npj | Quantum Information

www.nature.com/npjqi

ARTICLE OPEN

Check for updates

Distribution and quantification of remotely generated Wigner negativity

Yu Xiang[1,2,5], Shuheng Liu[1,5], Jiajie Guo[1], Qihuang Gong[1,2,3], Nicolas Treps[4], Qiongyi He[1,2,3] ✉ and Mattia Walschaers[4]

Wigner negativity, as a well-known indicator of nonclassicality, plays an essential role in quantum computing and simulation using continuous-variable systems. The conditional preparation of Wigner-negative states through appropriate non-Gaussian operations on an auxiliary mode is common procedure in quantum optics experiments. Motivated by the demand of real-world quantum network, here we investigate the remote creation and distribution of Wigner negativity in the multipartite scenario from a quantitative perspective. By establishing a monogamy relation akin to the generalized Coffman-Kundu-Wootters inequality, we show that the amount of Wigner negativity cannot be freely distributed among different modes. Moreover, for photon subtraction—one of the main experimentally realized non-Gaussian operations—we provide an intuitive method to quantify remotely generated Wigner negativity. Our results pave the way for exploiting Wigner negativity as a valuable resource for numerous quantum information protocols based on non-Gaussian scenario.

npj Quantum Information (2022)8:21 ; https://doi.org/10.1038/s41534-022-00533-3

INTRODUCTION

Continuous-variable (CV) systems have attained impressive success in quantum information processing[1]. As an important platform that has been widely studied, Gaussian systems and operations are extensively used in quantum teleportation[2], quantum key distribution[3], and quantum-enhanced sensing[4,5]. These protocols come with the advantage of deterministically producing resource states and being analytically tractable due to the Gaussian properties of the states. However, non-Gaussian states and operations have irreplaceable advantages in some CV protocols[6], such as entanglement distillation[7,8], error correction[9], secure quantum communication[10], and the verification of Bell nonlocality[11]. Considerable progress in the controllable generation of multimode non-Gaussian states has been made in recent experiments[12,13], which also provide support for the implementation of universal CV quantum computation in the long term[14].

For some non-Gaussian states, the Wigner function can reach negative values. This Wigner negativity has been seen as a necessary ingredient in CV quantum computation and simulation to outperform classical devices[15–17]. A common approach to generate Wigner negativity is by means of the action of a conditional operation on initially prepared Gaussian states. In the pursuit of long-distance quantum technologies, it is crucial to develop efficient methods to produce Wigner negativity in a distant node. Recently, it was proven that a necessary requirement for such scheme is the existence of Einstein-Podolsky-Rosen (EPR) steering[18,19]—a particular type of quantum correlation where local measurements performed on one party can adjust (steer), instantaneously, the state of the other remote party[20–22]. Based on this kind of nonlocal effect, one can remotely produce negativity in the steering mode by applying a set of appropriate operations on the steered mode.

In consideration of the real-world quantum network in the multipartite scenario, it is a worthwhile objective to deeply explore the remote generation and distribution of Wigner negativity over many nodes in an entanglement-based network. As an intermediate type of quantum correlation between entanglement and Bell nonlocality, multipartite quantum steering[23] has received extensive attention in recent developments of quantum information theory[24,25]. It has been successfully implemented in CV optical network[26–29], photonic network[30–32], and atomic ensembles[33]. Inspired by the shareability of EPR steering, known as monogamy[34–40], it is interesting to explore how can the remotely generated Wigner negativity be distributed over different modes? Is there any monogamy relations imposing quantitative constraints on that negativity? Does stronger steerability generate more negativity?

Here we present a quantitative investigation of Wigner negativity that is remotely created via multipartite EPR steering, in which non-Gaussian operations performed on one steered node of quantum network produce Wigner negativity in different distant nodes, as shown in Fig. 1. We first investigate to what extent Wigner negativity can be shared by establishing a monogamy relation. This constraints the degree of distributed negativity akin to the Coffman-Kundu-Wootters (CKW) monogamy inequality for steerability[36]. Basic examples including two typical schemes, i.e. photon subtraction and Fock-state projection, for the remote preparation of Wigner-negative states are given. Then we focus on photon subtraction, one of the commonly used non-Gaussian operations, and find an intuitive measure for the amount of induced Wigner negativity in the steering modes. This allows us to find out the different behaviors of the steerability and the created negativity depending on the squeezing and purities of initial Gaussian states.

RESULTS

We begin by briefly introducing the theoretical framework of multimode CV quantum optics. The noninteracting quantized

[1]State Key Laboratory for Mesoscopic Physics, School of Physics, Frontiers Science Center for Nano-optoelectronics, & Collaborative Innovation Center of Quantum Matter, Peking University, Beijing 100871, China. [2]Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China. [3]Peking University Yangtze Delta Institute of Optoelectronics, Nantong 226010 Jiangsu, China. [4]Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL, Collège de France, 4 place Jussieu, F-75252 Paris, France. [5]These authors contributed equally: Yu Xiang, Shuheng Liu. ✉email: qiongyihe@pku.edu.cn

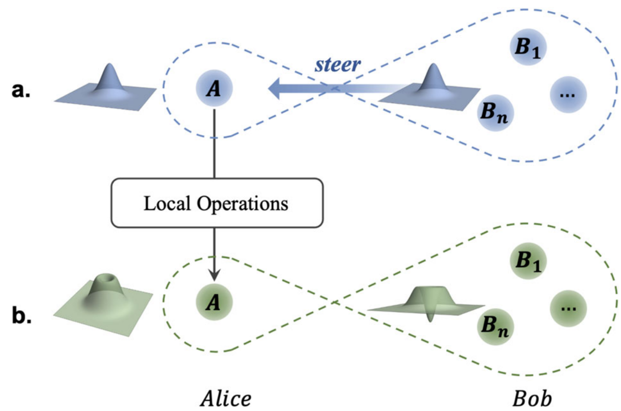

Fig. 1 Scheme of the remote generation of Wigner negativity through EPR steering in a multipartite scenario. a The initial Gaussian steerable system; **b** After some appropriate local operations on the steered mode hold by Alice, the steering subsystem hold by Bob becomes non-Gaussian with Wigner negativity.

electromagnetic field can be treated as a number N of optical modes that behave as quantum harmonic oscillators with different frequencies described by $\hat{H}=\sum_{k=1}^{N}2\omega_k(\hat{a}_k^{\dagger}\hat{a}_k+\frac{1}{2})$. Here, $\hat{a}_k$ and $\hat{a}_k^{\dagger}$ are the annihilation and creation operators of a photon in mode k, satisfying the bosonic commutation relation $[\hat{a}_k,\hat{a}_{k'}^{\dagger}]=\delta_{kk'}$. The corresponding quadrature phase operators for each mode are defined as $\hat{x}_k=\hat{a}_k+\hat{a}_k^{\dagger}$ and $\hat{p}_k=(\hat{a}_k-\hat{a}_k^{\dagger})/i$. Collecting the quadrature operators for all the modes into a vector $\hat{\boldsymbol{\xi}}\equiv(\hat{x}_1,\hat{p}_1,...,\hat{x}_N,\hat{p}_N)^{\top}$, the covariance matrix (CM) σ is given with elements $\sigma_{ij}=\langle\hat{\xi}_i\hat{\xi}_j+\hat{\xi}_j\hat{\xi}_i\rangle/2-\langle\hat{\xi}_i\rangle\langle\hat{\xi}_j\rangle$. If the system is prepared in a Gaussian state, the properties can be completely determined by its CM. Otherwise, the first and second-order statistical moments are not enough to characterize the non-Gaussian system, and we must resort to a more complete description. Here we choose the Wigner function as a preferred phase space representation for an arbitrary state with density matrix $\hat{\rho}$,

$$W(\boldsymbol{\xi})=\int_{\mathbb{R}^{2N}}\frac{d^{2N}\boldsymbol{\alpha}}{(2\pi)^{2N}}\exp\left(-i\boldsymbol{\xi}^{\top}\boldsymbol{\Omega}\boldsymbol{\alpha}\right)\chi(\boldsymbol{\alpha}),\tag{1}$$

where $\boldsymbol{\Omega}=\bigoplus_1^N\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ is the symplectic form and the Wigner characteristic function $\chi(\boldsymbol{\alpha})=\mathrm{Tr}\,[\hat{\rho}\exp(i\hat{\boldsymbol{\xi}}^{\top}\boldsymbol{\Omega}\boldsymbol{\alpha})]$. A particular attribute of non-Gaussian states is the possibility for this Wigner function to attain negative values that can be quantified as $\mathcal{N}\equiv\int|W(\boldsymbol{\xi})|d\boldsymbol{\xi}-1$[41].

