PROPAGATION OF EXTREMELY LOW FREQUENCY WAVES THROUGH THE IONOSPHERE

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Abstract. The propagation features of extremely low frequency electromagnetic waves through the multicomponent ionospheric plasma are studied. It is shown that at relatively lower frequencies refractive index for right hand mode is higher than the left-hand mode, which is reversed at higher frequencies. The thermal temperature of plasma particle causes decrease in phase and group velocities of both right and left-hand modes. The crossover frequencies for different plasma models are computed and variation with ion concentration and thermal velocity is studied. Explicit expression for group velocity and travel time has been derived and studied numerically. Finally, we have presented simulation of the ion whistler spectrograms for Hydrogen, Helium and Oxygen ions present in the ionospheric plasma. The results are compared with the experimentally detected hydrogen and helium ion whistlers. The importance of the present study in the exploration of ionospheric plasma is illustrated.

Keywords: Crossover frequency, dispersion, ion gyrofrequency, ionospheric plasma, refractive index, wave propagation

1. Introduction

The propagation of extremely low frequency electromagnetic waves through a model ionosphere containing several types of ions have been studied by several workers (Hines, 1957; Buchbaum, 1960; Stix, 1962; Yakimenko, 1962; Ginzburg, 1963; Smith and Brice, 1964; Singh, 1995; Singh et al., 1998; Orsolya, 1999). The presence of ions in the plasma supports the propagation of left hand polarized mode for frequencies smaller than ion gyrofrequency (ω_{Hi}). In fact a band of frequencies between each ion gyrofrequency and the lower cutoff frequency can propagate in the left-hand mode. The lower cutoff frequency for each band is greater than the next lower ion gyrofrequency. The available bandwidth depends on the relative concentration of various ions. Gurnett et al. (1965) have shown that polarization reversal occurs when ELF waves propagate through inhomogeneous medium and this polarization reversal is the main source of ion whistlers observed in the ionosphere, onboard rockets and satellites. The frequency at which polarization reversal takes place is known as crossover frequency at which inter mode coupling takes place

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and part of right hand polarized wave energy is converted into left hand polarized mode (Wang, 1971).

Ray tracing method has been used to study the wave spectrum of ELF waves (Terry, 1978; Cserepes et al., 1983), but this method has limited validity due to involved approximation of exact wave theory (Terry, 1978). In fact the wave component of electron whistler comes out more quickly from the coupling process near crossover frequency than is expected from the low values of group velocities. Because of this reason ray tracing method has limited validity. This method also fails because near the crossover frequency the group velocities for the two modes drop sharply. Cserepes (1987) has investigated the problem of mode coupling by the exact method of full wave analysis and extended it to the computation of spectrograms. He concluded that the frequency-time diagram of the ion cyclotron branch of whistlers can be accurately calculated by the ray tracing technique but the same does not hold for the electron branch. Further, the spectrogram is usually diffused and hence it is not an accurate tool to read the crossover frequency.

In this paper, we have studied the propagation of ELF waves including the effect of ion thermal velocity. Dispersion relation for the right hand and left hand polarized modes are discussed in Section 2. The effect of concentration of different types of ions and their thermal velocity on the phase refractive index has been studied. The expression for group velocity and group travel time as a function of $\Delta\omega = \omega_{\mathrm{Hi}}(s) - \omega$; where $\omega_{\mathrm{Hi}}(s)$ is ion gyrofrequency at the location s and ω is wave frequency. The correction in arrival time due to proton thermal velocity is included in the expression. Numerical computations have been carried out to study the frequency spectrum of Hydrogen, Helium and Oxygen whistlers. To compare our results we have also presented the measured spectrum of hydrogen and helium whistlers. It is also discussed that the present study is capable of estimating the ion density and temperature.

2. Dispersion Relation of Elf Waves

The finite temperature of electrons and ions of the plasma modifies the phase and group velocity of the waves apart from introducing Landau and cyclotron damping. Also, ion acoustic and electrostatic ion cyclotron modes arise due to finite temperature of the plasma. In the present paper we would be discussing the role of thermal velocity of various ions on electromagnetic modes only. Following Stix (1962), the dispersion relation for the right and left hand polarized modes for parallel propagating waves can be written as

$$\frac{k^2c^2}{\omega^2} = R = 1 - \sum_j \frac{\omega_{Pj}^2}{\omega(\omega + \epsilon_j \Omega_j)} - \sum_j \frac{\omega_{Pj}^2 k^2 v_{thj}^2}{2\omega(\omega + \epsilon_j \Omega_j)^3}, \tag{1}$$

$$\frac{c^2k^2}{\omega^2} = L = 1 - \sum_j \frac{\omega_{Pj}^2}{\omega(\omega - \epsilon_j \Omega_j)} - \sum_j \frac{\omega_{Pj}^2 k^2 v_{thj}^2}{2\omega(\omega - \epsilon_j \Omega_j)^3}, \tag{2}$$

where ϵ_j, $v_{thj} = (2\kappa T_j/m_j)^{1/2}$ and T_j are the sign of the charge, thermal velocity and temperature of the jth species of the plasma respectively. κ is Boltzmann constant. In the above derivation, the equilibrium velocity distribution function is assumed to be isotropic Maxwellian and only first order thermal correction terms were retained. Considering plasma consisting of electrons, H^+, He^+ and O^+ ions and neglecting 1 in Equations (1) and (2), we obtain (Singh and Tolpadi, 1975; Singh et al., 1998)

$$\begin{aligned} R \approx \frac{\omega_{Pe}^2}{\Omega_e \Omega_i} \Bigg[& \frac{1}{\lambda} - \frac{\alpha_1}{\lambda(1+\lambda)} - \frac{\alpha_2}{\lambda(1+4\lambda)} - \frac{\alpha_3}{\lambda(1+16\lambda)} \\ & - \frac{\alpha_1 \lambda}{2(1+\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 - \frac{\alpha_2 \lambda}{8(1+4\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 \\ & - \frac{\alpha_3 \lambda}{32(1+16\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 \Bigg] \end{aligned} \tag{3}$$

and

$$\begin{aligned} L \approx \frac{\omega_{Pe}^2}{\Omega_e \Omega_i} \Bigg[& \frac{1}{\lambda} - \frac{\alpha_1}{\lambda(1-\lambda)} - \frac{\alpha_2}{\lambda(1-4\lambda)} - \frac{\alpha_3}{\lambda(1-16\lambda)} \\ & - \frac{\alpha_1 \lambda}{2(1-\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 - \frac{\alpha_2 \lambda}{8(1-4\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 \\ & - \frac{\alpha_3 \lambda}{32(1-16\lambda)^3} \left(\frac{v_{th}}{v_{Ph}}\right)^2 \Bigg], \end{aligned} \tag{4}$$

where

$$\lambda = \frac{\omega}{\Omega_1}, \quad \alpha_1 = \frac{N_1}{N_e}, \quad \alpha_2 = \frac{N_2}{N_e}, \quad \alpha_3 = \frac{N_3}{N_e}.$$

