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Electron-phonon decoupling in two dimensions

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In order to observe many-body localisation in electronic systems, decoupling from the lattice phonons is required, which is possible only in out-of-equilibrium systems. We show that such an electron-phonon decoupling may happen in suspended films and it manifests itself via a bistability in the electron temperature. By studying the electron-phonon cooling rate in disordered, suspended films with two-dimensional phonons, we derive the conditions needed for such a bistability, which can be observed experimentally through hysteretic jumps of several orders of magnitude in the nonlinear current-voltage characteristics. We demonstrate that such a regime is achievable in systems with an Arrhenius form of the equilibrium conductivity, while practically unreachable in materials with Mott or Efros–Shklovskii hopping.

Tremendous experimental progress in isolating quantum many-body systems from the environment (see[1] and[2] for reviews) led to the observation of many-body localization (MBL) in ultracold atomic systems[3,4]. The question remains, however, whether MBL can be observed in disordered electronic systems for which it was originally predicted[5,6]. In the absence of interaction, disorder localizes all electron states in low-dimensional systems[7] so that the dc electronic current vanishes without inelastic processes. The essence of MBL is that inelasticity due to the electron-electron (e-e) interaction alone does not lead to thermal equilibration of the system, as was first suggested for interacting electrons in a chaotic quantum dot[8]. Hence in the absence of other mechanisms of inelasticity all states would remain localized so that finite-temperature conductivity would remain zero.

The main obstacle to the observation of this effect in electronic systems lies in the coupling of the electron system to the environment via the electron-phonon (e-ph) interaction. In equilibrium, such a coupling equilibrates all electron states with the underlying lattice leading to their delocalization. This results in nonzero finite-temperature conductivity, which is driven, in the absence of the electron-electron interaction, by Mott's variable-range hopping[9,10] and given, at temperatures lower than some constant T_0, by

$$\sigma(T) = \sigma_0 \exp\left[-(T_0/T)^\gamma\right], \tag{1}$$

where $\gamma = 1/(d+1)$ for a d-dimensional system, and σ_0 is a constant, temperature-independent prefactor. The presence of an electron-electron interaction changes the mechanism of equilibration at sufficiently low temperatures due to the emergence of a so-called Coulomb gap in the single-electron density of states[11] resulting in the change of the exponent in Eq. (1) to $\gamma = 1/2$, independent of dimensionality.

Although the electron-phonon coupling makes it impossible to observe MBL in electronic systems equilibrated with the lattice, in out-of-equilibrium systems electrons and phonons might decouple even in the presence of a weak electron-phonon interaction. For MBL to be observable, the interacting electrons should be at internal equilibrium but not equilibrated with the underlying lattice. It has been suggested[12] that such an out-of-equilibrium decoupling could manifest itself via a bistability in the nonlinear current-voltage (I-V) characteristics. It has been shown later[13] that such a bistability, caused by the electrons overheating, occurs at low temperatures, $T \lesssim 0.1T_0$, provided that the equilibrium conductivity is close to the Arrhenius law, i.e. $\gamma \approx 1$ in Eq. (1). The quantitative description of this bistability[13], based on an earlier developed analysis of the electron-phonon equilibration rate in bulk disordered systems[14–16], allowed a full explanation of giant jumps (up to six orders in magnitude) of resistivity experimentally observed[17–20] in various materials with the Arrhenius equilibrium conductivity where T_0 is of order of a few kelvins.

The presence of a bistability in the I-V characteristics below a critical temperature is not, by itself, necessarily a signature of MBL but its absence would mean electron-phonon equilibration and hence the absence of MBL. Intuitively, it seems that the electron-phonon decoupling would be easier to achieve in suspended disordered 2d films. Hence, such films might be promising for observing MBL provided that they are sufficiently disordered for the one-electron Anderson localization on the length scale smaller than the film dimensions.

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In this paper, we derive the electron-phonon equilibration rate in such films and use it to analyze a possible bistability of the *I-V* characteristics on the insulating side where the equilibrium conductivity is governed by Eq. (1). We found that in suspended films with the Arrhenius equilibrium conductivity, the bistability occurs at lattice temperatures $T \lesssim 0.1 T_0$ similar, albeit quantitatively different, to bulk systems or thick multilayered films. On the other hand, for disordered films with $\gamma \lesssim 1/2$, i.e. those with either Mott[9,10] or Efros-Shklovskii[11] conductivity, the bistability could take place at much lower temperatures. Hence only materials with the Arrhenius resistivity at low temperatures could be potentially promising for detecting MBL. While the origin of a small Arrhenius gap, $T_0 \sim 1$K, is quite an interesting problem by itself, we do not consider it here noticing only that there is a variety of materials with such a gap[21-25] which typically have granular disorder.