In order to effectively generate and distribute Wigner negativity, an indirect scheme was proposed based on EPR steering[18]. In a two-mode Gaussian system, when there exists steering from Bob to Alice, then an appropriate local Gaussian transformation together with photon subtraction on the steered mode A can remotely generate Wigner negativity in the steering mode B, i.e. $\mathcal{N}_B>0$. The bipartite Gaussian steerability can be quantified by the parameter $\mathcal{G}^{B\to A}=\max\{0,\frac{1}{2}\ln\frac{\mathrm{Det}\sigma_B}{\mathrm{Det}\,\sigma_{AB}}\}$, where σ_B and σ_{AB} denote the CM for mode B, and the group (AB), respectively[42]. This formalism was developed for arbitrary conditional operations on an arbitrary number of modes, showing that EPR steering is still necessary to prepare a Wigner-negative state in the steering modes[19]. The remotely generated Wigner negativity was not quantified, nor are its multimode properties such as the shareability of the negativity among steering modes understood. Especially, since EPR steering is a prerequisite for remote preparation of Wigner negativity, one may intuitively expect that stronger steerability in the initial Gaussian states creates more Wigner negativity. With our quantitative investigation, we show that such a joint increase of steering and Wigner negativity does not always exist.

Monogamy of remotely generated Wigner negativity

First, we study the multimode character of the remotely generated Wigner negativity by deriving constraints on the distribution of this negativity among various modes in the steering party $(B_1B_2\ldots B_n)$ for a $(1+n)$-mode Gaussian state $\sigma_{AB_1B_2\ldots B_n}$. As a fundamental property of EPR steering, the CKW-type monogamy relation reveals that the sum of Gaussian steerability between any two modes cannot exceed their intergroup steerability, i.e., $\mathcal{G}^{B_1B_2\ldots B_n\to A}\geq\sum_{i=1}^{n}\mathcal{G}^{B_i\to A}$, which bounds the achievable key rate of quantum secret sharing[36]. In analogy with this steering constraint, we establish a monogamy relation for the amount of the remotely generated Wigner negativity:

$$\mathcal{N}_{B_1B_2\ldots B_n}(\mathcal{L}_{A|B_1B_2\ldots B_n})\geq\sum_{i=1}^{n}\mathcal{N}_{B_i}(\mathcal{L}_{A|B_i}).\tag{2}$$

Here, $\mathcal{L}_{A|B_j\ldots B_k}$ represents the optimal set of local operations on mode A to induce the largest amount of Wigner negativity in the group of modes $(B_j\ldots B_k)$, where the subscript "$A|B_j\ldots B_k$" represents that the choice of $\mathcal{L}$ depends on the initial Gaussian steering from the group $(B_j\ldots B_k)$ to mode A. Thus generating negativities in different modes requires different optimal operations on the steered mode A. For instance, inducing Wigner negativity in the steering mode B_j, or B_k, or their joint (B_jB_k), requires different local Gaussian transformations prior to a non-Gaussian operation (e.g. photon subtraction).

Proof. Without loss of generality, let us focus on a tripartite scenario, in which the steering party B contains two modes B_1 and B_2. Now we use the fact that $\mathcal{N}_{B_1}(\mathcal{L}_{A|B_1})>0$ and $\mathcal{N}_{B_2}(\mathcal{L}_{A|B_2})>0$ cannot be true simultaneously, which is a consequence of another type of Gaussian steering monogamy relation: modes B_1 and B_2 cannot simultaneously steer mode A under Gaussian measurements[34,35]. Assuming that mode B_1 can steer mode A, negativity can be generated only in the Wigner function of mode B_1 under Alice's local operation $\mathcal{L}_{A|B_1}$, such that Eq. (2) takes the simpler form $\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1B_2})\geqslant\mathcal{N}_{B_1}(\mathcal{L}_{A|B_1})$ (or the analogous expression with swapped $B_1\leftrightarrow B_2$). The local operation $\mathcal{L}_{A|B_1}$ in the right side is chosen to generate the largest negativity $\mathcal{N}_{B_1}$ in mode B_1, determined by the Gaussian steering of mode A by individual mode B_1. Because the Wigner negativity is nonincreasing under partial trace within the group (B_1B_2)[16], it is straightforward as

$$\begin{aligned}\mathcal{N}_{B_1}(\mathcal{L}_{A|B_1})&=\int d\mathbf{r}_{B_1}\left|W[\mathrm{Tr}_{B_2}[\rho_{B_1B_2}]](\mathbf{r}_{B_1})\right|-1\\&=\int d\mathbf{r}_{B_1}\left|\int d\mathbf{r}_{B_2}W[\rho_{B_1B_2}](\mathbf{r}_{B_1},\mathbf{r}_{B_2})\right|-1\\&\leqslant\int d\mathbf{r}_{B_1}\int d\mathbf{r}_{B_2}\left|W[\rho_{B_1B_2}](\mathbf{r}_{B_1},\mathbf{r}_{B_2})\right|-1\\&=\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1}).\end{aligned}$$

Here, $W[\rho_{B_1B_2}](\mathbf{r})$ represents the Wigner distribution of the joint state $\rho_{B_1B_2}$. Note that the negativity $\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1})$ in the above inequality is also generated by the same local operation $\mathcal{L}_{A|B_1}$, which may not be optimal for the group (B_1B_2). To create the largest negativity $\mathcal{N}_{B_1B_2}$, we need to choose optimal local operations $\mathcal{L}_{A|B_1B_2}$ based on joint steering of mode A by group (B_1B_2), leading to $\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1B_2})\geq\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1})$. Afterwards it is promptly verified that $\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1B_2})\geq\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1})\geq\mathcal{N}_{B_1}(\mathcal{L}_{A|B_1})$.

An example is given in Fig. 2. In this linear optical network, a pure three-mode entangled Gaussian state can be generated in the local station by mixing three squeezed inputs, then two of the output modes B_1 and B_2 are sent to the remote nodes, as illustrated in Fig. 2a. When the first beam splitter is fixed at R_1: $(1-R_1)=50{:}50$ and the second beam splitter is adjustable, the

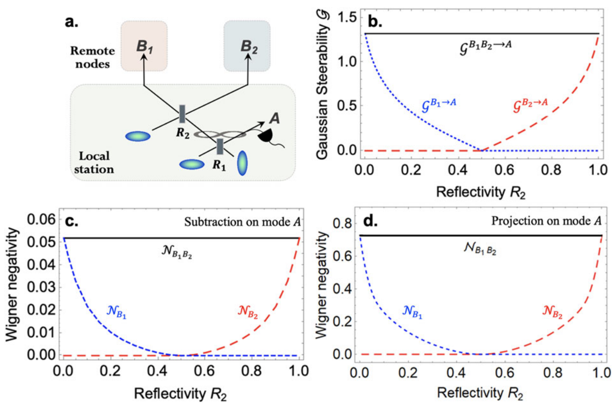

Fig. 2 Remotely generated Wigner negativity and initial Gaussian steering in a tripartite scenario. a A feasible linear optical network to remotely generate Wigner negativity: the local station produces a pure three-mode Gaussian state, then sends two outputs B_1 and B_2 to remote nodes. By performing some appropriate local operations on mode A, one can successfully prepare mode B_1, or B_2, or their joint (B_1B_2) to a Wigner-negative state. **b** Fixing $R_1:(1-R_1)=50:50$ and input squeezing levels $r=1$ (corresponding to -8.7 dB quadrature noise), the initial Gaussian steerability changes with a variable R_2. **c** After a single-photon subtraction on mode A, the remotely generated Wigner negativity of mode B_1, mode B_2, and the group (B_1B_2), respectively. **d** After a photon-number-resolving measurement with two photons in mode A, the remotely generated Wigner negativity of mode B_1, mode B_2, and the group (B_1B_2), respectively.

steerabilities distributed among three modes in the initial Gaussian state are given in Fig. 2b. Here we choose two typical types of non-Gaussian operations for Alice. One is the single-photon subtraction, which can be effectively realized in experiments[12,43]. The corresponding Wigner negativities remotely created through photon subtraction on mode A are shown in Fig. 2c. The other conditional preparation of Wigner negativity by local Fock-state projection can be found in panel Fig. 2d, where mode A projects to $|2\rangle$[44,45]. It is clear that in both cases the two-mode and three-mode Gaussian steering $\mathcal{G}^{B_1\to A}$, $\mathcal{G}^{B_2\to A}$ and $\mathcal{G}^{B_1B_2\to A}$ are necessary to induce Wigner negativity for individual mode B_1, B_2, and for the group (B_1B_2), respectively. However, constrained by the monogamy relation of Gaussian steering[34], i.e., modes B_1 and B_2 cannot steer mode A simultaneously, the negativities generated in the individual mode $\mathcal{N}_{B_1}>0$ and $\mathcal{N}_{B_2}>0$ can never be satisfied at the same time. This means that when B_1 receives a Wigner-negative state, it automatically guarantees that B_2 did not acquire Wigner negativity. Meanwhile, the joint Wigner negativities created on the group (B_1B_2) are significantly higher than the negativity of either individual mode in both cases.