$N_e = N_1 + N_2 + N_3$, is electron density. N_1, N_2 and N_3 are the densities of hydrogen, helium and oxygen ions respectively. Ω_1 is the gyrofrequency of hydrogen ion. Further, different types of ions have same temperature and contribution of electron thermal term is much smaller compared to ion thermal terms. $v_{Ph} = \omega/k$ is the phase velocity of the wave. The above dispersion relation reduces to the cold plasma dispersion relation when v_{th} is considered as zero.

From the cold plasma dispersion relation it can easily be shown that the polarization of the wave fields is given by

$$\frac{iE_x}{E_y} = \frac{n^2 - S}{D}, \tag{5}$$

where $S = \frac{1}{2}(R + L)$ and $D = \frac{1}{2}(R - L)$. For $\theta = 0$, substituting $n^2 = R(L)$ for right (left) hand polarized mode, we find

$$\frac{iE_x}{E_y} = \pm 1, \tag{6}$$

where $+$ve sign is for $n^2 = R$ and $-$ve sign is for $n^2 = L$. Equation (6) suggests that the polarization of the mode is circular with a right or left-hand sense of rotation. To have deeper insight in the propagation features of these waves, we present numerical computations for different plasma models valid in the outer ionosphere. We consider four types of plasma models whose constituents are defined by (a) H^+ = 70%, He^+ = 15% and O^+ = 15%; (b) H^+ = 80%, He^+ = 10% and O^+ = 10%; (c) H^+ = 70%, He^+ = 20% and O^+ = 10%; (d) H^+ = 80%, He^+ = 15% and O^+ = 5%. Apart from these major ionic constituents some trace elements are also present in the ionosphere. Since the concentration of trace elements is very small, their effect on the phase and group velocity is negligibly small and we ignore their presence. To estimate quantitative changes caused by thermal velocity of plasma, we have considered three cases, i.e., $v_{th} = 0$, $0.2v_{ph}$ and $0.5v_{ph}$.

Equations (3) and (4) have been used to compute the phase refractive indices of right hand and left hand polarized electromagnetic waves through cold and thermal ionospheric plasma and results are presented in Figures 1A–C. The figures show resonance for L mode at λ = 1 (Hydrogen ion), λ = 1/4 (Helium ion) and λ = 1/16 (Oxygen ion). No resonance is observed for R mode at lower frequencies. The refractive index of R mode decreases as wave frequency increases whereas the refractive index of L mode increases with the increase of the frequencies. The inclusion of thermal effect causes further decrease in refractive indices of R and L modes.

Figure 1A shows the variation of phase refractive indices for v_{th} = 0 and ion densities as stated in plasma models [(a) $\alpha_1 = 0.7$, $\alpha_2 = 0.15$, $\alpha_3 = 0.15$; (b) α_1 = 0.8, $\alpha_2 = 0.1$, $\alpha_3 = 0.1$; (c) $\alpha_1 = 0.7$, $\alpha_2 = 0.2$, $\alpha_3 = 0.1$ and (d) $\alpha_1 = 0.8$, $\alpha_2 = 0.15$ and $\alpha_3 = 0.05$] with normalized wave frequency. In the inset of the figure the phase refractive index variation with frequency for smaller values of normalized wave frequency is shown. From the figure it is noted that there are two crossover frequencies, one lying between Ω_1 and Ω_2, and the other between Ω_2 and Ω_3. At much lower wave frequencies the refractive indices for two modes coincide with each other which means that the two modes propagate with the same phase velocity. They can be distinguished only by measuring their polarization. The

TABLE I

Crossover frequencies for different plasma models

Crossover frequencies	Thermal velocity (v_{th}/v_{Ph})	Plasma models (a)	(b)	(c)	(d)
ω_{12}	0	66.6	65.6	55.9	50.5
ω_{23}		230.8	190.0	228.4	196.8
ω_{12}	0.2	66.4	65.6	53.3	50.4
ω_{23}		221.4	184.9	221.8	190.2
ω_{12}	0.5	66.0	64.4	54.9	49.4
ω_{23}		206.2	175.5	207.0	177.9

relative variation in ion concentration causes variation in crossover frequencies. For the above four models (a, b, c, d) and $v_{th} = 0.2v_{Ph}$, the computed phase refractive indices and its variation with normalized wave frequencies are shown in Figure 1B. Figure 1C show similar variation for $v_{th} = 0.5v_{Ph}$. The crossover frequencies for different models are given in Table I.

It is clearly seen that the crossover frequency decreases as the fractional concentration of hydrogen ion increases. It is also observed that there is appreciable change in ω_{23} where as change in ω_{12} due to relative variation of ion concentration is small. The crossover frequency in the case of thermal plasma $(v_{th}/v_{ph} = 0.5)$ lying between Ω_1 and Ω_2 is decreased by 1.5% and that lying between Ω_2 and Ω_3 decreased by 9.3% relative to its cold plasma value. The plasma models which are used in the above computations are representative of the topside ionosphere where hydrogen ions are dominant constituents. Thus, accurate measurement of wave spectrum including crossover frequency, will yield information about the plasma structure of the ionosphere.

Above the topside ionosphere, the effect of oxygen ions can be ignored and we may consider $\alpha_3 = 0$. Considering $\alpha_1 = 0.8$ and $\alpha_2 = 0.2$, the phase refractive indices for $v_{th}/v_{Ph} = 0$ and 0.5 are calculated and shown in Figure 1D. It is found that $\omega_{12} = 205$ Hz and 180.4 Hz for $v_{th}/v_{Ph} = 0$ and 0.5 respectively. Thus, it is observed that crossover frequency lying between Ω_1 and Ω_2 decreases by 12% for thermal plasma as compared to cold plasma model.

