MODELLING OF SHAPE CHANGES OF THE NUCLEI OF COMETS C/1995 O1 HALE–BOPP AND 46P/WIRTANEN CAUSED BY WATER ICE SUBLIMATION

BEATA DZIAK-JANKOWSKA
University of Warsaw, Institute of Geophysics, ul. Pasteura 7, 02-093 Warszawa, Poland

JACEK LELIWA-KOPYSTYŃSKI
University of Warsaw, Institute of Geophysics, ul. Pasteura 7, 02-093 Warszawa, Poland, and Space Research Centre of the Polish Academy of Sciences, ul. Bartycka 18A, 00-716 Warszawa, Poland

MAŁGORZATA KRÓLIKOWSKA
Space Research Centre of the Polish Academy of Sciences, ul. Bartycka 18A, 00-716 Warszawa, Poland

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Abstract. The aim of this modelling work is to assess shape changes of cometary nuclei caused by sublimation of ices. The simplest possible model is assumed with the nucleus being initially spherical and its thermal conductivity being neglected. We have calculated the time-dependent sublimation flux versus cometographic latitude. If the rotation axis of the comet is inclined to the orbital plane, then sublimation leads to non-symmetrical changes of the nucleus shape. Calculations were performed for the nuclei of comets Hale–Bopp and Wirtanen.

Keywords: C/1995 O1 Hale–Bopp, 46P/Wirtanen, cometary nuclei, shape changes, sublimation

1. The Model Description

1.1. Model assumptions

To calculate the sublimation of a comet nucleus we have introduced several assumptions related to the nucleus itself as well as to the sublimation mechanism. The essential assumptions are the following.

1. The nucleus is spherical (initially) and homogeneous (permanently), i.e., we do not consider stratification of the nucleus neither due to the solar radiation flux, nor due to the release of material from the nucleus.
2. The nucleus is composed of water ice and dust. The mass ratio

$$C = (mass\ of\ dust)/(total\ mass) \tag{1}$$

is constant throughout the whole nucleus.

3. During sublimation the water vapor blows away the dusty material, so the composition of the surface layer of the comet's nucleus remains unchanged with time.

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4. The relative change of the solar distance $r(t)$ is negligible during one nucleus rotation period P_{rot}, i.e.:

$$\frac{r(t + P_{rot}) - r(t)}{r(t)} \ll 1. \tag{2}$$

5. Thermal conductivity within the nucleus is neglected (Desvoivres et al., 2000). Our calculations thus concern the maximum sublimation flux, since the whole influx of energy is used for sublimation and none for heating the sub-surface layers of the nucleus. Comparing with the results of Möhlmann (2002) who calculated several mass and energy fluxes involved in the physical processes in the surface layer of the cometary nucleus, we estimate that our assumption does not introduce errors larger than about 30%.

1.2. Model equations

The local sublimation flux $Z = Z[T(z, \varphi)]$ depends only on the local, time-dependent temperature T. Here φ is the cometographic latitude. The zenith distance z is given by the formula

$$\cos z = \cos\theta \cos\varphi \cos\delta + \sin\varphi \sin\delta, \tag{3}$$

where θ is the hour angle and δ is the declination of the Sun.

Z is given by the temperature T of the nucleus surface and by the temperature-dependent water-vapor saturation pressure $p(T)$:

$$Z(T) = p(T)\sqrt{\frac{m_{H_2O}}{2\pi kT}} \quad [kg\ m^{-2}\ s^{-1}]. \tag{4}$$

Here: $m_{H_2O} = 2.988 \cdot 10^{-26}$ kg denote the molecular mass of H_2O, and $k = 1,38 \cdot 10^{-23}$ J K^{-1} is the Boltzmann constant. Following Fanale and Salvail (1984) the water-vapor saturation pressure is

$$p(T[inK]) = 3.56 \cdot 10^{12} \exp\frac{-6141.667}{T} \quad [Pa]. \tag{5}$$

The energy balance equation for the nucleus surface is

$$S\frac{1}{r^2}(1 - A)\cos z = \epsilon\sigma T^4 + H(T)Z(T) \quad [J\ m^{-2}\ s^{-1}]. \tag{6}$$

