LONG-TERM STABILITY OF GEOIDAL GEOPOTENTIAL FROM TOPEX/POSEIDON SATELLITE ALTIMETRY 1993–1999

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Abstract. The TOPEX/POSEIDON (T/P) satellite altimeter data from January 1, 1993 to October 24, 1999 (cycles 11–261) was used for investigating the long-term variations in the geoidal geopotential W_0 and/or in the geopotential scale factor $R_0 = GM/W_0$ (GM is the adopted geocentric gravitational constant). The mean values determined for the whole period covered are: $W_0 = (62\ 636\ 856.161 \pm 0.002)$ m^{2}s^{-2}, $R_0 = (6\ 363\ 672.5448 \pm 0.0002)$ m. The actual accuracy is limited by the altimeter calibration error (2–3 cm) and it is estimated to be about $\pm$ 0.5 m^2 s^{-2} ($\pm$ 5 cm). The yearly variations of the above mean values are at the formal error level. No long-term trend in W_0, representing the ocean volume change, was found for the seven years period 1993–9 on the basis of T/P altimeter (AVISO) data. No sea surface topography model was used in the solution.

Keywords: Geoidal geopotential, TOPEX/POSEIDON altimetry

1. Introduction

The geopotential value W_0 on the geoid as specified by Gauss, Bessel and Listing will be treated. Gauss (1828) first stipulated that the boundary surface representing

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the Earth should be an equipotential surface, as well as that the portion covering the world oceans should be identical with the world ocean level. Later on, Bessel (1837) defined in detail that it should be an equipotential surface approximating the calm ocean levels, i.e., the mean sea surface (MSS) and/or in recent terminology the sea surface topography (SST). Finally, Listing (1873) has named this surface geoid.

Since the equipotential surface $W = W_0$ is defined as the best fit of MSS covering the world ocean surface (S), the W_0-value can be determined by the TOPEX/POSEIDON (T/P) satellite altimetry corrected for all the usual physical, geophysical and geodetic corrections, see Ménard et al. (1994). This is the main aim of the paper, however, with the emphasis on the stability of the W_0-value during the period of 1993–1999. No SST model was required and used in the solution.

2. Methodology

After adopting the classical definition of the Gauss-Listing geoid, the geoidal geopotential can be determined as follows. First, we express the actual geopotential W_S on S (computed at the T/P altimetry points from the adopted geopotential model such as EGM96, see Lemoine et al. (1997)) as the following series of spherical harmonics:

$$W_S = \sum_{n=0}^{\bar{n}} \sum_{k=0}^{n} (w_n^{(k)} \cos k\Lambda + v_n^{(k)} \sin k\Lambda)\ P_n^{(k)}(\sin\Phi); \tag{1}$$

Φ and Λ stand for the geocentric latitude and longitude respectively, $P_n^{(k)}$ is the associated Legendre function of the degree n and order k and $\bar{n}$ is the maximum degree of the retained harmonics; $w_n^{(k)}$ and $v_n^{(k)}$ are the harmonics coefficients. However, because S represents about 69% of the Earth's surface only, the orthogonality of harmonics over S is not satisfied. In that case

$$\frac{1}{S} \int_S W_S \mathrm{d}S = w_0^{(0)} + \delta w_0^{(0)}, \tag{2}$$

where $\delta w_0^{(0)}$ is the correction due to the non-orthogonality, equal to

$$\delta w_0^{(0)} = \frac{1}{S} \int_S \sum_{n=1}^{\bar{n}} \sum_{k=0}^{n} (w_n^{(k)} \cos\ k\Lambda + v_n^{(k)} \sin\ k\Lambda) P_n^{(k)}(\sin\phi)\ \mathrm{d}S. \tag{3}$$

