market. The insurance company subscribes a portfolio of

⁵ Cf. Artzner *et al.* (1999). However, Dhaene *et al.* (2008), argue that the VaR could be a good risk measure for regulatory purposes due to the coherent risk measures that may be “too subadditive”.

⁶ Cf. Basak and Shapiro (2001); Alexander and Baptista (2006); Dowd and Blake (2006).

⁷ Cf., among others, Cuoco and Liu (2006); Leippold *et al.* (2006).

⁸ Bernard and Tian (2010).

insurance contracts at time $t=0$ which gives a random cash outflow for claims and expenses equal to $\tilde{L}_1$ at time $t=1$. $\tilde{L}_1$ is normally distributed with the expected value $E(\tilde{L}_1)$ and the standard deviation equal to $\sigma(\tilde{L}_1)$. The sum that the insurer should set aside at time $t=0$ to fulfil the insurance obligation, that is, the technical provision, is denoted with L_0 . Under the Solvency II framework, which is of interest here, a fair value measure for technical provisions is required (“the value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking”, art. 76.2 EC directives 138/2009). Practically, if random cash outflows are replicable by marketable financial instruments, technical provisions are equal to the value of the replicating portfolio. For non-replicable cash flows, technical provisions are equal to the best estimate plus a risk margin computed through the cost of capital approach. For convenience, only the replicable cash flows case is addressed hereinafter. Therefore L_0 is the market value of the $\tilde{L}_1$ replicating portfolio.

The sum $A_0=L_0$ is invested in financial instruments that give at time $t=1$ a random cash inflow equal to $\tilde{A}_1$, which is again normally distributed with $E(\tilde{A}_1)$ and standard deviation equal to $\sigma(\tilde{A}_1)$. A_0 is the value of assets to cover technical provisions. The cash flows are not necessarily invested in the $\tilde{L}_1$ replicating portfolio. However, in this situation, that is if $\tilde{A}_1=\tilde{L}_1$, the insurer has immunized its asset-liability risk. In normal situations $\tilde{A}_1 \neq \tilde{L}_1$ and some asset-liability risks emerge.

The insurer has an additional amount F_0 , named free assets, which is again invested in marketable financial instruments that give the normally distributed random cash inflow $\tilde{F}_1$ at time $t=1$. The insurance company net cash flows at the end of the period are:

$$\tilde{S}_1 = \tilde{A}_1 + \tilde{F}_1 - \tilde{L}_1$$

which is, due to the normal distribution properties, once again normally distributed, whose value at time $t=0$ is named net asset value and is equal to:

$$S_0 = A_0 + F_0 - L_0 = F_0.$$

In the Solvency II framework, S_0 corresponds to available own funds deriving from the full fair value economic balance sheet.

Let us assume that all financial assets are negotiated in frictionless and complete markets whose returns follow the following market model:

$$\tilde{r}_i = E(\tilde{r}_i) + \beta_i[\tilde{r}_m - E(\tilde{r}_m)] + \tilde{\varepsilon}_i \quad \text{for each financial instrument } i, \quad (1)$$

where β_i and $\tilde{\varepsilon}_i$ are respectively, the beta coefficient and the specific risk of the financial instrument i and $\tilde{r}_m$ is the return on market portfolio.⁹

⁹ In the classic market model (1), it is assumed that $E(\tilde{\varepsilon}_i)=0$, $E(\tilde{\varepsilon}_i \tilde{\varepsilon}_j)=0$ and $E(\tilde{\varepsilon}_i \tilde{r}_m)=0$.

Under this framework, if no arbitrage opportunities on the financial market are allowed, the Capital Asset Pricing Model could be assumed to hold, that is:

$$E(\tilde{r}_i) = r_f + \beta_i[E(\tilde{r}_m) - r_f] \quad \text{for each financial instrument } i, \quad (2)$$

where $[E(\tilde{r}_m) - r_f]$ is the market risk premium.

Clearly, Eqs. (1) and (2) also hold for the covering asset portfolio (A), the free asset portfolio (F) and the replicating portfolio (L). In this situation the net asset cash flow ($\tilde{S}_1$) is normally distributed with¹⁰:

$$\tilde{S}_1 \sim \varphi(E(\tilde{S}_1), \sigma(\tilde{S}_1)) = \varphi(S_0 \cdot E(\tilde{r}_s), S_0 \cdot \sigma(\tilde{r}_s)), \quad (3)$$

$$E(\tilde{r}_s) = r_f + \beta_S[E(\tilde{r}_m) - r_f], \quad (4)$$

$$\sigma(\tilde{r}_s) = \sqrt{[\beta_S \cdot \sigma(\tilde{r}_m)]^2 + \sigma^2(\tilde{e}_S)}, \quad (5)$$

$$\beta_S = \beta_F + \frac{L_0}{S_0} \cdot (\beta_A - \beta_L), \quad (6)$$

$$\sigma(\tilde{e}_s) = \sqrt{\sigma^2(\tilde{e}_F) + \frac{L_0^2}{S_0^2} \cdot (\sigma^2(\tilde{e}_A) + \sigma^2(\tilde{e}_L))}. \quad (7)$$

β_S , β_F , β_A and β_L are the net assets, free assets, covering assets and replicating assets beta coefficients respectively, and $\tilde{e}_S$, $\tilde{e}_F$, $\tilde{e}_A$ and $\tilde{e}_L$ are the net assets, free assets, covering assets and replicating assets specific risks, respectively. The symbol $E(\cdot)$ and $\sigma(\cdot)$ indicate the expected value and the standard deviation operators. The symbol “ $\sim$ ” means “distributed as” and the function $\varphi(\mu, \sigma)$ is a normal distribution function with the mean μ and the standard deviation σ .

In this model, the expected net asset return is only affected by the systematic risk β_S , the only risk priced in a frictionless financial market. The net asset systematic risk is affected (cf. Eq. (6)) by the free asset systematic risk β_F , the mismatch between the asset-liability systematic risk, that is, $(\beta_A - \beta_L)$, and by leverage ratio L_0/S_0 , which is a multiplier of the asset-liability risk.

The total risk on the net asset (cf. Eq. (5)) is not only affected by systematic risk but also by the diversifiable asset specific risk. The latter (cf. Eq. (7)) depends on the free asset specific risk $\sigma(\tilde{e}_F)$, the covering asset specific risk $\sigma(\tilde{e}_A)$, the replicating portfolio specific risk $\sigma(\tilde{e}_L)$ and the leverage ratio L_0/S_0 .