In a more extreme case with $R_2=0.5$, no Wigner negativity is created in either mode individually, but their joint Wigner function can be negative, i.e., $\mathcal{N}_{B_1B_2}(\mathcal{L}_{A|B_1B_2})>0$. In Fig. 4 in the following section, we show that in presence of loss such a scenario can be quite common. This reminds of a context of quantum secret sharing where neither B_1 nor B_2 can acquire Wigner negativity, but only when both cooperate to achieve it. Moreover, such setups where Wigner negativity appears globally but not locally have an appealing interpretation: the operation in mode A created Wigner negativity before beamsplitter R_2 and the latter delocalized it over mode B_1 and B_2. If neither B_1 nor B_2 locally manifest this Wigner negativity, it must be hidden in a non-trivial correlation between both modes. When the joint state for B_1 and B_2 is pure—as is the case in Fig. 2d—this is a clear signature of entanglement between the two modes. However, in the more general case where B_1 and B_2 share a mixed state such conclusion no longer holds. Nevertheless, as shown in Section "Distillation of nonlocal Wigner negativity", the global negativity of $W[\rho_{B_1B_2}](\mathbf{r})$ implies that one can always distill Wigner negativity in mode B_1 or B_2 by performing a suitable measurement on the other mode.

Quantification of the generated Wigner negativity

EPR steering refers to the ability of one system to adjust the state of another distant system by local measurements. Since remotely generated Wigner negativity is both enabled and constrained by Gaussian steering, one may intuitively expect that stronger steerability induces more Wigner negativity. However, we show that this is not the case by providing an intuitive quantification for some experimentally prominent bipartition (multimode) Gaussian states. We quantify the amount of Wigner negativity in the steering modes, via the purities of initial Gaussian states, and show that purity, rather than steerability, governs the amount of Wigner negativity that can be created. In some cases studied below, less squeezing in the initial Gaussian state (for both pure and mixed cases) produces weaker steerability but remotely creates stronger negativity.

It is well known that any two-mode Gaussian state can be transformed into a *standard form*[46] through local linear unitary Bogoliubov operations (LLUBOs), so that the CM σ_{AB} reads

$$\sigma_{AB,sf}=\begin{pmatrix}\sigma_A & \gamma_{AB}\\ \gamma_{AB}^{\top} & \sigma_B\end{pmatrix}=\begin{pmatrix}a & 0 & c_1 & 0\\ 0 & a & 0 & c_2\\ c_1 & 0 & b & 0\\ 0 & c_2 & 0 & b\end{pmatrix} \tag{3}$$

with $a,b\geq 1$ and $ab-c_{1(2)}^2\geq 0$. The two local purities $\mu_{A(B)}\equiv 1/\sqrt{\operatorname{Det}\sigma_{A(B)}}=1/a(b)$ and the global purity $\mu_{AB}\equiv 1/\sqrt{\operatorname{Det}\sigma_{AB}}=1/\sqrt{(ab-c_1^2)(ab-c_2^2)}$, are invariant under LLUBOs[47]. Furthermore, because LLUBOs are local and symplectic, the standard form $\sigma_{AB,sf}$ manifests equal quantity of Gaussian steerability possessed in the initial states.

We now restrict our scope to two-mode Gaussian states with $c_1=-c_2=c$, which include the major experimentally realized CV EPR resources such as the two-mode EPR state with phase-insensitive losses and the two-mode squeezed thermal state. By focusing on the experimentally relevant case where a single-photon subtraction $\mathcal{S}$ is performed on the steered mode, we derive that the amount of remotely generated Wigner negativity $\mathcal{N}_B$ is determined by the purities of initial Gaussian state μ_A, μ_B, and μ_{AB}:

$$\mathcal{N}_B(\mathcal{S}_{A|B})=2\left[\frac{e^{\frac{\mu_A\mu_B-\mu_{AB}\mu_A}{\mu_{AB}-\mu_A\mu_B}}(\mu_A\mu_B-\mu_{AB})}{\mu_{AB}(\mu_A-1)}-1\right]. \tag{4}$$

Exchanging $\mu_A\leftrightarrow\mu_B$, we can obtain the result for the other direction $\mathcal{N}_A(\mathcal{S}_{B|A})$. The derivation of the above relation is detailed in the section of "Methods". Gaussian steerability and remotely created Wigner negativity are both determined by the local and global purities, but the dependence is very different. EPR steering provides a necessary bridge to induce Wigner negativity, but it is insufficient to unambiguously quantify the created Wigner negativity.

We explicitly show this point in an example where a two-mode EPR state is distributed over a single lossy channel characterized by η_A (the other channel is assumed ideal), as shown in Fig. 3, which is often used to demonstrate one-way steering[26,27,48]. The CM of this kind of state is in the standard form (3) with $a=\eta_A(\cosh 2r-1)+1$, $b=\cosh 2r$, and $c_1=-c_2=\sqrt{\eta_A}\sinh 2r$, where r is the squeezing parameter. The asymmetric Gaussian steerabilities in two directions are indicated in Fig. 3b, where the Gaussian steerability $\mathcal{G}^{A\to B}>0$ when $\eta_A>0.5$ while the other direction $\mathcal{G}^{B\to A}>0$ happens for any $\eta_A>0$. It also shows that

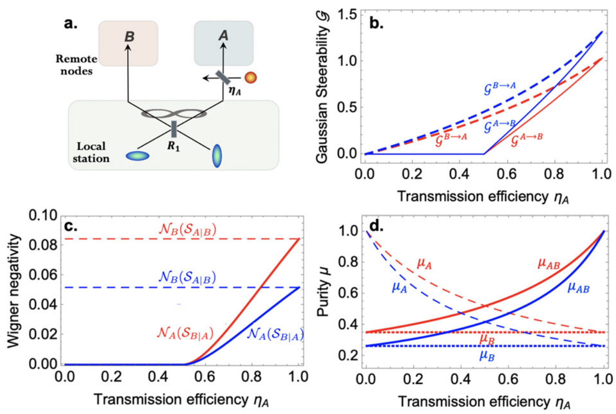

Fig. 3 Remotely generated Wigner negativity and initial Gaussian steering and purity in an asymmetric scenario. a Scheme of a two-mode squeezed vacuum state with one lossy channel on mode A, where R_1 is a balanced beam splitter. **b** The initial asymmetric Gaussian steerability with different squeezing levels $r = 1$ (blue) and $r = 0.85$ (red), corresponding to a quadrature noise reduction of -8.7 dB and -7.4 dB, respectively. Loss has a more significant effect on the steering mode A. **c** After the single-photon subtraction on the steered mode, the amount of induced Wigner negativity on the steering mode corresponding to Gaussian steerability given in **b**. **d** The local and global purities of the initial Gaussian states with different squeezing levels.

