

TOWARD A TAXONOMY OF THE EDGEWORTH–KUIPER OBJECTS: A MULTIVARIATE APPROACH

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Abstract. The principal component (PC) and G-mode multivariate statistics have been used in analysing the set of the 34 Edgeworth–Kuiper objects (EKOs) – 23 Trans Neptunian Objects (TNO) and 11 Centaurs – for which B, V, R, I, J homogeneous photometry were available. The results obtained show that V-I and V-Js are the key parameters in structuring the sample in homogeneous groups. The PC1 axis (which contains ~93% of the sample total variance) spans five times more than the PC2 (which contains ~6% of the sample total variance). The extremes of the PC1 axis contain the objects having a flat spectrum (low PC1 values) and a very red spectrum respectively. Independently, the G-mode analysis allows us to distinguish six homogeneous groups of objects which confirm and extend the results obtained with the PC analysis. In addition to these groups, a few objects remain not included in any group (i.e., does not have significant similitude with other objects) and yet give an indication of a more complex compositional structure of the sample. These preliminary results will have to be confirmed and completed when a larger sample is available, but they provide some interesting hints for understanding the – mainly collisional – evolution of the EKOs.

1. Introduction

Ten years after the discovery of the first object wandering outside the orbit of Neptune, the number of detected Edgeworth–Kuiper Objects is close to eight hundred: more or less the same number of asteroids that was found in a century after the discovery of 1 Ceres. The available information concerning the broad band photometry of EKOs: B, V, R, I data are available to date for a hundred of objects and for fifty or so of them J data are also available; while only forty or so asteroids had the U-B and B-Vs determined in the sixties, a hundred and seventy years after the discovery of Ceres. Despite this small number of asteroids (~2% of the population known at that time) Wood and Kuiper (1963) suggested, on the base of the observed distribution, the existence of two different compositional classes clustering around the indices of the Moon and of the Sun. These groups were the ancestors of the S and C classes respectively. Tegler and Romanishin (1998) obtained two groups of objects by analysing the B-V and V-Rs of 13 EKOs (~20% of the known EKOs population at that time) one very red and the other quite neutral with respect to the Sun. In the early eighties a few thousand asteroids were known, a large survey (ECAS, Eight Colors Asteroid Survey, Zellner et al., 1985) was devoted to measure in eight filters (range 0.35–1.0 μm) the sunlight reflected by several hundred asteroids (~10% of the known objects). The IRAS data then (Tedesco et al., 1992)



provided albedo data for most of these objects. Tholen (1984) and Barucci et al. (1987) used these data bases in classifying the asteroids by means of independent multivariate statistical techniques (the principal components analysis based on the ECAS data, and the G-mode analysis based on the ECAS data and IRAS albedo data, respectively). Both methods provide a similar classification scheme which constitutes the bulk of the current asteroid taxonomy, separating the asteroid population into a dozen spectrophotometrically homogeneous groups. Barucci et al. (2001) applied the same techniques to a sample of 22 EKO's (~6% of the known EKO's population at that time) characterised by 4 colours (B-V, V-R, V-I and V-J). The results indicate a clear compositional trend within the examined sample and suggest the possible existence of some homogeneous groups.

Nowadays about 800 EKO's have been discovered; three-colour photometry is available for about 80 objects and for 40 of them V-J is also available. In this paper we discuss the results of the multivariate analyses carried out on a subset of 34 EKO's for which homogeneous data (Table I), obtained in the B, V, R, I and J bands by the same observers, during the same run, or inter-calibrated through the V measurement.

2. Multivariate Statistical Analyses

We analysed this highly homogeneous EKO's group (hereafter "*the sample*") using both the Principal Component Analysis (PCA) (Reyment and Joreskog, 1993) and the G-mode analysis (Coradini et al., 1977; Fulchignoni et al., 2000). We refer the reader to the quoted literature for details on these statistics.

