JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 117, No. 1, pp. 217–218, April 2003 (© 2003)

ERRATA CORRIGE

General Ekeland's Variational Principle for Set-Valued Mappings

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Abstract. The authors correct Theorem 4.1 of Ref. 1.

Key Words. Set-valued optimization, variational principles, Hausdorff maximality principle.

We thank Dr. Andreads Hamel for pointing out an error in the proof of Theorem 4.1 of Ref. 1. See the last few lines of page 161. Specifically, we cannot conclude from $(v^*, y^*_\mu) \prec (v_\mu, y_\mu) \in Z_0$ that $(v^*, y^*_\mu) \in Z_0$ by the maximality of Z_0 because (v^*, y^*_μ) may depend on μ.

Theorem 4.1 and its proof can be corrected in the following way. First, an additional assumption should be made.

Assumption A2. For each $v \in X$, $F(v)$ is K-semicompact, $F(v) \cap (y - K)$ is compact for each $y \in Y$.

Clearly, if K is closed and F is compact-valued on X, then $F(v)$ is K-semicompact for each $v \in X$. Then, condition (II) of Theorem 4.1 should be revised as follows.

(II) K is closed and Assumptions A1 and A2 hold.

The contents from Line 3 of Page 161 to Line 7 of Page 162 of Ref. 1 can be revised as follows.

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"For each $\mu \in I$, define

$$B_\mu = \{(v^*, z) \in \{v^*\} \times E(v^*): (v^*, z) \prec (v_\mu, y_\mu)\}.$$

Clearly,

$$B_{\mu_2} \subset B_{\mu_1}, \qquad \forall \mu_1 < \mu_2. \tag{1}$$

We show that each B_μ is nonempty and compact."

"The nonemptiness of B_μ has been proved already. Now, we prove that B_μ is closed. Let $\{(v^*, z_\alpha)\} \subset B_\mu$ be a net, and let $z_\alpha \to z^*$. Then, $(v^*, z_\alpha) \prec (v_\mu, y_\mu)$, i.e.,

$$z_\alpha - y_\mu + \epsilon r(v^*, v_\mu) \in -K.$$

By the closedness of K, we deduce that

$$z^* - y_\mu + \epsilon r(v^*, v_\mu) \in -K,$$

namely, $(v^*, z^*) \in B_\mu$. Hence, B_μ is closed."

"Note that $B_\mu \subset \{v^*\} \times [F(v^*) \cap (y_\mu - K)]$. By Assumption A2, $F(v^*) \cap (y_\mu - K)$ is compact; hence, $\{v^*\} \times [F(v^*) \cap (y_\mu - K)]$ is compact. This fact, combined with the closedness of B_μ, yields that B_μ is compact."

"By the maximality of Z_0, we conclude $(v^*, y^*) \in Z_0$. Furthermore, we have that $v^* \in X_0$ by the definition of X_0 and (v^*, y^*) is the minimum of Z_0."

Reference

1. Chen, G. Y., Huang, X. X., and Hou, S. H., *General Ekeland's Variational Principle for Set-Valued Mappings*, Journal of Optimization Theory and Applications, Vol. 106, pp. 151–164, 2000.