In the outer ionosphere (in the vicinity of F_2 peak) oxygen ions are dominant. To study the effect of dominance of oxygen ions on the propagation of low frequency waves, we consider $\alpha_1 = 0.1$, $\alpha_2 = 0.15$ and $\alpha_3 = 0.75$. Computations are made for $v_{th}/v_{Ph} = 0$ and 0.5 and the results are shown in Figure 1E, ω_{12} and ω_{23} are 90.2, 89 Hz and 382.1, 334.2 Hz respectively. It is seen that crossover frequencies are in the neighborhood of the ion gyrofrequencies. The values are shifted towards

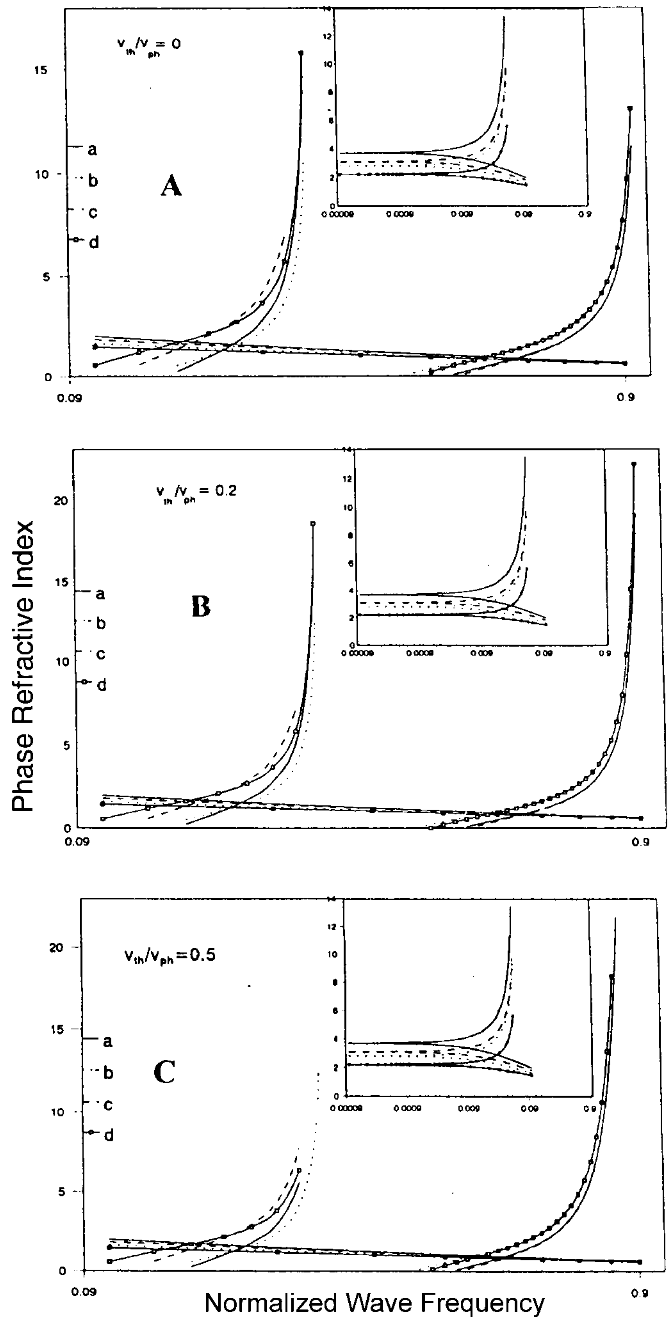

Figure 1. (A–C) Variation of phase refractive index for left and right hand polarized modes for different concentrations and thermal temperatures of the plasma.