The geoidal geopotential

$$W_0 = w_0^{(0)} \tag{4}$$

can now be expressed as

$$W_0 = \frac{1}{S}\int_S W_S \, \mathrm{d}S - \delta w_0^{(0)}. \tag{5}$$

However, all the harmonic coefficients $w_n^{(k)}$, $v_n^{(k)}$ should be known for determination of $\delta w_0^{(0)}$, so a successive approximation procedure has to be applied. The coefficients are determined by the following relation:

$$\left\{ \begin{matrix} w_n^{(k)} \\ v_n^{(k)} \end{matrix} \right\} = \left\{ \int_S W_S P_n^{(k)}(\sin\phi) \left\{ \begin{matrix} \cos k\Lambda \\ \sin k\Lambda \end{matrix} \right\} \mathrm{d}S - \int_S \sum_{l=0}^{\bar{n}} \sum_{j=0}^{l} \right.$$

$$\left. (w_l^{(j)} \cos j\Lambda + v_l^{(j)} \sin j\Lambda) P_l^{(j)}(\sin\phi) P_n^{(k)}(\sin\phi) \left\{ \begin{matrix} \cos k\Lambda \\ \sin k\Lambda \end{matrix} \right\} \mathrm{d}S \right\}$$

$$\times \left\{ \int_S [P_n^{(k)}(\sin\phi)]^2 \left\{ \begin{matrix} \cos^2 k\Lambda \\ \sin^2 k\Lambda \end{matrix} \right\} \mathrm{d}S \right\}^{-1}; \quad (l, j) \neq (n, k). \tag{6}$$

Note that the exact determination of W_0 by the above methodology does not require global coverage by the W_S-data. So the fact that the area S of the world oceans amounts to about 69% of the Earth's surface does not diminish the exactness of the solution. Moreover, according to the theory of Molodensky et al. (1962), the determination of W_0 by this procedure requires no spatial description of the surface $W = W_0$.

3. Numerical Solution

There is no problem with numerical application of Equations (5) and (6). We have applied it to the TOPEX/POSEIDON altimeter data 1993–1999 AVISO (1999) cycles 11–261. The altimeter data points were used as preprocessed within AVISO (1999), i.e., after corrections for usual physical, geophysical and geodetic corrections (including the inverted barometer (IB) model to correct for atmospheric pressure). The altimeter data processing has been described in detail by many papers dealing with the AVISO Project, see e.g., Ménard et al. (1994). The geopotential model EGM96 Lemoine et al. (1997) was used for computing the discrete W_S-values at T/P altimeter points. In order to evaluate the surface integrals, the applied weights were proportional to the areas corresponding to the altimeter points. The rms about mean crossover center position was found as ± 1.8 cm. For more details see Burša et al. (1997). Note that for the degree $n > 250$ the harmonics retained in (1) do not contribute significantly to W_0, see Figure 1. This is easy to understand, because, if S were the whole reference sphere and uniformly covered

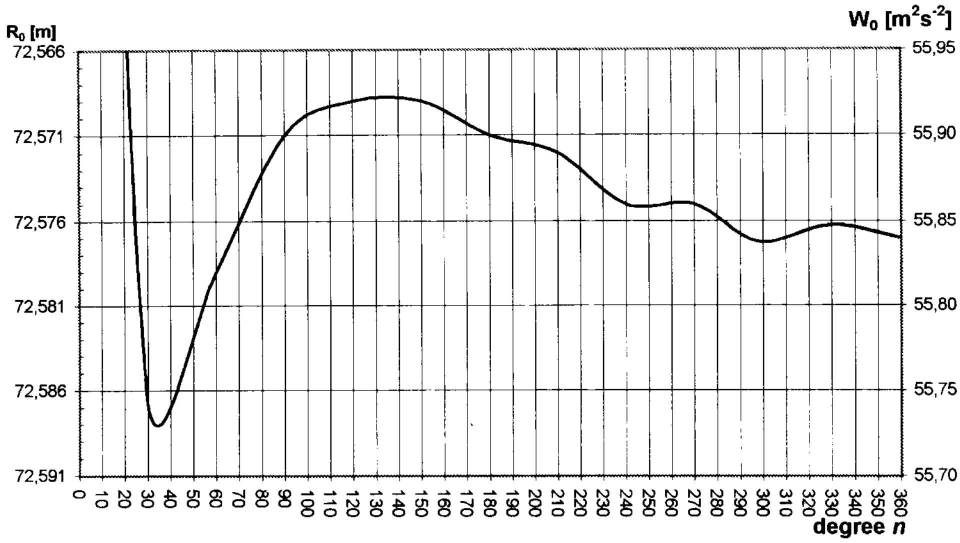

Figure 1. Dependence of W_0 (62 636 800 m^2 s^2 subtracted) and R_0 (6 363 600 m subtracted) on degree n of harmonics retained; geopotential model EGM96.

by a "very large" number of discrete data points, then only the zero-degree harmonic term $n = 0$ of W_S would be responsible for $\int W_S \, dS$. This is due to the exact harmonics orthogonality over the reference sphere. The nonorthogonality correction (3) amounted to (-0.041 ± 0.006) m^2 s^{-2}.