¹⁰ Eqs. (3)–(7) are obtained with simple algebra, assuming that the market model and the CAPM hold. This and all subsequent mathematical derivations, which are omitted or summarised hereinafter, are available upon request by the author.

The insurance company's activities and the shareholder's risk-return profile

At time 0 the insurance company could be assumed to be involved in the usual insurance company activities:

- The direct insurance and the insurance risk management activities;
- The investment and financial risk management activities.

The direct insurance activity involves subscribing to the insurance contract portfolio. Given the insurance contract portfolio $\tilde{L}'_1$ emerging from the direct insurance activity, the insurance risk management activity is devoted to acting in indirect insurance markets (for example reinsurance) in order to obtain the net insurance portfolio $\tilde{L}_1$. Although we have assumed that financial markets are frictionless, complete and without arbitrage opportunities and that insurance cash outflows may be replicated by marketable securities, no hypothesis is required on the direct and indirect insurance markets between the insurer and policyholders or between the insurer and the reinsurer. Even if the analysis of the reinsurance activity is of interest, this issue is not addressed in this paper as only the (net) insurance portfolio is considered hereinafter. Therefore, the insurance activities are summarised by P^* , the (net) premium obtained in $t=0$ for the portfolio $\tilde{L}_1$, and by the portfolio characteristics $E(\tilde{L}_1)$, β_L and $\sigma(\tilde{\varepsilon}_L)$, which are proxies for the portfolio size, the systematic liability risk and the diversifiable (but not diversified) insurance risk, respectively. The value of technical provisions is $L_0 = E(\tilde{L}_1) / [1 + E(\tilde{r}_L)]$, where, according to CAPM, $E(\tilde{r}_L) = r_f + \beta_L[E(\tilde{r}_m) - r_f]$.

The investment and financial risk management activities are aimed at managing investments in order to reach the desired risk-reward profile. In our model, the risk-return profile on net assets is univocally determined by the systematic risk on net assets β_S and the insurer is able to act on β_A , β_F or the leverage ratio L_0/S_0 in order to reach the desired risk-return profile.

To complete this model, it should be considered that if the insurance company's top management operates in the shareholders' interests, owing to the shareholders' limited liability, the sum given to shareholder at time $t=1$ is:

$$\tilde{E}_1 = \text{MAX}[\tilde{A}_1 + \tilde{F}_1 - \tilde{L}_1; 0] = \text{MAX}[\tilde{S}_1; 0]. \quad (8)$$

With a perfect and normally distributed financial market, it is relatively simple to compute the equity value. For instance, following Brennan's¹¹ equation (39), we obtain:

$$E_0 = S_0 \cdot N(d) + \frac{\sigma(\tilde{S}_1) \cdot n(d)}{r_f}, \quad (9)$$

$$d = \frac{r_f \cdot S_0}{\sigma(\tilde{S}_1)}, \quad (10)$$

¹¹ Brennan (1979).

where $N(\cdot)$ and $n(\cdot)$ are standard normal cumulative and density functions, respectively. The shareholders' risk-return profile depends on the equity beta which is equal to:

$$\beta_E = \beta_S \cdot N(d). \quad (11)$$

The equity risk-return profile is lower than the net asset risk-return profile ($N(d) < 1$), seeing as the limited liability protects shareholders from loss in the case of default.

The expected return on equity is:

$$E(\tilde{r}_E) = \frac{E(\tilde{E}_1)}{E_0} = r_f + \beta_E [E(\tilde{r}_m) - r_f]. \quad (12)$$

If insurance markets are also frictionless and perfectly competitive, the premium P^* obtained for the insurance contract portfolio L_1 should be equal to:

$$P^* = P_0 = L_0 - (E_0 - S_0), \quad (13)$$

where P_0 is the value of insurance liabilities considering the shareholders' limited liability. In this perfect competitive situation there is no opportunity for shareholder value creation from insurance or financial activities and the risk-reward profile which best reflects shareholders' interests could be assumed to be the objective that is pursued by top management.

If insurance markets are not perfectly competitive, allowing some insurers to gain extra-profit from competitive advantages (or lose extra-loss from competitive disadvantages), the premium effectively raised would be $P^* > P_0$ (or $P^* < P_0$), the amount that shareholders should invest at time 0 in the insurance company would be $I_0 = E_0 - P^* + P_0$ and the expected return on shareholder investment could be greater (or less) than $E(\tilde{r}_E)$, that is:

$$E(\tilde{r}_I) = \frac{E(\tilde{E}_1)}{I_0} = E(\tilde{r}_E) \cdot \frac{E_0}{I_0} = \left[\frac{P^* - P_0}{E_0} + r_f + \beta_E [E(\tilde{r}_m) - r_f] \right] \cdot \frac{E_0}{I_0}. \quad (14)$$

In this situation, the objective for top management—which acts in shareholders' interests—could be complex but quite realistic. Top management should exploit all insurance business opportunities while considering the risk-reward profile which emerges from investment and financial risk management activities. Hereinafter, no explicit management objective functions are introduced except for a classic non-satiety and risk aversion hypothesis. More specifically, it could be assumed that top management prefers a higher $E(\tilde{r}_I)$ given the systematic and/or total risk, or prefers less systematic and/or total risk given $E(\tilde{r}_I)$.

Lastly, it can be observed that, in the absence of regulation, the insurer depicted in this model may be able to act with a very high leveraged financial structure (which implies a very high default probability) and, if an insurer is able to gain enough extra-profit, it could operate with no shareholder investments. This is the case, for example, if $P^* \geq L_0$. In this situation the insurers' default probability could be very high (e.g., if $P^* = L_0$, $I_0 = 0$ and asset-liability risk is absent, the default probability is equal to 50 per cent) and some prudential regulation measures are strictly required.

Conversely, if no extra-profit is allowed and policyholders have perfect information about insurers' default risk, $P^*=P_0$ and the need for a regulatory minimal capital requirement could be questionable.¹²

The Solvency II Capital Requirement and the shareholder's risk-return profile threshold

In this section, the Solvency Capital Requirement based on the Solvency II directive is introduced. More precisely, Solvency II requires a Solvency Capital Requirement (SCR) that prevents default in 99.5 per cent of cases over a one-year time horizon.

Formally, the insurer default probability at time $t=1$ is:

$$Prob(\tilde{S}_1 \leq 0) = Prob(\tilde{r}_S \leq 0) = p = N\left[-\frac{E(\tilde{S}_1)}{\sigma(\tilde{S}_1)}\right] = N\left[-\frac{E(\tilde{r}_S)}{\sigma(\tilde{r}_S)}\right], \quad (15)$$

where $N(\cdot)$ is the standard normal cumulative function.

