Inflation and Interest Rates A Research Study Using the ASIR Model

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Summary

The underwriting of a class of insurance can be regarded from two aspects. Firstly, there is the contribution made by the risk class to the company's overall profits. Here there are two components, namely the underwriting profit as measured by conventional accounting methods, and the investment income obtained by investing the technical reserves of the risk class. The second aspect is the increase in the amount of shareholders' funds needed to maintain a solvency margin. Again there are two components: the minimum amount required by statutory regulations, and any further amount deemed necessary by the company to act as a cushion against adverse fluctuations.

The effects of different interest and inflation rates on the above are complex, and depend also on many other factors specific to the risk class, such as premium rates, and the average and maximum times to settlement of a claim. Clearly an investigation of these effects using computer simulation is only feasible if a number of assumptions are made.

For this study, it was decided to simulate a Motor Vehicle Class (Risk Group Private Cars) since this is an important type of risk for many general insurers, and also an almost complete set of claims settlement analysis data was available. Most basic parameters of the risk class, such as the average amounts of small and large claims at constant prices, were estimated from this data using the method described in Appendix A; others, such as the number of policies, and expenses, had to be allocated values consistent with what is generally known in this field. A basic assumption was that these parameters remained constant during the simulation, and also before the start of the simulation, thus excluding any effects due to business growth or underlying trends in real claims amounts and incidence.

Investments were assumed to be held simply in one year term, fixed interest, bonds whose yield at the time of investment is equal to the prevailing interest rates.

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The study was carried out in two main stages.

The first part investigated what may be called *static situations* in which the rates of interest and inflation are constant. The ASIR model was used to calculate the expected total profit resulting from a year's underwriting, where total profits are defined as the underwriting profit plus the interest received from the investment of technical reserves and funds built up during the year. In static situations, if premium rates are increased each year by the inflation rate then the ratio of expected profits to gross premium income is constant. This ratio is therefore a convenient financial measure, and is a function of the interest rate, the inflation rate, and the initial premium rate.

Inflation rates of 0 % to 30 %, in 5 % intervals, were simulated, together with rates of 40 %, 50 %, 75 % and 100 %, making a total of eleven different inflation rates in all. For each inflation rate, three different interest rates were calculated, equal to 80 %, 90 %, and 100 % of the inflation rate respectively, making a total of thirty-three different combinations of interest and inflation rates. Finally, for each of these combinations, three simulations were run with three different initial premium rates ; these were calculated as those rates giving expected underwriting *losses* of 0 %, 5 % and 10 % of gross premium income respectively. Thus a total of ninety-nine simulation runs was carried out for this part of the study.

The results are set out in detail in Section I. They show that, at low inflation rates, premium rates are the major factor affecting total profits, but as inflation rates increase, premium rates become less important, and at very high inflation rates the ratio of expected total profits to gross premium income actually increases if premium rates decrease. This somewhat paradoxical result naturally calls into question the validity of the basic assumptions, and in particular the applicability of conventional accounting procedures such as the non-discounting of technical reserves.

The second part of the study investigated some *dynamic situations* in which interest and inflation rates are not constant. Again it was necessary to choose from among the infinite number of possible variations.

It is known that increasing inflation causes problems for insurers. On the underwriting side, there is the difficulty of raising premium rates sufficiently rapidly either because the increase is unanticipated or because of market or regulatory constraints. On the investments side, interest rates may not start rising in line with inflation so that there may be a time lag between the peaks of the respective rates. Furthermore, the insurer may have some of his investments tied up in long-term, low yielding fixed interest bonds. It was therefore decided to concentrate on two simple models involving rising inflation rates, the *ramp* model and the *chimney* model.

The two models are essentially shifts from an initial static situation *I* to a final static situation *F*, with the shift occurring over a relatively short period of time. In the *ramp* model, interest and inflation rate in *F* are always significantly higher than in *I*, and they increase steadily from their initial to their final values. A ramp environment is specified by seven numerical values : the initial and final values of interest and inflation, the lengths of time taken to change from the first to the second, and the time

lag between changes in the two rates. A schematic representation of such an environment is in Figure 1 (a). In the *chimney* model, rates in *F* are not necessarily higher than in *I*; the important feature is that during the shift period, rates reach a higher level than in either *I* or *F*. These higher rates are maintained for a (short) time, thus giving a chimney shape as depicted in Figure 1 (b). In addition to the numerical values specified for a ramp, a chimney environment needs four further numerical specifications, to specify the lengths of time in the sub-sections of the transition period.