Here, A is the albedo, ϵ is the emissivity, $S = 1360$ J m^{-2} s^{-1} is the solar constant, and $\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant. The latent heat of sublimation H depends linearly on the surface temperature T (Delsemme and Miller, 1971):

$$H(T) = 2.888 \cdot 10^6 - 1116T \quad [J\ kg^{-1}]. \tag{7}$$

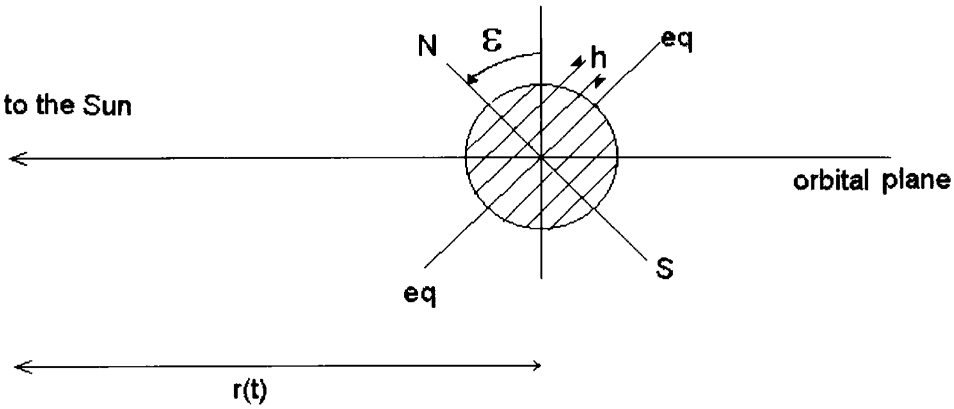

Figure 1. Spin axis orientation and division of the nucleus surface into the latitudinal strips of equal surface area; thus, the "thickness" of all strips is the same.

The sublimation flux Z is calculated versus time t elapsed from perihelion passage. Z is a function of the zenith distance z. The declination δ in Equation (3) depends on the inclination of the rotation axis ε to the normal of the orbital plane (see Figure 1). According to formula (4) the calculation of the local temperature T is crucial for estimating the sublimation flux Z. In order to calculate T, we divided the nucleus surface in N parallel strips of equal surface area, corresponding to different intervals of $\triangle\varphi$. The surface area of such a strip is given by

$$B = 2\pi Rh. \tag{8}$$

Here, R is the radius of the sphere and h is the axial "thickness" of the strip. Hence the strips of equal surface area should have equal "thickness" h (see Figure 1). The solar energy flux is assumed to be φ-independent within each strip. We take its value at mid-h.

The value of the sublimation flux Z can be found numerically by solving the system of Equations (4)–(7), for Z, p, T, and H. This system is supplemented by Equation (3) and by the equations of motion of the nucleus and of its rotation. They contain the angles z, δ, and θ as well as the solar distance r. All of them are time-dependent. To obtain the time-dependent mass loss due to sublimation, $Z_t(\varphi)$ kg m^{-2}, we have to perform the integration of Z over time t from the perihelion passage ($t = 0,\ r = q$) to the position $(t, r(t))$ in the comet orbit: the mass loss $Z_P(\varphi)$ over the whole orbital period P, we integrate the sublimation flux $Z(\varphi)$ until $t = P/2$ and multiply the result by 2 (since in our model the sublimation is symmetrical with respect to perihelion passage).

The decrease of the nucleus radius, $\triangle R(\varphi)$, follows from the sublimation loss of mass, $Z_P(\varphi)$ through formula:

$$\triangle R(\varphi) = \frac{Z_P(\varphi)}{\rho} \cdot \frac{1}{1-C} \quad [m]. \tag{9}$$

TABLE I

Model parameters of comets C/1995 O1 Hale–Bopp and 46P/Wirtanen

Parameter	Symbol	Value	Reference
C/1995 O1 Hale–Bopp			
Eccentricity	e	0.995124	Marsden and Williams, 1997
Semi-major axis	a	187.4820 AU	Marsden and Williams, 1997
Orbital period	P	2500 yrs	Assumed
Rotational period	P_{rot}	10 h	Assumed
Radius	R	20 km	Weaver and Lamy, 1999
Albedo	A	0.03	Assumed from Prialnik, 1997
Emissivity	ϵ	1	Kührt and Keller, 1994
Density	ρ	500 kg m^{-3}	Kührt and Keller, 1994
Composition factor	C	0.5	Assumed
Inclination of spin axis	ε	4 different values assumed	
46P/Wirtanen			
Eccentricity	e	0.656770	Marsden and Williams, 1997
Semi-major axis	a	3.0993 AU	Marsden and Williams, 1997
Orbital period	P	5.46 yrs	Marsden and Williams, 1997
Rotational period	P_{rot}	6 h	Assumed
Radius	R	600 m	Lamy et al., 1998
A, ϵ, ρ, C and ε as for 1995 O1 Hale–Bopp			