Model

We consider electron-phonon relaxation in a suspended disordered film where both electron and phonon degrees of freedom are two-dimensional. Electrons can thermally decouple from phonons when a finite source-drain voltage, *V*, drives the system out of equilibrium. The decoupling might reveal itself in a nonlinear, non-Ohmic regime when the electron-phonon interaction is too weak to effectively dissipate the power supplied to the electron system. Assuming the electron-electron interaction to be sufficiently strong for thermalizing electrons between themselves at a temperature T_{el}, the energy dissipation from the electronic system to the phonon bath (or equivalently the lattice), which is at a temperature T_{ph}, can be described by the phenomenological heat balance equation[13],

$$\frac{V^2}{R(T_{el})} = \frac{d\mathcal{E}}{dt} = \frac{\mathcal{E}(T_{el})}{\tau_{e\text{-}ph}(T_{el})} - \frac{\mathcal{E}(T_{ph})}{\tau_{e\text{-}ph}(T_{ph})}. \tag{2}$$

Here the temperature-dependent part of the total electron energy is given by $\mathcal{E}(T) = \pi^2 \nu \mathcal{A} T^2/6$ (where $\mathcal{A}$ is the sample area and ν is the density of states at the Fermi surface), and $R(T_{el})$ is the sample resistance at equilibrium, which is equal to the inverse conductivity $\sigma^{-1}(T_{el})$, see Eq. (1), assuming for simplicity a square shape of the film. As the electron energy is conserved in e-e collisions, the heat balance is fully determined by the e-ph interaction with the scattering time $\tau_{e\text{-}ph}(T)$ which is energy-independent at the low temperatures at which MBL might occur, as the relevant part of the dispersion for both the electrons and phonons is linear. In the presence of disorder, the e-ph interaction is modified by the effect of phonon-induced impurity displacements[14-16,26]. This can occur in two possible ways depending on whether the phonons directly affect the impurities. In the case of a suspended film, the impurities oscillate with the lattice so that the Hamiltonian becomes

$$\mathcal{H} = \frac{1}{\sqrt{\mathcal{A}}} \sum_{p,q,k} c^\dagger_{p+q+k} c_p \big(g_q \cdot u_q \, \delta_{k0} + g^{imp}_k \cdot u_q \big). \tag{3}$$

Here $c^\dagger$, c are the electron creation and annihilation operators, u_q is the the Fourier transform of the lattice displacement (corresponding to either transverse or longitudinal phonons), $g_q = iCq$ is the standard electron-phonon vertex with the deformation potential C equal to the Fermi energy ε_F for two-dimensional phonons, and $g^{imp}_k = -iU(k)k$ is the vertex corresponding to the phonon-displaced impurities, with $U(k)$ being the Fourier transform of the impurity potential. For electron scattering from impurities we assume the standard model of uncorrelated *s*-scatterers[27], which is equivalent to the Gaussian potential with zero average and δ-correlations,

$$\langle U(r)U(r') \rangle = \frac{1}{2\pi\nu\tau}\delta(r - r'), \tag{4}$$

where τ is the mean scattering time.

Results

We show that electrons can decouple from the phonon bath in thin suspended films provided that the equilibrium finite-temperature conductivity is close to the Arrhenius law, i.e. $\gamma \approx 1$ in Eq. (1), and the bath temperature is much lower than the Arrhenius "gap" T_0. This conclusion is based on our analysis of the electron-phonon cooling rate for 2d phonons similar to that for the phonons in bulk materials (see, e.g.,[14-16]). Using the quantum kinetic equation derived in the Keldysh formalism (see, e.g.,[28]), we derive the following expression for the electron-phonon cooling rate due to transverse phonons:

$$\dot{\mathcal{E}} = \frac{\alpha^2 k_F \ell n_{el} \mathcal{A}}{\hbar \Delta_0^3} \big(T_{el}^5 - T_{ph}^5 \big), \quad \Delta_0^3 = \hbar^2 \rho_{2d} u_t^4, \quad \alpha^2 = \frac{3}{\pi}\zeta(5) \approx 0.99, \tag{5}$$

