

# Weak-value amplification of the nonlinear effect of a single photon

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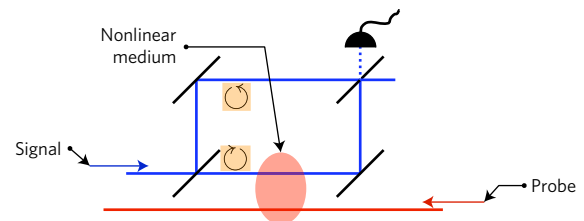
**In quantum mechanics, the concept of weak measurements allows for the description of a quantum system both in terms of the initial preparation and the final state (post-selection)<sup>1</sup>. This paradigm has been extensively studied theoretically and experimentally, but almost all of weak-measurement experiments carried out to date can be understood in terms of the classical (electromagnetic wave) theory of optics. Here, we present a quantum version in which the measurement apparatus deterministically entangles two distinct optical beams. We show that a single photon, when properly post-selected, can have an effect equal to that of eight photons: that is, in a system where a single photon has been calibrated to write a nonlinear phase shift of  $\phi_0$  on a probe beam, we measure phase shifts as large as  $8\phi_0$  for appropriately post-selected single photons. This opens up a new regime for the study of entanglement of optical beams, as well as further investigations of the power of weak-value amplification for the measurement of small quantities.**

Measurement of a property of a system generally proceeds by coupling the system to a probe in such a way that the change of state of the probe depends on the value of this property. For example, a galvanometer is constructed so that its needle deflects by an amount proportional to the potential difference across the system being studied. A subsequent observation of the final state of the probe provides information (often incomplete) about the value of the observable. In quantum mechanics, this information is gained at the price of disturbing the system through the interaction. There is a strict trade-off between the minimum disturbance and the amount of information which can be gained<sup>2–4</sup>. In weak measurement, the disturbance to the system is reduced at the cost of a similar reduction in the amount of information provided by the measurement. This minimal disturbance makes it reasonable to consider conditioning the read-out of the probe on finding the system in a particular final state after the interaction (post-selection). In this case, the pointer shift, averaged over many measurement repetitions, has been shown<sup>1</sup> to have a magnitude which would correspond to what is termed the ‘weak value’ of the observable:  $\langle f | \hat{A} | i \rangle / \langle f | i \rangle$ , where  $\hat{A}$  is the observable, and  $|i\rangle$  and  $|f\rangle$  are the pre- and post-selected states of the system, respectively. Evidently, the weak value depends equally on both pre-selected (initial) and post-selected (final) states. This feature of weak measurement makes it a powerful tool for exploring fundamental questions in quantum mechanics<sup>3,5–16</sup>, specifically the properties of post-selected subensembles ranging from particles transmitted through tunnel barriers to measurement-based quantum-computing systems<sup>17–19</sup>.

Strangely, the weak value is not constrained to be within the eigenvalue spectrum of the observable  $\hat{A}$ , and is not even in general a real number. In particular, as the overlap between the initial

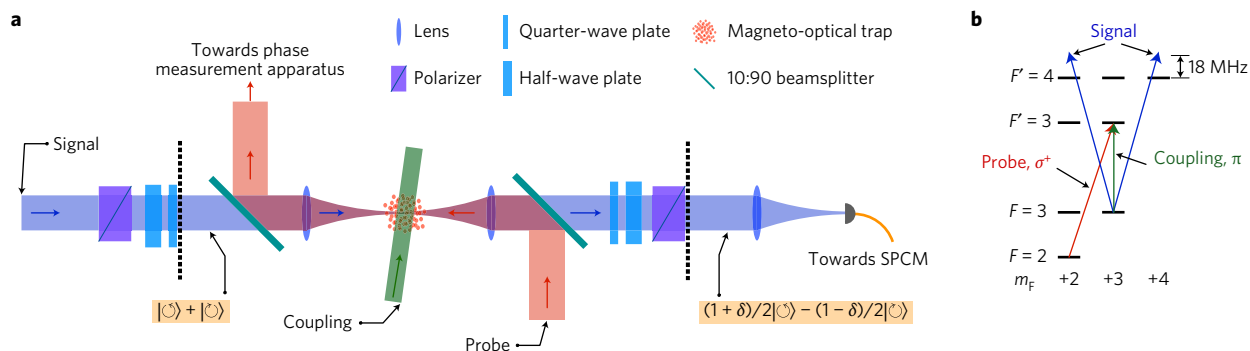
and final states becomes very small,  $\langle f | i \rangle \rightarrow 0$ , the weak value can become (almost) arbitrarily large (as long as the post-selection success is dominated by the overlap of pre- and post-selected states and not the measurement back-action; see the Supplementary Information.) Indeed, the founding paper of the field<sup>1</sup> appeared under the unwieldy but provocative title ‘How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100’. This has led to the idea of using ‘weak-value amplification’ (WVA) to improve the detection or measurement of small effects<sup>20–26</sup>. Interest in this application of weak measurement has grown in the past few years alongside an ongoing debate on the usefulness of WVA<sup>27–31</sup>. Even the quantum mechanical nature of WVA has been challenged<sup>32</sup>, and attempts have been made to describe the effect classically based on measurement disturbance. In 2011, we proposed that WVA of the small optical nonlinearity at the single-photon level<sup>33</sup> was possible and could, under some conditions, improve the signal-to-noise ratio. Here, we present an experiment implementing this idea, asking a question directly analogous to that of the original Aharonov, Albert and Vaidman paper: In a two-arm interferometer containing one photon in total, can the result of a measurement of the photon number in one arm turn out to be greater than 1? We find the answer is yes.