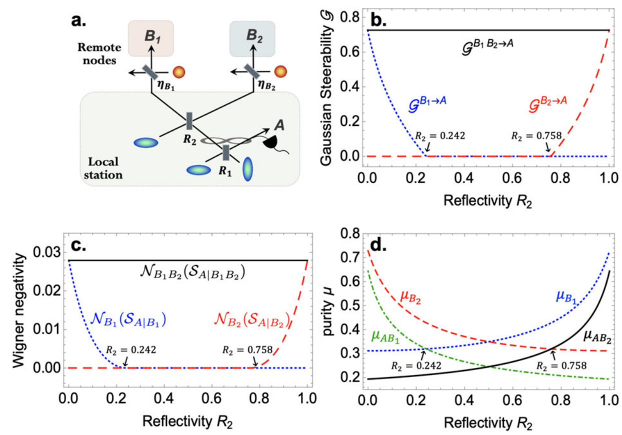

Fig. 4 Remotely generated Wigner negativity and initial Gaussian steering and purity in a lossy tripartite scenario. a A similar scheme as that shown in Fig. 2 but now both two output modes are distributed over lossy channels. **b** Fixing R_1: $(1 - R_1) = 50$: 50, $\eta_{B_1} = \eta_{B_2} = 0.8$ and input squeezing levels $r = 1$, the initial Gaussian steerabilities change with a variable R_2. **c** After a single-photon subtraction on mode A, the remotely generated Wigner negativities of mode B_1, mode B_2 and the group (B_1B_2), respectively. **d** Some local purities of the initial Gaussian state. The remaining cases are constants with the current setting, i.e., $\mu_A = 0.266$, $\mu_{B_1B_2} = 0.227$ and $\mu_{AB_1B_2} = 0.469$.

higher squeezing level creates stronger steerability (blue lines). By performing a single-photon subtraction on the steered mode B, the Wigner negativity can be created in mode A ($\mathcal{N}_A(\mathcal{S}_{B|A}) > 0$) when $\eta_A > 0.5$ as well and becomes larger with increasing efficiency η_A (solid lines), as shown in Fig. 3c. Interestingly, for the other direction, by performing a single-photon subtraction on the steered mode A, the generated negativity $\mathcal{N}_B(\mathcal{S}_{A|B})$ does not vary with η_A. This observation can be understood as a consequence of losses in the mode of photon subtraction commute with the subtraction operation itself[49]. As derived in Eq. (11) in Section "Quantifying the remotely created Wigner negativity", it can be seen that the Wigner negativity generated in mode B is only determined by the loss in its own channel (i.e. η_B, here it is considered as unity). This highlights the asymmetry of the induced Wigner negativity and suggests a way to remotely generate negativity that is robust to channel loss on the photon-subtraction side. Figure 3c also shows, unlike the Gaussian steerability plotted in Fig. 3b, the lower squeezing level leads to larger negativities (red lines). This can be explained by the purities of initial states given in Fig. 3d. The purities μ_A, μ_B, and μ_{AB} of the initial states with higher squeezing level are more sensitive to the loss (blue lines). Since purity plays a bigger role than squeezing in generating Wigner negativity[50], the case with higher squeezing but lower purities can only lead to weaker Wigner negativities (blue lines).

Besides the examples discussed above, we also analyze the asymmetric Gaussian steerability and the properties of induced Wigner negativity for another important CV EPR resource—two-mode squeezed thermal states[51], which are detailed in the section of "Methods". Moreover, as shown in Section "Necessity of the additional local Gaussian transformation", we also emphasize that producing EPR resources with CM in a standard form (3) can significantly simplify the procedure for remote generation of Wigner negativity and makes the resulting non-Gaussian state readily available for further applications.

Furthermore, for any three-mode globally pure Gaussian state, i.e., $\mu_{AB_1B_2} = 1$, the standard form of CM with respect to $A - (B_1B_2)$ splitting is locally equivalent to

$$\sigma^{\text{pure}}_{A-(B_1B_2),sf} = \left(\begin{array}{cc|cccc} a & 0 & c_1 & 0 & 0 & 0 \\ 0 & a & 0 & -c_1 & 0 & 0 \\ \hline c_1 & 0 & a & 0 & 0 & 0 \\ 0 & -c_1 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \tag{5}$$

through Williamson and Bloch-Messiah decompositions[47], where $c_1 = \sqrt{a^2 - 1}$. We can see that the system is qualitatively equivalent to a product of a two-mode squeezed vacuum state tensor an uncorrelated vacuum mode, such that the steering property between mode A and modes (B_1B_2) can be unitarily reduced to the two-mode case. Thus, the generated Wigner negativity of the group (B_1B_2) is equivalently indicated by

$$\mathcal{N}_{B_1B_2}(\mathcal{S}_{A|B_1B_2}) = 2\left[e^{-\frac{\mu}{1+\mu}}(1 + \mu) - 1\right], \tag{6}$$

where $\mu = \mu_A = \mu_{B_1B_2} = 1/a$. By the same method, this result (6) is also valid for $(1 + n)$-mode pure Gaussian states with respect to $A - (B_1B_2 \ldots B_n)$ splitting $(n > 2)$.

Finally, as the CM for a more general mixed state contains more parameters, here we numerically show that the amount of remotely generated negativity is still related to the purities of initial Gaussian state. As shown in Fig. 4, we provide an evidence to show this conclusion still holds by a three-mode mixed state. In this scheme, the output modes B_1 and B_2 are both distributed over lossy channels characterized by η_{B_1} and η_{B_2}, respectively, such that the global purity $\mu_{AB_1B_2} < 1$ when $\eta_{B_1}, \eta_{B_2} \neq 1$. For simplicity, we assume that the distance between two remote nodes and the local station is the same, thus $\eta_{B_1} = \eta_{B_2} = \eta$. As shown in Fig. 4b, due to the existence of channel losses (for a fixed value of $\eta = 0.8$), the area where neither mode B_1 nor mode B_2 can individually steer mode A has expanded to $0.242 < R_2 < 0.758$ compared with the scheme discussed in Fig. 2. The corresponding Wigner negativities remotely created in mode B_1 or B_2, or the group (B_1B_2) through a photon subtraction on mode A are given in

Fig. 4c. It is clear that both Gaussian steerability and Wigner negativity appear in the same condition. Comparing with Fig. 4c, d, we can find that the amount of remotely created Wigner negativities is still determined by the purities of initial states.

DISCUSSION

We develop the scheme for remote generation of Wigner negativity through EPR steering to multimode scenario, and show the presence of constraints for distributing Wigner negativity over different modes. So far, multipartite steering has been demonstrated in various Gaussian systems, e.g., linear optical networks[26,27], quantum frequency comb[29], and Bose–Einstein condensates[33]. These experimental developments lay a favorable foundation for implementing remote generation of multipartite non-Gaussian states through photon subtraction or other appropriate operations. Furthermore, we present an intuitive and computable quantification of the generated Wigner negativity for bipartition (multimode) system in terms of the local and global purities of initial Gaussian states. Our results deepen the understanding of Wigner negativity as a resource and provide an important framework of non-Gaussian quantum information theory.

Our work also triggers several interesting questions to stimulate further research. For instance, as Gaussian steerability $\mathcal{G}^{A\to B_1}>0$ and $\mathcal{G}^{A\to B_2}>0$ can happen simultaneously, then by performing a single-photon subtraction on each mode B_1, B_2, can we achieve more significant increase of the negativity in mode A? In addition, for this direction the Gaussian steerability still follows the CKW-type monogamy constraint, however, this constraint does not hold any more for the generated negativity. We have observed a violation in a pure three-mode state (see Methods), i.e., $\mathcal{N}_A(\mathcal{L}_{B_1B_2|A})<\mathcal{N}_A(\mathcal{L}_{B_1|A})+\mathcal{N}_A(\mathcal{L}_{B_2|A})$. Moreover, after non-Gaussian operations on the steered mode, the resulting system cannot be fully captured by the second-order correlations given in CM. To this day, relatively little is known about the characteristics of non-Gaussian steering[52].

METHODS

Quantifying the remotely created Wigner negativity

It is of particular interest to us is whether stronger steerability in the initial Gaussian states induces more Wigner negativity, as it is enabled and constrained by Gaussian steering. To answer this, we need first quantify the amount of Wigner negativity. In this part, we aim to derive the qualitative measure of Wigner negativity Eq. (4) in the main text by focusing on the experimentally relevant case where a photon subtracted from the steered mode in two-mode Gaussian states $c_1=-c_2=c$, which include the major experimentally realized CV EPR states.