Several sets of values were chosen for ramp and chimney simulations, and these are shown in Table 1. These figures approximate some actual rates and their changes that have occurred within the last twenty years in the UK and elsewhere. Inflation rates started rising in year 9 of each simulation, and each basic set was run three times, with time lags of 0, 1, and 2 years respectively between the start of the rise in inflation and the start of the rise in interest rates. The difficulties experienced by insurers in estimating loss reserves when changes in inflation are unanticipated were simulated by specifying that loss reserves for outstanding claims would be underestimated by

Figure 1 : Dynamic models showing shifts from a static situation I to a static situation F. Inflation rates are depicted by continuous lines, interest rates by broken lines

(a) Ramp model

T_1 = transition period for inflation rates

T_2 = transition period for interest rates

L = lag between starts

T_1 = total transition period for inflation rates

T_2 = total transition period for interest rates

L = lag between starts

increasing proportions up to year 10, then gradually returning to the expected values. Constraints on setting premiums were simulated by specifying a low value for the credibility of experience rating.

There is, of course, no single constant financial ratio which characterises the results of a dynamic simulation ; it is only possible to investigate and compare the values of some of the key variables. It was expected that in the specified simulations there would be a sag in profits after year 8, possibly followed by a recovery. Bearing in mind two major objectives of an insurer, namely to remain solvent, and to make profits for the shareholders, it was decided to simulate the risk class with its own shareholders funds. These funds would be maintained (if possible) at 17.5 % of gross premium income, assuming that the required solvency margin is 16 % of premium income and that a cushion of a further 1.5 % is aimed for. Any surplus funds after tax at 50 % are declared as final dividends at the end of each simulated year, and it is the stream of these dividends which is the main variable of interest in the results. These results are presented in detail in Section 2.

Table 1 : Parameters for dynamic models

(a) Ramp models

	<u>Initial values %</u>		<u>Final values %</u>		<u>Transition Times (years)</u>	
	Interest	Inflation	Interest	Inflation	Interest	Inflation
R1	5	5	10	10	2	2
R2	5	5	15	15	2	2
R3	15	15	25	25	2	2

(b) Chimney models

	<u>Initial values %</u>		<u>Maximum %</u>		<u>Final values %</u>		<u>Total transition time (years)</u>	
	Interest	Inflation	Interest	Inflation	Interest	Inflation	Interest	Inflation
C1	5	5	15	15	5	5	6	6
C2	10	10	25	25	10	10	6	6
C3	10	10	30	30	15	15	6	6

Notes :

1. Each of the six basic models was simulated three times, with lags of 0, 1, and 2 years respectively between the start of the rise in inflation and the start of the rise in interest rates.
2. In each chimney model, the total transition time comprised 2 years of rising rates, 2 years at the maximum rate and 2 years decreasing rates.

1. Static situation simulations

As described in the summary, the main variable of interest was the ratio of expected total profits before tax to premium income. This is constant over time for any given combination of premium rate, inflation rate and interest rate. These ratios are tabulated in Table 2. They are also illustrated graphically in Figures 2, 3, and 4 which show their variation with inflation in the three cases of fixed interest to inflation ratios.

The striking features of these results were :

- (a) the rapid increase of the ratios with inflation ;
- (b) their convergence (at a given interest to inflation ratio) at high inflation rates ;
- (c) the reversal of the expected results at the very highest inflation rates.

*Table 2 : Expected total profits before taxation
as a percentage of premium income in static situations*

<i>Underwriting profit/premium income</i>	0			-5 %			-10 %		
<i>Interest/ Inflation Inflation %</i>	1.0	0.9	0.8	1.0	0.9	0.8	1.0	0.9	0.8
0	0	0	0	-5.00	-5.00	-5.00	-10.00	-10.00	-10.00
5	5.88	5.27	4.67	1.05	0.43	-0.25	-3.71	-4.35	-5.17
10	12.30	11.01	9.73	7.66	6.34	5.03	3.03	1.67	0.32
15	19.35	17.28	15.24	14.96	12.83	10.72	10.57	8.38	6.21
20	27.07	24.12	21.23	22.99	19.94	16.95	18.91	15.76	12.67
25	35.21	31.30	27.48	31.49	27.44	23.48	27.77	23.57	19.47
30	44.11	39.13	34.28	40.85	35.68	30.64	37.55	32.19	26.96
40	63.20	55.82	48.68	60.87	53.34	45.74	58.52	50.53	42.77
50	82.74	72.78	63.21	81.48	71.05	61.05	80.22	69.33	58.89
75	125.93	109.66	94.30	127.01	109.93	93.81	128.09	110.20	93.32
100	159.87	137.93	117.51	162.59	139.54	118.11	165.30	141.16	118.71

Figure 2 : Expected total profits as a percentage of premium income when interest = inflation (shown by dotted line). The three curves connect the results of simulations in which underwriting profits are 0, - 5 % and - 10 % of premium income respectively.