Anomalous weak values observed to date<sup>20,21</sup> have typically utilized two different degrees of freedom (such as polarization and propagation direction) of a photon as the ‘system’ and the ‘probe’, obviating the need for any inter-photon interaction; the effects can thus be explained perfectly in terms of linear optics, without resorting to quantum theory. There have been two exceptions. In one, a probabilistic quantum logic gate was implemented, so that although there was no deterministic entanglement of system and probe, an additional post-selection step projected the system onto an entangled state some fraction of the time<sup>14,34</sup>. In the other, deterministic WVA was implemented in a transmon



**Figure 1 | Conceptual schematic of the interferometer.** The signal beam is split into two paths, labelled  $\oplus$  and  $\ominus$  (to make a connection with the actual, polarization-based, interferometer shown in Fig. 2). A probe beam measures the number of photons in one path through a nonlinear interaction. The interferometer is made slightly imbalanced so that there is a small chance for a signal photon to be detected in the nearly dark port.

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**Figure 2 | Experimental setup and level scheme. a**, The experimental set-up and our implementation of the polarization interferometer.

Counter-propagating probe and signal beams are focused to a waist of  $13 \pm 1 \mu\text{m}$  inside a cloud of laser-cooled  $^{85}\text{Rb}$  atoms confined in a magneto-optical trap (MOT). 90%-reflectivity beamsplitters are used to overlap the probe beam with and separate it from the signal beam. The phase of the probe is then measured using frequency-domain interferometry. A collimated coupling beam, propagating perpendicularly to the probe and signal beams, generates EIT for the probe. The polarization of the signal beam is set to be linear before the interaction—that is, an equal superposition of right-handed and left-handed circular polarizations  $|\odot\rangle + |\ominus\rangle$ . After the interaction the polarization of the signal beam is post-selected to be in the final state  $(1 + \delta)/\sqrt{2}|\odot\rangle - (1 - \delta)/\sqrt{2}|\ominus\rangle$ , where  $\delta$  denotes the overlap between the two polarization states. A single-photon counting module (SPCM) is used to detect the presence of a photon in the dark port. The vertical dashed lines mark the beginning and the end of the interferometer. **b**, Atomic level structure used for the experiment. Only the levels with the highest interaction strengths are shown. See the Supplementary Information for the full level structure.

qubit system<sup>35</sup>. Here, we present the first observation of WVA via deterministic entanglement of two distinct optical systems, amplifying the number of photons in a signal beam by measuring the nonlinear phase shift it writes on a separate probe beam. From the perspective of the development of quantum nonlinear optics, this is exciting evidence for, and an application of, the deterministic entanglement between two independent light beams at the quantum level, which can now be generated thanks to modern techniques for enhancing nonlinearities. By contrast with the transmon work, in which entanglement occurs between two stationary qubits on a single chip, our observations rely on entanglement between two distinct light beams propagating in opposite directions, which are eventually measured on two different detectors which are about a metre apart, and could in principle be arbitrarily distant from each other.

