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ERRATA CORRIGE

Jacobi Condition for Elliptic Forms in Hilbert Spaces[1]

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Abstract. A definition given in Ref. 1 is corrected together with its consequences.

Key Words. Conjugate points, Jacobi condition, necessary and sufficient optimality conditions.

In Ref. 1, we developed a Jacobi theory for elliptic quadratic forms in a Hilbert space by means of an auxiliary function v, which is defined in two different ways. One of the two definitions is not correct, while the other is correct and leads to the stated results.

The given definition is: let H be a Hilbert space, and let $J: H \times H \to \mathbb{R}$ be a quadratic form; define

$$v(\lambda) = \inf_{x \in \Omega_\lambda} J(x, x),$$

where

$$\Omega_\lambda = \{x \in H_\lambda : \mathscr{H}(x, x) = 1\},$$

H_λ is an increasing family of subspaces of H, and $\mathscr{H}$ is a positive quadratic form, which is either weakly continuous or positive definite.

The form $\mathscr{H}$ must be assumed to be weakly continuous; otherwise, the infimum is not a minimum; hence, the proof of Part 1 of Lemma 2.1 is not correct. For clarity, the lemma and its proof are restated below.

Lemma 2.1. Assume that the form $J(x, x)$ is elliptic on H_λ. If $\Omega_\lambda \neq \emptyset$, then the infimum is a minimum.

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Proof. By Theorem 2.2, there are positive constants κ, M such that

$$J(x, x) \geq \kappa \|x\|^2 - M\mathscr{H}(x, x).$$

Let $x_n \in \Omega_\lambda$ be a minimizing sequence; from the above inequality, it follows that it has to be bounded; hence, without loss of generality, we can assume that $x_n \rightharpoonup x_0$. From the weak continuity of $\mathscr{H}$, it follows that $\mathscr{H}(x_0, x_0) = 1$; moreover, the weak lower semicontinuity of J yields

$$v(\lambda) = \liminf_{n \to \infty} J(x_n, x_n) \geq J(x_0, x_0),$$

and hence

$$J(x_0, x_0) = v(\lambda).$$ □

All the statements concerning the case when $\mathscr{H}$ is positive definite should be disregarded as useless or incorrect, because the minimum property is crucial. In Example 4.5, the forms $\mathscr{H}_3$, $\mathscr{H}_4$ do not have the required properties.

Reference

1. Zezza, P., *Jacobi Condition for Elliptic Forms in Hilbert Spaces*, Journal of Optimization Theory and Applications, Vol. 76, pp. 357–380, 1993.