where k_F is the Fermi wave vector, ℓ is the electron mean free path, u_t is the transverse phonon speed of sound, $n_{el} = k_F^2/(2\pi)$ is the 2d electron density, ρ_{2d} is the 2d material density and ζ is the Riemann-zeta function. This result corresponds to the τ-approximation for the e-ph relaxation rate in Eq. (2) with the temperature dependence $1/\tau_{e\text{-}ph}(T) \propto T^3$ and is similar to that for the case of 3d phonons[13], where $\dot{\mathcal{E}} \propto T_{el}^6 - T_{ph}^6$, with the difference being caused by the weaker dependence of the phonon density of states on the phonon frequency, which goes as ω^{d-1}. As in the 3d case, the leading contribution to the cooling rate is due to the impurity-facilitated interaction of electrons with transverse phonons, which is absent in a clean metal. The contribution from the interaction with longitudinal phonons has the same form as Eq. (5) with the change $u_t \to u_l$. Since the longitudinal speed of sound, u_l, is typically a few times larger than its transverse counterpart[29], the longitudinal-phonon contribution contains a small factor of $(u_t/u_l)^4$ in comparison to the leading contribution given by Eq. (5). It is worth noting
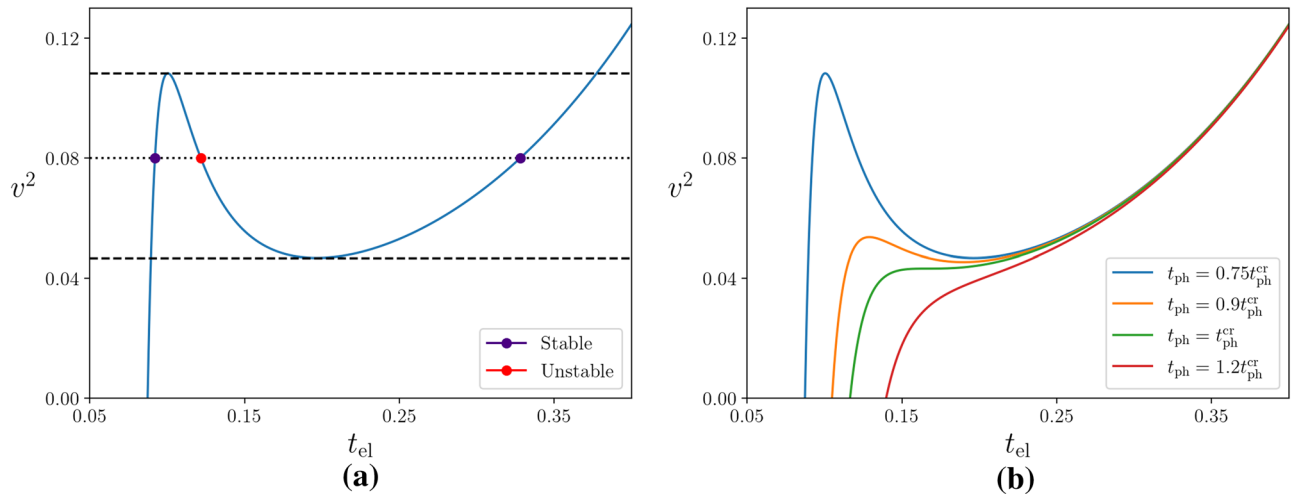
Figure 1. (a) The bistability region, where two stable solutions for t_{el} exist in a certain range of the applied voltage, is shown for $t_{ph} = 0.75 t_{ph}^{cr}$ for the Arrhenius equilibrium resistance, $\gamma = 1$. The blue dots correspond to cold and hot electron states at temperatures $t_{el}^{<}$ and $t_{el}^{>}$, respectively, and the red dot to an unstable solution. (b) The dependence of t_{el} on v^2 for different phonon bath temperatures is shown as follows from Eq. (7). Above the critical bath temperature this corresponds to the actual $t_{el}(v)$ dependence while below t_{ph}^{cr} the electronic system will fall either to $t_{el}^{<}$ or to $t_{el}^{>}$, making temperatures in between experimentally inaccessible.

that the overall low-temperature suppression of the e-ph relaxation rate in disordered semiconductors, as compared to a clean metal, is given by a factor of $n^* T \ell / \hbar u_t (u_l / u_t)^3$, reflecting Pippard's ineffectiveness condition[30]. Here n^* is the number of electrons per unit cell, which is small in semiconductors most promising for MBL so that, with a typical u_t of order of 10^3 m/s, the cooling rate could be several orders in magnitude smaller than in a dense clean metal in spite of the factor $(u_l / u_t)^3 \sim 10$.

