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## Bogolon-mediated light absorption in atomic condensates of different dimensionality

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In the case of structureless bosons, cooled down to low temperatures, the absorption of electromagnetic waves by their Bose-Einstein condensate is usually forbidden due to the momentum and energy conservation laws: the phase velocity of the collective modes of the condensate called bogolons is sufficiently lower than the speed of light. Thus, only the light scattering processes persist. However, the situation might be different in the case of composite bosons or the bosons with an internal structure. Here, we develop a microscopic theory of electromagnetic power absorption by a Bose-Einstein condensates of cold atoms in various dimensions, utilizing the Bogoliubov model of a weakly-interacting Bose gas. Thus, we address the transitions between a collective coherent state of bosons and the discrete energy levels corresponding to excited internal degrees of freedom of non-condensed individual bosons. It is shown, that such transitions are mediated by one and two-bogolon excitations above the condensate, which demonstrate different efficiency at different frequencies and strongly depend on the condensate density, which influence varies depending on the dimensionality of the system.

Radiation pressure generates a flow of particles, for instance, electrons, molecules, or atoms due to the momentum transfer from the photons into the system. The direction of the resultant, generated by the light field current of particles coincides (up to the sign) with the direction of the wave vector of the light field,  $\mathbf{j}(\omega) \sim \mathbf{k}\alpha(\omega)I$ , where  $\mathbf{k}$  is the photon wave vector,  $I$  is the intensity of the electromagnetic (EM) wave and  $\alpha(\omega)$  is the absorption coefficient with  $\omega$  the EM field frequency. This phenomenon is often referred to as the photon drag effect. It has been widely studied in generic two-dimensional (2D) electron gas and graphene systems<sup>1,2</sup>, metal films<sup>3,4</sup>, topological insulators<sup>5,6</sup>, van der Waals structures and Dirac materials<sup>7,8</sup>, and cavities<sup>9</sup>.

In cold atoms, the role of radiation pressure-based techniques cannot be overestimated. The primary reason why the radiation pressure is so important there is the possibility of particle manipulation and trapping by external fields<sup>10,11</sup>, resulting in their confinement and focusing or acceleration<sup>12–16</sup>. This allows for, in particular, effective laser cooling<sup>17–20</sup> by utilizing several coherent laser beams with the possibility of subsequent formation of an atomic Bose-Einstein condensate (BEC)<sup>21</sup>.

In the normal state of atomic gas, it experiences two fundamental processes of light-matter interaction, i.e. light scattering and light absorption. However, once the BEC is formed, the interaction of light by the system gets suppressed. Indeed, according to, e.g., the Bogoliubov model of a weakly interacting Bose gas, since the over-BEC excitations of condensate density fluctuations, bogolons, possess linear dispersion with their velocity much smaller than the speed of light, the system absorption violates the conservation laws. That is why only the light scattering processes have been actively studied in cold atom BECs.

However, if we consider Bose particles with internal degrees of freedom, then finite absorption can happen involving the transitions between the energy states of internal particle motion. Then, the system still experiences the influence of the external light field after the formation of the BEC. Indeed, the spectrum of a single atom with an eigenfunction  $|\eta, \mathbf{p}\rangle$  reads  $E_\eta(\mathbf{p}) = E(\mathbf{p}) + \Delta_\eta$ , where  $E(\mathbf{p}) = \mathbf{p}^2/2m$  is a kinetic energy of the particle center-of-mass motion, and  $\Delta_\eta$  is the energy spectrum of the internal motion of the atom; the index  $\eta$  stands for the full set of quantum numbers characterizing the internal spectrum of the particle, thus, the value  $\eta = 0$  refers to the ground state of the internal spectrum of Bose particles. Due to the interaction between the internal motion of the atomic gas and photon gas, photon bubbles become unstable<sup>22</sup>. In absence of the coupling between

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photon gas and atomic gas, the dispersion gives two independent modes. A purely decaying photon mode is characterized by damping, resulting from absorption and diffusion. An oscillating fluid mode is characterized by decay due to viscosity. However, with the coupling between the two fluids, the two modes become unstable, resulting in the formation of static and dynamic photon bubbles. The photon bubble instability is experimentally observed in two different regimes<sup>23</sup>.

In our subsequent analysis, we assume that initially, the system is in the BEC state,  $|\eta = 0, \mathbf{p} = 0\rangle$ , and study a particular type of radiation pressure phenomena to occur in atomic condensates of different dimensionalities. To quantify the effect, we calculate the absorption probability and the absorption coefficient of the system and analyze their behavior. Obviously, the frequency dependence of the radiation pressure is determined by the spectrum of the absorption coefficient. This dependence is usually either monotonous or resonant: If the external EM field frequency  $\omega$  approaches the energy of a quantum transition in the system, the response might experience resonant behavior.