Figure 3 : Expected total profits as a percentage of premium income when interest = 90 % of inflation (shown by dotted line). The three curves connect the results of simulations in which underwriting profits are 0, - 5 % and - 10 % of premium income respectively.

Figure 4 : Expected total profits as a percentage of premium income when interest = 80 % of inflation (shown by dotted line). The three curves connect the results of simulations in which underwriting profits are 0, - 5 % and - 10 % of premium income respectively.

*Table 3 : Profits available for shareholders
after deduction of tax at 50 % and increase in solvency margin,
as a percentage of premium income in static situations*

<i>Underwriting profit/premium income</i>	0				- 5 %				-10 %			
<i>Interest/ Inflation</i>	1.0				1.0				1.0			
<i>Inflation %</i>	0	0.9	0.8		0	0.9	0.8		0	0.9	0.8	
0	0	0	0		-5.00	-5.00	-5.00		-10.00	-10.00	-10.00	
5	2.18	1.87	1.57		-0.24	-0.55	-0.74		-4.47	-5.11	-5.93	
10	4.70	4.05	3.41		2.38	1.72	1.06		0.06	-0.62	-1.29	
15	7.59	6.55	5.53		5.39	4.33	3.27		3.20	2.10	1.02	
20	10.87	9.39	7.95		8.83	7.30	5.81		6.79	5.21	3.67	
25	14.41	12.45	10.54		12.55	10.52	8.54		10.69	8.59	6.54	
30	18.36	15.87	13.45		16.73	14.15	11.63		15.08	12.40	9.79	
40	27.03	23.33	19.77		25.86	22.10	18.20		24.69	20.69	16.81	
50	36.04	31.06	26.27		35.41	30.19	25.19		34.78	29.33	24.11	
75	56.11	47.97	40.29		56.65	48.11	40.05		57.19	48.24	39.80	
100	71.94	60.97	50.76		73.30	61.77	51.06		74.65	62.58	51.36	

These effects are due mainly to the rapid rise in the amount of loss reserves for outstanding claims. It is clear that as long as loss reserves are calculated in the conventional manner, and not by a discounting method, the effects of high inflation and interest rates far outweigh the relative effects of premium rates. In contrast, at the very lowest rates of interest and inflation, the setting of premium rates is crucial.

A second ratio was also calculated, which took into account taxation and increases in solvency margins, and this is tabulated in Table 3. Tax is assumed to be at a rate of 50 % on overall profits, and the required minimum solvency margin is assumed to be 16 % of premium income. Thus, each year the margin increases by 16 % of the increase in premium income. Total profits remaining after these deductions is, of course, available for distribution to shareholders, or for augmentation of the free reserves.

It can be seen from Table 3 that at the lowest rates of inflation, interest and premiums, the insurer would not even be able to maintain the solvency margin without recourse to some other funds, but for inflation rates of 15 % and upwards the increase in solvency margin is well exceeded and rapidly increasing amounts of profits (as a percentage of premium income) are available for shareholders.

Of course, these results depend on the validity of the basic assumptions made in the ASIR model, and in particular on the chosen parameters. They must be considered increasingly speculative for the very highest rates of inflation considered in this study, since UK domestic insurers have never experienced such high rates. However, even if consideration is limited to the results for, say, inflation of 40 % and below, the general picture given by the static situation simulations does seem to accord with the experience of direct insurers, in that investment income becomes an increasingly more important part of total income as inflation rises. Conversely, it becomes more important to set premium rates which give an underwriting profit in inflation drops below 10 %.

2. Dynamic situation simulations

As described in the summary, six basic sets of dynamic situations were simulated, namely three ramp models (*R1*, *R2* and *R3*) and three chimney models (*C1*, *C2* and *C3*). The parameters specifying the environment of each set are given in Table 1.