Next, we substitute the cooling rate (5) into the heat balance equation (2). Assuming the usual Drude prefactor for the equilibrium resistance,

$$R(T_{el}) = R_0 \exp \left[\left(\frac{T_0}{T_{el}} \right)^\gamma \right] \equiv \frac{\hbar k_F}{n_{el} e^2 \ell} \exp \left[\left(\frac{T_0}{T_{el}} \right)^\gamma \right], \tag{6}$$

we find that the heat balance equation is independent of the mean free path, ℓ. This allows us to extend the results for the electron-phonon cooling rate we have obtained in the metallic regime, $k_F \ell \gg 1$, to the transition regime, $k_F \ell \sim 1$, and beyond. This is empirically justified by experiments[20] made in the vicinity of the superconducting-insulating transition, where $k_F \ell < 1$, as the results obtained were in excellent quantitative agreement with the results for the bistability[13] obtained using the cooling rate via interactions with bulk phonons which had been calculated in the metallic regime[14–16].

It is convenient to represent the heat balance equation, obtained by substituting the equilibrium resistance (6) and the cooling rate (5) into Eq. (2), in terms of a dimensionless temperature and voltage, defined by $t_{el,ph} = T_{el,ph}/T_0$ and $v = V/V_0$ with $V_0^2 = \alpha^2 k_F^2 \mathcal{A} T_0^5 / (e^2 \Delta_0^3)$, as follows:

$$v^2 = \left[t_{el}^5 - t_{ph}^5 \right] \exp \left[(1/t_{el})^\gamma \right]. \tag{7}$$

For any given voltage, the electron temperature must be higher than the bath temperature to satisfy this equation. By itself this does not signify the electron-phonon decoupling. On the other hand, we can see clear evidence of decoupling in the presence of a bistability where, below a critical bath temperature and in a certain range of the applied voltage, electrons can mutually equilibrate at two distinct temperatures, "cold" $t_{el}^{<}$ and "hot" $t_{el}^{>}$. It is in the regime of overheating, at temperature $t_{el}^{>}$ which is practically independent of the lattice temperature t_{ph}, that the electrons become fully decoupled from the phonons.

Such a bistability occurs when Eq. (7) has, for a given voltage and bath temperature, two stable solutions for t_{el}. This happens below the critical phonon bath temperature t_{ph}^{cr} when the r.h.s. of this equation becomes a non-monotonic function of the electron temperature. An elementary analysis shows that the critical bath temperature is given by

$$t_{ph}^{cr} \equiv \frac{T_{ph}^{cr}}{T_0} = \left(1 + \frac{5}{\gamma} \right)^{-\left(\frac{1}{\gamma} + \frac{1}{5} \right)}. \tag{8}$$

For $t_{ph} < t_{ph}^{cr}$ Eq. (7) has three solutions in a certain region of the source-drain voltage, as illustrated in Fig. 1 for the Arrhenius case, $\gamma = 1$. For a given voltage within this region, both the "cold" and "hot" states, at electronic temperatures $t_{el}^{<}$ and $t_{el}^{>}$ respectively, are stable. The middle solution, however, corresponds to an unstable electronic state.
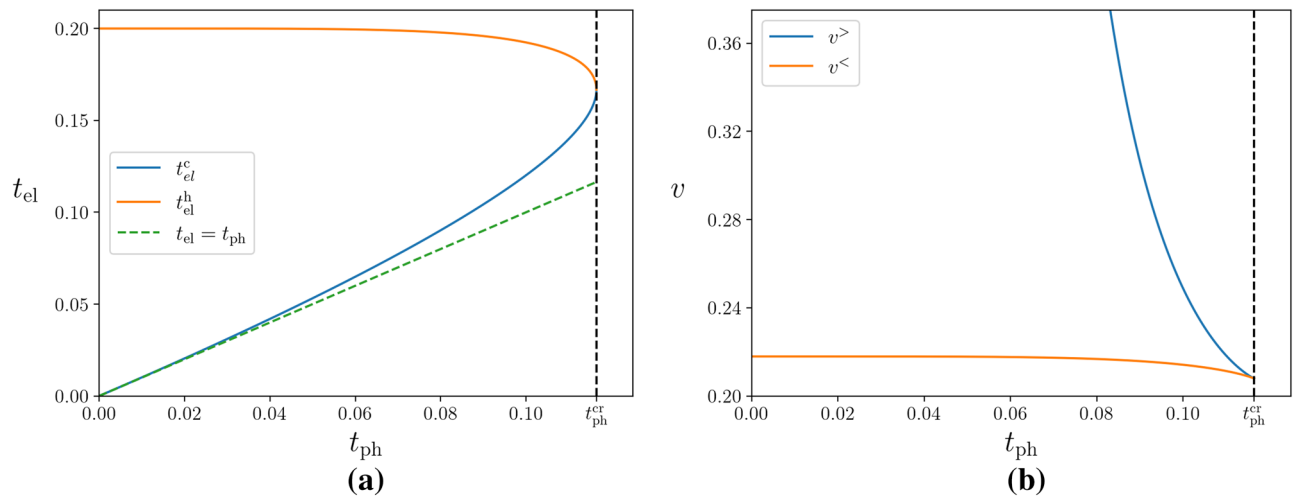
Figure 2. Dependence of the bistability boundaries on the phonon temperature for **(a)** the electron temperature and **(b)** the source-drain voltage, for $\gamma = 1$. The region of electron temperatures inside the curve **(a)** is experimentally inaccessible as it corresponds to the unstable states.