This Letter is organized as follows: In “**System Hamiltonian and light–bosons interaction**”, we start with the Hamiltonian of the system describing the interaction between the bosonic particles and the external electromagnetic field and we employ the Bogoliubov model of a weakly-interacting bose gas in order to investigate the emission of bogolons in this system. In “**Absorption probability**”, we show two different types of processes that arise by the absorption of a photon, resulting in the creation of an excited particle and several bogolons and we calculate the absorption coefficients of the processes in various dimensions with different condensate densities.

### System Hamiltonian and light–bosons interaction

Let us consider a bosonic field operator as a composition of the center of mass term,  $\psi_\eta(\mathbf{r}, t)$ , and an internal motion term,  $\chi_\eta$ , respectively. Thus, the full operator for the bosonic particle is  $\psi_\eta(\mathbf{r}, t)\chi_\eta$ , where  $\mathbf{r}$  is the center of mass coordinate and  $\eta$  is the quantum number to represent the particle’s internal degrees of freedom. Then, the total Hamiltonian of the system reads<sup>24</sup>,

$$\hat{H} = \hat{H}_0 + \hat{V}, \quad (1)$$

where

$$\hat{V} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}} = -\sum_{\eta\eta'} \mathbf{d}_{\eta'\eta} \int d\mathbf{r} \psi_{\eta'}^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_\eta(\mathbf{r}, t) \quad (2)$$

describes interaction between the bosons and light. In Eq. (2),  $\hat{\mathbf{E}}(\mathbf{r}, t) = \hat{\mathbf{E}}_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}}t) + \text{c.c.}$  according to the classical representation of the light field with  $\omega_{\mathbf{k}} = c|\mathbf{k}|$ , and, the matrix elements of the dipole moment operator read as  $\mathbf{d}_{\eta'\eta} = \langle \chi_{\eta'} | \hat{\mathbf{d}} | \chi_\eta \rangle$ .

Integrating out the internal particle motion variables yields bare Hamiltonian of the system,

$$\begin{aligned} \hat{H}_0 = \int d\mathbf{r} \psi_0^\dagger(\mathbf{r}, t) \left[ \frac{\hat{\mathbf{p}}^2}{2m} - \mu + g|\psi_0(\mathbf{r}, t)|^2 \right] \psi_0(\mathbf{r}, t) \\ + \sum_{\eta \neq 0} \int d\mathbf{r} \psi_\eta^\dagger(\mathbf{r}, t) \left[ \frac{\hat{\mathbf{p}}^2}{2m} + \Delta_\eta \right] \psi_\eta(\mathbf{r}, t), \end{aligned} \quad (3)$$

where  $\mu$  is the chemical potential and  $g$  is the particle–particle interaction strength for the ground state of the bosonic system; for simplicity and clarity of results, let us assume that the spectrum of the internal motion of particles is equidistant,  $\Delta_\eta = \eta\Delta$ . The employment of a realistic spectrum of particular atoms, such as hydrogen-like spectrum,  $\Delta_\eta = \Delta/\eta^2$ , is a trivial complication.

The first line in Eq. (3) represents the Gross-Pitaevskii (GP) equation describing atomic BEC in the ground state with  $\eta = 0$  and  $\mathbf{p} = 0$ , and the second line describes the bosons in the excited internal states  $\eta \neq 0$ . We will assume that most of the particles are in the condensate with the quantum number  $\eta = 0$ , thus, the interaction of non-condensed particles with  $\eta \neq 0$  with each other and with the BEC is weak and can be disregarded.

The system described by the Hamiltonian (3) possesses two types of low-energy excitations. The BEC described by the GP equation is characterized by sound-like excitations of its density (bogolons) and single-particle excitations describing the motion of individual atoms with  $\eta \neq 0$ , as indicated by the second term in Eq. (3). The field operator also contains two terms,  $\psi_0$  and  $\psi_{\eta \neq 0}$ , corresponding to these two types of excitations in the system, respectively. We assume that  $|\psi_0|^2 \gg |\psi_{\eta \neq 0}|^2$ , thus indicating that at low temperatures most of particles are in the BEC state.

For legibility, it is convenient to separate the terms containing  $\eta = 0$  and  $\eta \neq 0$  in Eq. (2),

$$\begin{aligned} \hat{V} = \hat{V}_1 + \hat{V}_2 + \hat{V}_3 + \hat{V}_4 \\ = -\mathbf{d}_{00} \int d\mathbf{r} \psi_0^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_0(\mathbf{r}, t) - \sum_{\eta \neq 0} \mathbf{d}_{0\eta} \int d\mathbf{r} \psi_0^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_\eta(\mathbf{r}, t) \\ - \sum_{\eta' \neq 0} \mathbf{d}_{\eta'0} \int d\mathbf{r} \psi_{\eta'}^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_0(\mathbf{r}, t) - \sum_{\eta, \eta' \neq 0} \mathbf{d}_{\eta'\eta} \int d\mathbf{r} \psi_{\eta'}^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_\eta(\mathbf{r}, t). \end{aligned} \quad (4)$$