In our case, the principal components are linear combinations of the original variables (B-V, V-R, V-I and V-J) whose coefficients reflect the relative importance of each variable (colour) within each principal component. These coefficients are the eigenvectors of the variance-covariance matrix of the colours. The sum of the eigenvalues of this matrix (which is equal to its trace) accounts for the total variance of the sample. Each eigenvalue reflects the percentage of the total variance contributed by each principal component. The eigenvectors, the percentages of total variance contributed by each eigenvector and the eigenvalues of the variance-covariance matrix of the sample are reported in Table II.

The first eigenvector accounts for most of the sample variance (92.93%) and the larger contribution comes from V-J (46%). The second eigenvector adds only 6.08% of the total variance and is weighted in quite equal measure by V-I (33%) and V-J (30%). The first and the second principal component account for more than 99% of the total variance, therefore the PC1 vs. PC2 plane contains practically all the information on the variance of the variables characterising the sample. It is possible to infer from this result that the degree of reddening is the main distinctive character of the EKO population. In Figure 1 the EKO sample is plotted in the PC1-PC2 plane. The predominance of PC1 (i.e., of V-J colour) in characterising

TABLE I
Colours of the 34 EKO and Centaurs analysed.

| Object/type* | B-V | V-R | V-I | V-J | References |
|--------------------|-------------|-------------|-------------|-------------|----------------------------|
| Sun | 0.67 | 0.36 | 0.68 | 1.08 | 1, 2 |
| 1996TQ66/R | 1.18 ± 0.11 | 0.65 ± 0.08 | 1.44 ± 0.14 | 2.43 ± 0.12 | 6, 7, 15, 10, 20 |
| 1996TS66/CI | 1.01 ± 0.08 | 0.63 ± 0.11 | 1.19 ± 0.12 | 1.82 ± 0.17 | 5, 6, 7, 10, 20 |
| 1997CS29/CI | 1.05 ± 0.08 | 0.67 ± 0.05 | 1.27 ± 0.07 | 2.07 ± 0.13 | 6, 7, 10, 12, 20, 22, 23 |
| 1998HK151/R | 0.64 ± 0.12 | 0.49 ± 0.05 | 0.87 ± 0.06 | 1.57 ± 0.09 | 16, 17, 22, 24 |
| 1998WU24/? | 0.78 ± 0.03 | 0.53 ± 0.04 | 0.99 ± 0.03 | 1.67 ± 0.04 | 41 |
| 1999CD158/CI | 0.86 ± 0.05 | 0.52 ± 0.05 | 1.09 ± 0.07 | 1.81 ± 0.08 | 12, 17 |
| 2060 Chiron/C | 0.68 ± 0.04 | 0.36 ± 0.02 | 0.73 ± 0.04 | 1.20 ± 0.11 | 30, 31, 32 |
| 5145 Pholus/C | 1.30 ± 0.09 | 0.79 ± 0.03 | 1.58 ± 0.05 | 2.61 ± 0.04 | 31,33, 34, 35 |
| 7066 Nessus/C | 1.09 ± 0.04 | 0.79 ± 0.04 | 1.50 ± 0.07 | 2.29 ± 0.04 | 3, 6, 14, 31 |