Let us recall that, any two-mode Gaussian state can be transformed through LLUBOs to the standard form (3). Using the formula derived from ref. [18], we obtain the reduced Wigner function of the steering mode B after the local Gaussian transformation $R_{A|B}$ combined with a single-photon subtraction applied on the steered mode A,

$$\begin{aligned}
W_B(\boldsymbol{\beta}_B) &= \frac{\exp\{-\frac{1}{2}(\boldsymbol{\beta}_B,\sigma_B^{-1}\boldsymbol{\beta}_B)\}}{2\pi\sqrt{\operatorname{Det}\sigma_B}\left[\operatorname{Tr}\left(R_{A|B}^{\top}\sigma_A R_{A|B}\right)-2\right]} \\
&\quad\times\left[\boldsymbol{\beta}_B^{\top}\sigma_B^{-1\top}\gamma_{AB}^{\top}R_{A|B}R_{A|B}^{\top}\gamma_{AB}\sigma_B^{-1}\boldsymbol{\beta}_B+\operatorname{Tr}\left(R_{A|B}^{\top}V_{A|B}R_{A|B}\right)-2\right] \\
&= \frac{\exp\{-\frac{1}{2}(\boldsymbol{\beta}_B,\sigma_B^{-1}\boldsymbol{\beta}_B)\}}{2\pi\sqrt{\operatorname{Det}\sigma_B}\left[\operatorname{Tr}\left(v\sigma_A V_{A|B}^{-1}\right)-2\right]} \\
&\quad\times\left(v\boldsymbol{\beta}_B^{\top}\sigma_B^{-1\top}\gamma_{AB}^{\top}V_{A|B}^{-1}\gamma_{AB}\sigma_B^{-1}\boldsymbol{\beta}_B+2v-2\right),
\end{aligned} \tag{7}$$

where $\boldsymbol{\beta}_B=(x_B,p_B)^{\top}$ is the coordinate in a multimode phase spaces of subsystem B, $V_{A|B}=\sigma_A-\gamma_{AB}\sigma_B^{-1}\gamma_{AB}^{\top}$ is the Schur complement of σ_B and v is the corresponding symplectic eigenvalue. The Schur complement $V_{A|B}$ can be decomposed through Williamson decomposition via $V_{A|B}=vS_{A|B}^{\top}S_{A|B}$, where $S_{A|B}$ is the corresponding symplectic matrix and a local Gaussian transformation $R_{A|B}=S_{A|B}^{-1}$. When it comes to our particular interest subclass $c_1=-c_2=c$, it is easy to find out that the Schur complement is a multiple of identity matrix so that there is no need to perform an additional local Gaussian operation. Then we get

$$W_B(x_B,p_B)=\frac{e^{-\frac{x_B^2+p_B^2}{2b}}\left[2b^2(a-1)-2bc^2+c^2(x_B^2+p_B^2)\right]}{4\pi b^3(a-1)}, \tag{8}$$

which is circularly symmetric. It is straightforward to calculate Wigner negativity using integral,

$$\mathcal{N}_B(\mathcal{S}_{A|B})=\frac{2c^2e^{\frac{b(a-1)}{c^2}-1}}{b(a-1)}-2. \tag{9}$$

By expressing $\mathcal{N}_B(\mathcal{S}_{A|B})$ in terms of purities $\mu_{AB}=1/(ab-c^2)$, $\mu_A=1/a$, $\mu_B=1/b$, Eq. (9) becomes

$$\mathcal{N}_B(\mathcal{S}_{A|B})=2\left[\frac{e^{\frac{\mu_A\mu_B-\mu_{AB}\mu_A}{\mu_{AB}-\mu_A\mu_B}}(\mu_A\mu_B-\mu_{AB})}{\mu_{AB}(\mu_A-1)}-1\right]. \tag{10}$$

Figure 3 showed the case of two-mode squeezed vacuum state transmitted with single lossy channel of mode A. To further understand the effect of channel losses, we also take into account the loss in mode B's channel, characterized by η_B. When both channels are nonideal, the generated negativity in mode B is

$$\mathcal{N}_B=\frac{4e^{\left(\frac{1}{2\eta_B}-1\right)\operatorname{sech}^2 r}\eta_B\cosh^2 r}{1-\eta_B+\eta_B\cosh 2r}-2, \tag{11}$$

which merely depends on η_B but still does not vary with the loss in the channel of the steered mode A.

In the following, we analyze another important kind of experimentally realized CV EPR states–two-mode squeezed thermal states. The CM elements of these states are $a=(n_A+n_B+1)\cosh(2r)+(n_A-n_B)$, $b=(n_A+n_B+1)\cosh(2r)-(n_A-n_B)$, $c_1=-c_2=(n_A+n_B+1)\sinh(2r)$, where n_A, n_B are the average number of thermal photons for each subsystem[51]. We set the thermal noise only on one input mode with n_A and leave $n_B=0$, as illustrated in Fig. 5a. The asymmetric Gaussian steerability in two directions varying with n_A is denoted in Fig. 5b, and as a consequence the induced Wigner negativity on the steering mode by applying single-photon subtraction on the steered mode is quantified in Fig. 5c. Note that the effect of thermal noise on the steered mode is more significant than that on the steering mode, which is opposite to the effect of losses on two modes in the main text. As there exists a thermal barrier in the direction $\mathcal{G}^{B\to A}$, and correspondingly, a nonzero $\mathcal{N}_B(\mathcal{S}_{A|B})$ can exist only when $n_A<0.682$. In the

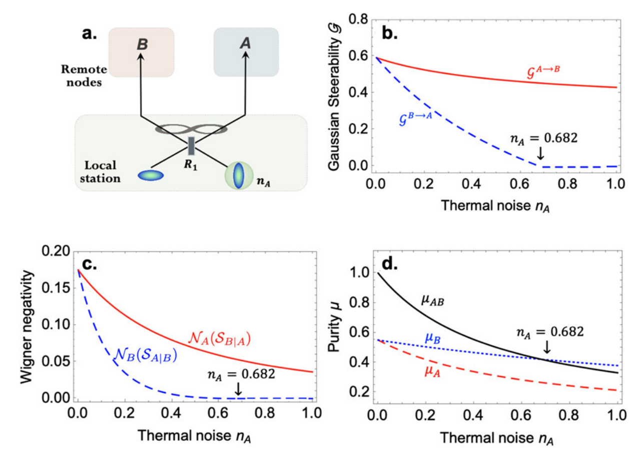

Fig. 5 Remotely generated Wigner negativity and initial Gaussian steering and purity in a noisy bipartite scenario. **a** Scheme of a two-mode squeezed thermal state with asymmetric thermal noise n_A and $n_B=0$. **b** The initial asymmetric Gaussian steerability with fixed squeezing level of $r=0.6$ (corresponding to -5.2 dB quadrature noise), where thermal noise has a more significant effect on the steered mode. **c** Corresponding to **b**, after the single-photon subtraction on one side, the remotely generated Wigner negativity of the other side. **d** The local and global purities of the initial Gaussian states.

opposite direction, $\mathcal{G}^{A\to B}>0$ and thus $\mathcal{N}_A(\mathcal{S}_{B|A})>0$ for arbitrarily large value of thermal noise n_A. It is clear that the amount of remotely created Wigner negativity is quantitatively determined by the purities of initial states, as plotted in Fig. 5d.

Distillation of nonlocal Wigner negativity

In Figs. 2 and 4 we observe cases where the local Wigner functions of B_1 and B_2 are fully positive, but nevertheless, Wigner negativity arises in the joint state for (B_1B_2). In this particular situation, one could argue that Wigner negativity is hidden in the nonlocal part of the Wigner function. Nevertheless, it turns out that such hidden negativity can always be unveiled by performing a well-chosen measurement on either B_1 or B_2. Here we prove this claim.