ρ denotes the uniform density of the nucleus. For the nucleus, made of water ice and dust, the density is given by:

$$(1-\psi)/\rho = (1-C)/\rho_{ice} + C/\rho_{dust}. \tag{10}$$

Here, ψ is the porosity (the fraction of the voids in the unit volume), ρ_{ice} is the bulk density of non-porous ice, ρ_{dust} is the bulk density of the dust grains, and C is defined by Equation (1). The denominator (1 - C) appearing in Equation (9) suggests that $\triangle R(\varphi)$ goes to infinity when C approaches to 1. However, this formal interpretation of Equation (9) has no physical meaning. One can imagine that for cometary nuclei C is equal to 0.5 or so, see Table I. Therefore the volume ratio *ice* : *dust* is $\approx 3:1$. Assuming that the 'typical' porosity of the nucleus is $\psi \geq 0.5$, we get the volume ratio *voids* : *ice* : *dust* = 4 (or more): 3 : 1. The sublimating water ice drags away the dust grains and therefore the surface of the nucleus recedes. However, this phenomenon cannot be effective for C close to 1, since in that case the volume ratio *ice* : *dust* $\ll 1$ and the gas drag forces are not sufficient to blow away the dust.

For a given (calculate) $\triangle R(\varphi)$ the total moment of inertia I is:

$$I = I_0 \left[1 - \sum_n \left(\frac{\triangle I}{I_0} \right)_n \right] \quad [kg\ m^2] \text{ where } I_0 = \frac{2}{5} M R^2, \tag{11}$$

where n is the number of the surface strip and $(\frac{\triangle I}{I_0})_n$ corresponds to the fractional change of the dimensionless moment of inertia calculated for the n-th strip.

2. Applications of the Model

We have applied our model to two comets: C/1995 O1 Hale–Bopp and 46P/Wirtanen. Their orbital and physical data are summarized in Table I.

It is evident that the long-period comet Hale–Bopp sublimates efficiently only during a small fraction of its orbital period. On the other hand, the sublimation of the nucleus of 46P/Wirtanen may occur during a large part of its orbit. One may expect that the mass-loss of 46P/Wirtanen will influence considerably the moment of inertia of its nucleus, and therefore the rotation properties, on time scale of several orbital periods.

3. Results

In all particular modeling runs we assumed that the winter solstice on the northern hemisphere happened when the comet was passing perihelion. The inclination angle ε of the rotation axis is a parameter. The number of the strips $N = 40$. Figure 2 presents the partial mass-loss due to sublimation Z_t (in kg m^{-2}) versus solar distance r (in AU). When the rotation axis is inclined to the orbital plane, the sublimation pattern is evidently asymmetric. Curves are labeled by the cometographic latitude φ of the middle of the n-th strip. For clarity the results are shown only for selected strips, $n = \pm 1, \pm 2, \pm 5, \pm 8, \pm 11, \pm 14, \pm 17$ and ± 20. The strips on the southern hemisphere have negative numbers. Note that the numbers $|n|$ increase as we go from the equator to the pole. Therefore, $n = \pm 1$ for the near-equatorial strip and $n = \pm 20$ for the near-polar strip.

In Figure 3 the mass losses over a complete orbital revolution are presented. The figure compares the sublimation losses for the nuclei of both comets Hale–Bopp and Wirtanen. The values of the radius decrease $\triangle R(\varphi)$ allowed us to calculate the changes of the moment of inertia of the nucleus. We note that sublimation mass loss can change the moment of inertia of the nucleus of comet Wirtanen much more than that of comet Hale–Bopp.