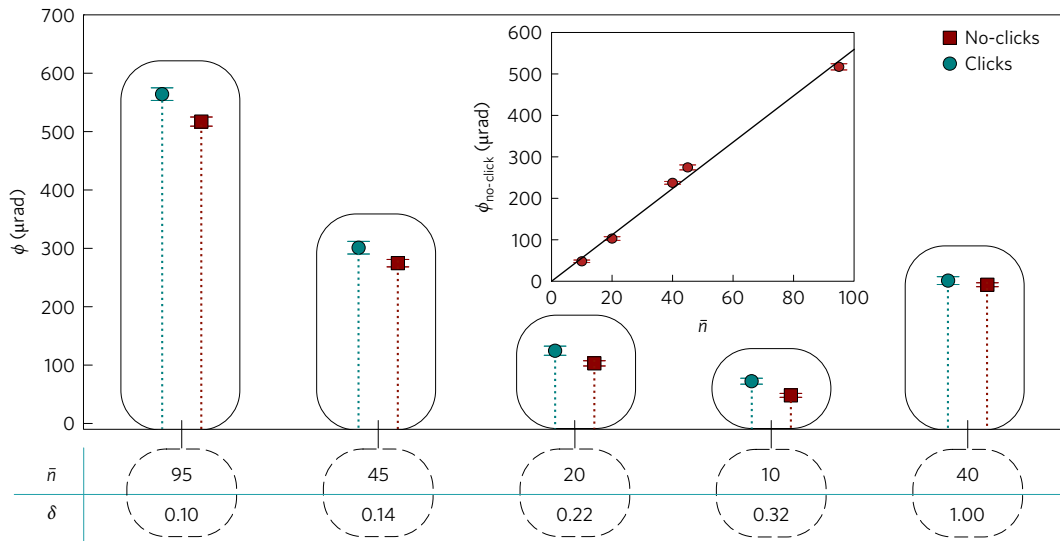
In 2011, we proposed that WVA could be used to amplify the (weak) nonlinear effect of photons in a signal pulse on a probe beam. The scheme begins by splitting the signal beam into two paths (Fig. 1), which are later interferometrically recombined such that there is strong constructive interference at one port, and very few photons exiting the other, ‘nearly dark’, port. By post-selecting on cases where a photon exits this nearly dark port, one can ‘amplify’ the weak value of the photon number in one arm of the interferometer. The physical content of this statement is that if an optical probe beam interacts, through a Kerr-type nonlinear medium, with the light in that arm of the interferometer, it will experience a phase shift proportional to the weak value of the photon number, which may be much larger than the nonlinear phase shift expected for a single signal photon—even if there was only ever a single signal photon present in the entire interferometer.

In our experiment, the nonlinear interaction between the signal and probe beams is mediated by a sample of laser-cooled  $^{85}\text{Rb}$  atoms in a magneto-optical trap (MOT). A coupling beam is used to set up electromagnetically induced transparency (EIT) for the probe beam. The frequencies of the probe and coupling beams are set so that each of them is individually on resonance with its corresponding atomic level (Fig. 2b). The EIT resonantly enhances the nonlinear interaction between the signal and the probe beams, while simultaneously minimizing the probe absorption. As a signal pulse passes through the medium, it alters the optical properties (the index of refraction) of the medium as seen by the probe beam. As a result of this change in the index of refraction, the probe picks up a phase shift relative to what the phase would have been

in the absence of the signal pulse. This cross phase shift (XPS) depends linearly on the number of photons (intensity) of the signal pulse<sup>36</sup>. This interaction thus constitutes a measurement of signal photon number, with the phase of the probe beam acting as a ‘pointer’. The typical phase shift per photon is on the order of  $10^{-5}$  rad (ref. 36), much smaller than the quantum uncertainty in our probe phase, meaning that a single measurement cannot provide enough information to determine the photon number to better accuracy than its initial uncertainty, which is the sense in which this measurement is ‘weak’.