Conceptually, each simulation may be regarded as a possible transition between one of three initial static situations (*I*) and one of four final static situations (*F*). This is shown schematically in Figure 5. There is, of course, a very large number of possible transitions so that the results of this section must be considered as a small sample which may indicate some general features.

These results are also a function of simulated company decisions, in particular the setting of premium rates and the estimation of the amounts of outstanding claims (loss reserves). For premium rating, the ASIR model allows two separate multiplicative factors to be specified depending respectively on past inflation rates and past claims experience. Premium rates are thus partly dependent on loss reserve estimation if a non-zero credibility rating is specified.

*Figure 5 : Transitions between initial and final static situations.
The pairs of numbers in brackets are the combinations of interest and inflation rates,
as percentages, defining the static situations*

A simple premium rating decision model was adopted for all the simulations. This assumed that :

- a) premiums were updated at the end of each year ;
- b) the inflation factor was always equal to the inflation rate during the year ;
- c) the credibility rating was .25 up to year 13 and zero thereafter.

Initial premium rates were chosen so as to give different initial ratios of underwriting loss to premium income, depending on the initial environment. These ratios, and the target loss ratios, are given in Table 4. The annual premium income for each of the six simulation sets is shown graphically in Figure 6.

Loss reserves were simulated assuming that an insufficient allowance (in years 1 to 9) was made for future inflation increases. In ramp models, the estimated values then caught up with the true values by year 13. In chimney models, underestimation was followed by overestimation, settling down to the true values by year 15. The actual and estimated amounts of loss reserves are shown graphically in Figures 7 and 8, for two pairs of simulation sets, each pair having the same initial environments. Note that the parameters were slightly different in each simulation so that some transitions were more severe than others ; for example, the relative increase in the estimated loss reserves from year 9 to year 12 is greater for R2 than for the other simulations.

Table 4 : Premium rates parameters

	<i>Initial inflation rate %</i>	<i>Initial underwriting loss as % of premium income</i>	<i>Target loss ratio</i>
<i>R1</i>	5	0	.69
<i>R2</i>	5	0	.69
<i>R3</i>	15	5.6	.72
<i>C1</i>	5	0	.69
<i>C2</i>	10	1.5	.70
<i>C3</i>	10	1.6	.70
		Note : Percentages are rounded off to one decimal place	Note : Ratios are rounded off to two decimal places

It was intended that the simulations would focus on the flow of "dividends" defined in the summary. This can be considered as the contribution of the simulated motor vehicle risk class to the financial results of an insurance company whose portfolio of risks includes this particular class. In order to calculate these values using the ASIR model it was necessary to simulate a hypothetical company comprising just one risk class, with all funds invested in fixed interest bonds. In practice, losses in one branch of an insurance company may be offset by gains in another ; moreover, a proportion of investments is held in property and equities, whose market values may perhaps rise in line with inflation rates. For these reasons, the level of shareholders funds below which the hypothetical simulated company would be declared insolvent by the ASIR model was arbitrarily taken to be 10 % of premium income. The current EEC solvency requirement is, of course, just over 16 % of premium income (if calculated on a premium basis), and this is reflected in the calculation of the "dividends" paid by the hypothetical company as equal to any profits remaining after tax at 50 % and maintenance of shareholders funds at 17.5 % of premiums.

A first glance at the simulation results shows that, out of 18 simulation runs, five were terminated prematurely because of "insolvency" ; as explained above this must be interpreted as indicating that an insurance company would run into severe difficulties. The details are given in Table 5. As expected, the majority (4) occurred when there was the greatest lag between the onsets of inflation increases and interest rate increases.

A more detailed survey of the values of the ratio of shareholders funds to premium income is given in Table 6, which gives the minimum ratio attained in each

simulation run. It can be seen that the aimed-for level of 17.5 % was maintained in each zero lag simulation except R2. In the one year and two year lag simulations, however, the level dropped below 16 % in eleven of the runs, including the five runs which were terminated prematurely. This suggests that non-zero lags can have detrimental effects on solvency margins, these effects becoming increasingly severe with increasing lags.