For the particles with  $\eta \neq 0$ , one can use the plain-wave ansatz:  $\psi_{\eta \neq 0}(\mathbf{r}, t) = \sum_{\mathbf{p}} c_{\eta\mathbf{p}}(t) \exp(i\mathbf{p}\mathbf{r})$ , where  $c_{\eta\mathbf{p}}(t) = c_{\eta\mathbf{p}}(0) \exp(-iE_\eta(\mathbf{p})t)$  with the corresponding energy  $E_\eta(\mathbf{p}) = p^2/2m + \Delta_\eta$ . For the condensate,  $\eta = 0$ ,

the Bogoliubov transformation reads  $\psi_0(\mathbf{r}, t) = \sum_{\mathbf{p}} \left[ \sqrt{n_c} \delta(\mathbf{p}) + u_{\mathbf{p}} b_{\mathbf{p}}(t) + v_{\mathbf{p}} b_{-\mathbf{p}}^\dagger(t) \right] \exp(i\mathbf{p}\mathbf{r})$ , where  $n_c$  is the density of the condensate,  $u_{\mathbf{p}}$  and  $v_{\mathbf{p}}$  are the Bogoliubov transformation coefficients, and  $b_{\mathbf{p}}(t) = b_{\mathbf{p}}(0) \exp(-i\varepsilon_{\mathbf{p}}t)$  is the annihilation operator for Bogoliubov quasi-particle (bogolon). The spectrum of bogolons is given by<sup>25</sup>  $\varepsilon_{\mathbf{p}} = sp\sqrt{1 + p^2\xi^2}$  with  $s = \sqrt{n_c g/m}$  being the sound velocity and  $\xi = (2ms)^{-1}$  being the healing length. Then, the first term in Eq. (4) reads:

$$\begin{aligned} \hat{V}_1 &= -\mathbf{d}_{00} \int d\mathbf{r} \psi_0^\dagger(\mathbf{r}, t) \hat{\mathbf{E}}(\mathbf{r}, t) \psi_0(\mathbf{r}, t) \\ &= -\mathbf{d}_{00} \sum_{\mathbf{p}'\mathbf{p}} \left[ \sqrt{n_c} \delta(\mathbf{p}') + u_{\mathbf{p}'} b_{\mathbf{p}'}^\dagger(t) + v_{\mathbf{p}'} b_{-\mathbf{p}'}(t) \right] \\ &\quad \times \int d\mathbf{r} \exp(-i\mathbf{p}'\mathbf{r}) \hat{\mathbf{E}}(\mathbf{r}, t) \exp(i\mathbf{p}\mathbf{r}) \left[ \sqrt{n_c} \delta(\mathbf{p}) + u_{\mathbf{p}} b_{\mathbf{p}}(t) + v_{\mathbf{p}} b_{-\mathbf{p}}^\dagger(t) \right]. \end{aligned} \tag{5}$$

In what follows, let us only focus on the light absorption processes, thus disregarding the terms containing  $\hat{\mathbf{E}}_0^\dagger$ . Moreover, at low temperatures only the processes accompanied by the emission (not absorption) of bogolons are considerable, thus only the terms containing  $b^\dagger$  and  $b^\dagger b^\dagger$  matter. After the integration, Eq. (5) reads:

$$\begin{aligned} \hat{V}_1 &= -(2\pi)^d \sum_{\mathbf{p}'\mathbf{p}} \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) \left[ \sqrt{n_c} \left( \delta(\mathbf{p}) u_{\mathbf{p}'} b_{\mathbf{p}'}^\dagger(t) + \delta(\mathbf{p}') v_{\mathbf{p}} b_{-\mathbf{p}}^\dagger(t) \right) \right. \\ &\quad \left. + u_{\mathbf{p}'} v_{\mathbf{p}} b_{\mathbf{p}'}^\dagger(t) b_{-\mathbf{p}}^\dagger(t) \right] \mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0(t), \end{aligned} \tag{6}$$

where the trivial case ( $\mathbf{k} = 0$ ) was disregarded. Thus,  $\hat{V}_1$  describes a possible direct light absorption by the BEC with a possible excitation of single or two bogolons in the BEC. It should be noted that these processes are possible only in the BEC of atoms having a nonzero dipole moment in its ground state,  $\mathbf{d}_{00} \neq 0$ .