Therefore, the Solvency Capital Requirement based on a Value-at-Risk measure with a default probability p and a level of confidence $1-p$ is the minimum level of net asset value S_0^* (or the minimum level of free asset F_0^*) that should be invested in order to prevent insolvency with probability p . Given the normal distributions of asset and liability, the Solvency Capital Requirement is the minimum level of net asset value S_0^* such that:

$$E(\tilde{S}_1) - N^{-1}(1-p) \cdot \sigma(\tilde{S}_1) \geq 0, \quad (16)$$

where $N^{-1}(\cdot)$ is the inverse of the standard normal cumulative function.

In order to obtain a relatively simple analytic solution for S_0^* , it could be useful to assume, without loss of generality, that the assets to cover the technical provisions are invested in order to immunize asset-liability risk, that is, $\beta_A = \beta_L$. In this situation, the insurer risk-reward profile is univocally defined by the investments of free assets, that is, $\beta_S = \beta_F$, and the Solvency Capital Requirement is¹³:

$$S_0^* = L_0 N^{-1}(1-p) \sqrt{\frac{\sigma^2(\tilde{e}_A) + \sigma^2(\tilde{e}_L)}{(r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f])^2 - (N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m))^2 - (N^{-1}(1-p) \cdot \sigma(\tilde{e}_F))^2}}. \quad (17)$$

¹² Cf. Rees *et al.*, 1999.

¹³ Considering Eqs. (3)–(7), Eq. (16) becomes:

$$r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f] - N^{-1}(1-p) \sqrt{(\beta_F \cdot \sigma(\tilde{r}_m))^2 + \sigma^2(\tilde{e}_F) + \frac{L_0^2}{S_0^2} \cdot (\sigma^2(\tilde{e}_A) + \sigma^2(\tilde{e}_L))} \geq 0.$$

Considering that the first two addendum and the quantities into the square root are positive:

$$(r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f])^2 - (N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m))^2 - (N^{-1}(1-p) \cdot \sigma(\tilde{e}_F))^2 \geq (L_0^2/S_0^2) \cdot [(N^{-1}(1-p)]^2 (\sigma^2(\tilde{e}_A) + \sigma^2(\tilde{e}_L)). \quad [*].$$

The minimum level of net asset value S_0^* that satisfies the inequality (16) could be obtained by [*] with simple algebra.

A brief discussion of the formula could be of interest here.

Firstly and intuitively, the minimum required solvency ratio S_0^*/L_0 is greater when:

- the confidence level $(1-p)$ is higher (or the maximum tolerable default probability level p is lower); in Solvency II the confidence level is 99.5 per cent ($p=0.5$ per cent);
- the diversifiable (but not diversified) risks on assets and liabilities are higher;
- the insurer's exposure to systematic risks (β_F) is higher.¹⁴

Secondly, when the minimum solvency ratio S_0^*/L_0 is set, a clear trade-off between systematic risks (β_F) and diversifiable risks ($\sigma(\tilde{\varepsilon}_A)$, $\sigma(\tilde{\varepsilon}_L)$ and $\sigma(\tilde{\varepsilon}_F)$) emerges. In particular, an increase in β_F requires a reduction in some diversifiable risks in order to preserve the same minimum required solvency ratio S_0^*/L_0 .

Third, given the liability portfolio characteristics ($E(\tilde{L}_1)$, β_L and $\sigma(\tilde{\varepsilon}_L)$), the lowest level in the Solvency Capital Requirement (or in the minimum required solvency ratio S_0^*/L_0) is obtainable if the financial risk management activities are capable of completely eliminating asset risks and asset-liability risks, that is $\beta_A=\beta_L$, $\beta_F=0$, $\sigma(\tilde{\varepsilon}_F)=0$, $\sigma(\tilde{\varepsilon}_A)=0$. In this situation, the Solvency Capital Requirement is equal to:

$$S_0^* = \frac{N^{-1}(1-p) \cdot L_0 \cdot \sigma(\tilde{\varepsilon}_L)}{r_f}. \quad (18)$$

Clearly, the expected return on surplus here is in line with the return on risk-free asset. If the insurance company wants to reach a higher risk-reward profile it should assume systematic risk.

Lastly and most importantly, the introduction of a VaR capital requirement establishes an important restriction on the assumption of systematic risk and consequently on the risk-reward profile that is feasible. Actually, it could be demonstrated that the superior limit for systematic risk assumption is:¹⁵

$$\beta_S^* = \frac{r_f}{N^{-1}(1-p) \cdot \sigma(\tilde{r}_m) - [E(\tilde{r}_m) - r_f]}. \quad (19)$$

This implies that the maximum feasible expected return on equity is:

$$E(\tilde{r}_E^*) = \frac{E(\tilde{E}_1)}{E_0} = r_f + \beta_S^* \cdot N(d)[E(\tilde{r}_m) - r_f] \quad (20)$$

¹⁴ This latter point is only effective for plausible values of the relevant parameters. Under particular market conditions, a marginal increase in betas could decrease the Solvency Capital Requirement as the increase in expected net assets overwhelms the effects of the increase in standard deviation.

¹⁵ In Eq. (17), the quantity in square root should be positive. Taking into account that the numerator is always non-negative, this implies that the denominator should be positive, that is:

$$(r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f])^2 - (N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m))^2 - (N^{-1}(1-p) \cdot \sigma(\tilde{\varepsilon}_F))^2 > 0).$$

Therefore, if free assets are invested in a fully diversified portfolio ($\sigma(\tilde{\varepsilon}_F)=0$) Eq. (19) is obtained with simple algebra. Clearly, if the free asset portfolio is not fully diversified, the systematic risk that the insurance company is able to achieve lowers.

and a maximum feasible expected return on investment equal to:

$$E(\tilde{r}_1^*) = \left[\frac{P^* - P_0}{E_0} + r_f + \beta_S^* \cdot N(d)[E(\tilde{r}_m) - r_f] \right] \cdot \frac{E_0}{I_0}. \quad (21)$$

An insurance company is able to reach this superior limit only if it is fully diversified, that is, the specific risk on investment ($\sigma(\tilde{\varepsilon}_F)$ and $\sigma(\tilde{\varepsilon}_A)$) and insurance contracts ($\sigma(\tilde{\varepsilon}_L)$) is negligible with respect to systematic risk on net asset value. In particular, this upper limit can be reached if the ratio between systematic risk and total risk tends to 1. Let the diversification degree (DG) be equal to:

$$DG = \frac{\beta_S \cdot \sigma(\tilde{r}_m)}{\sigma(\tilde{r}_S)} = \frac{\beta_S \cdot \sigma(\tilde{r}_m)}{\sqrt{[\beta_S \cdot \sigma(\tilde{r}_m)]^2 + \sigma^2(\tilde{\varepsilon}_S)}}. \quad (22)$$

If $DG \rightarrow 1$, then the maximum systematic risk exposure tends to be equal to β_S^* .