The geometry and polarizations of the coupling and probe beams (Fig. 2) are chosen such that the strength of this nonlinear interaction also depends on the polarization of the signal beam, with the interaction being strongest for right-handed circular ( $\sigma^+$ ) polarization and weakest for left-handed circular ( $\sigma^-$ ) polarization of the signal; see Supplementary Information. As mentioned before, an interferometer is necessary to implement WVA of signal photon number. We chose this configuration so as to use the polarization dependence of XPS to build an interferometer for the signal beam, in which the two ‘paths’ of the interferometer are in fact two polarizations, to one of which the probe beam is much strongly coupled than to the other (Fig. 2). The incident signal photons are linearly polarized—that is, in an equal superposition of  $\sigma^+$  and  $\sigma^-$ . After they interact with the probe beam, we use a waveplate and a polarizer to transmit photons with a nearly orthogonal polarization (corresponding to the nearly dark interferometer port, Fig. 1), which subsequently impinge on a single-photon counting module (SPCM). When this detector fires, constituting a successful post-selection, a large weak value of photon number (and hence large nonlinear phase shift) is expected; for each signal pulse, we record whether or not the detector fires, to measure the average nonlinear phase shift written on the probe separately for the cases of successful post-selection (which we term ‘click’) and the cases of failed post-selection (‘no-click’).

Although our original proposal concerned a signal pulse that contained a single-photon Fock state<sup>33</sup>, a similar effect can be observed even if one uses a coherent state, with an average photon number larger than one. In that proposal we showed that the weak value of photon number in one arm of the interferometer, when the photon is post-selected to be in the nearly dark port, is given by  $\langle n \rangle_{\text{wk}} = 1/2 + 1/2\delta$ . The parameter  $\delta$  is defined as the overlap between the initial and final state of the photon, ranging from 0 (completely dark port) to 1 (bright port). However, here  $\delta$  is assumed



**Figure 3 | Measured XPS for click events and no-click events.** Red squares and green circles are the measured phase shift conditioned on the SPCM not firing (no-click) and firing (click), respectively. The horizontal axis shows the mean signal photon number  $\bar{n}$  and the post-selection parameter  $\delta$  used for each case. The inset plots the measured phase shift for the no-click cases versus the average photon number in the signal beam  $\bar{n}$ . The linear fit reveals a per-photon XPS of  $\phi_0 = 5.59 \pm 0.02 \mu\text{rad}$ . The error bars represent the standard deviation of the mean as described in the Methods.

to be very small, which corresponds to a nearly dark port. In another work<sup>36</sup>, we showed that, in the limit of low detection efficiency, the (mean) inferred photon number in a coherent state containing  $\bar{n}$  photons, when conditioned on a successful photon detection, increases by one photon: the inferred photon number after a successful post-selection,  $n_{\text{click}}$ , turns out to be one more than the inferred photon number in the absence of a click,  $n_{\text{no-click}}$ . Now, if one combines these ideas, sending a coherent state in the interferometer instead of a single-photon Fock state and conditioning the photon number measurement of one arm on detection of a photon in the nearly dark port, the added photon due to the photon detection will undergo weak-value amplification. Using B and D to denote bright and nearly dark ports, one can approximate the initial signal coherent state  $|i\rangle \approx |\alpha\rangle_B |\alpha\delta\rangle_D \approx |\alpha\rangle_B (|0\rangle + \alpha\delta|1\rangle)_D$ , and the final state post-selected when the detector fires as  $|\alpha\rangle_B |1\rangle_D$  (occurring with probability  $|\alpha|^2|\delta|^2$ , multiplied by the experimental collection and detector efficiency.) One can then calculate the weak value of the number of photons in each arm of the interferometer,  $\langle n_{\pm} \rangle_{\text{wk}}$ , with ‘plus’ and ‘minus’ signs denoting the  $\sigma^+$  and  $\sigma^-$  polarizations, by

$$\langle n_{\pm} \rangle_{\text{wk}} = \frac{\langle f | \hat{n}_{\pm} | i \rangle}{\langle f | i \rangle} \quad (1)$$

The number operators  $\hat{n}_{\pm}$  can be written  $a_{\pm}^{\dagger} a_{\pm}$ , where the field operators  $a_{\pm} \approx (a_B \pm a_D) / \sqrt{2}$ . The numerator can thus be written

$$\frac{1}{2} \langle \alpha |_B \langle 1 |_D [a_{\pm}^{\dagger} \pm a_{\pm}^{\dagger}] \cdot [a_B \pm a_D] \{ |0\rangle_D + \alpha\delta |1\rangle_D \} | \alpha \rangle_B = \frac{1}{2} [ \langle \alpha |_B \langle 1 |_D \alpha^* \pm \langle \alpha |_B \langle 0 |_D ] \cdot [ (\alpha \pm \alpha\delta) | \alpha \rangle_B | 0 \rangle_D + \alpha^2 \delta | \alpha \rangle_B | 1 \rangle_D ] \quad (2)$$