Figure 4 illustrates the result for the fractional dimensionless moment of inertia over the whole nucleus. We believe that due to simplifying assumptions of our model the results give upper limits for the sublimation flux and therefore also upper

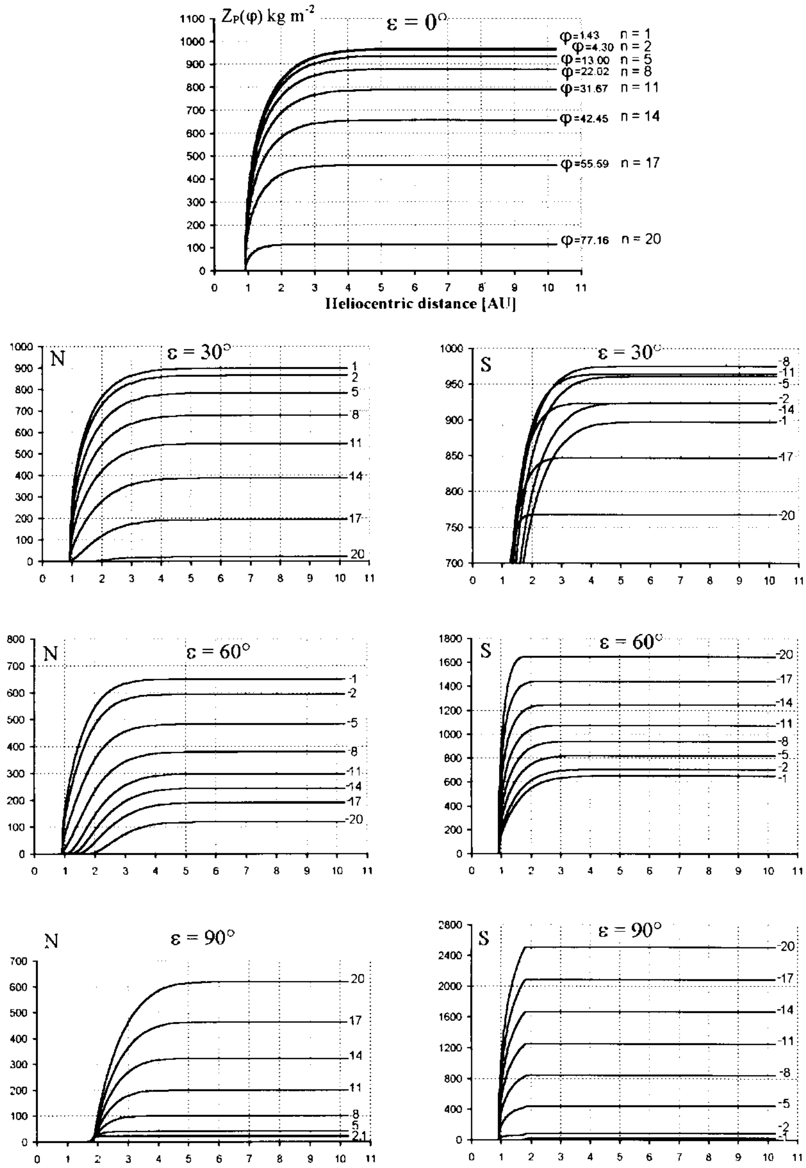

Figure 2. Hale–Bopp nucleus: The partial mass loss due to sublimation Z_t (in kg m^{-2}, vertical axis) versus solar distance r (in AU, horizontal axis). Cometographic latitude φ (i.e., the strip number n) is given as line label. Upper panel is for rotation axis of nucleus perpendicular to the orbital plane, $\varepsilon = 0°$. Lower panels are for $\varepsilon = 30°$, $60°$, and $90°$. The left column is for northern hemisphere, the right column is for southern hemisphere. Winter solstice on the northern hemisphere is assumed to be when the comet passes perihelion. When the rotation axis is inclined, the sublimation pattern is evidently asymmetric.