Formally, a similar bistability takes place also for the Mott ($\gamma = 1/3$) and Efros-Shklovskii ($\gamma = 1/2$) hopping regimes. However, a faster than exponential dependence of the critical phonon bath temperature on $1/\gamma$, Eq. (8), pushes the bistability in these regimes to very low temperatures: while $t_{ph}^{cr} \approx 0.12$ in the Arrhenius case, it is about $5 \cdot 10^{-3}$ in the Efros-Shklovskii regime, and 10^{-4} in the Mott regime. With $T_0 \sim 1$K in materials of interest, the bistability regime would be practically unreachable in the systems with the Mott or Efros-Shklovskii conductivity, while the experimentally observed bistability in an Arrhenius material[20] was in a quantitative agreement with the theoretical description[13] similar to that developed here but with the electrons interacting with bulk phonons. Due to this fact we conclude that for $\gamma \approx 1$ the bistability occurs for $T \lesssim 0.1 T_0$.

At the bistability boundaries for a given t_{ph}, the derivative of the r.h.s. of Eq. (7) vanishes, so that the boundaries are determined in the Arrhenius case by the following equation

$$5t_{el} = 1 - (t_{ph}/t_{el})^5, \tag{9}$$

which for $t_{ph} < t_{ph}^{cr}$ has two solutions, hot, t_{el}^h, and cold, t_{el}^c, depicted in Fig. 2(a). The corresponding temperature dependence of the voltage boundaries of the bistability, $v^>$ for the cold state and $v^<$ for the hot one, obtained by substituting $t_{el}^{c,h}$ into Eq. (7), is shown in Fig. 2(b). As previously mentioned, in order to satisfy the heat balance, Eq. (2), the electron temperature must always be higher than the phonon bath. However, while in the cold state t_{el}^c almost follows t_{ph}, in the overheated hot state t_{el}^h is almost independent of the bath temperature, and so is the voltage boundary of this state, $v^<$. Since the electrons in the overheated state are practically decoupled from the phonon bath, it is the state most suitable for a possible observation of MBL.

It is important to understand the experimental signatures of the bistability as this will confirm with certainty that electron-phonon decoupling is present. A relatively simple experimental signature is the presence of a region of excluded temperatures corresponding to the unstable states, which are those enclosed by the curve in Fig. 2(a). Such a region was experimentally observed in[20] and turned out to be in quantitative agreement with the theoretical prediction[13] made for films on a substrate with electrons interacting with bulk phonons. But the most striking feature due to bistability is giant hysteretic jumps in the I-V characteristics: due to the exponential dependence of resistance on the inverse electron temperature, a switch between the cold and hot electron states under a given voltage V leads to abrupt changes in the current I that can be of many orders in magnitude.

To see this, we solve numerically the equation for the non-linear conductance in the Arrhenius regime,

$$G = \frac{I}{V} = \frac{1}{R(t_{el})} = \frac{1}{R_0} e^{-1/t_{el}(v)}, \tag{10}$$

where R_0 is the Drude resistivity, Eq. (6). The solution has an S-shape, as shown in Fig. 3(a), with the dotted part being unstable. This makes hysteretic jumps between the low conductance (cold electron) state and the high conductance (hot electron) state inevitable.