Following a similar procedure gives other interaction terms,

$$\hat{V}_2 = -(2\pi)^d \sum_{\mathbf{p}'\mathbf{p}} \sum_{\eta} \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) \times u_{\mathbf{p}'} b_{\mathbf{p}'}^\dagger(t) c_{\eta\mathbf{p}}(t) \mathbf{d}_{0\eta} \cdot \hat{\mathbf{E}}_0(t) \tag{7}$$

$$\hat{V}_3 = -(2\pi)^d \sum_{\mathbf{p}'\mathbf{p}} \sum_{\eta} \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) \times c_{\eta\mathbf{p}'}^\dagger(t) \left[ \sqrt{n_c} \delta(\mathbf{p}) + v_{\mathbf{p}} b_{-\mathbf{p}}^\dagger(t) \right] \mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0(t) \tag{8}$$

$$\hat{V}_4 = -(2\pi)^d \sum_{\mathbf{p}'\mathbf{p}} \sum_{\eta\eta'} \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) \times c_{\eta'\mathbf{p}'}^\dagger(t) c_{\eta\mathbf{p}}(t) \mathbf{d}_{\eta'\eta} \cdot \hat{\mathbf{E}}_0(t). \tag{9}$$

Before the system being irradiated, all the particles are in BEC. Therefore,  $\hat{V}_2$  and  $\hat{V}_4$  terms can be disregarded in later consideration.

Let us now discuss the physical meaning of the processes incorporated in  $V_3$  term. The first term in  $V_3$ , which is  $\propto \sqrt{n_c}$ , describes the absorption of the photon with energy  $\omega_{\mathbf{k}}$  and momentum  $\mathbf{k}$  accompanied by the direct excitation of the atom from the BEC state  $\eta = 0$ ;  $\mathbf{p} = 0$  to the noncondensed state with  $\eta' \neq 0$  and momentum  $\mathbf{p}' = \mathbf{k}$  with the energy  $E_{\eta'}(\mathbf{p}' = \mathbf{k}) = \omega_{\mathbf{k}}$ .

The second term in  $V_3$  describes the transition of the BEC atom to the final state with  $\eta' \neq 0$  and momentum  $\mathbf{p}' = \mathbf{p} + \mathbf{k}$  with an arbitrary value of  $\mathbf{p}$ . To conserve the total momentum, a single bogolon with energy  $\varepsilon_{-\mathbf{p}}$  is excited in the BEC carrying away the missing momentum  $-\mathbf{p}$  in such a way that the total momentum in the system is conserved. Below, the processes described by  $V_1$  and  $V_3$  terms are analyzed in more detail.

### Absorption probability

According to the Fermi golden rule, the absorption probability<sup>26</sup> for different interaction channels reads as

$$\alpha = \frac{2\pi}{\hbar} | \langle f | \hat{V} | i \rangle |^2 \delta(E_f + E_i - \omega), \tag{10}$$

where  $|i\rangle$  is the initial (unperturbed) state. The perturbation, which is the EM field, results in the transitions from the initial state to the final state  $|f\rangle$ . The  $\hat{V}_1$  part of BEC-light interaction Hamiltonian contains two different processes. In the first term in Eq. (6), the initial state of the system is the unperturbed BEC and the photon with energy  $\omega_{\mathbf{k}}$ , whereas the final state corresponds to the presence of single bogolon in the system with energy  $\varepsilon_{\mathbf{k}}$ . Thus, the corresponding probability can be written as

$$\alpha_{1b}^1 = \frac{2\pi}{\hbar} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 |u_{-\mathbf{k}} + v_{\mathbf{k}}|^2 \delta(\varepsilon_{\mathbf{k}} - \omega_{\mathbf{k}}). \tag{11}$$

here, 1b stands for one-bogolon-mediated processes, the  $\delta$ -function indicates the energy conservation law for the direct process, photon transforming its energy into a bogolon.

The probability due to the second term in Eq. (6) describes the transitions from the BEC accompanied by two bogolons (2b),

$$\alpha_{2b}^1 = \frac{2\pi}{\hbar} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 \sum_{\mathbf{p}} |u_{\mathbf{p}+\mathbf{k}} v_{\mathbf{p}}|^2 \delta(\varepsilon_{\mathbf{p}+\mathbf{k}} + \varepsilon_{-\mathbf{p}} - \omega_{\mathbf{k}}). \quad (12)$$

Expressions (11) and (12) describe light absorption with a direct transfer of the photon energy to the excitations of the BEC. Since the phase velocity of bogolons is much lower than the speed of light,  $\varepsilon_{\mathbf{k}} \neq \omega_{\mathbf{k}}$  for any finite values of  $\mathbf{k}$ . Thus, Eq. (11) does not give a contribution. In contrast, the probability (12) can be finite. Its value for typical parameters characterizing atomic BECs of different dimensionalities is analyzed below. In order to observe the absorption of the two bogolons process, the bogolons should be in the linear regime of nearly  $k = 0$ . Both  $\mathbf{k} + \mathbf{p}$  and  $\mathbf{k}$  should be less than  $1/\xi$  and this leads to also  $k_{\parallel} \ll 1/\xi$ . The linear regime can be achieved by using the light being nearly perpendicularly incident to the system.