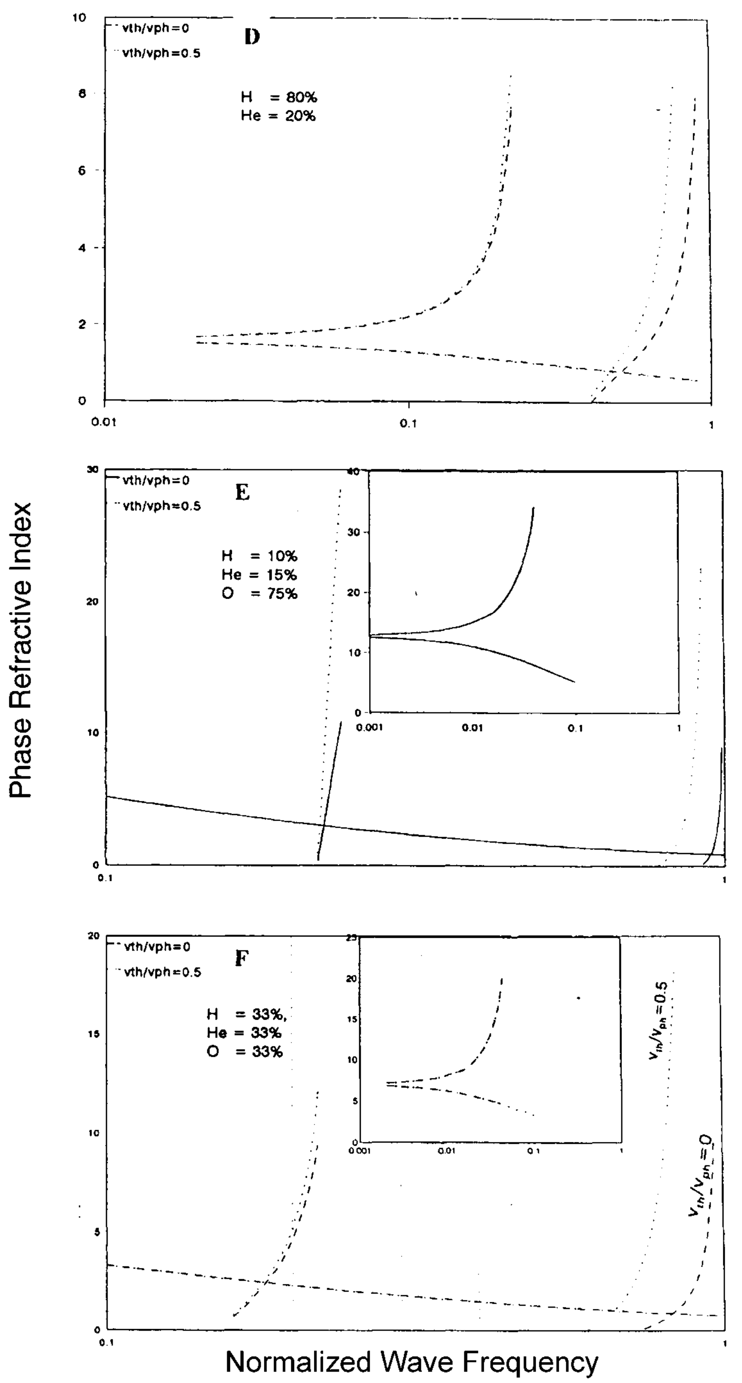

Figure 1. Continued. (D–F) Variation of phase refractive index for left and right hand polarized modes for different concentrations and thermal temperatures of cold plasma.

lower side when thermal effects are taken into account. In fact ω_{12} decreases by 1.33% and ω_{23} decreases by 12.54% in the thermal plasma.

We consider a peculiar case when all the three ions are in equal proportion ($\alpha_1 = 0.33 = \alpha_2 = \alpha_3$) and compute refractive indices for $v_{th}/v_{Ph} = 0$ and 0.5. The results are shown in Figure 1F. It is found that ω_{12} = 69.8, 68.9 Hz and ω_{23} = 335.4, 262.6 Hz respectively for cold and thermal plasma. Thermal effects are much more significant for ω_{23} because it changes from 335.4 Hz to 262.6 Hz.

3. Group Velocity and Travel Time of ELF Waves

In the previous section we have shown that three resonances at $\omega = \Omega_1$, $\omega = \Omega_2$ and $\omega = \Omega_3$ appeared in the phase refractive index curve indicating that there exist three regions of wave propagation: (i) $0 < \omega < \Omega_1$, (ii) $0 < \omega < \Omega_2$ and (iii) $0 < \omega < \Omega_3$. The waves in these regions are named as proton whistlers, helium whistlers and oxygen whistlers respectively. The approximate expression for phase refractive indices for frequencies in the neighborhood of ion gyrofrequencies can be written as

$$\mu_1^2 = \frac{\omega_{P1}^2}{\Omega_1(\Omega_1 - \omega)} + \frac{\omega_{P1}^2 k^2 V_{th1}^2}{2\omega(\Omega_1 - \omega)^3} \qquad \text{for Proton whistlers,} \tag{7}$$

$$\mu_2^2 = \frac{\omega_{P2}^2}{\Omega_2(\Omega_2 - \omega)} + \frac{\omega_{P2}^2 k^2 V_{th2}^2}{2\omega(\Omega_2 - \omega)^3} \qquad \text{for Helium whistlers,} \tag{8}$$

$$\mu_3^2 = \frac{\omega_{P3}^2}{\Omega_3(\Omega_3 - \omega)} + \frac{\omega_{P3}^2 k^2 V_{th3}^2}{2\omega(\Omega_3 - \omega)^3} \qquad \text{for Oxygen whistlers.} \tag{9}$$

The expression for group refractive index is obtained from the relation

$$\mu_g = \mu + \omega \frac{\mathrm{d}\mu}{\mathrm{d}\omega}. \tag{10}$$

Using the above set of equations the expression for group refractive index is obtained as

$$\begin{aligned}\mu_{gi} \; = \; & \frac{\omega_{Pi}^{1/2}}{\left[1 + \frac{1}{2}\frac{k_i^2 V_{thi}^2}{(\Omega_I - \omega)^2}\right]^{1/2}} \left[\frac{\omega_{Pi}(\Omega_i - \omega/2)}{\Omega_I(\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi} k_i^2 V_{thi}^2}{4(\Omega_I - \omega)^{5/2}}\right. \\ & \left. \times \left(\frac{1}{\omega} + \frac{3}{(\Omega_i - \omega)}\right) + \frac{\omega_{Pi} V_{thi}^2 2k_i}{4(\Omega_i - \omega)^{5/2}} \frac{\partial k_i}{\partial \omega}\right],\end{aligned} \tag{11}$$

where

$$k_i^2 = \frac{\omega_{Pi}^2 \omega^2}{c^2 \Omega_i (\Omega_i - \omega)},$$

$$2k_i \frac{\partial k_i}{\partial \omega} = \frac{\omega_{Pi}^2 2\omega (\Omega_i - \omega/2)}{c^2 \Omega_i (\Omega_i - \omega)^2}.$$

From this relation, the expression for group velocity of the corresponding wave (Hydrogen, Helium and Oxygen ion mode whistler waves) is given by

$$\begin{aligned} V_{gi} &= \frac{c}{\Omega_i^{1/2}} \left[1 + \frac{k_i^2 V_{thi}^2}{2(\Omega_i - \omega)^2} \right]^{1/2} \left[\frac{\omega_{Pi} (\Omega_i - \omega/2)}{(\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi} k_i^2 V_{thi}^2}{4(\Omega_i - \omega)^{5/2}} \right. \\ &\quad \left. \times \left(\frac{1}{\omega} + \frac{3}{(\Omega_i - \omega)} \right) + \frac{\omega_{Pi} V_{thi}^2 2k_i}{4(\Omega_i - \omega)^{5/2}} \frac{\partial k_i}{\partial \omega} \right]^{-1} . \end{aligned} \tag{12}$$

The group travel time from the source to the observation point (satellite or rocket placed in the ionosphere) is given by

$$t_i(\omega) = \int_s \frac{\mathrm{d}s}{V_{gi}}. \tag{13}$$

Here it should be noted that these waves could not reach the earth surface because the waves damp whenever wave frequency approaches the respective ion gyrofrequency.