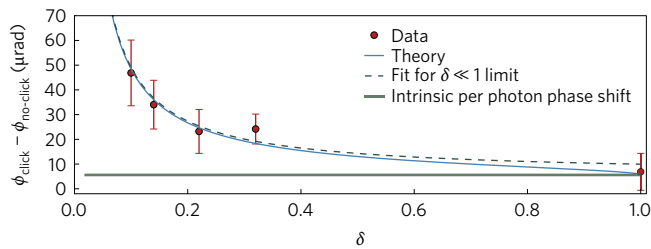
As the denominator simply evaluates to  $\delta$ , we find

$$\langle n_{\pm} \rangle_{\text{wk}} = \frac{1}{2} \frac{\alpha |\alpha|^2 \delta + \alpha \delta \pm \alpha}{\alpha \delta} = \frac{1}{2} (|\alpha|^2 + 1 \pm 1/\delta) \quad (3)$$

(See the Supplementary Information for a more detailed derivation of equation (3).) It is instructive to note that the total number of photons inside the interferometer is  $\langle n_{+} \rangle_{\text{wk}} + \langle n_{-} \rangle_{\text{wk}} = \bar{n} + 1$ , where  $\bar{n}$  is the average number of photons sent into the interferometer, and  $\bar{n} + 1$  is the revised estimate of the mean photon number

based on detection of one ‘additional’ photon in the nearly dark port. What equation (3) demonstrates is that this added photon (‘the’ photon which causes the SPCM in the nearly dark port to fire, so to speak) undergoes WVA, giving rise to the term  $1/2\delta$ . This occurs even in the presence of a ‘background’ of  $\bar{n}$  non-post-selected photons. The firing of the detector alerts us to the fact that a single photon reached the post-selection port, and the effect of that photon on the probe is consequently amplified; on the other hand, it provides no additional information about the behaviour of the other  $n$  photons, each of which therefore exhibits a 50% probability of being in the correct arm to affect the probe beam. This 50% is determined entirely by the preparation, as the undetected photons undergo no post-selection, and their effect is not amplified. In this context, when the probe beam interacts with the one arm of the interferometer, the nonlinear phase shift written on it will be proportional to the weak value of photon number in that arm. Due to the spatial overlap between the probe beam and the two arms, which are the two circular polarizations of the signal beam, the total XPS written on the probe is  $\phi = \langle n_{+} \rangle_{\text{wk}} \phi_{+} + \langle n_{-} \rangle_{\text{wk}} \phi_{-}$ , where  $\phi_{\pm}$  are the per-photon phase shifts for the corresponding signal polarizations. On detection of a photon in the nearly dark port (a click event), these are to be replaced by the weak values from equation (3), leading to  $\phi_{\text{click}} = (\bar{n} + 1)(\phi_{+} + \phi_{-})/2 + (1/\delta)(\phi_{+} - \phi_{-})/2$ . The amplification shows up in the second term, which is proportional to the difference between the XPS for the two polarizations. In cases when the detector fails to detect a photon (no-click events), there is no ‘added photon’ and the weak value of photon number in each arm is simply  $\bar{n}/2$ , corresponding to an XPS of  $\phi_{\text{no-click}} = \bar{n}(\phi_{+} + \phi_{-})/2$ .

Figure 3 shows the measured XPS for the click and no-click events versus a range of values of post-selection parameter  $\delta$ ; for each  $\delta$ , the mean input photon number  $\bar{n}$  and the overall detection efficiency  $\eta$  are adjusted so as to keep the probability of photon detection ( $P_{\text{click}} = \eta \delta^2 \bar{n}$ ) low, which is the necessary condition for a photon detection to add one photon to the inferred photon number<sup>36</sup> (for more technical details see the Supplementary Information.) The measured XPS for the click events is manifestly always larger than the measured XPS for the no-click cases. It is worth noting that had we not utilized post-selection to separate the click from no-click events, the expected XPS of the probe would have been  $\phi = \bar{n}(\phi_{+} + \phi_{-})/2$ , which is the same as  $\phi_{\text{no-click}}$  when the overall detection efficiency is low. Therefore, we can define