Two fundamental constants were used in the solution: the geocentric gravitational constant of Ries et al. (1992)

$$GM = (398\,600\,441.8 \pm 0.8) \times 10^6 \,\mathrm{m}^3\,\mathrm{s}^{-2} \tag{7}$$

and the nominal mean angular velocity of the Earth's rotation, see IAG SC3 Rep. (1995)

$$\omega = 7292115 \times 10^{-11} \mathrm{rad\ s}^{-1}. \tag{8}$$

The yearly means are listed in Table I. They were computed by collecting and processing all data within one year in one solution; the rms about mean listed in Table I are the formal standard deviations of the means. Application of the IB correction is considered problematic (J. C. Ries, personal communication, 2000), that is why all the AVISO altimeter data were reprocessed with no Inverted Barometer (IB) correction applied in our solution. Note that the IB corrections would have changed the W_0 value by about 0.341 m^2 s^{-2} and correspondingly the R_0-value by about 3.41 cm. This term is largely due to the use of the traditional value of 1013 mb for the mean pressure in the AVISO's IB model, rather than the actual mean pressure value of about 1010.6 mb (J. C. Ries, personal communication, 2000). This difference seems to be quite significant when compared with the rms

TABLE I
Yearly mean values of geoidal geopotential W_o and geopotential scale factor $R_o = GM/W_o$; no IB correction applied

Year	Number of points	W_o [$m^2\ s^{-2}$]	rms [$m^2\ s^{-2}$]	R_o [m]	rms [m]
1993	203 856	62 636 856.157	0.005	6 363 672.5452	0.0005
1994	206 973	62 636 856.168	0.005	6 363 672.5440	0.0005
1995	205 746	62 636 856.163	0.005	6 363 672.5445	0.0005
1996	203 960	62 636 856.158	0.005	6 363 672.5450	0.0005
1997	216 757	62 636 856.157	0.005	6 363 672.5451	0.0005
1998	206 803	62 636 856.162	0.005	6 363 672.5446	0.0005
1999	202 172	62 636 856.159	0.005	6 363 672.5449	0.0005
1993–1999	1 446 267	62 636 856.161	0.002	6 363 672.5448	0.0002

of the yearly means in Table I and the nonorthogonality correction. However, the actual accuracy of the solution is limited mainly by the altimeter calibration error which has been estimated as 2–3 cm (J. C. Ries, personal communication, 2000) that corresponds to (0.2–0.3) $m^2\ s^{-2}$ in W_0. Furthermore, there may be additional biases such those caused by T/P satellite orbit dynamics, troposphere modeling errors, etc. That is why, the 1993–1999 mean estimates and formal errors were adopted as follows

$$W_0 = (62\ 636\ 856.2 \pm 0.5)\ \mathrm{m^2\ s^{-2}}, \tag{9}$$

$$R_0 = GM/W_0 = (6\ 363\ 672.54 \pm 0.05)\ \mathrm{m}. \tag{10}$$

The differences between the yearly mean sea surface (MSS) levels came out as follows (see Table I):

$$\begin{aligned} &1993\text{–}4: && -(1.2 \pm 0.7)\ \mathrm{mm},\\ &1994\text{–}5: && (0.5 \pm 0.7)\ \mathrm{mm},\\ &1995\text{–}6: && (0.5 \pm 0.7)\ \mathrm{mm},\\ &1996\text{–}7: && (0.1 \pm 0.7)\ \mathrm{mm},\\ &1997\text{–}8: && -(0.5 \pm 0.7)\ \mathrm{mm},\\ &1998\text{–}9: && (0.3 \pm 0.7)\ \mathrm{mm}. \end{aligned}$$

The corresponding rate of change in the MSS level (or R_0) during the whole 1993–1999 period covered is

$$1993 - -1999 : (0.03 \pm 0.08)\ \mathrm{mm/y}.$$

The above rate and formal error are with respect to the reference frame implied by the T/P orbits. Currently the best, state-of-art, International Terrestrial Frames show apparent stability at the 0.5–1.0 mm/y level, see IERS (1998). Consequently, the above formal error should be increased to about 0.5–1.0 mm/y.