The introduction of the Solvency Capital Requirement may or may not affect the management of the insurance company. It is not affected if, given insurance portfolio $\tilde{L}_1$ and available own funds F_0 , the risk-reward profile required by shareholders could be reached without infringing on the Solvency Capital Requirement constraints. Otherwise, the insurer is affected by regulation. There are two different possible situations. On the one hand, the risk-reward required by the shareholder is feasible since it is lower than $E(\tilde{r}_E^*)$ in Eq. (20). However, the insurer is required to increase the diversification degree in Eq. (22). This could be done either through a reduction in diversifiable asset risks ($\sigma(\tilde{\varepsilon}_A)$ and $\sigma(\tilde{\varepsilon}_F)$) or diversifiable liability risks ($\sigma(\tilde{\varepsilon}_L)$) or, less likely still, by increasing its own funds. On the other hand, the risk-reward required by the shareholders may not be reached since it is equal or higher than $E(\tilde{r}_E^*)$ in Eq. (20). In this situation, the management of the insurance company has a marked inducement towards fully diversified investments and insurance activities (i.e. to reach a diversification degree close to 1) obtaining the highest feasible risk-reward profile.

Numerical examples

At this stage it could be useful to introduce a simple numerical specification:

- Market conditions: risk-free rate=4 per cent ($r_f=1.04$); market risk premium= $[E(\tilde{r}_m)-r_f]=6$ per cent; market risk volatility= $\sigma(\tilde{r}_m)=20$ per cent;
- Insurance contract portfolio: $E(\tilde{L}_1)=100$; $\beta_L=-0.1$, $\sigma(\tilde{\varepsilon}_L)=8$ per cent (this implies that $L_0=A_0=96.7118$);
- Investments covering technical provisions are fully diversified ($\sigma(\tilde{\varepsilon}_A)=0$) and asset-liability risk is immunized ($\beta_A=-0.1$);
- Available free assets= $F_0=S_0=20$; free assets are invested in a fully diversified portfolio ($\sigma(\tilde{\varepsilon}_F)=0$) with β_F set in order to reach, where possible, the risk-reward profile required by shareholders.

- Regulators require a Solvency Capital Requirement based on a Value at Risk measure with a confidence level equal to 99.5 per cent; therefore, the superior limit for systematic risk (Eq. (19)) is $\beta_S^*=2.28488$ which corresponds to a maximum shareholder rate of return on equity equal to 17.55 per cent ($E(\tilde{r}_E^*)=1.1755$).

If the shareholder required rate of return on equity is low (for example 6 per cent), the optimal risk-reward profile may be reached by simply investing the free assets in a fully diversified portfolio with $\beta_F=0.336$. The Solvency Capital Requirement here is 19.0538, less than the available net assets ($S_0=20$).

Should the shareholder require a higher rate of return on equity (for example 14 per cent), the insurer is to invest its free assets in a fully diversified portfolio with $\beta_F=1.682$. However, the Solvency Capital Requirement is 26.8513 and to fulfil it new own funds of 6.8513 have to be raised or, more likely, part of the insurance portfolio risk needs to be diversified (for instance $\sigma(\tilde{\varepsilon}_L)$ from 8 per cent to 5.959 per cent).

Where the shareholder rate of return on equity is higher than feasible (for example 22 per cent), the maximum risk-reward profile (17.55 per cent) could be reached raising an unlimited quantity of own funds (and investing such a sum in a fully diversified portfolio with $\beta_F=2.28488$) or, more realistically, to fully diversify the insurance contract portfolio ($\sigma(\tilde{\varepsilon}_L)=0$).

In this example, the shareholder return on equity (Eqs. (12) and (19)) is considered. If the insurer has no competitive advantage or disadvantage, the expected return on shareholder investments (Eq. (14)) is equal to the expected return on equity. Whereas, if some competitive advantage or disadvantage emerges, that is, $P^* \neq P_0$, the incentive to diversification may be lower, in particular when insurers are able to gain some extra profit ($P^* > P_0$). Intuitively, if the insurance activity permits value creation, expected return on shareholder investment may be optimised through the insurance activity and the investment or financial risk management activities could be devoted lowering unnecessary risks (diversifiable investment risks) to the minimum level while assuming a not too high systematic risk.

As a simple example, let us consider the previous numerical specification, while allowing an extra profit from insurance activities of five for every 100 expected claims at time one. For an insurance portfolio with $E(\tilde{L}_1)=100$; $\beta_L=-0.1$, $\sigma(\tilde{\varepsilon}_L)=8$ per cent and without investment and asset-liability risks, the Solvency Capital Requirement is equal to 19.1625, the fair value of the insurance portfolio (P_0) is 96.7000, the premium raised is 101.7000 ($P^*=P_0+5$), the minimum capital invested by the shareholder which supports the SCR is 14.1743 and the expected rate of return on shareholder investment is 40.69 per cent ($E(\tilde{r}_I)=1.4069$, cf. Eq. (21)). With the same insurance activities and some systematic investment risks, for example $\beta_F=1$, an SCR equal to 20.5051 would be required, with a minimum shareholder investment of 15.5257 and the expected rate of return on shareholder investment increases to 45.366 per cent. However, for higher systematic risk exposure (for example $\beta_F=2$), the minimum shareholder investment is 32.4528 and the expected rate of return on shareholder investment drops to 33.727 per cent.

This example illustrates how for an insurer with some competitive advantage, the expected return on shareholder investment could decrease with an increase in systematic risk and the incentive to fully diversify insurance activities may not be as high as for an insurer with no competitive advantage.