**Figure 4 | Differential phase shift versus post-selection parameter.** Red circles are the phase shift difference between click and no-click cases. The black dashed line is a fit to  $(\phi_+ + \phi_-)/2 + (\phi_+ - \phi_-)/2\delta$ , where the measured value of  $5.59 \mu\text{rad}$  (shown as solid green line) is used for  $(\phi_+ + \phi_-)/2$  and  $\phi_+ - \phi_-$  is left as a fitting parameter. The best fit corresponds to a value of  $8.7 \pm 0.6 \mu\text{rad}$  for  $\phi_+ - \phi_-$ . The error bars represent the standard deviation of the mean as described in the Methods. The fit is valid only for  $\delta \ll 1$ . The blue solid line is the full theoretical calculation assuming the numbers mentioned above.

$\phi_0 = (\phi_+ + \phi_-)/2$  as the expected per-photon phase shift for this experiment. The inset of Fig. 3 plots  $\phi_{\text{no-click}}$  versus the input photon number. From a linear fit to this data, the per-photon phase shift  $\phi_0$  is measured to be  $5.59 \pm 0.02 \mu\text{rad}$ .

To directly observe the WVA of the added photon due to photon detection, we plot  $\phi_{\text{click}} - \phi_{\text{no-click}}$  versus the post-selection parameter  $\delta$  in Fig. 4. It is easy to see that this quantity, which we term ‘the differential phase shift’, is independent of  $\bar{n}$ , and contains only the XPS of the added photon and its amplified effect:  $\phi_{\text{click}} - \phi_{\text{no-click}} = (\phi_+ + \phi_-)/2 + (1/\delta)(\phi_+ - \phi_-)/2$ . The plot clearly shows that, as  $\delta$  becomes smaller, the effect of the post-selected single photon becomes larger. For  $\delta = 0.1$ , the smallest post-selection parameter used in our experiment, we measure a differential phase shift of  $47.0 \pm 13.5 \mu\text{rad}$ , which is  $8.4 \pm 2.4$  times larger than the per-photon phase shift  $\phi_0$ . Hence, a single post-selected photon can act like eight photons. In an ideal case, where only the photon number in one arm of the interferometer is measured,  $\delta = 0.1$  corresponds to an amplification factor of ten. For the points with  $\delta = 0.14$  and  $\delta = 1$ , we used nearly the same mean photon number (45 and 40, respectively), so that the results would be directly comparable even without knowledge of the independence of the differential phase shift on  $\bar{n}$ . For the  $\delta = 1$  case, we observed  $6.7 \pm 7.5 \mu\text{rad}$ , consistent with the unamplified value of  $5.59 \pm 0.02 \mu\text{rad}$  expected in the absence of WVA, while for the  $\delta = 0.14$  case we found  $34 \pm 10 \mu\text{rad}$ , three standard deviations above  $\phi_0$ . From a fit to the data in Fig. 4 we estimate  $\phi_+ - \phi_-$  to be  $8.7 \pm 0.6 \mu\text{rad}$ .

Given the extensive discussion in recent years over the possible merits of WVA for making sensitive measurements of small parameters, it is interesting to contrast the present experiment with an earlier one, in which we measured the nonlinear phase shift due to post-selected single photons, but without any weak-value amplification<sup>36</sup>. In our previous experiment, a total of approximately 1 billion trials (300 million events with post-selected photons, and 700 million without) were used to measure the XPS due to  $\sigma^+$ -polarized photons. By looking at the difference between the XPS measured for ‘click’ and ‘no-click’ events, we measured peak XPS  $\phi_+$  of  $18 \pm 4 \mu\text{rad}$ . In this experiment, where we use the WVA technique, we used a total of around 830 million trials (200 million successful post-selections) to extract an average XPS  $\phi_+$  of  $10.0 \pm 0.6 \mu\text{rad}$  (for more information regarding the reported average XPS see the section entitled ‘Probe phase measurement’ in the Supplementary Information). Note that this number agrees well with our classical calibration of the peak XPS of  $13.0 \pm 1.5 \mu\text{rad}$ <sup>36</sup>. It is evident that the WVA technique yielded a better signal-to-noise ratio (SNR). This may seem surprising at first, given that under statistical-noise conditions, WVA is known to have the same SNR as a brute-force measurement<sup>33</sup>; this is because the amplification of