| 8405 Asbolus/C | 0.75 ± 0.04 | 0.51 ± 0.06 | 0.99 ± 0.06 | 1.66 ± 0.06 | 9, 14, 31, 36, 37, 38 |
| 10199 Chariklo/C | 0.80 ± 0.05 | 0.48 ± 0.02 | 1.01 ± 0.03 | 1.74 ± 0.02 | 6, 31, 39, 40 |
| 10370 Hylonome/C | 0.67 ± 0.08 | 0.44 ± 0.06 | 0.91 ± 0.09 | 1.31 ± 0.10 | 3, 6, 17, 20, 31 |
| 15789 1993SC/R | 1.01 ± 0.10 | 0.67 ± 0.06 | 1.40 ± 0.09 | 2.43 ± 0.15 | 3, 4, 5, 6, 7, 8, 9, 10 |
| 15820 1994TB/R | 1.08 ± 0.13 | 0.71 ± 0.08 | 1.45 ± 0.12 | 2.48 ± 0.15 | 3, 7, 8, 9, 10, 11, 12, 13 |
| 15874 1996TL66/S | 0.69 ± 0.05 | 0.33 ± 0.05 | 0.69 ± 0.05 | 1.45 ± 0.11 | 5, 6, 7, 10, 13, 20 |
| 15875 1996TP66/CI | 0.98 ± 0.10 | 0.65 ± 0.07 | 1.33 ± 0.09 | 2.31 ± 0.06 | 5, 6, 7, 10, 13, 22 |
| 19308 1996TO66/CI | 0.66 ± 0.06 | 0.38 ± 0.04 | 0.74 ± 0.07 | 1.00 ± 0.10 | 5, 6, 10, 13, 17, 20, 21 |
| 19521 Chaos/CI | 0.92 ± 0.05 | 0.61 ± 0.04 | 1.19 ± 0.08 | 1.77 ± 0.07 | 7, 12, 18, 22 |
| 20000 Varuna/CI | 0.92 ± 0.03 | 0.61 ± 0.02 | 1.22 ± 0.02 | 2.01 ± 0.05 | 17, 28, 29 |
| 24835 1995SM55/CI | 0.65 ± 0.03 | 0.40 ± 0.04 | 0.72 ± 0.05 | 1.02 ± 0.05 | 11, 12, 15, 16, 17 |
| 26181 1996GQ21/S | 1.01 ± 0.06 | 0.73 ± 0.04 | 1.42 ± 0.05 | 2.44 ± 0.06 | 12, 16, 18, 19 |
| 26308 1998SM165/CI | 0.97 ± 0.09 | 0.69 ± 0.07 | 1.29 ± 0.07 | 2.37 ± 0.06 | 18, 16, 12 |
| 26375 1999DE9/S | 0.92 ± 0.04 | 0.58 ± 0.03 | 1.15 ± 0.04 | 1.89 ± 0.06 | 10, 11, 12, 16, 17 |
| 29981 1999TD10/S | 0.73 ± 0.04 | 0.50 ± 0.03 | 0.97 ± 0.03 | 1.79 ± 0.05 | 11, 16, 17, 27 |
| 32929 1995QY9/R | 0.70 ± 0.12 | 0.52 ± 0.09 | 0.86 ± 0.08 | 2.03 ± 0.20 | 3, 7, 13, 15 |
| 33128 1998BU48/C | 1.04 ± 0.07 | 0.64 ± 0.04 | 1.19 ± 0.05 | 2.07 ± 0.06 | 12, 17 |
| 35671 1998SN165/CI | 0.71 ± 0.09 | 0.44 ± 0.08 | 0.82 ± 0.09 | 1.27 ± 0.05 | 10, 15, 11, 16, 24 |
| 38628 2000EB173/R | 0.95 ± 0.04 | 0.58 ± 0.08 | 1.19 ± 0.04 | 1.97 ± 0.05 | 10, 16, 19, 27 |
| 44594 1999OX3/C | 1.12 ± 0.10 | 0.70 ± 0.06 | 1.42 ± 0.06 | 2.11 ± 0.08 | 11, 16, 17, 18, 19, 24 |
| 47171 1999TC36/R | 1.01 ± 0.05 | 0.69 ± 0.04 | 1.31 ± 0.05 | 2.29 ± 0.07 | 11, 12, 16, 22, 24 |
| 47932 2000GN171/R | 0.92 ± 0.05 | 0.62 ± 0.04 | 1.24 ± 0.04 | 1.76 ± 0.06 | 16, 19 |
| 48639 1995TL8/S | 0.82 ± 0.18 | 0.55 ± 0.15 | 0.95 ± 0.33 | 2.41 ± 0.05 | 12, 17, 18, 19 |
| 52975 1998TF35/C | 1.14 ± 0.08 | 0.70 ± 0.05 | 1.38 ± 0.07 | 2.37 ± 0.06 | 12, 17, 22 |
| 54598 2000QC243/C | 0.69 ± 0.05 | 0.44 ± 0.04 | 0.89 ± 0.05 | 1.69 ± 0.07 | 12, 17 |

* Type: C= centaur, CI = classical, R = resonant, S = scattered disk, ? = unusual, Halley family comet orbit.