Substituting for V_{gi} in Equation (13), the expression for travel time is obtained as

$$\begin{aligned} t_i(\omega) &= \int \frac{\mathrm{d}s}{c} \frac{\Omega_i^{1/2}}{\left[1 + \frac{1}{2} \frac{k_i^2 V_{thi}^2}{(\Omega_i - \omega)^2} \right]} \left[\frac{\omega_{Pi} (\Omega_i - \omega/2)}{\Omega_i (\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi} k_i^2 V_{thi}^2}{4(\Omega_i - \omega)^{5/2}} \right. \\ &\quad \left. \times \left(\frac{1}{\omega} + \frac{3}{(\Omega_i - \omega)} \right) + \frac{\omega_{Pi} V_{thi}^2 2k_i}{4(\Omega_i - \omega)^{5/2}} \frac{\partial k_i}{\partial \omega} \right]. \end{aligned} \tag{14}$$

Usually $\{\frac{k_i^2 V_{thi}^2}{2(\Omega_i - \omega)^2}\} \ll 1$. Under this condition we can use binomial expansion. Neglecting the terms containing v_{thi}^4/c^4 and higher orders, we obtain

$$\begin{aligned} t_i(\omega) &= \int \frac{\mathrm{d}s \Omega_i^{1/2}}{c} \left[\frac{\omega_{Pi} (\Omega_i - \omega/2)}{\Omega_i (\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi}^3 V_{thi}^2}{4c^2 \Omega_i (\Omega_i - \omega)^{9/2}} \omega (\omega + 3\Omega_i) \right] \\ &= \int \frac{\mathrm{d}s}{c} \omega_{Pi} \Omega_i^{1/2} \left[\frac{\Omega_i - \omega/2}{\Omega_i (\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi}^2 V_{thi}^2 \omega (\omega + 3\Omega_i)}{4c^2 \Omega_i (\Omega_i - \omega)^{9/2}} \right]. \end{aligned} \tag{15}$$

According to Equation (15), $t(\omega) \to \infty$ as $\omega \to \Omega$ because in that case $V_{gi} \to 0$. In order to derive analytical expression for travel time t for frequencies near the ion gyrofrequency, we replace ω by Ω_i in the numerator of Equation (15) to obtain

$$t_i(\omega) = \int \frac{\mathrm{d}s}{c} \omega_{Pi} \Omega_i^{1/2} \left[\frac{1}{2(\Omega_i - \omega)^{3/2}} + \frac{\omega_{Pi}^2 V_{thi}^2 \Omega_i}{c^2(\Omega_i - \omega)^{9/2}} \right]. \tag{16}$$

For further simplification we assume that ion gyrofrequency varies linearly with distance (Gurnett and Brice, 1966) and write

$$\Omega_i(s) = \Omega_i(0) - \Omega_i'(0)s, \tag{17}$$

where $\Omega_i(0)$ and $\Omega_i(s)$ are the ion gyrofrequency at the observation point and at a distance s along the ray path from the observation point. $\Omega_i'(0)$ is the gradient of ion gyrofrequency along the ray path for the ith type of ion. Let us define

$$\Delta\omega_i(s) = \Omega_i(s) - \omega. \tag{18}$$

Hence

$$\mathrm{d}\Delta\omega_i(s) = \mathrm{d}[\Omega_i(0) - \omega - \Omega_i'(0)s] = -\Omega_i'(0)\,\mathrm{d}s,$$

and

$$\mathrm{d}s = -\mathrm{d}\Delta\omega_i(s)/\Omega_i'(0).$$

Thus Equation (16) is rewritten as

$$\begin{aligned} t_i(\omega) \;=\; & -\int_{-\Delta\omega_i(h)}^{\Delta\omega_i(0)} \frac{\omega_{Pi}(s)\Omega_i^{1/2}(s)}{2c(\Delta\omega_i(s))^{3/2}} \cdot \frac{\mathrm{d}(\Delta\omega_i(s))}{\Omega_i'(0)} \\ & -\int_{-\Delta\omega_i(h)}^{\Delta\omega_i(0)} \frac{\omega_{Pi}^3(s) V_{thi}^2 \Omega_i^{3/2}(s)}{c^3 \Omega_i'(0)(\Delta\omega_i(s))^{9/2}} \,\mathrm{d}\Delta\omega(s). \end{aligned} \tag{19}$$

Assuming slow variation of plasma density and magnetic field, we can write $\omega_{Pi}(s) = \omega_{Pi}(0)$, $\Omega_i(s) = \omega_i(0)$. Further, let us assume that the thermal velocity of ions does not change appreciably in the vicinity of the observation location. With these assumptions we can perform the integration in Equation (19) to obtain

$$\begin{aligned} t_i(\omega) \;=\; & \frac{\omega_{Pi}(0)\Omega_i^{1/2}(0)}{c\Omega_i'(0)} \left[\frac{1}{(\Delta\omega(0))^{1/2}} - \frac{1}{(\Delta\omega(h))^{1/2}} \right] \\ & + \frac{2}{7} \frac{\omega_{Pi}^3(0)\Omega_i^{3/2}(0) V_{thi}^2}{c^3 \Omega_i'(0)} \cdot \left[\frac{1}{(\Delta\omega(0))^{7/2}} - \frac{1}{(\Delta\omega(h))^{7/2}} \right]. \end{aligned} \tag{20}$$

Since $\Delta\omega(0)$ at the observation point would be very small relative to $\Delta\omega(h)$, terms containing $[\Delta\omega(h)]^{-n}$ can be ignored [n = 1/2, 7/2] and we obtain

$$\begin{aligned} t_i(\omega) &= \frac{\omega_{Pi}(0)\Omega_i^{1/2}(0)}{c\Omega_i'(0)} \frac{1}{(\Delta\omega(0))^{1/2}} + \frac{2}{7} \\ &\quad \times \frac{\omega_{Pi}^3(0)\Omega_i^{3/2}(0)V_{thi}^2}{c^3\Omega_i'(0)} \cdot \frac{1}{(\Delta\omega(0))^{7/2}}. \end{aligned} \tag{21}$$

In Equation (21) substituting values for Hydrogen, Helium and Oxygen ions, we obtain time delay for three ion whistler waves. The first term is dominant term. The second term arises due to thermal effect and is proportional to ion temperature and gradient of ion gyrofrequency. Similar relation was derived by Gurnett and Shawhan (1966) and Gurnett and Brice (1966) to interpret the observed spectra of proton whistlers. In their derivation they have ignored the effect of thermal contribution. The ratio of contribution of temperature correction term to the total group delay time is written as

$$R_{tc} = \frac{2}{7}\frac{\omega_{Pi}^3(0)\Omega_i^{3/2}V_{thi}^2}{c^3\Omega_i'(0)(\Delta\omega(0))^{7/2}} \cdot \frac{1}{t_i(\omega)}. \tag{22}$$