the signal only comes at the price of a post-selection which reduces the size of the data set just enough to cancel out any advantage one might have hoped for. In our case, however, while the differential phase shift grows as  $1/\delta$ , the size of the post-selected data set is determined by  $\delta^2 \bar{n}$ . Hence, we were able to maintain a substantial set of post-selections even for small  $\delta$  simply by adjusting  $\bar{n}$  accordingly. This allowed us to amplify the per-event signal while keeping the number of events high, thereby achieving a precise measurement of the differential phase shift. Furthermore, in the no-click cases, the large photon number provided an excellent determination of  $\phi_0$ . Therefore, we were able to estimate the value of  $\phi_+$  more precisely with fewer trials. It is essential to note that the WVA technique in this experiment is advantageous only if one considers the number of trials (and not the number of photons) as the measurement resource.

Future work will address the ongoing discussions about the advantages of WVA over traditional strong measurement. To tackle this question, future iterations of the experiment can be carried out with artificial noise added, conforming to a variety of noise models. It is expected that in cases where the noise has long time correlations, WVA will prove superior to standard measurement for the estimation of small parameters such as the intrinsic per-photon phase shift or  $\chi^{(3)}$  of a sample.

## Methods

Methods, including statements of data availability and any associated accession codes and references, are available in the [online version of this paper](#).

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### Author contributions

All authors contributed equally to the results, interpretation and presentation of this Letter.

### Additional information

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### Competing financial interests

The authors declare no competing financial interests.

## Methods

**Atom preparation.** A cloud of  $^{85}\text{Rb}$  atoms is prepared in a magneto-optical trap (MOT). Three pairs of beams are used for cooling while a magnetic gradient of approximately  $20\text{ G cm}^{-1}$  (along the quadrupole axis) provides confinement in space. Each beam contains a trapping beam, tuned 20 MHz to the red of the  $F = 3 \rightarrow F' = 4$  transition, and a repumper, which is tuned close to the  $F = 2 \rightarrow F' = 3$  resonance. Each measurement cycle lasts for 52 ms. For the first 50 ms, the MOT beams and magnetic field gradient cool and prepare the cloud. They are then turned off and the atoms are probed for 1.5 ms with a  $500\text{ }\mu\text{s}$  gap between cooling and probing to ensure the absence of residual magnetic field gradients.

**Probe phase measurement.** To measure the phase and amplitude of the probe beam, we use frequency-domain interferometry. The probe beam is comprised of two frequency components that co-propagate through the interaction region. One frequency component is tuned to resonance with  $F = 2 \rightarrow F' = 3$ . A second frequency component is prepared by frequency-modulating an acousto-optic modulator (AOM) at 100 MHz, and is used as a probe reference. The beating of these two components is detected on a fast avalanche photodiode. Any change in the phase and/or amplitude of the probe beam results in a change in phase and/or amplitude of this 100 MHz beating signal. We use an IQ-demodulator to then read off the phase ( $\arctan(I/Q)$ ) and amplitude ( $\sqrt{I^2 + Q^2}$ ) of the beat signal. The resulting phase and amplitude are digitized with a sampling period of  $200/3\text{ ns}$  (sampling frequency of 15 MHz). Each measurement cycle contains 22,501 samples (corresponding to 1.5 ms of phase measurement), which we call a trace. The first 4,500 samples ( $300\text{ }\mu\text{s}$ ) in each trace are ignored to avoid contamination from the residual probe phase dynamics. The last 180 samples are used to measure the final OD of the atomic cloud (by turning the coupling beam off, hence eliminating the EIT, and measuring the probe absorption). The remaining 1.188 ms is divided into 495 shots,  $2.4\text{ }\mu\text{s}$  (36 samples) each. Each shot contains a nonlinear phase shift due to the interaction between the probe and the signal pulse. The nonlinear phase shift has a full-width at half-maximum width of around 500 ns (ref. 37). We average the value of this phase shift for seven samples (466.6 ns) in each shot. To eliminate slow drifts in our phase measurement, the phase of the probe is averaged for three samples (200 ns) with a 266 ns gap before and after the seven samples. This value is then subtracted from the average of the probe phase in the interval when nonlinear phase shift is expected. The resulting number represents the average phase shift due to the signal pulses, and is reported as the measured XPS. It is worth noting that the reported values for XPS in this report underestimate the peak phase shift because of the averaging and background subtraction. The probe beam contains about 2,000 photons, which corresponds to a 11 mrad quantum limit on the phase uncertainty (shot-noise). We determine our phase uncertainty experimentally by calculating the standard deviation and the standard deviation of the mean from our data sets. The standard deviation is about  $100\text{ }\mu\text{rad}$ , which is about ten times larger than the quantum limit on the phase uncertainty. To measure the phase shift down to a few  $\mu\text{rad}$  precision, we repeat the measurement about half a billion times with around 200 shots in each 1.5 ms measurement window. The final reported uncertainty is the standard deviation of the mean of this full data set.