Table 5 : Years in which simulation runs were terminated prematurely because the level of shareholders funds fell below 10 % of premium income

	<i>Lag 0</i>	<i>Lag 1</i>	<i>Lag 2</i>
R1	—	—	—
R2	—	Year 10	Year 10
R3	—	—	—
C1	—	—	Year 11
C2	—	—	Year 11
C3	—	—	Year 11

Of course, a company decision which would affect these results to some degree would be the maintenance of shareholders funds at a higher level than 17.5 % of premium income, so that a larger "cushion" is available to fall back on if unanticipated adverse situations occur. The most severe test of this decision was the R2 group of simulations, probably because the relatively large underestimation of loss reserves in years 1 to 9 led to very heavy losses in years 10 to 12. This group was therefore re-run with different aimed-for levels of shareholders funds, up to 20.5 % of premium income. The simulations with non-zero lags still terminated prematurely, however, and the question of what is a prudent and desirable level of free reserves obviously needs further investigation.

The flows of dividends in each simulation run are depicted graphically in Figures 9 to 14. Note that, for each basic set, the dividend flow in years 1 to 9 is identical for each of the three lags, and the dividend in year 10 is the same for lags 1 and 2 years. After year 10, the flows for lags 1 and 2 either terminate, or are separate until they converge to a common value with the zero lag simulation towards the end of the run.

The results for the three ramp models show a typical sag in dividends in years 10 to 12, followed by a rapid rise (in those runs which did not terminate prematurely). The results for the chimney models show a similar sag, followed by a rapid rise, followed by a relatively small sag due to the drop in interest rates. The striking feature of all these results is that once the difficult years 9 to 12 had been passed, dividends increased, and by the end of each simulation run, the level of dividends was substantially higher than it would have been if no surges in inflation and interest rates had taken place. This is true even in the chimney models C1 and C2, in which the final environment is the same as the initial environment. This is because the loss reserves were

substantially increased in years 10 to 12 thus giving a larger amount of technical reserves from which investment income was generated.

However it must again be emphasised that these results depend on many assumptions, and in particular that different premium rates might lead to less striking dividend flows. Indeed an interesting follow-up to a study such as this might be to investigate the effect of taking interest rates into account when setting premium rates, as well as inflation rates and claims experience. This would act in two ways : firstly, premiums would be increased more rapidly in difficult times such as years 10 to 12 in the two years lag simulations ; secondly, premiums would be increased less rapidly in favourable times such as high interest rate environments. This would, however, require an additional module for the existing ASIR model.

Table 6 : Years in which the ratio of shareholders funds to premium income fell below 17.5 % and the minimum ratio attained

	<i>Lag</i>	<i>Years below 17.5 %</i>	<i>Minimum Ratio (%)</i>	<i>Year in which attained</i>
<i>R1</i>	0	—	—	—
	1	10, 11	15.9	11
	2	10, 11, 12, 13	13.1	11
<i>R2</i>	0	10, 11	16.3	10
	1	10	8.9	10
	2	10	8.8	10
<i>R3</i>	0	—	—	—
	1	10	16.3	10
	2	10, 11, 12	13.3	11
<i>C1</i>	0	—	—	—
	1	10, 11	12.3	10
	2	10, 11	7.1	11
<i>C2</i>	0	—	—	—
	1	10, 11	14.1	10
	2	10, 11	8.8	11
<i>C3</i>	0	—	—	—
	1	10, 11	10.4	10
	2	10	9.9	10

Note : Percentages are rounded to one decimal place.

Figure 6 : Premium income in the first 16 years of each basic set of simulations, in units of £ 000. The continuous curves are, from top to bottom, R3, R2 and R1. The dashed curves are, from top to bottom, C3, C2 and C1.

Figure 7 : Actual and estimated loss reserves at the beginning of each year, in units of £ 000, for simulations R2 (upper curves) and C1 (lower curves). Estimated reserves are indicated by the dotted lines.

Figure 8 : Actual and estimated loss reserves at the beginning of each year, in units of £ 000, for simulations C2 (lower curves) and C3 (upper curves). Estimated reserves are shown by the dotted lines.

Figure 9 : Flow of dividends, in units of £ 000, for simulation R1. The upper curve connects the values generated by the zero lag simulation. The flows for lags 1 and 2 years separate after year 10, and converge to the zero lag values in year 15.

Figure 10 : Flow of dividends, in units of £ 000, for simulation R2.
The flows for lags 1 and 2 years stop after year 10.

Figure 11 : Flow of dividends, in units of £ 000, for simulation R3.
 The upper curve connects the values generated by the zero lag simulation.
 The flows for lags 1 and 2 separate after year 10 and converge to a common value
 with lag 0 after year 14.