1, Hardorp (1980); 2, Hartmann et al. (1982); 3, Luu and Jewitt (1996a); 4, Luu and Jewitt (1996b); 5, Jewitt and Luu (1998); 6, Romanishin and Tegler (1999); 7, Davies et al. (2000); 8, Romanishin et al. (1997); 9, Tegler and Romanishin (1997); 10, Jewitt and Luu (2001); 11, Delsanti et al. (2001); 12, Delsanti et al. (2003); 13, Barucci et al. (1999); 14, Davies (2000); 15, Gil-Hutton and Licandro (2001); 16, Mac Bride et al. (2003); 17, Doressoudiram et al. (2002); 18, Tegler and Romanishin (2000); 19, Boehnhardt et al. (2002); 20, Tegler and Romanishin (1998); 21, Hainaut et al. (2000); 22, Boehnhardt et al. (2001); 23, Barucci et al. (2000); 24, Doressoudiram et al. (2001); 25, Barucci et al. (2001); 26, Consolmagno et al. (2000); 27, Ferrin et al. (2001); 28, Olivier R. Hainaut, private communication; 29, Jewitt and Sheppard (2002); 30, Hartmann et al. (1981); 31, Davies et al. (1998); 32, Parker et al. (1997); 33, Mueller et al. (1992); 34, Binzel (1992); 35, Davies et al. (1993); 36, Weintraub et al. (1997); 37, Brown and Luu (1997); 38, Romon et al. (2002); 39, Jewitt and Kalas (1998); 40, MacBride et al. (1999); 41, Davies et al. (2001).

TABLE II

Eigenvectors, eigenvalues and percentage of total variance contributed by each eigenvalue.

| Variable | Eigenvector 1 | Eigenvector 2 | Eigenvector 3 | Eigenvector 4 |
|------------------|---------------|---------------|---------------|---------------|
| B-V | 0.3029 | 0.4733 | 0.8208 | -0.1025 |
| V-R | 0.2171 | 0.2266 | -0.3223 | -0.8931 |
| V-I | 0.4469 | 0.6236 | -0.4701 | 0.4365 |
| V-J | 0.8133 | -0.5794 | 0.0386 | 0.0367 |
| Eigenvalues | 0.2828 | 0.0185 | 0.0023 | 0.0007 |
| % total variance | 92.93 | 6.08 | 0.75 | 0.24 |

the EKO's behaviour is shown by the PC1 scores, which span three times more than the PC2 ones. The objects having a neutral colour with respect to the Sun have the lower values of the PC1 scores and fall in the left part of the plot, for larger PC1 scores the objects are redder and redder.

We analysed the same sample with the G-mode multivariate statistics (Coradini et al., 1977), which allowed us to investigate the existence of a finer structure of the sample. The G-mode statistics analyse our sample of 34 objects described by 4 variables, the colours B-V, V-R, V-I and V-J. The total number of degrees of freedom (136) allows us to use this kind of statistics. The goal of the analysis is to find groups of objects which have an homogeneous behaviour in terms of their variables content, if any. The method provides a quantitative estimation of the weight of each variable in separating the groups.

The G-mode analysis has been carried out transforming the colour data into reflectance values by $R_{c_\lambda} = 10^{\pm 0.4(c_\lambda - c_{\lambda, \text{sun}})}$ are the λ -V colours of the object and of the Sun, respectively. The method grouped the 34 objects into seven groups at a significance level of 99%. In Figure 2 the average broad band reflectance spectra (normalised to the V bands) for the groups found by the G-mode analysis are reported.