**Probe and coupling fields.** Electromagnetically induced transparency (EIT) is a coherent effect in which destructive interference prevents the two laser beams from being absorbed by the atoms. To generate EIT, the probe and coupling lasers should be phase-locked. To generate the probe beam, some power is first extracted from a master laser beam that is locked approximately 30 MHz red of the  $F = 2 \rightarrow F' = 3$  transition. By using an AOM that is driven at +130 MHz (double-passing at +65 MHz), the off-resonance component of the probe beam is generated. This off-resonance beam is then sent through another AOM at  $-100\text{ MHz}$  to generate the on-resonance probe component. The two beams are then combined on a

beam splitter and sent towards the interaction region. We use an electro-optic modulator (EOM), which is driven at around 3 GHz, to frequency modulate the remaining portion of the master laser; this writes frequency sidebands on the laser. The frequency modulated beam is then used to seed an injection-locked diode laser and we lock this diode laser to the first lower sideband. As a result, the second diode laser, the coupling laser, is phase-locked to the master laser. The frequency of the coupling beam is set to be on resonance with the  $F = 3 \rightarrow F' = 3$  transition. We use another AOM (single-pass driven at +103 MHz) to switch the coupling beam on and off. The intensities of the probe and coupling beams are chosen so that the resulting EIT width is 2 MHz. The polarizations of the probe and coupling beams are set to be  $\sigma^+$  and  $\pi$  respectively.

**Signal pulses.** A portion of the injection-locked diode laser, mentioned above, is sent through two AOMs. These AOMs are used to set the frequency of the pulses to be around +18 MHz from the  $F = 3 \rightarrow F' = 4$  transition. One of the AOMs is also used to amplitude-modulate the signal beam to create 40 ns pulses. A neutral density (ND) filter is used to attenuate the signal pulses, preparing pulses with low average photon numbers. A polarizer followed by a half-wave plate and a quarter-wave plate is used to set the polarization of the signal pulse before its interaction with the probe. For all WVA measurements reported here, the signal pulse is initially linearly polarized.

**Signal post-selection.** After its interaction with the probe beam, the signal beam is sent through a half-wave plate, a quarter-wave plate and a Glan–Thompson polarizer. With this combination, the polarization of the signal beam is fully characterized with and without the atoms (by doing tomography). Once the initial polarization of the signal beam is well understood, and any polarization rotations due to the presence of atoms are corrected for, the half- and quarter-wave plates are set so that the signal polarization in the output of the Glan–Thompson polarizer is projected onto a polarization almost orthogonal to the initial polarization with real overlap with the initial polarization. The signal pulses are then collected in a multi-mode fibre and are detected on a SPCM. On detecting a photon, the SPCM sends a signal which triggers our tagging module to expose the probe detector to a 100 ns flash of light. This pulse shows itself as a spike in the probe amplitude and phase. This spike tags the corresponding shot as a successful post-selection. A time delay between when the tag appears in a shot and when the XPS is expected to happen is introduced and carefully adjusted to avoid any incursion of the tags to the measured XPS in that shot. The tags, however, affect the XPS in the next shot and, therefore, the shot after each tag is discarded. We then use these tags, and the absence thereof, to group the shots into successful (click) and unsuccessful (no-click) post-selection bins.

**Background photons.** Any residual photon that hits the SPCM results in a photon detection which will falsely be counted as a successful post-selection. To reduce the chances of getting a false positive, we time-gate the SPCM in the 40 ns windows where we expect the signal pulses to arrive. With this gating, 6% of the measured shots are still falsely tagged as 'click'. These background detections cause deterioration in the desired effect. Therefore, to be less sensitive to these detections, we attempt to operate in regimes where the total detection rate is 20–30%.

**Data availability.** The data that support the plots within this paper and other findings of this study are available from the corresponding author upon request.

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