Equation (21) has been used to evaluate group travel time as a function of $\Delta\omega$ and plasma temperature. Group travel time as a function of $\Delta\omega$ for different temperature is shown in Figure 2A. Figure 2B shows group travel time as a function of frequency for different temperature. Figure 2C shows the variation of R_{tc} as a function of $t(\omega)$ and temperature. The thermal correction due to Hydrogen ion is larger as compared to Helium and Oxygen ions. Group travel time increases as $\Delta\omega$ decreases that is as the wave frequency approaches towards the respective ion gyrofrequency. The effect of thermal correction is to decrease the group velocity and hence to increase the travel time. The travel time increases by increasing the thermal temperature of the ion component of the plasma. The departure from cold plasma approximation is insignificant at lower wave frequency but it becomes appreciable and finite as wave frequency approaches towards the ion gyrofrequency. Thus, the contribution of thermal term becomes significant when wave frequency approaches the ion gyrofrequency. Comparing Figures 2A–C it is clearly evident that the deviation near ion gyrofrequency caused by thermal effects is large for the hydrogen ion and small for the oxygen ions. In fact there is insignificant deviation caused by oxygen ions. Thus, it can be safely argued that the thermal effect is dominant as wave frequency approaches the ion gyrofrequency.

The theoretically computed frequency time spectrum is identical to the observed hydrogen and helium whistlers (Figure 3). By varying the proton temperature a set of frequency spectrum of proton whistlers have been generated. Comparing the actual spectrum with the computed spectrum, one can evaluate proton temperature. Here, it should be remembered that in the derivation of group travel time

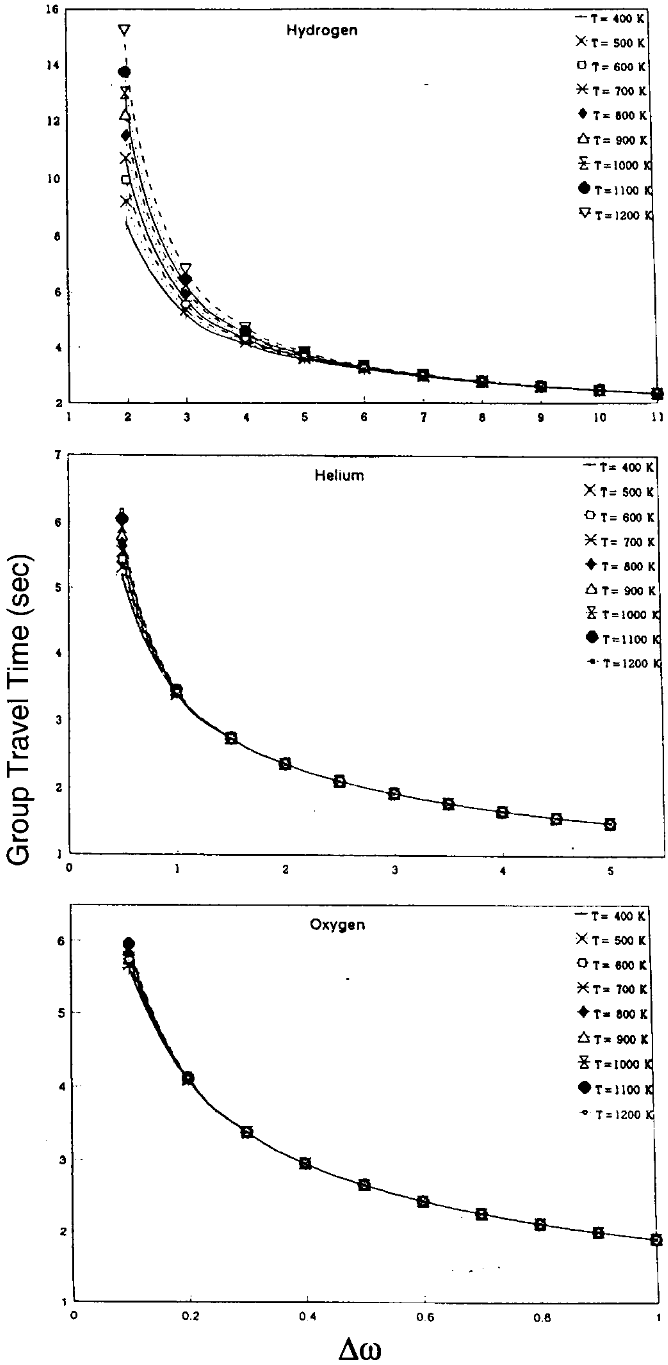

Figure 2A. Variation of group travel time with $\Delta\omega$ for different temperature and plasma species.