First of all, we introduce the displaced parity operator

$$\hat{\Pi}(\mathbf{r}_{B_1},\mathbf{r}_{B_2}) := \hat{D}^{\dagger}(\mathbf{r}_{B_1},\mathbf{r}_{B_2})(-\mathbb{1})^{\hat{n}_{B_1B_2}}\hat{D}(\mathbf{r}_{B_1},\mathbf{r}_{B_2}), \tag{12}$$

where the displacement operator $\hat{D}(\mathbf{r}_{B_1},\mathbf{r}_{B_2})=\hat{D}(\mathbf{r}_{B_1})\otimes\hat{D}(\mathbf{r}_{B_2})$, with $\hat{D}(\mathbf{r}_{B_k}) := \exp(i\hat{\boldsymbol{\xi}}_{B_k}^{\top}\boldsymbol{\Omega}\mathbf{r}_{B_k}/2)$. Furthermore, $\hat{n}_{B_1B_2}$ is the number operator for the joint system (B_1B_2). Because this number operator can be written as a sum of the local number operators $\hat{n}_{B_1B_2}=\hat{n}_{B_1}+\hat{n}_{B_2}$, we can now write that

$$\hat{\Pi}(\mathbf{r}_{B_1},\mathbf{r}_{B_2}) = \hat{\Pi}(\mathbf{r}_{B_1})\otimes\hat{\Pi}(\mathbf{r}_{B_2}). \tag{13}$$

The values of the Wigner function are given by[53,54]

$$\begin{aligned} W[\rho_{B_1B_2}](\mathbf{r}_{B_1},\mathbf{r}_{B_2}) &= \tfrac{1}{4\pi^2}\langle\hat{\Pi}(\mathbf{r}_{B_1},\mathbf{r}_{B_2})\rangle_{B_1B_2} \\ &= \tfrac{1}{4\pi^2}\langle\hat{\Pi}(\mathbf{r}_{B_1})\otimes\hat{\Pi}(\mathbf{r}_{B_2})\rangle_{B_1B_2}, \end{aligned} \tag{14}$$

where we introduce the shorthand notation $\langle\hat{X}\rangle_{B_1B_2} := \mathrm{Tr}\,[\rho_{B_1B_2}\hat{X}]$.

The identity (14) is now particularly useful to express the correlation between displaced parity measurements on modes B_1 and B_2 as

$$\begin{aligned} &\langle\hat{\Pi}(\mathbf{r}_{B_1})\otimes\hat{\Pi}(\mathbf{r}_{B_2})\rangle_{B_1B_2} - \langle\hat{\Pi}(\mathbf{r}_{B_1})\rangle_{B_1}\langle\hat{\Pi}(\mathbf{r}_{B_2})\rangle_{B_2} \\ &= 4\pi^2\{W[\rho_{B_1B_2}](\mathbf{r}_{B_1},\mathbf{r}_{B_2}) - W[\rho_{B_1}](\mathbf{r}_{B_1})W[\rho_{B_2}](\mathbf{r}_{B_2})\}. \end{aligned} \tag{15}$$

In the specific case where $\mathcal{N}_{B_1B_2}>0$ and $\mathcal{N}_{B_1}=\mathcal{N}_{B_2}=0$ the above identity implies the existence of values $\mathbf{r}_{B_1}$ and $\mathbf{r}_{B_2}$ for which $\langle\hat{\Pi}(\mathbf{r}_{B_1})\otimes\hat{\Pi}(\mathbf{r}_{B_1})\rangle_{B_1B_2} - \langle\hat{\Pi}(\mathbf{r}_{B_1})\rangle_{B_1}\langle\hat{\Pi}(\mathbf{r}_{B_2})\rangle_{B_2}<0$. In other words, there is always a case where the displaced parity measurements are anti-correlated.

A state which has Wigner negativity is always a state where for some phase space coordinates $\mathbf{r}_{B_k}$ measurements of the observable $\hat{\Pi}(\mathbf{r}_{B_k})$ provide the outcome -1 with a higher probability than the outcome $+1$. This observation is important in the light of (15). Let us now fix $\mathbf{r}_{B_1}$ and $\mathbf{r}_{B_2}$ such that $W[\rho_{B_1B_2}](\mathbf{r}_{B_1},\mathbf{r}_{B_2})<0$. For joint measurements of $\hat{\Pi}(\mathbf{r}_{B_1})$ and $\hat{\Pi}(\mathbf{r}_{B_2})$, we are most likely to obtain opposite parities due to the anti-correlation in (15). More formally phrased, the only way of obtaining this anti-correlation is through

$$\begin{aligned} &\mathrm{Prob}\,[\hat{\Pi}(\mathbf{r}_{B_2})=-1|\hat{\Pi}(\mathbf{r}_{B_1})=+1] \\ &\quad>\mathrm{Prob}[\hat{\Pi}(\mathbf{r}_{B_2})=+1|\hat{\Pi}(\mathbf{r}_{B_1})=+1], \\ &\mathrm{Prob}[\hat{\Pi}(\mathbf{r}_{B_1})=-1|\hat{\Pi}(\mathbf{r}_{B_2})=+1] \\ &\quad>\mathrm{Prob}[\hat{\Pi}(\mathbf{r}_{B_1})=+1|\hat{\Pi}(\mathbf{r}_{B_2})=+1]. \end{aligned} \tag{16}$$

However, this means that when we measure $\hat{\Pi}(\mathbf{r}_{B_1})$ and post-select on measurement outcomes $+1$, we find that for the state in B_2 is given by

$$\begin{aligned} W[\rho_{B_2}](\mathbf{r}_{B_2}) &= \tfrac{1}{2\pi}\langle\hat{\Pi}(\mathbf{r}_{B_2})\rangle_{B_2} \\ &= \tfrac{1}{2\pi}\big(\mathrm{Prob}[\hat{\Pi}(\mathbf{r}_{B_2})=+1|\hat{\Pi}(\mathbf{r}_{B_1})=+1] \\ &\quad-\mathrm{Prob}[\hat{\Pi}(\mathbf{r}_{B_2})=-1|\hat{\Pi}(\mathbf{r}_{B_1})=+1]\big) \\ &< 0. \end{aligned} \tag{17}$$

In other words, when the global Wigner function in (B_1B_2) is non-positive, whereas the local Wigner functions in B_1 and B_2 are positive, conditioning on a positive outcome for a displaced parity measurement on either B_1 or B_2 allows to prepare a Wigner-negative state in B_2 or B_1, respectively. Even though this proof shows the existence of some measurement to prepare Wigner negativity, in many cases, one can probably find more convenient measurements to unveil the Wigner negativity.

We note finally that this is a very complementary setting to the one discussed in the remainder of this article. We emphasized that to create and distribute Wigner negativity when the global state is Gaussian, we require quantum steering in this Gaussian state. Yet, when the global state is already Wigner negative, the requirements on the level of the correlations are far less stringent. A good example to keep in mind is that of the state $\rho_{B_1B_2}=(|0,1\rangle\langle 0,1|+|1,0\rangle\langle 1,0|)/2$. This state is globally Wigner negative, but locally leads to positive Wigner functions. Conditioning on a measurement outcome associated with the projector $|0\rangle\langle 0|$ on either subsystem will project the other subsystem into a state $|1\rangle\langle 1|$. As such, a simple Gaussian measurement on a global Wigner negative state with only classical correlations can lead to local Wigner negativity. This illustrates that delocalized Wigner negativity is a very strong resource to create local Wigner negativity.

Necessity of the additional local Gaussian transformation

Here, we prove that the additional local Gaussian transformation $R_{A|B}$ required prior to photon subtraction in ref. [18] to optimize the induced Wigner negativity is no longer needed *if and only if* the CM of two-mode Gaussian states in the standard form $\sigma_{AB,sf}$ satisfies $c_1=-c_2=c$. Thus producing EPR resource with CM in a *standard form* significantly simplifies the procedure for remote generation of Wigner negativity and makes the resulting non-Gaussian state readily available for further applications.