Figure 12 : Flow of dividends, in units of £ 000, for simulation C1.
The solid line connects values generated by the zero lag simulation.
The flow for lag 2 stops after year 10, while the flow for lag 1 is depicted
by the dashed line after year 11.

Figure 13 : Flow of dividends, in units of £ 000, for simulation C2.
The solid line connects values generated by the zero lag simulation.
The flow for lag 2 stops after year 10, while the flow for lag 1 after year 11
is shown by the dashed line.

Figure 14 : Flow of dividends, in units of £ 000, for simulation C3.
The solid line connects values generated by the zero lag simulation.
The flow for lag 2 stops after year 10, while the flow for lag 1 after year 11
is shown by the dashed line.

Appendix : Determination of risk class parameters

A1 Times between incurrence and settlement of claims, and the average proportion of "large" claims

The ASIR model requires three numerical values for time parameters : the minimum time to settlement u , the average time to settlement a , and the maximum time to settlement M . It also requires the average proportion p , of claims that are "large", i.e. sampled from a log normal distribution.

The data on which the numerical estimates were based is a claim settlement analysis of claims incurred for 1970 in a Motor Vehicle (Private Cars) risk class of a large UK insurer. The total number of claims attributable to the year of origin was given as 92135, and the numbers of claims settled each year are set out in Table A1.1.

The first step was to estimate a value for M . In ASIR, M may only be assigned one of ten values ranging from 3.5 to 15 years. Although the data in Table A1.1 is incomplete, it seems reasonable to assume that the 34 claims outstanding at the end of year 9 will be settled by the end of year 11, and thus the most appropriate value of M is 10.5 years.

Table A1.1 : Number of claims settled, from 1970 claims settlement analysis

<i>Financial Year</i>		<i>Number of claims closed in year</i>		<i>Number of claims outstanding at the end of the year</i>
		<i>at no cost</i>	<i>at some cost</i>	
1970	1	9,019	37,137	41,472
1971	2	9,776	28,115	8,063
1972	3	1,399	3,368	3,248
1973	4	757	1,668	864
1974	5	71	444	377
1975	6	23	179	163
1976	7	9	86	75
1977	8	1	26	50
1978	9	3	20	34

Next, the total number of non-zero claims had to be estimated, since the ASIR parameters are related to these. The total number of claims closed at some cost by the end of year 9 is 71,043, so the required estimate lies between 71,043 and 71,077 ($= 71,043 + 34$). The actual number chosen has very little significance, and was taken to be 71,070. Thus 7 of the 34 outstanding claims were assumed to be closed at no cost, while the remaining 27 claims were arbitrarily assumed to be settled in year 10 (15 claims) and year 11 (12 claims).

There is no information about the minimum time to settlement, and so u was temporarily assigned the value of .05 years. This value of u did in fact give a good fit to the data once u and p had been calculated, so it was the final value for u ; if necessary, different values of u could have been tried.

The next step was to estimate a rough value for a , the average time to settlement, bearing in mind that ASIR assumes that claims are incurred at a uniform rate throughout the year. With such aggregated data, it is impossible at this stage to obtain anything more than an approximation. One approach is to take each year's settled claims and assume they were all settled at the end of the first quarter of the year, and all incurred in the middle of year 1. This works for each year of the run-off except year 1, for which an average value between incurrence and settlement has to be guessed. This was taken to be .25 years, and the rough value for a was calculated as

$$\begin{aligned} & (37137 \times .25 + 28115 \times .75 + 3368 \times 1.75 + 1668 \times 2.75 \\ & + 444 \times 3.75 + 179 \times 4.75 + 86 \times 5.75 + 26 \times 6.75 \\ & + 20 \times 7.75 + 15 \times 8.75 + 12 \times 9.75) / 71070 \end{aligned}$$

which is approximately .6 years.

Next, a rough value for p was estimated. One approach is to use the fact that virtually all the claims settled in years 1 and 2 are "small", so that

$$\begin{aligned} 1 - p & < (37137 + 28115) / 71070 = .918 \\ \text{i.e. } p & < .082 \end{aligned}$$

whereas virtually all the claims settled in years 5 onwards are "large", so that

$$\begin{aligned} p & > (444 + 179 + 86 + 26 + 20 + 15 + 12) / 710170 \\ \text{i.e. } p & > .011 \end{aligned}$$

(Years 3 and 4 are likely to contain a mixture of "small" and "large" claims.)

The simplest approximation is to take the average of the two bounds for p , which is .0465 (or 4.65 %).