Group I contains the objects having neutral reflectance spectra with respect to the Sun. The objects in group II have a higher value of V-J, which separates this group from the previous one. Group III and group IV are clearly distinguished by the V-I colour, while V-J separates these two groups from all the other groups, their spectra are redder than those of group I and group II. Group V contains objects still redder and group VI is formed by the reddest objects of the solar system. V-R and V-I distinguish the two last groups, which have the same dramatic reddening, completely different from the trend of all the other spectra.

(32929) 1995QY₉ and 48639 1995TL₈, characterised by the lower PC2 scores, remain isolated, they could form two "single object groups" as well as (4) Vesta,

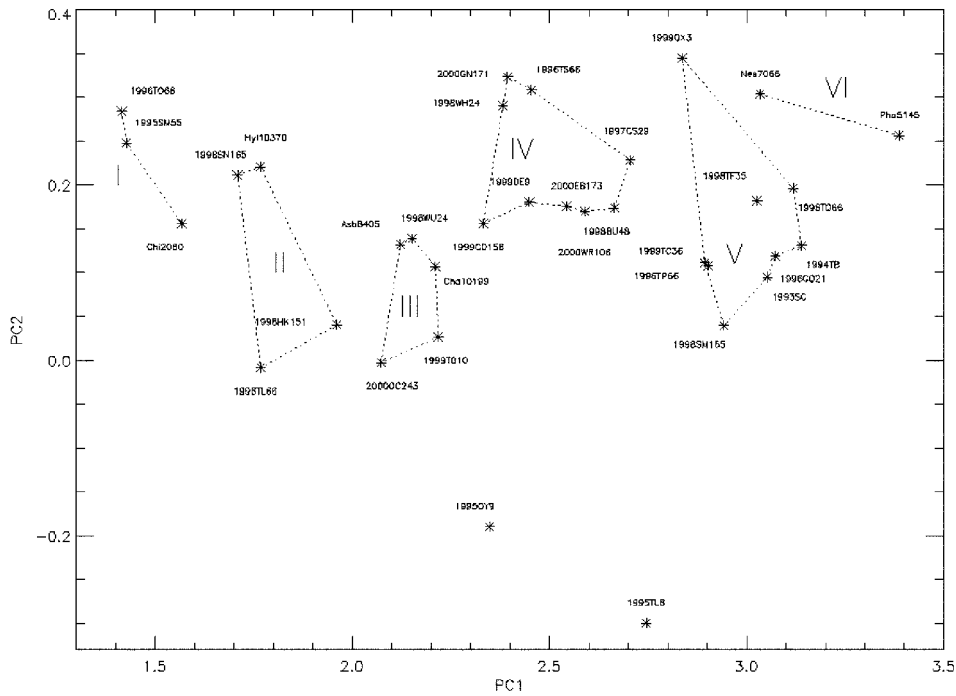


Figure 1. EKOs and Centaurs designation for each object in the PC1-PC2 plot are reported. The dotted lines individuate the groups found the G-mode analysis.

(1862) Apollo and (349) Dembowska formed the V, Q and R classes in the Tholen (1984) asteroid taxonomy [dozens of small asteroids populate the V class and several new objects have been added to the R and Q classes recently (Binzel and Xu, 1993; Bus and Binzel, 2002)].

The relative weights of the variables in structuring the EKO sample in these groups are 38% for V-J, 30% V-I, 17% V-R and 15% B-V. The V-J colour discriminates the groups from each other at a high significance level ($>3\sigma$), V-I plays the same role at a slightly lower level. Minor contributions are provided by V-R and B-V.

These groups constitute the finer structure, overlapping the general trend from neutral to very red spectra resulting from the PC analysis, as shown in Figure 1.

3. Conclusions

The PC analysis of a sample of 34 EKOs for which high quality colour data were available provided a quasi continuous trend from neutral to very red spectra. G-mode suggests the presence of a finer structure superimposed on this trend, separating six groups of homogeneous objects and possibly two single class objects, as far as the colour behaviour is concerned.