From the standard form $\sigma_{AB,sf}$, the Schur complement of σ_B is given by

$$V_{A|B} = \begin{pmatrix} a-c_1^2/b & 0 \\ 0 & a-c_2^2/b \end{pmatrix}, \tag{18}$$

whose symplectic eigenvalue is $v=\sqrt{(a-c_1^2/b)(a-c_2^2/b)}$. When there exists Gaussian steering $\mathcal{G}^{B\to A}$, the symplectic eigenvalue v must be smaller than 1[42]. Without any local Gaussian transformation $R_{A|B}$ prior to the photon subtraction on mode A, the condition for $W_B(\boldsymbol{\beta}_B)<0$ should be $\mathrm{tr}\,[V_{A|B}]<2$. Note that every CM σ_{AB} that corresponds to a physical quantum state has to satisfy the bona fide condition $a-c_1^2/b>0$ and $a-c_2^2/b>0$[42], then we have

$$\mathrm{Tr}\,[V_{A|B}] = \left(a-\frac{c_1^2}{b}\right)+\left(a-\frac{c_2^2}{b}\right) \geqslant 2\sqrt{\left(a-\frac{c_1^2}{b}\right)\left(a-\frac{c_2^2}{b}\right)} = 2v. \tag{19}$$

The above inequality can be saturated *if and only if* $c_1^2=c_2^2$. With this condition, $\mathrm{tr}\,[V_{A|B}]<2$ is equivalent to $v<1$, i.e., the photon subtraction on mode A can always generate Wigner negativity in mode B as long as $\mathcal{G}^{B\to A}>0$ without any prior local Gaussian transformation. Otherwise, if $c_1^2\neq c_2^2$, then $\mathrm{tr}\,[V_{A|B}]>2v$, which means an additional local Gaussian transformation $R_{A|B}$ is necessary to make EPR steering sufficient for remotely generating Wigner negativity. This complements the results of ref. [18], where it was shown that an additional local Gaussian transformation prior to photon subtraction is necessary to make EPR steering sufficient for remotely generating Wigner negativity. This Gaussian transformation requires inline squeezing and is experimentally challenging. Our results can be used as a recipe to prepare resourceful Gaussian states for the remote generation of Wigner negativity without the need for inline squeezing.

Distribution of Wigner negativity in the other direction

Steering is a directional form of nonlocality, related to the Einstein "spooky" paradox, which is fundamentally defined differently from entanglement. In previous work[36], we have derived the CKW-type monogamy inequalities for multipartite Gaussian steering in two directions. We then wonder whether the CKW-type monogamy inequality holds for the distribution of Wigner negativity created in the opposite direction, i.e., $\mathcal{N}_A(\mathcal{L}_{B_1B_2|A})\geq\mathcal{N}_A(\mathcal{L}_{B_1|A})+\mathcal{N}_A(\mathcal{L}_{B_2|A})$.

We present a three-mode entangled Gaussian state that is similar to the case shown in Fig. 2a, but with the first beamsplitter being adjustable R_1: $(1-R_1)$ and the second fixed as a balanced one. In particular, when the first beamsplitter is adjusted at $R_1=1/3$, the initial Gaussian state is produced as a GHZ-like state. For the direction where mode A acts as the steering party to steer the modes B_1,B_2, the Gaussian steerability distributed among three modes and the corresponding Wigner negativities remotely created by a single-photon subtraction on the individual or joint modes B_1,B_2 are denoted in Fig. 6a, b. It is clear that the two-mode and three-mode Gaussian steerability $\mathcal{G}^{A\to B_1(B_2)}>0$ and $\mathcal{G}^{A\to B_1B_2}>0$ are necessary to induce negativities $\mathcal{N}_A(\mathcal{S}_{B_1(B_2)|A})$ and $\mathcal{N}_A(\mathcal{S}_{B_1B_2|A})$ in the Wigner functions of the steering mode A, respectively. Interestingly, we observe that $\mathcal{N}_A(\mathcal{S}_{B_1B_2|A})<\mathcal{N}_A(\mathcal{S}_{B_1|A})+\mathcal{N}_A(\mathcal{S}_{B_2|A})$ when R_1 approaches to 1 (Fig. 6d), even though the Gaussian steerability still follows the monogamy constraint, i.e., $\mathcal{G}^{A\to B_1B_2}-\mathcal{G}^{A\to B_1}-\mathcal{G}^{A\to B_2}>0$ presented in

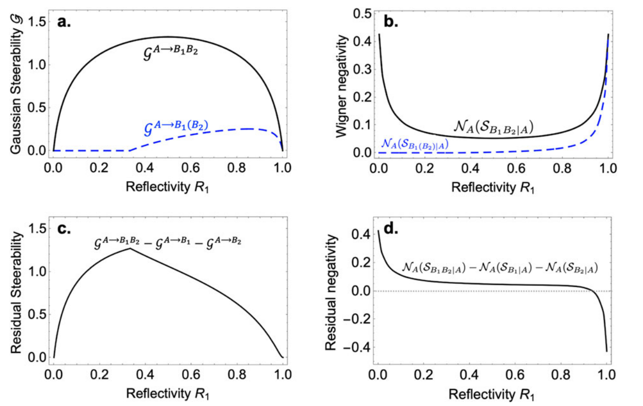

Fig. 6 A violation of CKW-type monogamy constraint of Wigner negativity. In the same scheme discussed in Fig. 2a, the first beamsplitter R_1 is adjustable and the second beamsplitter is fixed at $R_2{:}(1-R_2)=50{:}50$. The input squeezing levels $r=1$ (-8.7dB quadrature noise). **a** The initial Gaussian steerabilities varying with R_1. **b** The corresponding transferred Wigner negativities in the steering mode A by applying single-photon subtraction on the individual mode $B_1(B_2)$, or their joint (B_1B_2), respectively. **c** The residual tripartite Gaussian steering $\mathcal{G}^{A\to B_1B_2}-\mathcal{G}^{A\to B_1}-\mathcal{G}^{A\to B_2}\geq 0$ which is stemming from the reverse CKW-type monogamy. **d** The residual tripartite Wigner negativity remotely created in mode A, $\mathcal{N}_A(\mathcal{S}_{B_1B_2|A})-\mathcal{N}_A(\mathcal{S}_{B_1|A})-\mathcal{N}_A(\mathcal{S}_{B_2|A})$, which is not always larger than zero.

Fig. 6c. This settles an open question for the shareability of generated Wigner negativity in this direction.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

CODE AVAILABILITY

The codes for numerical simulation and data processing are available from the corresponding authors upon reasonable request.

Received: 20 August 2021; Accepted: 26 January 2022;
Published online: 02 March 2022