These jumps are illustrated in Fig. 3(b). Let us stress that exact positions of the jumps are random as the boundaries here are simply bounds on the true jumps; where the actual jumps occur depends on the decay mechanisms of the states, as discussed in[13,31]. Moreover, we do not estimate numerical values for these boundaries, because in order to obtain an accurate value for the voltage scale, V_0, we would also need to include the effects of localisation into the electron-phonon cooling rate[32], which goes beyond the aim of this work. Despite this, the temperature dependence of the positions of the jumps should be experimentally observable, as in the case of electrons interacting with bulk phonons[17-20]. Namely, one expects to see a strong temperature dependence
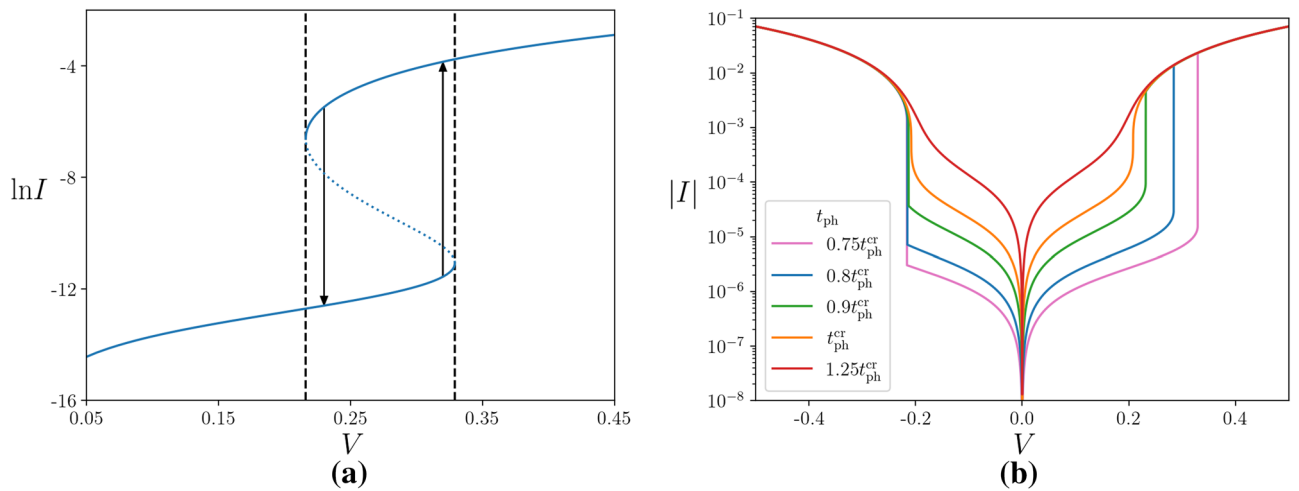
Figure 3. (a) The S-shape solution to the equation for the non-linear conductance, Eq. (10), for $t_{ph} = 0.75t_{ph}^{cr}$. The dotted part corresponds to unstable states, resulting in hysteretic jumps, denoted by the arrows. Note that the jumps do not necessarily occur at the boundaries of the bistability (dashed lines). (b) The numerically predicted I-V characteristics for various lattice temperatures. The jumps here are shown to be at the bistability boundaries, though this may not be the case in reality. The $V > 0$ side of the graph illustrates the transition from the cold electron (low conductance) state to the hot electron (high conductance) state, which occurs when the source-drain voltage is increased. The $V < 0$ side displays the opposite transition when the voltage is decreased, going from the hot to cold electron states. In both (a) and (b) the voltage is measured in units of V_0 and the current is in units such that the resistance is measured in units of R_0.

of the boundary for the cold electron states ($v^>$) and almost no temperature dependence of the boundary for the hot states ($v^<$), as well as the inaccessible region of electron temperatures as in Fig. 2(a).

Discussion

To summarise, we have shown that for films with an equilibrium conductivity exhibiting an Arrhenius (or Arrhenius-like) law, $R(T) = R_0 \exp\left[(T_0/T)^\gamma\right]$ with $\gamma \approx 1$, electrons can decouple from phonons in a nonlinear regime. Such an electron-phonon decoupling manifests itself as a bistability in the electron temperature that can be observed via the I-V characteristics. This bistability occurs in a certain region of source-drain voltages for a lattice temperature $T \lesssim 0.1T_0$, while T_0 is known to be of order of a few kelvins in numerous materials. On the contrary, in systems where the equilibrium conductivity is of the form of either Mott ($\gamma = 1/3$) or Efros-Shklovskii ($\gamma = 1/2$) hopping, the lattice temperature required for the bistability is much lower and practically not accessible. Therefore, for achieving the electron-phonon decoupling necessary for MBL, materials with an Arrhenius conductivity, such as those recently seen in[33], are most promising.

The bistability in the electron temperature means that there exist stable 'cold' and 'hot' electron states. The former have a temperature proportional to (but slightly higher than) that of the lattice while the latter have a temperature which is almost independent of the lattice temperature. It is in this state that the electrons are fully decoupled from the phonons, making it most promising for observing MBL. The most significant experimental signature of the bistability is giant jumps in the non-linear I-V characteristics between the cold (low conductance) and the hot (relatively high conductance) states. Such jumps have been previously associated, in the 3d case, with a possible transition to MBL[34]. We emphasise, however, that while these jumps provide the evidence for electron-phonon decoupling, further evidence would be needed to confirm the existence of the MBL state.