Sensitivity analysis

Introducing a VaR capital requirement, the maximum risk-reward profile attainable by shareholders—that is, the maximum feasible systematic risk assumption expressed by β_S —depends on financial market conditions (r_f , $[E(\tilde{r}_m) - r_f]$, $\sigma(\tilde{r}_m)$), insurance contract portfolio characteristics ($E(\tilde{L}_1)$, $\sigma(\tilde{\varepsilon}_L)$), insurer competitive advantage or disadvantage ($P^* - P_0$), available free assets (F_0), investment portfolio characteristics ($\sigma(\tilde{\varepsilon}_A)$ and $\sigma(\tilde{\varepsilon}_F)$), and the regulatory level of confidence ($1 - p$). Fixing the other variables, the maximum shareholder expected return on investment is a monotonically increasing function of r_f , $[E(\tilde{r}_m) - r_f]$ and $(P^* - P_0)$, while it is a monotonically decreasing function of $\sigma(\tilde{r}_m)$ and specific risk parameters ($\sigma(\tilde{\varepsilon}_L)$, $\sigma(\tilde{\varepsilon}_A)$ and $\sigma(\tilde{\varepsilon}_F)$). These relationships are all intuitive: an increase in one of the first set of variables raises the return side of the risk-return relationship, without affecting the risk side; conversely, an increase in one of the second set of variables, affects positively the risk of insurance companies and the solvency capital requirement without affecting expected return. Clearly, also the confidence level $1 - p$ is negatively related to the maximum feasible expected return. An increase in $E(\tilde{L}_1)$ (the size of the insurance contract portfolio) affects negatively the maximum feasible β_S (and the maximum expected return on equity), since it increases the total risks of the insurer. However, the effect of an increase in size on maximum expected return on investment also depends on the insurer's competitive position ($P^* - P_0$). If the insurer has no competitive advantage or disadvantage ($P^* = P_0$), an increase in size reduces the maximum feasible risk-return profile. With some competitive advantage ($P^* > P_0$) the relationship could be reversed. Finally, an increase in available free assets (F_0) affects positively the maximum feasible β_S and—without any competitive advantage—increases the maximum feasible expected return on investment. In case of some competitive advantage, an increase in available free assets could decrease the maximum feasible expected return on investment.

Even if a complete sensitivities analysis is beyond the objective of this paper, Figure 1 shows some of the relationships described above. More precisely, there is a clear representation of how the maximum expected rate of return on investment varies as a function of the financial market risk premium (panel A), the insurance contract portfolio specific risk (panel B), the insurer extra-profit $P^* - P_0$ (panel C), the insurer contract portfolio size with and without competitive advantage (panel D), the available free capital (part E) and the regulatory level of confidence (panel F).¹⁶

¹⁶ The maximum expected rate of return on investment is directly obtainable from Eq. (21) only if the insurer is perfectly diversified. In Figure 1, this is the case only for $\sigma(\varepsilon_L) \rightarrow 0$ in panel C and for $F_0 \rightarrow \infty$ in panel E. In both cases the expected rate of return on investment approaches 17.55 per cent. Apart from these two limiting cases, the graphs in Figure 1 are computed numerically from Eq. (17).

Figure 1. Maximum feasible risk-reward profile under Solvency II capital requirement: sensitivity analysis. This figure depicts the maximum feasible expected rate of return on shareholder investment (Eq. (14) minus 1) as a function of: the financial market risk premium (panel A), the insurance contract portfolio-specific risk (panel B), the insurance company's competitive position (panel C), the insurance contract portfolio size (panel D) in case of no competitive advantage (lower line) and in case of a positive competitive advantage (upper line), the insurance company's available free capital (panel E) and the confidence level required by regulator (panel F). The initial parameters of the model are: $r_f = 1.04$; $[E(\tilde{r}_m) - r_f] = 6$ per cent; $\sigma(\tilde{r}_m) = 20$ per cent; $E(\tilde{L}_1) = 100$; $\beta_L = -0.1$; $\sigma(\tilde{e}_L) = 8$ per cent; $\sigma(\tilde{e}_A) = 0$; $\beta_A = -0.1$; $F_0 = S_0 = 20$; $\sigma(\tilde{e}_F) = 0$; $p = 0.5$ per cent ($1 - p = 99.5$ per cent); $P^* = P_0$. The dotted lines in some panels indicate a superior (panels B D—F) or inferior (panel E) limit in the variable.

Insurer default probability in case of a market shortfall

The analysis in the previous section illustrates that the Solvency Capital Requirement based on a Value-at-Risk measure places an upper limit on the risk-reward profile that shareholders are able to reach. In addition, to reach this maximum feasible risk-reward profile, the insurer should diversify investment and insurance risks. In this section, the theoretical analysis is concluded, considering the insurance default probability in case of a market shortfall. It is shown that if a large proportion of insurers wishes to diversify its liability risk, in order to raise their risk-reward profile, a very strong macro-systemic effect is induced by regulation.

Let us introduce the return on net asset, given a specific market return as:

$$(\tilde{r}_S | \tilde{r}_m = \bar{r}_m) \sim \varphi(r_f + \beta_S[\bar{r}_m - r_f], \sigma(\tilde{\varepsilon}_S)).$$

Hence, the conditional default probability given a specific market return is:

$$\begin{aligned} \text{Prob}(\tilde{r}_S < 0 | \tilde{r}_m = \bar{r}_m) &= q(\bar{r}_m) = N \left[-\frac{E(\tilde{r}_S | (\tilde{r}_m = \bar{r}_m))}{\sigma(\tilde{r}_S | (\tilde{r}_m = \bar{r}_m))} \right] = N \left[-\frac{r_f + \beta_S[\bar{r}_m - r_f]}{\sigma(\tilde{\varepsilon}_S)} \right] \\ &= N \left[-\frac{r_f + \left(\beta_F + \frac{L_0}{S_0} \cdot (\beta_A - \beta_L) \right) [\bar{r}_m - r_f]}{\sqrt{\sigma^2(\tilde{\varepsilon}_F) + \frac{L_0^2}{S_0^2} \cdot (\sigma^2(\tilde{\varepsilon}_A) + \sigma^2(\tilde{\varepsilon}_L))}} \right]. \end{aligned} \quad (23)$$

We are interested in analysing this conditional default probability specifically within a VaR-SCR constraint. It can be demonstrated that if the insurer holds net assets S_0 exactly equal to the VaR-SCR S_0^* and, for simplicity, investment risk is perfectly diversified and asset-liability risk is immunized, the conditional default probability given a specific market return is:¹⁷

$$q(\bar{r}_m) = N \left[-N^{-1}(1-p) \cdot \frac{r_f + \beta_F[\bar{r}_m - r_f]}{[r_f + \beta_F[E(\tilde{r}_m) - r_f]]^2 - (N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m))^2} \right]. \quad (24)$$

Clearly, this conditional default probability inversely depends on market return. The better the performance of the market, the less the default probability is and vice versa. More importantly, the function $q(\bar{r}_m)$ depends on systematic risk β_F . There are two limiting cases depicted in Figure 2. In the first case, the insurer has no systematic risk, that is, $\beta_F=0$. Here, the default probability is equal to p , irrespective of the market return, that is:

$$q(\bar{r}_m) = N[-N^{-1}(1-p)] = p.$$

¹⁷ Eq. (24) is obtained with simple algebra considering that $\sigma(\tilde{\varepsilon}_F)=0$ and $\beta_L=\beta_A$ and substitute for S_0^* of Eq. (17) in Eq. (23).