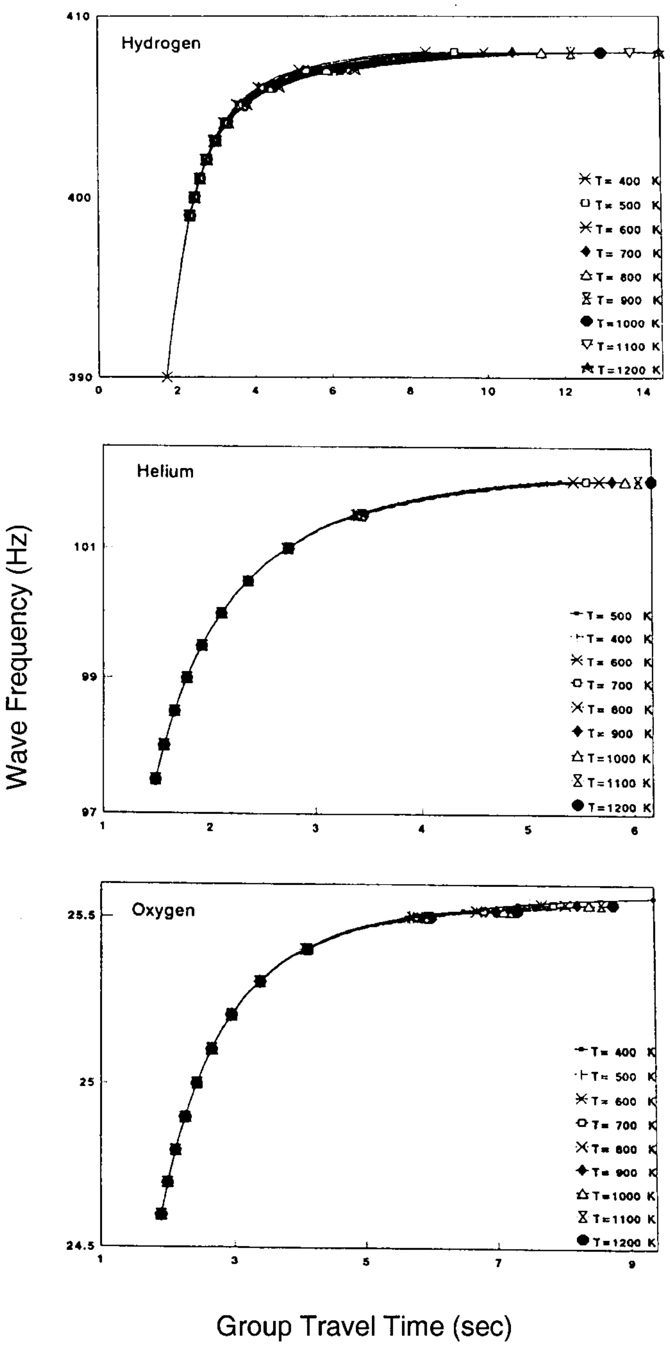

Figure 2B. Variation of group travel time as a function of frequency for different temperature.

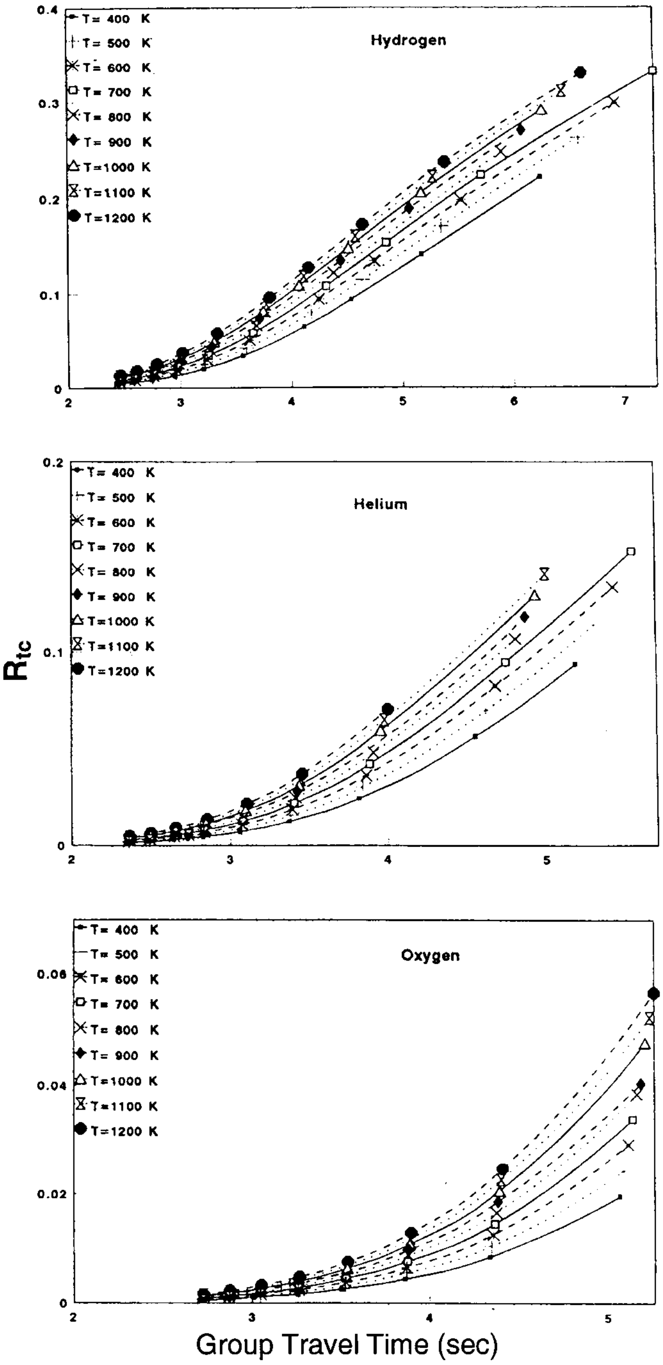

Figure 2C. Variation of R_{tc} with travel time.

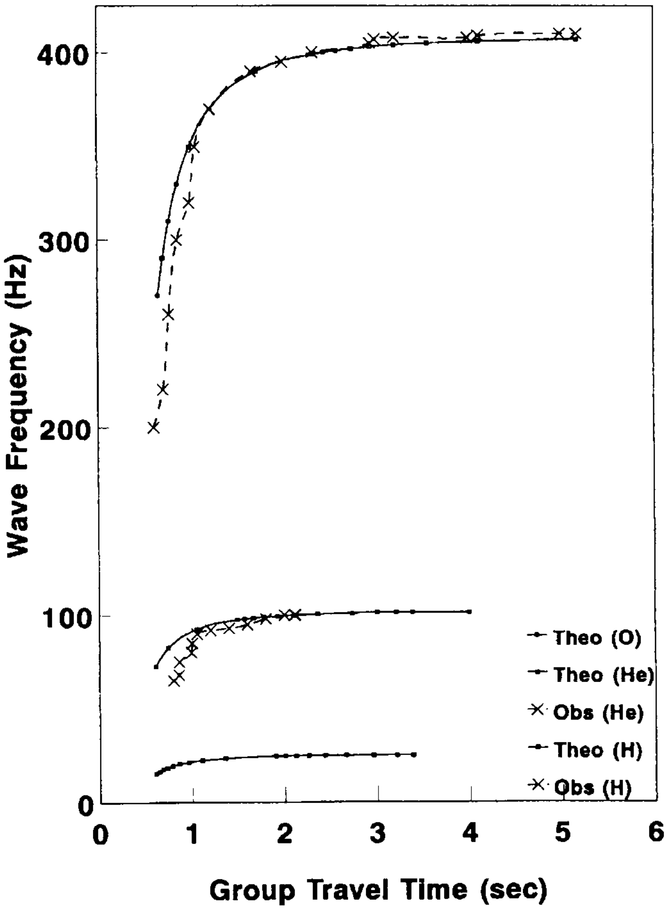

Figure 3. Spectrograms of experimentally observed Hydrogen and Helium whistlers (after Gurnett and Brice, 1966), and simulated whistlers for Hydrogen, Helium and Oxygen ions.

field aligned propagation of ion-cyclotron waves has been considered. For waves propagating at certain angles, the actual proton temperature should be larger by a factor $\frac{1}{1+\cos^2\theta}$, where θ is wave-normal angle (Gurnett and Brice, 1966). Thus, during the evaluation of proton temperature from dispersion analysis of whistler waves, it is essential to have exact knowledge of the wave propagation direction. In the absence of measurement of wave propagation direction there is likely to occur an error in the estimated proton temperature.