Methods

In order to calculate the electron-phonon cooling rate in two-dimensional systems, Eq. (5), we used the Keldysh formalism (see, e.g.,[28]) in the form similar to that used in calculating the cooling rate in 3D systems[16]. The quantum kinetic equation can be written as

$$\partial_t f_\varepsilon(t) = I[f],$$ (11)

where (after setting $\hbar = 1$) the collision integral for the electron-phonon interaction modified by disorder is given by[16],

$$I[f] = \frac{i}{4\pi \nu \mathcal{A}} \left\langle \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int d\mathbf{r} d\mathbf{r}' \Delta G(\mathbf{r}, \mathbf{r}', \varepsilon) \hat{g}_\alpha(\mathbf{r}') \Delta G(\mathbf{r}', \mathbf{r}, \varepsilon - \omega) \Delta D_{\alpha\beta}(\mathbf{r}' - \mathbf{r}, \omega) \hat{g}_\beta(\mathbf{r}) \right. $$

$$\left. \times \left[(f_\varepsilon - f_{\varepsilon-\omega})N_\omega + f_\varepsilon f_{\varepsilon-\omega} - 1 \right] \right\rangle.$$ (12)

Here the brackets $\langle \cdots \rangle$ stand for averaging over the disorder potential, Eq. (4), $N_\omega = 1 + 2n_B(\omega)$ and $f_\varepsilon = 1 - 2n_F(\varepsilon)$, with $n_B(\omega)$ and $n_F(\varepsilon)$ being the standard Bose and Fermi distributions respectively; $\hat{g}_{\alpha,\beta}$ can
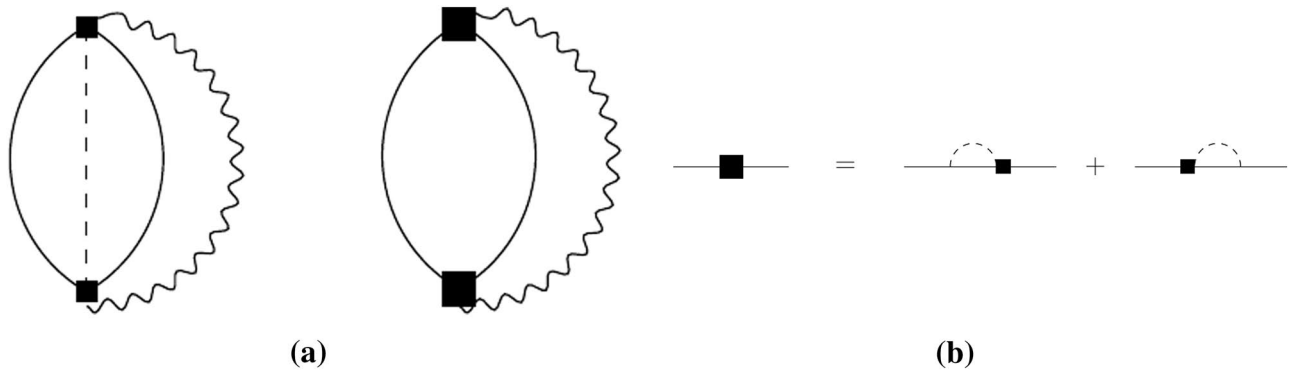
Figure 4. The two lowest-order diagrams that contribute to the collision integral in Eq. (12) due to the interaction of electrons with transverse phonons via impurity scattering: the smaller squares correspond to g_k^{imp}, the straight lines are the electron Green's functions, the wavy lines are the phonon Green's functions and the dashed lines describe the standard averaging over impurities.

be either g_q or g_k^{imp}, see Eq. (3); $\Delta G \equiv G^R - G^A$ and $\Delta D \equiv D^R - D^A$ are the differences between the retarded and advanced Green's functions for electrons and phonons, respectively.

The phonon Green's functions are not directly affected by impurities so that their Fourier transforms, which include contributions from the longitudinal, $j = \text{l}$, and transverse, $j = \text{t}$, phonons, $\Delta D_{\alpha\beta}(\boldsymbol{q}, \omega) = \sum_j \Delta D_{\alpha\beta}^{(j)}(\boldsymbol{q}, \omega)$, are given by the standard expressions

$$\Delta D_{\alpha\beta}^{(j)}(\boldsymbol{q}, \omega) = \left[D_{\alpha\beta}^R(\boldsymbol{q}, \omega) - D_{\alpha\beta}^A(\boldsymbol{q}, \omega) \right]^{(j)} = -\frac{\pi i \eta_{\alpha\beta}^{(j)}}{\rho_{2d}\omega_j(q)} \left[\delta(\omega - \omega_j(\boldsymbol{q})) - \delta(\omega + \omega_j(\boldsymbol{q})) \right], \quad (13)$$

where $\eta_{\alpha\beta}^{(l)} = q_\alpha q_\beta / q^2$ and $\eta_{\alpha\beta}^{(t)} = \delta_{\alpha\beta} - q_\alpha q_\beta / q^2$, and we assume the Debye model for the phonon dispersion, $\omega_j(\boldsymbol{q}) = u_j|\boldsymbol{q}|\Theta(q_0 - |\boldsymbol{q}|)$, where q_0 is the Debye momentum.