Figure 2. Insurer default probability as a function of market return: the two limiting cases.

This figure depicts the default probability of a fully diversified insurer (black line) and of an insurer without systematic risk (grey line) as a function of market return. The insurer operates with a net asset value equal to the Value at Risk requirement with a confidence level equal to $1-p$. The market return density distribution is depicted in the upper part of the figure.

The second limiting case is where the insurer is fully diversified, that is only systematic risk is assumed. As illustrated in the previous section, this situation occurs when the insurer wishes to reach a risk-reward profile higher than feasible and the diversifiable risk is eliminated in order to reach the maximum allowed risk-return profile.

Allowing $\hat{r}_m$ to be the p -percentile of the market return distribution, for a fully diversified insurer, the insurer default probability is:

- equal to zero when the market return is greater than $\hat{r}_m$;
- equal to 0.5 when the market return is equal to $\hat{r}_m$; and
- equal to one when the market return is less than $\hat{r}_m$.

The effects of these limiting situations are of utmost importance. On the one hand, when the market drop exceeds the level of confidence fixed by the regulator, all fully diversified insurance companies enter into bankruptcy simultaneously. This is a very extreme macro-systemic effect induced by capital requirements. On the other hand, in a deep market scenario, the less diversified insurance companies have more resilience as a whole, since the capital requirement is not only based on systematic risks but also on diversifiable risks.

Furthermore, the strength of the VaR capital requirement—that is, the rise in the confidence level $1-p$ —makes the insurers' default wave less likely but consistently stronger. Thus, a stronger capital requirement reduces the maximum expected return on equity that could be reached by the insurers and this makes the capital requirement binding for a larger number of insurers, providing them with incentives for higher diversification.

Figure 3 depicts the insurer's conditional default probability given a specific market return, that is, $q(\bar{r}_m)$ in Eq. (24), as a function of the diversification degree for five different levels of market shortfall. The preceding two limiting cases correspond to a

Figure 3. Insurer conditional default probability given the market return as a function of the diversification degree.

This figure shows the insurer default probability (left-y-axis scale) as a function of the diversification degree (cf. Eq. (24)) for five different market shortfalls (worst case scenarios). The parameters of the model are: $r_f = 1.04$; $[E(\tilde{r}_m) - r_f] = 6$ per cent; $\sigma(\tilde{r}_m) = 20$ per cent; $E(\tilde{L}_1) = 100$; $\beta_L = -0.1$; $\sigma(\tilde{\varepsilon}_A) = 0$; $\beta_A = -0.1$; $F_0 = S_0 = 20$; $\sigma(\tilde{\varepsilon}_F) = 0$. The Solvency Capital Requirement of the insurance company is the VaR with a 99.5 per cent confidence level and is computed by using Eq. (17). $\sigma(\tilde{\varepsilon}_L)$ and β_F are variable across the x-axis in order to obtain an SCR exactly equal to the existing net asset value. For example, on the left-hand side of the graph (degree of diversification = 0) $\sigma(\tilde{\varepsilon}_L) = 8.35$ per cent and $\beta_F = 0$, while on the right hand side (full diversification) $\sigma(\tilde{\varepsilon}_L) = 0$ per cent and $\beta_F = 2.28480$. The expected rate of return on equity is also depicted as a function of the diversification degree (right-y-axis).

diversification degree equal to zero and one respectively. In the first limiting case, the insurer default probability is equal to 0.5 per cent (i.e. one unit less than the confidence level), independently to market return. In the second limiting case, the insurer:

- defaults with certainty when the market return is lower than in the 0.5 per cent worst case scenario (which corresponds to a one-year market return of less than 0.5848 or a market rate of return less than -41.52 per cent);
- has a fifty/fifty default probability when the one-year market return is exactly equal to 0.5848; and
- has zero default probability when the market shortfall is less than -41.52 per cent.

For the less diversified insurers (for example with a diversification degree lower than 40 per cent), the default probability remains relatively low even in a very deep market shortfall (for instance in the case of the 0.01 per cent worst case scenario which corresponds to a one-year market shortfall of -64.38 per cent, the default probability is around 10 per cent). On the contrary, the more diversified insurers (for example

with a DG between 80 per cent and 95 per cent) have significant default probability (i.e. more than 10 per cent) also in “normal” market shortfalls (for example in the 1 per cent worst case scenario which corresponds to a one-year market rate of return equal to -36.53 per cent).

On the technical hypothesis of the model

The simple model hereby outlined is based on several assumptions that are essential to discuss at this stage:

1. the normality of insurance claims and asset returns;
2. the replicability of insurance claims through financial assets;
3. financial markets that are frictionless, complete and without arbitrage opportunity; and
4. a one-period model.

The normal distribution hypothesis

The normal hypothesis on asset returns and insurance contract cash outflows is quite unrealistic, above all in the latter case. The lognormal hypothesis better describes the distribution of the claims by far. However, there are several reasons here to take the normal hypothesis into consideration.

Firstly, the market model, CAPM and normality are all closely connected to a one-period discrete model.¹⁸ The lognormal distribution hypothesis could be introduced in a multiperiod continuous settings as in the Merton ICAPM or in the classical Option Pricing Theory. Unfortunately, the sums of lognormal distributions are not lognormal themselves and the analytical tractability of the model disappears without any further hypothesis.¹⁹ In order to solve the lognormal case analytically, only insurance contract liability risk needs to be admitted. Nevertheless, the aim of this paper is to also consider asset and asset-liability risks, not only the insurance liability risk.

Secondly, as previously noted, the VaR measure is recognised as an acceptable risk measure only under normality. This paper is not a critique of the VaR measure itself but rather the total risk measures used as a benchmark for Solvency Capital Requirement. Therefore, in order to bring this aspect to light, it is desirable that the VaR measure has the same properties of the better total risk measures (such as Tail-VaR) and this occurs only in the normal distribution case.

Thirdly, the Solvency II framework employs a risk aggregation formula based on the classic variance and covariance approach which is again theoretically founded only in the case of multivariate normal distribution.