A similar analysis can be applied to expression (8). The first term in  $\hat{V}_3$  describes the direct transition of individual atom energy from the ground to an excited state under photon absorption. This interaction channel does not involve any bogolons,

$$\alpha_{0b}^3 = \frac{2\pi}{\hbar} n_c \sum_{\eta \neq 0} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 \delta(E_{\eta}(\mathbf{k}) - \omega_{\mathbf{k}}). \quad (13)$$

The second term in Eq. (8) involves 1b processes, and the corresponding probability reads

$$\alpha_{1b}^3 = \frac{2\pi}{\hbar} \sum_{\eta, \mathbf{p}} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 |v_{\mathbf{p}}|^2 \delta(E_{\eta}(\mathbf{p} + \mathbf{k}) + \varepsilon_{-\mathbf{p}} - \omega_{\mathbf{k}}). \quad (14)$$

Thus, in zero-temperature limit, photons can only be absorbed by the BEC in three different channels: (1) by generating an excited boson; (2) by generating a single bogolon and an excited boson; (3) by generating a pair of bogolons from the condensate.

In the most interesting case  $p\xi \ll 1$  when the dispersion of the Bogoliubov excitations of the BEC can be approximated by the linear dispersion as  $\varepsilon_p \approx sp$ , analytical results for the probabilities of these processes can be found. Under this approximation, the Bogoliubov transformation coefficients can be written as

$$u_{\mathbf{p}} \approx \sqrt{\frac{ms}{2|\mathbf{p}|}}, \quad v_{\mathbf{p}} \approx -\sqrt{\frac{ms}{2|\mathbf{p}|}}. \quad (15)$$

Furthermore, we analytically calculate the absorption probabilities given by Eqs. (12), (13) and (14) in various dimensions.

**Absorption probability  $\alpha_{2b}^1$ .** Let us, first, find  $\alpha_{2b}^1$  in different dimensionalities by substituting Eq. (15) in Eq. (12), yielding

$$\alpha_{2b}^1 = \frac{\pi m^2 s}{2\hbar} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 \frac{1}{(2\pi\hbar)^d} \int d\mathbf{p} \frac{1}{|\mathbf{p}||\mathbf{p} + \mathbf{k}|} \delta\left(|\mathbf{p} + \mathbf{k}| + |\mathbf{p}| - \frac{\omega_{\mathbf{k}}}{s}\right). \quad (16)$$

In case of 1D BEC, the absorption probability is

$$\alpha_{2b,1D}^1 = \frac{m^2 s}{4\hbar^2} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 I_{2b,1D}^1 = \frac{2m^2 s}{\hbar^2} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 \frac{\Theta[\omega_{\mathbf{k}}/s - k_{\parallel}]}{(\omega_{\mathbf{k}}/s)^2 - k_{\parallel}^2}, \quad (17)$$

where

$$I_{2b,1D}^1 = \int dp \frac{1}{|p||p + k_{\parallel}|} \delta\left(|p + k_{\parallel}| + |p| - \frac{\omega_{\mathbf{k}}}{s}\right). \quad (18)$$

In Eq. (18), the momentum  $k_{\parallel}$  inherent from the photon is just the component which is parallel to the 1D sample. It should be noted, that the frequency of light  $\omega_{\mathbf{k}}$  depends on both the parallel and vertical components of  $\mathbf{k}$  ( $k_{\parallel}, \mathbf{k}_{\perp}$ ).

In the 2D case, the absorption probability reads (See the details in ‘‘Supplemental material’’),

$$\alpha_{2b,2D}^1 = \frac{sm^2}{8\pi\hbar^3} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 I_{2b,2D}^1 = \frac{sm^2}{4\hbar^3} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 \frac{\Theta[\omega_{\mathbf{k}}/s - k_{\parallel}]}{\sqrt{(\omega_{\mathbf{k}}/s)^2 - k_{\parallel}^2}}. \quad (19)$$

In 3D, we have

$$\alpha_{2b,3D}^1 = \frac{sm^2}{16\pi^2\hbar^4} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 I_{2b,3D}^1 = \frac{sm^2}{8\pi\hbar^4} |\mathbf{d}_{00} \cdot \hat{\mathbf{E}}_0|^2 \Theta[\omega_{\mathbf{k}}/s - k]. \quad (20)$$

**Absorption probability  $\alpha_{1b}^3$ .** By plugging Eq. (15) into Eq. (14), we attain

$$\alpha_{1b}^3 = \frac{m\pi}{\hbar} \frac{s}{(2\pi\hbar)^d} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 \int d\mathbf{p} \frac{1}{p} \delta(\Delta_{\eta} + sp - \omega_{\mathbf{k}}). \tag{21}$$

here, several approximations have been assumed. First, the bogolon dispersion is linear,  $\epsilon_{\mathbf{p}} \approx sp$ . This approximation implies the dropping of the  $p^2$  and other higher-order terms in the bogolon dispersion. At the same level of approximation, we must disregard the kinetic energy of individual atom,  $(\mathbf{p} + \mathbf{k})^2/2m$ , assuming  $E_{\eta}(\mathbf{p} + \mathbf{k}) \approx \Delta_{\eta}$  in Eq. (14).