REFERENCES

1. Braunstein, S. L. & van Loock, P. Quantum information with continuous variables. *Rev. Mod. Phys.* **77**, 513–577 (2005).
2. Braunstein, S. L. & Kimble, H. J. Teleportation of continuous quantum variables. *Phys. Rev. Lett.* **80**, 869 (1998).
3. Grosshans, F. & Grangier, P. Continuous variable quantum cryptography using coherent states. *Phys. Rev. Lett.* **88**, 057902 (2002).
4. Tan, S. H. et al. Quantum illumination with Gaussian states. *Phys. Rev. Lett.* **101**, 253601 (2008).
5. Zhuang, Q., Zhang, Z. & Shapiro, J. H. Optimum mixed-state discrimination for noisy entanglement-enhanced sensing. *Phys. Rev. Lett.* **118**, 040801 (2017).
6. Andersen, U. L., Neergaard-Nielsen, J. S., van Loock, P. & Furusawa, A. Hybrid discrete- and continuous-variable quantum information. *Nat. Phys.* **11**, 713–719 (2015).
7. Eisert, J., Scheel, S. & Plenio, M. B. Distilling Gaussian states with Gaussian operations is impossible. *Phys. Rev. Lett.* **89**, 137903 (2002).
8. Takahashi, H. et al. Entanglement distillation from Gaussian input states. *Nat. Photonics.* **4**, 178–181 (2010).
9. Niset, J., Fiurášek, J. & Cerf, N. J. No-go theorem for Gaussian quantum error correction. *Phys. Rev. Lett.* **102**, 120501 (2009).
10. Lee, J., Park, J. & Nha, H. Quantum non-Gaussianity and secure quantum communication. *npj Quantum Inform.* **5**, 49 (2019).
11. Chen, Z. B., Pan, J. W., Hou, G. & Zhang, Y. D. Maximal violation of Bell's inequalities for continuous variable systems. *Phys. Rev. Lett.* **88**, 040406 (2002).
12. Ra, Y. S. et al. Non-Gaussian quantum states of a multimode light field. *Nat. Phys.* **16**, 144–147 (2020).
13. Chang, C. W. S. et al. Observation of three-photon spontaneous parametric down-conversion in a superconducting parametric cavity. *Phys. Rev. X* **10**, 011011 (2020).
14. Menicucci, N. C. et al. Universal quantum computation with continuous-variable cluster states. *Phys. Rev. Lett.* **97**, 110501 (2006).
15. Takagi, R. & Zhuang, Q. Convex resource theory of non-Gaussianity. *Phys. Rev. A* **97**, 062337 (2018).
16. Albarelli, F., Genoni, M. G., Paris, M. G. A. & Ferraro, A. Resource theory of quantum non-Gaussianity and Wigner negativity. *Phys. Rev. A* **98**, 052350 (2018).
17. Mari, A. & Eisert, J. Positive Wigner functions render classical simulation of quantum computation efficient. *Phys. Rev. Lett.* **109**, 230503 (2012).
18. Walschaers, M. & Treps, N. Remote generation of Wigner negativity through Einstein-Podolsky-Rosen steering. *Phys. Rev. Lett.* **124**, 150501 (2020).
19. Walschaers, M., Parigi, V. & Treps, N. Practical framework for conditional non-Gaussian quantum state preparation. *PRX Quantum* **1**, 020305 (2020).
20. Schrödinger, E. Discussion of probability relations between separated systems. *Proc. Camb. Phil. Soc.* **31**, 555–563 (1935).
21. Wiseman, H. M., Jones, S. J. & Doherty, A. C. Steering, entanglement, nonlocality, and the Einstein-Podolsky-Rosen paradox. *Phys. Rev. Lett.* **98**, 140402 (2007).
22. Reid, M. D. et al. Colloquium: The Einstein-Podolsky-Rosen paradox: from concepts to applications. *Rev. Mod. Phys.* **81**, 1727–1751 (2009).
23. He, Q. Y. & Reid, M. D. Genuine multipartite Einstein-Podolsky-Rosen steering. *Phys. Rev. Lett.* **111**, 250403 (2013).
24. Cavalcanti, D. & Skrzypczyk, P. Quantum steering: a review with focus on semi-definite programming. *Rep. Prog. Phys.* **80**, 024001 (2017).
25. Uola, R., Costa, A. C. S., ChauNguyen, H. & Gühne, O. Quantum steering. *Rev. Mod. Phys.* **92**, 015001 (2020).
26. Armstrong, S. et al. Multipartite Einstein-Podolsky-Rosen steering and genuine tripartite entanglement with optical networks. *Nat. Phys.* **11**, 167–172 (2015).
27. Deng, X. W. et al. Demonstration of monogamy relations for Einstein-Podolsky-Rosen steering in Gaussian cluster states. *Phys. Rev. Lett.* **118**, 230501 (2017).
28. Wang, M. H. et al. Deterministic distribution of multipartite entanglement and steering in a quantum network by separable states. *Phys. Rev. Lett.* **125**, 260506 (2020).
29. Cai, Y., Xiang, Y., Liu, Y., He, Q. Y. & Treps, N. Versatile multipartite Einstein-Podolsky-Rosen steering via a quantum frequency comb. *Phys. Rev. Res.* **2**, 032046 (R) (2020).
30. Cavalcanti, D. et al. Detection of entanglement in asymmetric quantum networks and multipartite quantum steering. *Nat. Commun.* **6**, 7941 (2015).
31. Li, C.-M. et al. Genuine high-order Einstein-Podolsky-Rosen steering. *Phys. Rev. Lett.* **115**, 010402 (2015).
32. Lu, H. et al. Counting classical nodes in quantum networks. *Phys. Rev. Lett.* **124**, 180503 (2020).
33. Kunkel, P. et al. Spatially distributed multipartite entanglement enables EPR steering of atomic clouds. *Science* **360**, 413–416 (2018).
34. Reid, M. D. Monogamy inequalities for the Einstein-Podolsky-Rosen paradox and quantum steering. *Phys. Rev. A* **88**, 062108 (2013).
35. S-W, J., Kim, M. S. & Nha, H. Quantum steering of multimode Gaussian states by Gaussian measurements: monogamy relations and the Peres conjecture. *J. Phys. A: Math. Theor.* **48**, 135301 (2015).
36. Xiang, Y., Kogias, I., Adesso, G. & He, Q. Y. Multipartite Gaussian steering: monogamy constraints and quantum cryptography applications. *Phys. Rev. A* **95**, 010101(R) (2017).
37. Adesso, G. & Simon, R. Strong subadditivity for log-determinant of covariance matrices and its applications. *J. Phys. A: Math. Theor.* **49**, 34LT02 (2016).
38. Lami, L., Hirche, C., Adesso, G. & Winter, A. Schur complement inequalities for covariance matrices and monogamy of quantum correlations. *Phys. Rev. Lett.* **117**, 220502 (2016).
39. Cheng, S., Milne, A., Hall, M. J. W. & Wiseman, H. M. Volume monogamy of quantum steering ellipsoids for multiqubit systems. *Phys. Rev. A* **94**, 042105 (2016).
40. Zhang, C. et al. Experimental validation of quantum steering ellipsoids and tests of volume monogamy relations. *Phys. Rev. Lett.* **122**, 070402 (2019).
41. Kenfack, A. & Życzkowski, K. Negativity of the Wigner function as an indicator of non-classicality. *J. Opt. B: Quantum Semiclass. Opt.* **6**, 396–404 (2004).
42. Kogias, I., Lee, A. R., Ragy, S. & Adesso, G. Quantification of Gaussian quantum steering. *Phys. Rev. Lett.* **114**, 060403 (2015).
43. Parigi, V., Zavatta, A., Kim, M. & Bellini, M. Probing quantum commutation rules by addition and subtraction of single photons to/from a light field. *Science* **317**, 1890–1893 (2007).

44. Khoury, G., Eisenberg, H. S., Fonseca, E. J. S. & Bouwmeester, D. Nonlinear Interferometry via Fock-State Projection. *Phys. Rev. Lett.* **96**, 203601 (2006).
45. Su, D., Myers, C. R. & Sabapathy, K. K. Conversion of Gaussian states to non-Gaussian states using photon number-resolving detectors. *Phys. Rev. A* **100**, 052301 (2019).
46. Duan, L. M., Giedke, G., Cirac, J. I. & Zoller, P. Inseparability criterion for continuous variable systems. *Phys. Rev. Lett.* **84**, 2722 (2000).
47. Adesso, G. & Illuminati, F. Entanglement in continuous-variable systems: recent advances and current perspectives. *J. Phys. A: Math. Theor.* **40**, 7821–7880 (2007).
48. Händchen, V. et al. Observation of one-way Einstein-Podolsky-Rosen steering. *Nat. Photonics* **6**, 596–599 (2012).
49. Walschaers, M., Ra, Y.-S. & Treps, N. Mode-dependent-loss model for multimode photon-subtracted states. *Phys. Rev. A* **100**, 023828 (2019).
50. Namekata, N. et al. Non-Gaussian operation based on photon subtraction using a photon-number-resolving detector at a telecommunications wavelength. *Nat. Photonics* **4**, 655–660 (2010).
51. He, Q. Y., Gong, Q. H. & Reid, M. D. Classifying directional Gaussian entanglement, Einstein-Podolsky-Rosen steering, and discord. *Phys. Rev. Lett.* **114**, 060402 (2015).
52. Xiang, Y. et al. Investigating Einstein-Podolsky-Rosen steering of continuous-variable bipartite states by non-Gaussian pseudospin measurements. *Phys. Rev. A* **96**, 042326 (2017).
53. Royer, A. Wigner function as the expectation value of a parity operator. *Phys. Rev. A* **15**, 449 (1977).
54. Walschaers, M. Non-Gaussian quantum states and where to find them. *PRX Quantum* **2**, 030204 (2021).

ACKNOWLEDGEMENTS

M.W. acknowledges enlightening discussions with C. Dross regarding Section “Distillation of nonlocal Wigner negativity”. This work is supported by the National Natural Science Foundation of China (Grants No. 12004011, No. 11975026, and No. 12125402), the National Key R&D Program of China (Grant No. 2019YFA0308702). Q. H. also thanks partial support from Beijing Natural Science Foundation (Grant No. Z190005) and the Key R&D Program of Guangdong Province (Grant No. 2018B030329001). N.T. and M.W. received funding from the European Union’s Horizon 2020 research and innovation program under grant agreement No 899587.

AUTHOR CONTRIBUTIONS

Y.X. and S.L. contributed equally to this work. All authors contributed to the research and the preparation of the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

Correspondence and requests for materials should be addressed to Qiongyi He.

Reprints and permission information is available at http://www.nature.com/reprints

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.