Use was now made of a computer program which calculated the expected values of the numbers of claims settled in each year of the run off, exactly as it would be done in an ASIR simulation, given u , M , and the total number of claims. Ranges of values for a and p , based on the rough approximations, were tested until the best fit (determined by the lowest chi-square statistic) was obtained. The final values for the mean time to settlement and the proportion of large claims were respectively

$$\begin{aligned} & .569 \text{ years} \\ \text{and} & \quad 5.32 \% \end{aligned}$$

The actual and expected number of claims settled are set out in Table A1.2, together with the cumulative percentages. It can be seen that there is a good fit between the statistical distribution and the actual data.

A2 Means and standard deviations of "large" and "small" claims

The ASIR model requires four numerical values for claims amount parameters; the means of "small" and "large" claims, and their standard deviations, at current prices.

The data given in the 1970 claims settlement analysis is set out in Table A2.1. It is immediately apparent that a substantial proportion of claims amounts is paid in a year previous to the year in which the claim is finally closed ; for example, over 34 % of the total amount paid in year 1 is for claims settled in year 2 onwards.

This posed two problems in the use of the ASIR model : firstly, there is no mechanism for partial payment of claims in Group risks and, secondly, even if such a mechanism were available it is not possible from the given data to deduce exactly how

Table A1.2 : Expected and actual numbers of non-zero claims settled, and their cumulative percentages

Year	Number settled		Cumulative Percentage settled	
	Actual	Expected	Actual	Expected
1	37,137	37,544	52.25	52.83
2	28,115	27,197	91.81	91.09
3	3,368	4,125	96.55	96.90
4	1,668	1,263	98.90	98.68
5	444	521	99.52	99.41
6	179	228	99.78	99.73
7	86	103	99.90	99.87
8	26	47	99.93	99.94
9	20	22	99.96	99.97
10	(15)	10	(99.98)	99.99
11	(12)	7	(100.00)	100.00
12	(0)	3	(100.00)	100.00

Note : Percentages are rounded to 2 decimal places.

much was paid as partial payments, and in what years. Any assumptions made about the timing of partial payments may make a significant difference to the estimation of claims payments in current prices, as illustrated by the following simple example.

Suppose that the total amounts paid for claims incurred in year 1 are as follows (including payments on account) :

Year 1	160
Year 2	40
Year 3	40

but that the total amounts for claims actually settled in each year are:

Year 1	100
Year 2	80
Year 3	60

This data is consistent with either of the following:

- (a) Year 1 : 100 is paid for claims settled in year 1
 plus 40 on account for claims settled in year 2
 plus 20 on account for claims settled in year 3
 Year 2 : 40 is paid for claims settled in year 2
 Year 3 : 40 is paid for claims settled in year 3
- (b) Year 1 : 100 is paid for claims settled in year 1
 plus 60 on account for claims settled in year 2
 Year 2 : 20 is paid for claims settled in year 2
 plus 20 on account for claims settled in year 3
 Year 3 : 40 is paid for claims settled in year 3

Suppose inflation is constant at 25 % per year, then the total amount of claims settled in years 2 and 3, in current prices (i.e. Year 1 prices) is :

- (a) Year 2 $40 + 40 / 1.25 = 72$
 Year 3 $40 / (1.25)^2 + 20 = 45.6$
- (b) Year 2 $60 + 20 / 1.25 = 76$
 Year 3 $40 / (1.25)^2 + 20 / 1.25 = 41.6$

Table A2.1 : Amounts of claims settled, from 1970 claims settlement analysis

Financial Year		Amount of payments made in the year in settlement or on account	X Aggregate payments made up to the end of year	Claims outstanding at the end of the year		Total amount paid and outstanding at the end
				Payments on account included in column X	Estimated payments remaining to be made	
1970	1	4,177,319	4,177,319	1,452,955	4,824,123	9,001,442
1971	2	2,649,892	6,827,211	924,580	2,130,244	8,957,455
1972	3	714,269	7,541,480	760,618	1,377,889	8,919,369
1973	4	709,267	8,250,747	637,843	892,018	9,142,765
1974	5	293,136	8,543,883	519,353	531,772	9,075,655
1975	6	270,514	8,814,397	490,097	335,146	9,149,543
1976	7	147,032	8,961,429	333,130	156,667	9,118,096
1977	8	91,579	9,053,008	125,146	59,802	9,112,810
1978	9	44,373	9,097,381	59,847	21,443	9,118,824

Table A2.2 : Total amounts paid each year, in 1970 prices

Year		Amount
1970	1	4,177,319
1971	2	2,455,920
1972	3	611,771
1973	4	562,023
1974	5	206,280
1975	6	158,494
1976	7	71,619
1977	8	38,417
1978	9	16,622
1979	10	(4,207)
1980	11	(2,792)
Total		8,305,464 Mean 116,863

Note: the amounts for years 10 and 11 were estimated by assuming that the payments outstanding at the end of year 9 (in Table A2.1) were actually paid in year 10 (12,433) and year 11 (9,000).