4. Discussion

In the present paper we have studied dispersion, group travel time and crossover frequency for ELF waves propagating along magnetic field lines. For numerical computations, we have used parameters appropriate to the night-time ionosphere at the altitude of 1000 km, which corresponds to the location of Injun 3 satellite on January 11, 1963 at 1059:38 UT when a number of proton whistlers were recorded. Similar other locations have also been chosen and computations have been made. Eventually, we have analyzed dispersive features of ELF waves propagating through four component ionospheric plasma. Five types of ionospheric plasma models have been used in the numerical study, which represent different regions of ionosphere.

It is observed that in contrast to VLF range, both right hand and left hand polarized modes propagate in the ELF range. In the VLF range propagation is possible only for right hand polarized waves. In the ELF range, the most important wave mode is the ion whistler mode, which is possible for a band of frequencies below each ion gyrofrequency. This mode is identical to electron whistler mode in the VLF range except the change of polarization. The lower cutoff frequency for each band is greater than the next lower ion gyrofrequency and is the function of ion concentration. Therefore, the left hand polarized wave associated with a particular ion can propagate only for a restricted altitude range. Because of this, ion whistlers of different types are observed at different locations in the ionosphere. Watanabe and Ondoh (1976) have reported deuteron whistler at the satellite height of 1366 km and explained it by considering ionospheric plasma consisting of H^+, D^+, He^+ and O^+ ions. Based on the computations, they have suggested transequatorial propagation of the ion cyclotron whistlers in the vicinity of the equatorial topside ionosphere.

The presence of ions having different charge to mass ratio exhibit multiple resonances and cut-offs lying in the ELF range. The measurement of these resonances and cut-offs provide useful diagnostic tools and can estimate relative concentration of each species if charge to mass ratio of species are known (Kikuchi, 1969; Narayan, 1998). During ionospheric high frequency modification experiment, electrostatic parametric resonances are excited which need further investigations (Fejer, 1979; Dubois et al., 1993; McKenzie and Hagfors, 1996). Further, numerical computations show that the group velocity changes if the concentration of ions is changed or thermal temperature of the plasma is changed. Thus, if thermal temperature is known then an exact measurement of dynamic spectrum can be used for the accurate measurements of ion distributions in the ionosphere.

The role of ion thermal velocity or phase velocity, crossover frequency and group travel time has been analyzed. It is shown that the thermal effect decreases the crossover frequency and hence accordingly the starting spectrum of ion whistler would shift in frequency. The thermal velocity also affects the group travel time and hence the shape of the dynamic spectrum. Therefore, for the correct interpretation of the spectrograms, the effect of thermal correction should be included in the theoretical modelling of the wave propagation. The dynamic spectrum is dependent on the proton temperature at the location of the satellite. Hence, by varying proton temperature, a set of spectrograms are generated and comparing the theoretically generated and experimentally observed spectrograms, we can easily decipher the proton temperature of the medium.

The above discussions are valid for homogeneous medium. The ionosphere and Magnetosphere behave like inhomogeneous medium and one may use WKB approximation considering wave packet propagating through a slowing varying medium. The wave packet can be presented as a set of plane waves near the central frequency ω and wave vector k ($\Delta\omega \ll \omega$, $\Delta k \ll k$). In the ionosphere large vertical ionization gradient exist which complicates the process of partial reflection, absorption, spreading and mode coupling (Laird, 1981). In the case of magnetosphere the inhomogeneity of magnetic field affect largely the propagation of whistler mode waves along the field lines and temperature gradient (Strangeways, 1986). Propagation at low latitudes is also affected by the curvature of magnetic field. The penetration and coupling process of elementary plane waves strongly depends on their incident directions and wavelengths, especially at very low latitude region where penetration is highly asymmetric. Nagano et al. (1982) has developed a full wave technique capable of treating the spatially confined whistler beam propagation in the horizontally stratified ionosphere where incident beam was resolved into a large number of elementary plane waves. The introduction of thermal correction along with inhomogeneities in medium parameter make it possible to analyze the propagation more correctly, especially near the resonance frequencies and to calculate the amplitude fluctuations. However, the problem becomes more complex and an analytical solution is beyond the scope of the present paper.

5. Conclusions

In this paper we have studied the propagation features of extremely low frequency waves through the multicomponent ionospheric plasma. The presence of ions in the plasma containing magnetic field facilitates the propagation of left-hand polarized mode in addition to usual right-hand polarized whistler mode waves. Following points emerge from the present study:

(i) At much lower wave frequencies (smaller than the gyrofrequency of heaviest ions), the two modes (left and right hand polarized) propagate with the same phase velocity. The effect of thermal velocity of ions is to increase the phase velocity of the wave.

(ii) The role of proton temperature on the dispersion characteristics of proton whistler is studied. It is shown that the estimated proton density in the absence of thermal effect is higher by at least 5%.

(iii) Between two-ion gyrofrequency, there is a crossover frequency at which coupling between two modes takes place. The crossover frequencies vary due to relative variation in ion concentration and variation in ion temperature. The change in lower crossover frequency caused by thermal effect is more as compared to the change in higher crossover frequency.

(iv) The expression for group travel time including the effect of thermal plasma is derived and studied numerically for model ionosphere. Temperature has a significant effect on the group travel time of the ion whistler as the wave frequency approaches ion gyrofrequency. The proton temperature near the observation site could be determined by fitting the observed spectra with numerically simulated spectra.

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