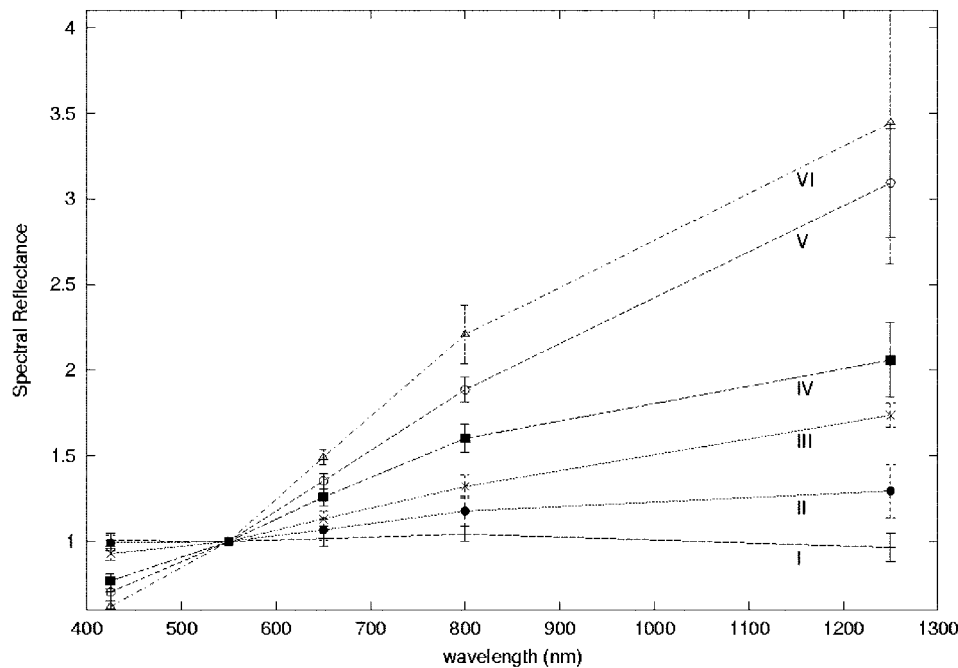


Figure 2. The average broad band reflectance spectra (normalized to the V band) for the groups found by the G-mode analysis. The error bar is the standard deviation of the reflectance mean value within each group. The group spectra spread from the neutral (group I) to the very red one (group VI).

The significance level (99%) of the grouping obtained with the G-mode is larger than the one (93%) obtained by Barucci et al. (1987) in classifying a sample of 438 asteroids with the same statistical technique. This may be an indication that colours could reveal real differences in the EKO's surface nature, probably caused by their physical-chemical evolution.

In our preliminary analysis (Barucci et al., 2001) we found a basically similar result when analysing a smaller sample of 22 EKO's: a quasi continuous trend with a finer structure allowing us to recognise four groups of objects. The 50% increase of the sample size allowed us to refine the description of the EKO's population, particularly concerning the intermediate groups. In fact, the two extreme groups (neutral and reddest spectra) have the same average spectra both in Barucci et al. and in the present analysis, while each of the two Barucci's intermediate groups splits into two more homogeneous groups.

We can conclude that the multivariate analysis of large band spectrophotometric data of EKO's provides strong indications for differences in the surface nature of these objects. The quasi continuous trend, evidenced by the principal component analysis, is probably a witness for the possible sequence of the alteration processes undergone by the surface of each object, while the different groups, obtained with the G-mode, indicate the present physico-chemical state of the objects analysed.

More data are needed to confirm these preliminary results of the multivariate statistical analysis and to allow us to provide a rigorous description of the meaning of the EKO colour differences.

With the aim to draw at least a rough, incomplete and foggy scenario, we could attempt to read Figure 1 as the EKO's evolution diagram: (1) the position of an object along the trend connecting the group of neutrals to the group of the reddest would indicate the time for which it has been exposed to the different alteration processes (collisions, cratering, energetic particles bombardment ...), starting from a given initial state (original or the consequence of a resetting event); (2) each group would represent an evolution stage of the population; (3) the relative number density of each group would account for how long that stage lasts; (4) the presence of single objects groups might imply the existence of a different (cyclic?) evolution paths.

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