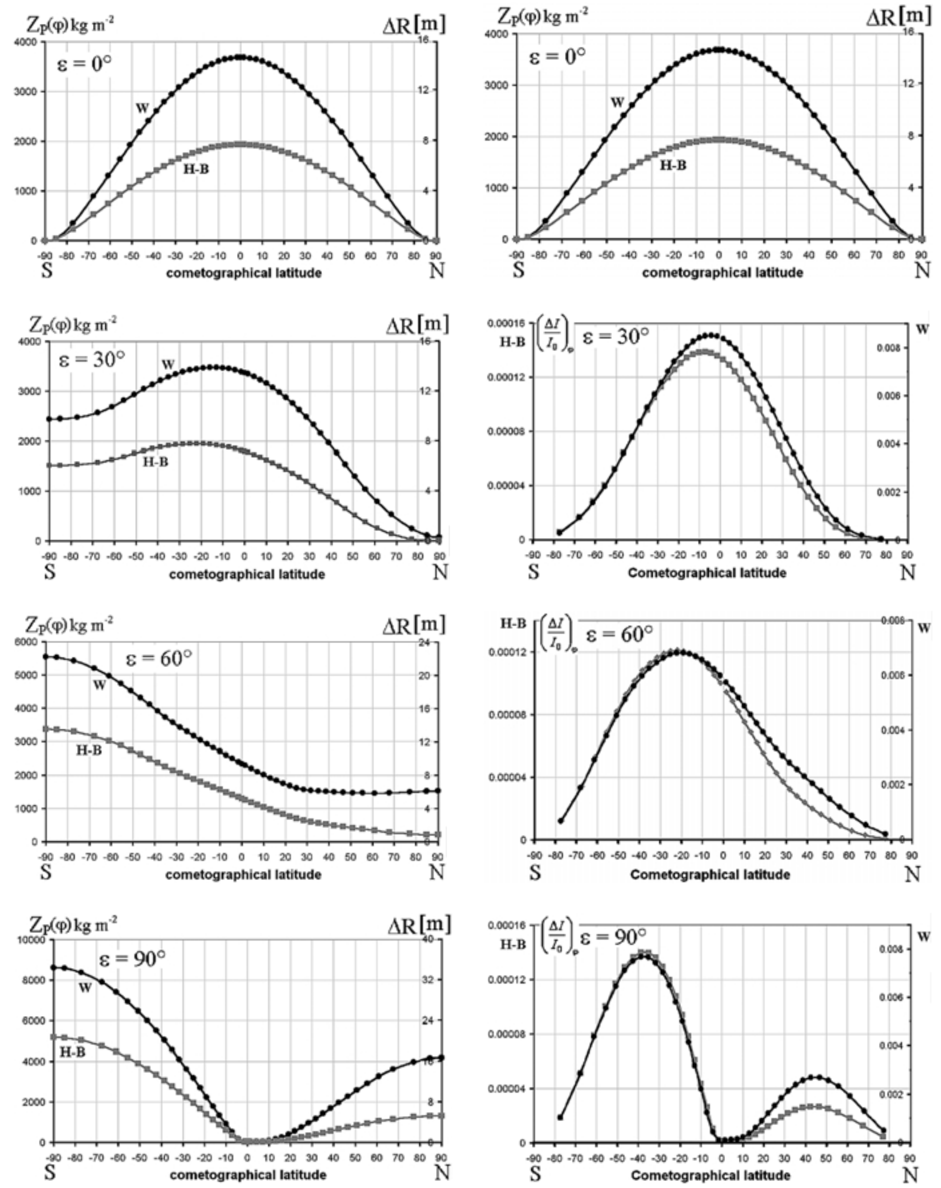

Figure 3. Modelling results for comets Hale–Bopp (line with squares) and Wirtanen (line with circles). The horizontal pairs correspond to the same inclination ε of the nucleus rotation axis. Left panels: Mass-loss due to sublimation Z_P (in kg m^{-2}, left-hand-side vertical axis) and decrease of radius $\triangle R$ (in meters, right-hand-side vertical axis) versus cometographic latitude. The symbols Z_P and $\triangle R$ correspond to the whole orbital period P. Right panels: The changes of the fractional dimensionless moment of inertia $(\frac{\triangle I}{I_0})_n$ of the n-th strip. The scale on the left-hand-side vertical axis is valid for Hale–Bopp, that on the right-hand-side for Wirtanen.