It was clear that, at best, only imperfect estimates could be made from the available data, and the choice lay between either assuming an arbitrary pattern of partial payments or taking the actual amounts paid in each year of the run off as the total amounts of claims settled in that year, converting these to 1970 prices, and using this data to estimate the claims parameters. The latter option was in fact adopted, and the deflated values (using actual U.K. inflation rates) are shown in Table A2.2.

With such a large number of claims, the sample mean is a very good approximation to the theoretical mean, so values of the means of "small" and "large" claims, a_s and a_l , were sought which satisfied the equation

$$(1 - p)a_s + pa_l = 116.863$$

The value of p had, of course, already been determined (see A.1) as .0532. It was then a question of trying out various combinations of a_s and a_l , together with guessed values for the standard deviations, and finding the best fit with the data. A computer program was used which calculated the expected values for the amounts of claims settled in each year of the run-off for ranges of values of the four claims parameters, keeping the total number of claims, and the time parameters and the proportion of large claims, fixed. The final estimates were as follows :

mean of "small" claims	110.54
standard deviation	41.00
means of "large" claims	230.10
standard deviation	810.00

The expected and actual claims amounts are set out in Table A2.3, together with the cumulative percentages. It can be seen that the fitted values underestimate the claims amounts for year 1, overestimate the cumulative claims amounts in years 2 and 3, and underestimate the *cumulative* claims amounts in the remaining years, because of overestimation of claims amounts in year 7 onwards. This is clearly an effect of the partial payments, and should be borne in mind when considering the results of simulations based on these parameters.

A3 Average incidence of claims, premium rate changes, expenses and time delay parameters

The remaining risk class parameters had to be assigned values based on general knowledge and a certain amount of guesswork.

Firstly, the average percentage of policies resulting in non-zero claims was set to 15 %, a figure which seems to be of the right order for the current experience of many

Table A2.3 : Expected and actual amounts of claims settled, and their cumulative percentages

Year	Amounts settled		Cumulative Percentages	
	Actual	Expected	Actual	Expected
1	4,177,319	3,861,515	50.30	46.68
2	2,455,920	3,094,156	79.87	83.72
3	611,771	471,817	87.23	89.40
4	562,023	167,366	94.00	91.41
5	206,280	150,314	96.48	93.22
6	158,494	139,498	98.39	94.90
7	71,619	116,058	99.25	96.30
8	38,417	88,777	99.72	97.37
9	16,622	64,797	99.92	98.15
10	4,207	46,084	99.97	98.70
11	2,792	66,694	100.00	99.51
12	0	41,059	100.00	100.00

Kolmogorov-Smirnov statistic (the greatest cumulative percentage difference between actual and expected) = 3.82 %.

Note : percentages are rounded to two decimal places.

insurers. The total number of policies was then set to 473,827, which gave the expected number of non-zero claims each year as 71074, very close to the number in the sample.

In simulations with inflation rates less than 25 % per year, premiums wer changed at the end of each year ; above this level, they were changed half yearly, since this is more likely at higher inflation rates.

As stated in the Summary, the number of policies was kept constant.

Expenses parameters were assigned values which would give an expenses ratio (in simulations with low inflation rates) of approximately 30 %. They were as follows :

Commission on renewals	22.5 %
Fixed expenses	150000
Average expenses per renewable policy	.75
Average expenses per claim reported	5.00

(The monetary items are, of course, at initial price levels.)

The minimum and average times to reporting of claims were assigned values less than the corresponding values for times to settlement, for consistency, and these were 0 and 0.2 years respectively.

The time lags between accounting for, and actually receiving, premiums and commissions were set to .06 years and .07 years respectively.

Expected values for claims numbers and amounts were calculated in all the simulations. In the static environment simulations, loss reserve estimation was exact. Loss reserve estimation in dynamic simulations is described in more detail in Section 2.