Lastly, the introduction of skewed insurance risk distribution strengthens rather than weakens the above-mentioned results. More precisely, with a skewed insurance risk distribution, the scenarios exceeding the pre-specified level of confidence are ever more frequent, rather than those predicted in the normal distribution case. Further-

¹⁸ Cf. Stapleton and Subrahmanyam (1983).

¹⁹ Cf. Cummins (1988) and Butsic (1994).

more, the scenario exceeding the pre-specified level of confidence could be worse than normal distribution predicts.

In brief, the removal of the normality hypothesis only slightly strengthens the results of the model and therefore adds technical complexity with no clear advantage. Moreover, it introduces further critiques to the Solvency II framework that this paper, for the sake of relevance, does not wish to address here.

The replicability of insurance claims and the systematic risk definition

The hypothesis of the existence of a replicating portfolio for insurance claims has two grounds.

First, with the replicability hypothesis, the value of insurance claims is determined simply by the replicating portfolio and therefore the market model and the CAPM may be applied without any further assumptions. In the opposite case, the best estimate plus the risk margin approach should be used. Even if the latter is compatible with the classic financial economics theory,²⁰ this introduces unnecessary complexities into the model.

Second, and more importantly, the replicability allows asset and claim distributions to be both governed by a simple market model, that is, there is only one risk source that is common to all tradable securities and claims. Therefore in the theoretical model, systematic risk has been defined in the simplest-classic manner, that is, as the risk related to market return.

There are two significant characteristics of systematic risk. On the one hand, it is a priced risk (i.e. it affects expected returns). On the other hand, it is a common risk factor across all financial and/or insurance contracts. Both characteristics are essential to this paper's model. The first affects the insurance company risk-return profile; the second influences the insurance company default probability in case of a market (or factor) shortfall.

Actually, systematic risk could be defined as the risk sources that have both of these two highlighted characteristics. Therefore, each risk source that is priced, that is, that affects the insurance company risk-reward profile, and is common across the insurance sector may be considered as systematic.

Numerous risk sources reflect these characteristics. They may refer to the global economy, such as economic growth, interest and inflation rates, credit spreads²¹ or only refer to the insurance sector (for example cat risk or longevity risk). More precisely, according to the Solvency II QIS 5 results,²² insurance companies are mainly affected by market risk sources (such as equity risk, interest rate risk and spread risk), life underwriting risk sources (such as mortality risk, longevity risk and lapse risk) and non-life underwriting risk sources (such as premium and reserve risk and cat non-life risk). Clearly, the market risk sources cited above all have an important systematic component, while underwriting risk sources may or may not. This is an empirical

²⁰ See Floreani (2011).

²¹ See for example Chen *et al.* (1986).

²² Cf. EIOPA (2011).

matter, even if longevity risk, lapse risk and cat non-life risk are valid candidates for detecting systematic risk sources.

Finally, systemic risk and systematic risk are two broadly different concepts. However, systematic risk assumption by insurers could expose the insurance sector to a macro-systemic risk if the insurers' degree of diversification is close to one. The theoretical model is focused on only one systematic risk source ("the market"). Moreover, the same reasoning could be extended to all other systematic risk sources that affect the insurance sector.

On the other hypothesis of the model

The frictionless, complete and the non-arbitrage financial markets hypothesis is standard and does not require specific discussion. As already observed, this hypothesis does not imply that direct or indirect insurance markets are perfectly competitive. Conversely, it is implicitly required that insurance markets show some friction, since with perfect insurance markets the entire system of prudential regulation based on capital requirement is questionable.²³ The characteristics of insurance markets are not critical, but the characteristics of the insurers may be. The theoretical model of this paper has shown that without any competitive advantage to the insurer, the VaR-capital requirement could encourage the insurer to diversify its diversifiable risk. For an insurer with a competitive advantage this inducement could be less forceful, while, even if not addressed in the theoretical model, in case of a competitive disadvantage some moral hazard issues may emerge.

Finally, with reference to the time horizon, in our single-period theoretical model insurers become bankrupted only when asset cash inflows are less than claim cash outflows. In real markets, Solvency II measures the financial strength of the insurers over a one-year horizon which is based on fair value rather than cash flows. Therefore, an insurance company is bankrupted when the fair value of assets become less than the fulfilment value of liability, not considering the fact that the fair value of assets and the fulfilment value of liabilities also depend on specific market conditions (i.e. current structure of interest rates, equity prices, credit spreads), even when unrelated to the fulfilment of contractual obligations in the long run.

Policy implications and conclusive remarks

The above depicted theoretical model suggests that the Solvency II regulation based on a total risk measure has some significant drawbacks. Moreover:

- it provides several insurance companies with incentives towards the *financialization* of insurance business;
- it amplifies the macro-systemic exposure of the insurance industry to market shortfall.

²³ Cf., among others, Rees *et al.* (1999).

The first adverse side effect is a consequence of the incentive towards diversification for insurance companies whose optimal risk-reward profile is higher than feasible. The complete diversification of insurance risks may be obtained in various ways. These include:

- growth (intra-LOB growth, inter-LOB growth or geographical diversification);
- a strategic refocusing of the business in the activities which implies a limited assumption of diversifiable insurance risks and allows for higher exposure to market risks (for example life business vs. non-life business);
- a shift towards insurance activities which implies an easier diversification of the risk insured (retail markets with highly homogeneous and insurable risks vs. corporate markets with less homogenous and insurable risks, such as cat risks);
- the revision of contractual clauses in insurance contracts in order to avoid or limit the insurance exposure to diversifiable insurance risks (for example, the study or promotion of insurance policies that transfer only nominal mortality or longevity risk to insurers).

The second adverse side effect is a consequence of the incentive to hold only systematic risks that, in case of a deep market shortfall (exceeding the confidence level required by regulators), lead all the diversified insurers—who are typically also the biggest—to bankruptcy.

Previous studies²⁴ have concluded that the systemic risk in the insurance sector is only marginally relevant. However, the fact that the insurance sector, especially the European insurance market, is now and was in the past less exposed to systemic risk does not imply that it can ever happen in the future, if the new regulatory system brings about this exposure.

Clearly, it is difficult to establish whether the severe prediction of this theoretical model would happen. However:

- the generalised banking crisis could be easily explained by this model (the systematic exposure to credit and market risks has led the “too-big-to-fail”-diversified and levered financial institutions to a deep financial crisis when a highly negative “market return” has been realised);
- the fact that the discussion on Solvency II regulation has not paid attention to the differences between diversifiable and systematic risk is not encouraging;
- in the presence of such deep drawbacks, which could lead to the crisis across a vast part of the European insurance sector in a near or distant future, a general precautionary principle needs to be applied.