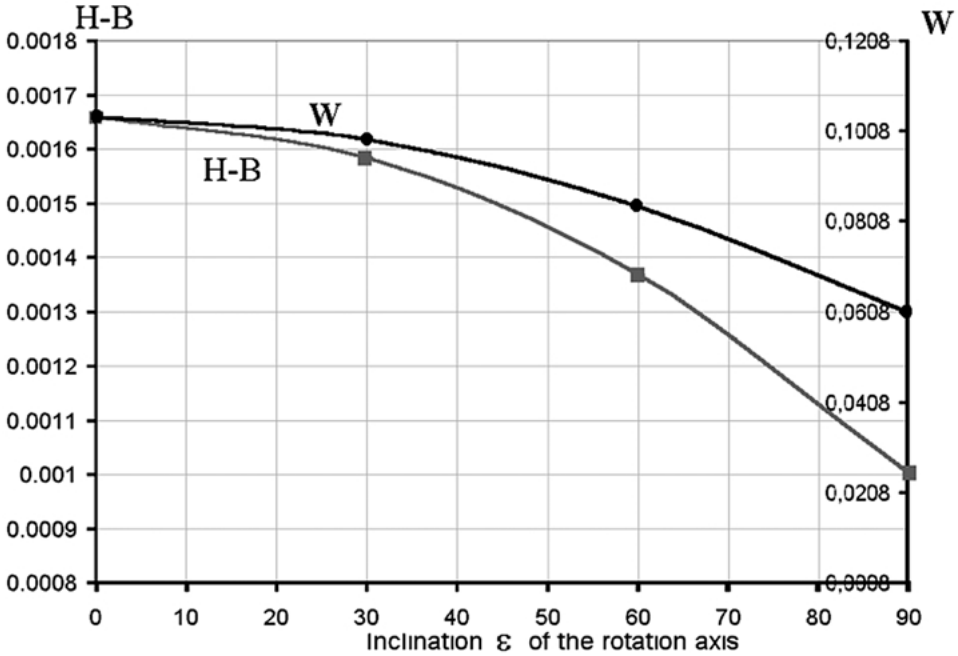

Figure 4. Sublimation induced changes of the total (dimensionless) moment of inertia $\sum(\triangle I/I_0)_n$ versus inclination ε of the rotation axis. Line with squares for Hale–Bopp (H–B, left-hand vertical scale), line with circles for Wirtanen (W, right-hand vertical scale).

limit for the changes of the moment of inertia. However, an important finding is that the sublimation of a small nucleus (e.g., Wirtanen) leads to significant changes of its moment of inertia. Therefore, the rotation parameters (orientation of the rotation axis and period) can be affected as well.

4. Conclusions

The integration of the sublimation rate $Z(\varphi)$ kg m^{-2} s^{-1} over the whole surface of a nucleus leads to the total production rate $Q(H_2O)$ kg s^{-1} or mol s^{-1} of water escaping from the nucleus. Our results of $Q(H_2O)$ for Hale–Bopp are in good agreement with the observations (see Table II). Thus, our simple model seems to be justified.

The calculations of $Z_P(\varphi)$ allow us to obtain the local, φ-dependent, value of the decrease of nucleus radius $\triangle R(\varphi)$. We assumed a nucleus density $\rho = 500$ kg m^{-3} and a nucleus composition with $C = 0.5$. From $\triangle R$ the moment of inertia can be calculated (see the left panels of Figure 3). Therefore, our results can be considered as a first step toward a more advanced analysis of the rotational behavior of cometary nuclei.

TABLE II

Sublimation rate $Q(H_2O)$ in molecules per second from the whole nucleus of Hale–Bopp

r	$Q(H_2O)$, observed		$Q(H_2O)$, this work
[AU]	mol s^{-1}	Reference	mol s^{-1}
4.8	1.5×10^{28}	Weaver et al., 1996	2.5×10^{28}
4.5	4.1×10^{28}	Weaver et al., 1996	5.2×10^{28}
2.88	4.7×10^{29}	Bockelée-Morvan et al., 1996	8.8×10^{29}
2.41	1.1×10^{30}	Bockelée-Morvan et al., 1996	1.7×10^{30}
1.37	2×10^{30}	Flammer et al., 1997	8.5×10^{30}
1.07	3.16×10^{30}	Flammer et al., 1997	1.5×10^{31}
0.914	2×10^{31}	Enzian et al., 1998; estimate	2.2×10^{31}

A comparison of the calculated sublimation pattern, $Z_P(\varphi)$ and $\triangle R(\varphi)$, for the nuclei of comets Hale–Bopp and Wirtanen is presented in Figure 3. However, we note that the assumption of uniform nucleus structure may be much more realistic for comet C/1995 O1 Hale–Bopp than for 46PWirtanen. The crust of the nucleus of comet Hale–Bopp most probably differs considerably from the crust of 46P/Wirtanen.

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