This formula, Eq. (21), produces different results for BECs of different dimensions (See the details in ‘‘Supplemental material’’),

$$\alpha_{1b,1D}^3 = \frac{m}{2\hbar^2} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 I_{1b,1D}^3 = \frac{ms}{\hbar^2} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 \frac{\Theta[\omega_{\mathbf{k}} - \Delta_{\eta}]}{\omega_{\mathbf{k}} - \Delta_{\eta}}, \tag{22}$$

$$\alpha_{1b,2D}^3 = \frac{m}{4\pi\hbar^3} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 I_{1b,2D}^3 = \frac{m}{2\hbar^3} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 \Theta[\omega_{\mathbf{k}} - \Delta_{\eta}], \tag{23}$$

$$\alpha_{1b,3D}^3 = \frac{m}{8\pi^2\hbar^4} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 I_{1b,3D}^3 = \frac{m}{2\pi s\hbar^4} \sum_{\eta} |\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0|^2 (\omega_{\mathbf{k}} - \Delta_{\eta}) \Theta[\omega_{\mathbf{k}} - \Delta_{\eta}]. \tag{24}$$

Let us compare the absorption probability for the 1b and 2b processes.

### Discussion

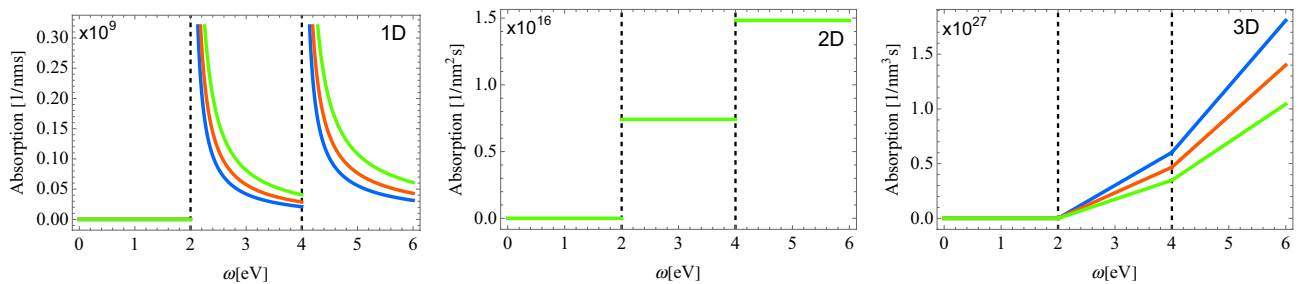
As concerns expression (13), it produces a series of resonances corresponding to direct transitions of the atoms from BEC without the excitation of the condensate density. Despite the absence of bogolons under this photon absorption processes, its magnitude is determined by the BEC density  $n_c$ .

Figure 1 shows the absorption probabilities in case of 1b processes for different dimensionalities of the system and for various condensate densities. Below certain threshold,  $\Delta = 2$  eV in our case, the absorption in all the 1D, 2D and 3D samples is absent. At higher frequencies  $\omega > \Delta$ , in 1D case, the absorption experiences a comb-like behavior with the peaks at  $\omega_{\mathbf{k}} = \eta\Delta$ . By increasing the condensate density, the absorption probability also increases. In the 2D case, the absorption reveals a step-like behavior, and it does not depend on the condensate density. In 3D, the absorption probability curve reminds a broken threshold. Interesting to note, that it decreases with the increase of the condensate density, which is opposite to the 1D case.

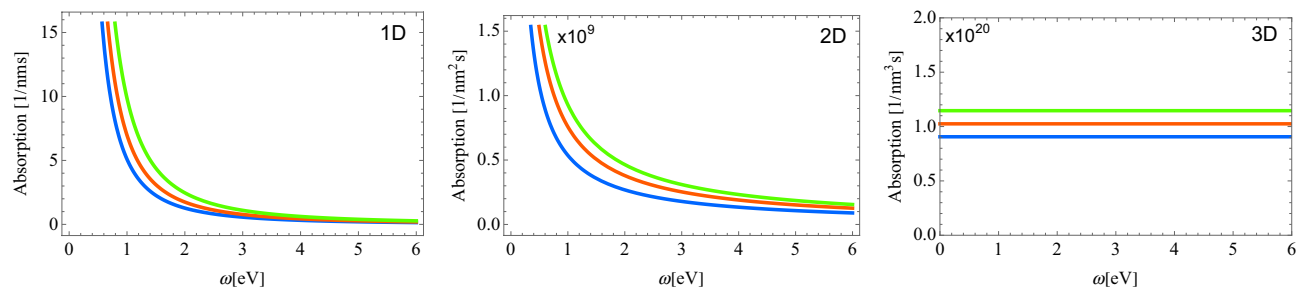
Figure 2 shows the absorption probabilities for 2b processes and for different condensate densities. By increasing the condensate density, the absorption probability increases in all dimensionalities. For 1D and 2D, we observe an exponential decay, and in 3D, the absorption probability is a constant.