How to improve the Solvency II framework

Finally, how can the Solvency II Framework be adjusted in order to minimise or avoid the consequences foreseen by this model?

²⁴ Cf. Geneva Association (2010); Baluch *et al.* (2011).

The most intuitive solutions do not seem to be easily viable or effective. Let us examine these findings a little more closely.

First, the model has shown a trade-off between competitiveness of insurance markets and financial stability. More specifically, if insurance activities (i.e. the effective assumption of underwriting insurance risks) are profitable (i.e. most insurers are able to earn some extra profit), the insurer's incentive to become fully diversified is compensated by the incentive to exploit business opportunities. However, requesting regulators to introduce some anti-competitive measures in order to avoid a potential systemic effect does not seem to be easily viable, at least before such an effect happens.

Second, the theoretical model suggests a different treatment between systematic and diversifiable risks. More specifically, systematic risk capital requirements should be strengthened and in turn, diversifiable risk capital requirements should be weakened. For instance, the Solvency II level of confidence on systematic risk could be raised to 99.9 per cent while the level of confidence on diversifiable risk could be decreased to 95 per cent. Even if the Solvency II framework does not differentiate between systematic and diversifiable risk, this solution seems viable in practice. The modular Solvency II standard formula requires the calibration of the parameters of each sub-module in order to obtain a proxy for the Value-at-Risk over a one-year time horizon with a 99.5 per cent level of confidence. As already marked, an analysis should be performed in order to detect the sub-modules that imply the assumption of significant systematic risks. Some market risk sub-modules—such as equity, interest rate and spread—credit risk and some life underwriting risk sub-modules (e.g. lapse, longevity, and non-life cat) may be considered as the first candidates.

Although this solution seems to be viable, with only minor technical modifications in the Solvency II framework, it does not appear to be effective in practice. On the one hand, the strengthening of systematic risk requirements reduces the probability of default from 0.5 per cent to a lower level (say for example 0.1 per cent). On the other hand, it lowers the maximum risk-reward profile reachable by the insurers and a larger number of insurers are expected to have the incentive to fully diversify their business. Therefore, in the case of a market shortfall that exceeds the new confidence level, the systemic financial crisis of the insurance sector is not deterred but rather encouraged. This solution reduces the probability of a systemic crisis but it does not mitigate the crisis of the insurance sector in the case of a deep market shortfall.

Whereas a practical differentiation between the Solvency Capital Requirement on systematic risk and diversifiable risk does not appear to be practically effective, the analyses of the systematic nature of insurance company risks should be performed carefully. This is in view of the fact that it is important to avoid a calibration that has a favourable treatment of systematic risk. One example of such a misconception is the calibration of the equity sub-module in the QIS5 specification. Hence, the basic capital requirement on equity is reduced to 39 per cent,²⁵ even if the parameter estimated and recommended by the EIOPA, which is 42 per cent, is based on hypothesis (normal distribution and long-run historical estimates) that already undermine future equity risk.²⁶

²⁵ Cf. European Commission (2010).

²⁶ Cf. Ceiops (2010).

A viable amendment for the Solvency II framework

A simple, viable and possibly effective solution to the theoretical issues posed by this paper is to set out that a significant part of the Solvency Capital Requirement be referred to diversifiable (insurance) risks. If the quota of diversifiable (insurance) risk drops below a predefined threshold, further diversification benefits must not be considered in the SCR calculation. The technical solutions in order to introduce such limits are not addressed in this paper. However, some comments should be made on the positive consequences of this proposal. To this end, it could be useful to illustrate the effects of this measure in Figure 4 which mirrors Figure 3 apart from the fact that when a diversification degree of 60 per cent is overwhelming, no further diversification effects are admitted to the SCR calculation. This avoids further assumption of systematic risk and as diversification continues, the insurance companies are less (not more) exposed to market shortfalls, since diversification reduces total risks but has no effects on the reduction of the SCR. Consequently, all the previously illustrated side effects are null and void. Moreover:

- this measure reduces the incentive towards the financialization of insurance activities;

Figure 4. Insurer conditional default probability given the market return as a function of the diversification degree: SCR calculation with limits in the recognition of the diversification effects.

This figure shows the insurer default probability (left-y-axis scale) as a function of the diversification degree (cf. Eq. (24)) for five different market shortfalls (worst case scenarios). Each parameter of the model is the same as Figure 3 apart from the Solvency Capital Requirement calculation. The Solvency Capital Requirement of the insurance company is the VaR with a 99.5 per cent confidence level and is computed by using Eq. (17) up to a diversification degree equal to 60 per cent (which corresponds to $\beta_F=1.29$ and $\sigma(\tilde{\varepsilon}_L)=7.21$ per cent). For a higher diversification degree ($\sigma(\tilde{\varepsilon}_L)>7.21$ per cent), the Solvency Capital Requirement is not lowered. Therefore the maximum systematic risk exposure compatible with an $SCR=S_0=20$ is constant and equal to $\beta_F=1.29$ even if diversifiable risk decreases.

- the bigger/better diversified insurance companies are required to be less risky than the smaller/worse diversified insurers; thus, it reduces the “too-big-to-fail” issue and the macro-systemic exposure of the insurance industry to market shortfalls, contributing to the effective achievement of the proportionality principle, while possibly permitting a higher credit rating for the bigger/better diversified insurer or better still, a reduction in the overall confidence level required by the regulators (for example from 99.5 per cent to 99 per cent or less).

Conclusive remarks

Solvency II introduces a revolutionary risk management framework for the European insurance industry. Its basic more risk/more capital principle is simple and unanimously shareable. However, the technical specification of the principle is far from simple and requires a careful risk measurement. Here, the metric used for risk measurement is the most important issue. This paper has shown that the metric used by regulators, which is based on a total risk measure such as the Value-at-Risk, is not a balanced solution between effectiveness and simplicity, but is simply wrong and could lead to significant adverse side effects, ultimately resulting in a generalised European insurance industry crisis in the case of a hard market shortfall. Therefore some adjustments to the Solvency II framework are necessary before the new regulatory system is enforced. Invariably, academic predictions and suggestions regarding the pitfalls in risk measurement and risk management systems are taken on board and endorsed only after the foreseen side effects have occurred, as in the case of Cassandra in the Greek myth. Let us hope that is not the case.

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