The disorder-averaged electron Green's functions $G^{R,A}(\boldsymbol{r}, \boldsymbol{r}', \varepsilon)$ depend only on the difference of their spatial arguments, and the appropriate Fourier transforms are given by

$$G^{R,A}(\boldsymbol{p}, \varepsilon) = \frac{1}{\varepsilon - \xi_{\boldsymbol{p}} \pm i/2\tau}, \xi_{\boldsymbol{p}} = \varepsilon_{\boldsymbol{p}} - \varepsilon_{\text{F}}. \quad (14)$$

A further contribution of disorder in the collision integral (12) is described by vertex corrections. Including only the leading transverse phonons contribution, these corrections are shown in the metallic regime, $k_{\text{F}}\ell \gg 1$, in Fig. 4. In the absence of disorder, transverse phonons do not alter the local charge density and so cannot couple directly to the electrons. However, in disordered materials they contribute via the vertices g_k^{imp}, Eq. (3), which describe the effect of phonon-induced impurity displacements.

The longitudinal-phonons contribution to the cooling rate turns out to be functionally the same as that of the transverse phonons, given in Eq. (5), with u_l substituted for u_t. It is much smaller as $(u_t/u_l)^4 \ll 1$. Note that this contribution, which exists also in clean systems, involves more cumbersome diagrams that include diffuson propagators similar to the 3d case[16]. We do not give any further detail of calculating the longitudinal-phonons contribution as it is not relevant for the final results.

The calculation of the diagrams depicted in Fig. 4 is relatively straightforward. We assume that the electron-electron interaction is sufficiently strong such that the electrons mutually thermalise and can be assigned a single temperature, T_{el} which is higher than the phonon bath (lattice) temperature, T_{ph}. This results in a quasi-equilibrium situation where $f_\varepsilon = \tanh(\varepsilon/2T_{\text{el}}(t))$ and $N_\omega(T_{\text{ph}}) = \coth(\omega/2T_{\text{ph}})$. Then the spatial integral in Eq. (12) is calculated after the Fourier transform and using the fact that $q_T\ell \ll 1$ (where $q_T \sim T/u_{\text{t}}$ is a typical phonon momentum at temperature T) and the identity $f_\varepsilon f_{\varepsilon-\omega} - 1 = -N_\omega(T_{\text{el}})(f_\varepsilon - f_{\varepsilon-\omega})$, one reduces the collision integral to

$$I[f] = \int \mathrm{d}\omega\, K(\omega) \left[N_\omega(T_{\text{ph}}) - N_\omega(T_{\text{el}}) \right] \left[(f_{\varepsilon+\omega} - f_\varepsilon) + (f_{\varepsilon-\omega} - f_\varepsilon) \right], \quad (15)$$

where $K(\omega)$ is expressed in terms of a dimensionless electron-phonon coupling constant, $\beta_t = \dfrac{\nu \varepsilon_{\text{F}}^2}{2\rho_{2d}u_{\text{t}}^2}$, as

$$K(\omega) = \frac{\beta_t \text{sgn}(\omega)}{8k_{\text{F}}\ell} \left(\frac{\omega\ell}{u_{\text{t}}} \right)^2. \quad (16)$$

Substituting the result of Eq. (15) into Eq. (11) and multiplying both sides by ε, one finds after integrating with to respect to ε that the cooling rate (restoring factors of $\hbar$) is given by

$$\dot{\mathcal{E}} = \frac{k_{\mathrm{F}} \ell n_{\mathrm{el}} \mathcal{A}}{\hbar \Delta_0^3} \int_0^\infty \frac{d\omega}{16\pi} \omega^4 \left[\coth\left(\frac{\omega}{2T_{\mathrm{el}}}\right) - \coth\left(\frac{\omega}{2T_{\mathrm{ph}}}\right) \right], \Delta_0^3 = \hbar^2 \rho_{2d} u_{\mathrm{t}}^4 \qquad (17)$$

Performing the integration leads to the result in Eq. (5).

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Competing interests

The authors declare no competing interests.

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