There are several differences between 1b and 2b processes. In 2D, 2b processes-mediated absorption does depend on the condensate density as compared to 1b processes, which do not experience any dependence on the condensate density. Another principle difference between 1b and 2b processes is that for 2b process, the absorption is finite even at small frequencies ( $\omega < \Delta$ ). Moreover, it is more pronounced at smaller frequencies.

The theory developed above is based on the model, which disregards the disorder and interaction between photo-excited particles and BEC. These effects result in finite lifetime of the particles, changing the density of state in the system, which, in order, might modify the frequency dependence of the light absorption coefficient, which is defined as the ratio of number of absorbed photons and number of incident photons,  $\alpha(\hbar\omega) = \hbar\omega W/P$ <sup>26</sup>, where  $W$  is absorption probability and  $P$  is the average of the Poynting flux for the light intensity,  $P = c\epsilon_0 E_0^2/2$ , where  $\epsilon_0$  is the vacuum permittivity.



**Figure 1.** Spectrum of the absorption probability in each dimensionality for 1b processes. The parameters used are:  $m = 1.443 \times 10^{-25}$  kg is the mass of atom.  $\Delta_{\eta} = \eta \cdot 2\text{eV}$  is the energy spectrum of the internal motion,  $|\mathbf{d}_{\eta 0} \cdot \hat{\mathbf{E}}_0| = 2 \times 10^{-3}\text{eV}$  is the transition matrix element with  $\mathbf{d}_{\eta 0} = 3.584 \times 10^{-29}\text{C} \cdot \text{m}$  the transition dipole matrix element;  $\hat{\mathbf{E}}_0 = 100\text{KV/cm}$  is electric field intensity.  $k = \omega/c$  is the wavevector of light. For 1D, the condensate densities are  $n_{1d} = 80\ \mu\text{m}^{-1}$  (blue),  $n_{1d} = 150\ \mu\text{m}^{-1}$  (red),  $n_{1d} = 300\ \mu\text{m}^{-1}$  (green). For 2D,  $n_{2d} = 100\ \mu\text{m}^{-2}$ . For 3D, the condensate densities are  $n_{3d} = 250\ \mu\text{m}^{-3}$  (blue),  $n_{3d} = 320\ \mu\text{m}^{-3}$  (red),  $n_{3d} = 400\ \mu\text{m}^{-3}$  (green).



**Figure 2.** Absorption probabilities for 2b processes in different dimensionalities with various condensate densities. For 2D, the condensate densities are  $n_{2d} = 100 \mu\text{m}^{-2}$  (blue),  $n_{2d} = 200 \mu\text{m}^{-2}$  (red),  $n_{2d} = 300 \mu\text{m}^{-2}$  (green). The other parameters are taken the same as in Fig. 1.

In the case of a disordered BEC or finite lifetime of bogolons, the  $\delta$ -functions in the equations for the absorption probability terms has to be widened into the Lorentz form. The microscopic analysis of the disorder or particle collisions is a separate question, which we leave beyond the scope of the present paper.

The other specific feature of the presented theory is that we consider BEC in different dimensions to be spatially uniform, such that  $n_c = \text{const}$ . In the case of 0D trapped or spatially-modulated BECs the condensate density in equilibrium becomes spatially-dependent,  $n_c(\mathbf{r})$ . Thus, the system preserves the translation invariance. In that case, the Bose particles momenta (and bogolons' ones)  $\mathbf{p}$  are not conserved violating the momentum conservation law under optical transitions. A careful analysis of light absorption in these BECs requires a separate consideration.

## Conclusions

We studied the response of cold atomic gas in the BEC phase to an external electromagnetic field by calculating the absorption probabilities. For that, we used the standard Bogoliubov theory, extending it to the case of Bose particles possessing internal degrees of freedom. We show, that several specific processes might occur if the atomic gas is in the BEC state. In particular, two types of transitions occur, which contribute to light absorption by the system. The processes of the first kind involve an excitation of an atom accompanied by an emission of a bogolon—the quantum of BEC density fluctuations. The processes of the second type involve creating two bogolons with different momenta. We demonstrate that the one-bogolon processes are dominant in a broad range of the external EM field frequencies except for the small-frequency region. Moreover, the light absorption in different dimensionalities depends differently on the condensate density. For one-bogolon processes, the absorption increases with larger condensate densities in 1D. However, in 3D, by increasing the condensate density, the absorption decreases. In 2D, the absorption does not depend on the condensate density. For the process involving pairs of bogolons, in all the dimensions, the absorption increases with the increases of condensate density.

## Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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## References

- Wieck, A. D., Sigg, H. & Ploog, K. Observation of resonant photon drag in a two-dimensional electron gas. *Phys. Rev. Lett.* **64**, 463–466 (1990).
- Glazov, M. & Ganichev, S. High frequency electric field induced nonlinear effects in graphene. *Phys. Rep.* **535**, 101–138 (2014).
- Strait, J. H. *et al.* Revisiting the photon-drag effect in metal films. *Phys. Rev. Lett.* **123**, 053903 (2019).
- Khichar, V., Sharma, S. C. & Hozhabri, N. New features in the surface plasmon induced photon drag effect in noble metal thin films. *J. Phys. Commun.* **5**, 055005 (2021).
- Lee, H. C. On the photon-drag effect of photocurrent of surface states of topological insulators. *Physica E Low-Dimensional Syst. Nanostruct.* **79**, 44–51 (2016).
- Plank, H. *et al.* Photon drag effect in  $(\text{bi}_{1-x}\text{sb}_x)_2\text{te}_3$  three-dimensional topological insulators. *Phys. Rev. B* **93**, 125434 (2016).
- Kiemle, J., Zimmermann, P., Holleitner, A. W. & Kastl, C. Light-field and spin-orbit-driven currents in van der waals materials. *Nanophotonics* **9**, 2693–2708 (2020).
- Ko, D., Morozov, A. V., Kovalev, V. M. & Savenko, I. G. Optical valleytronics of impurity states in two-dimensional dirac materials. *Phys. Rev. B* **103**, L161301 (2021).
- Berman, O. L., Kezerashvili, R. Y. & Lozovik, Y. E. Drag effects in a system of electrons and microcavity polaritons. *Phys. Rev. B* **82**, 125307 (2010).
- Raab, E. L., Prentiss, M., Cable, A., Chu, S. & Pritchard, D. E. Trapping of neutral sodium atoms with radiation pressure. *Phys. Rev. Lett.* **59**, 2631–2634 (1987).
- Ashkin, A. Trapping of atoms by resonance radiation pressure. *Phys. Rev. Lett.* **40**, 729–732 (1978).
- Chu, S., Hollberg, L., Bjorkholm, J. E., Cable, A. & Ashkin, A. Three-dimensional viscous confinement and cooling of atoms by resonance radiation pressure. *Phys. Rev. Lett.* **55**, 48–51 (1985).
- Bjorkholm, J. E., Freeman, R. R., Ashkin, A. & Pearson, D. B. Observation of focusing of neutral atoms by the dipole forces of resonance-radiation pressure. *Phys. Rev. Lett.* **41**, 1361–1364 (1978).
- Ashkin, A. Acceleration and trapping of particles by radiation pressure. *Phys. Rev. Lett.* **24**, 156–159 (1970).



15. Hemmerich, A. & Hänsch, T. W. Radiation pressure vortices in two crossed standing waves. *Phys. Rev. Lett.* **68**, 1492–1495 (1992).
16. Guibal, S. *et al.* Radiation pressure in a rubidium optical lattice: An atomic analog to the photorefractive effect. *Phys. Rev. Lett.* **78**, 4709–4712 (1997).
17. Lett, P. D. *et al.* Observation of atoms laser cooled below the doppler limit. *Phys. Rev. Lett.* **61**, 169–172 (1988).
18. Rooijakkers, W., Hogervorst, W. & Vassen, W. Two-color excitation of metastable helium atoms for efficient laser cooling. *Phys. Rev. Lett.* **74**, 3348–3351 (1995).
19. Harris, G. I. *et al.* Laser cooling and control of excitations in superfluid helium. *Nat. Phys.* **12**, 788–793 (2016).
20. Cohen-Tannoudji, C. N. Nobel lecture: Manipulating atoms with photons. *Rev. Mod. Phys.* **70**, 707–719 (1998).
21. Anderson, M. H., Ensher, J. R., Matthews, M. R., Wieman, C. E. & Cornell, E. A. Observation of Bose–Einstein condensation in a dilute atomic vapor. *Science* **269**, 198–201 (1995).
22. Mendonça, J. T. & Kaiser, R. Photon bubbles in ultracold matter. *Phys. Rev. Lett.* **108**, 033001. <https://doi.org/10.1103/PhysRevLett.108.033001> (2012).
23. Giampaoli, R., Rodrigues, J. D., Rodrigues, J.-A. & Mendonça, J. T. Photon bubble turbulence in cold atom gases. *Nat. Commun.* **12**, 3240 (2021).
24. Kovalev, V. M., Miroshnichenko, A. E. & Savenko, I. G. Photon drag of a Bose–Einstein condensate. *Phys. Rev. B* **98**, 165405 (2018).
25. Landau, L. & Lifshitz, E. *Chapter III—The Gibbs Distribution (In Statistical Physics)* 3rd edn. (Butterworth-Heinemann, 1980).
26. Chuang, S. L. *Physics of Photonic Devices* Vol. 80 (Wiley, 2012).

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## Author contributions

D.K. wrote the main manuscript and prepared all the figures and all authors reviewed the manuscript.

## Competing interests

The authors declare no competing interests.

## Additional information

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