4. Quantities Responsible for Theoretical Variations of W_0

There are three quantities defining $W_0 : GM, \omega$ and the volume τ enclosed by surface $W = W_0$.

The rms of the recent value (7) of GM (i.e., $\pm 8 \times 10^5$ m^3 s^{-2}) contributes to the rms of W_0 by about $\pm$ 0.13 m^2 s^{-2}. Furthermore, currently there is no evidence that the variation of GM due to the variation in G exceeds the rate of 800 m^3 s^{-2}/year. Consequently, the annual variation in W_0 due to GM can be estimated as $<$ 0.0002 m^2 s^{-2} (0.02 mm).

Regarding ω, there is a large spectrum of variations. However, from the point of view of the long-term stability of W_0 which is being considered here, only the long-term variation, see IAG SC3 Rep. (1995)

$$\mathrm{d}\omega/\mathrm{d}t = (-4.5 \pm 0.1) \times 10^{-22}\,\mathrm{rad\ s}^{-2} \tag{11}$$

is to be discussed. It yields only $\mathrm{d}W_0/\mathrm{d}t = -4 \times 10^{-3}\,\mathrm{m}^2\,\mathrm{s}^{-2}$/cy and it is not practically significant at present.

The third quantity defining W_0, the volume τ, requires the following question: Which phenomena cause the long-term variations in τ?

First, let us ask, whether any harmonics $n \neq 0$ in the disturbing potential does change τ. Let us have an auxiliary reference sphere S_{ref} with the radius $\bar{R}$, say the mean Earth's radius ($\bar{R} \approx$ 6 371 km). The actual geopotential W on S_{ref} is not constant. It can be expressed with the use of the Stokes geopotential coefficients $J_n^{(k)}$, $S_n^{(k)}$ (e.g., model EGM96) as

$$W = \frac{GM}{\bar{R}} \Bigg\{ 1 + \sum_{n=2}^{\bar{n}} \sum_{k=0}^{n} \left(\frac{a_0}{\bar{R}}\right)^n (J_n^{(k)} \cos k\Lambda + \\ + S_n^{(k)} \sin k\Lambda) P_n^{(k)}(\sin\phi) + \frac{1}{3} q \left(\frac{a_0}{R}\right)^{-3} [1 - P_2^{(0)}(\sin\phi)] \Bigg\}, \tag{12}$$

$$q = \frac{\omega^2 a_0^3}{GM};$$

a_0 = 6 378 136.3 m is the scaling factor rendering $J_n^{(k)}$ and $S_n^{(k)}$ dimensionless. The mean value $\bar{W}$ of W on S_{ref} is

$$\bar{W} = \frac{1}{S_{\mathrm{ref}}} \int_{S_{\mathrm{ref}}} W \mathrm{d}S = \frac{1}{4\pi \bar{R}^2} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} W \bar{R}^2 \cos\phi \,\mathrm{d}\Lambda \mathrm{d}\phi, \tag{13}$$

and, because of orthogonality of the spherical harmonics on $S_{\rm ref}$

$$\bar{W} = \frac{GM}{\bar{R}}\left[1 + \frac{1}{3}q\left(\frac{a_0}{\bar{R}}\right)^{-3}\right] = \frac{GM}{\bar{R}} + \frac{1}{3}\,\omega^2\bar{R}^2. \tag{14}$$

It is a function of GM, ω and $\bar{R}$ only, no harmonic term $n \neq 0$ comes into account. It remains constant at any harmonic term $n \neq 0$. Consequently, radius $\bar{R}$ of $S_{\rm ref}$

$$\bar{R} = \frac{GM}{\bar{W}}\left(1 + \frac{1}{3}\,\frac{\omega^2\bar{R}^3}{GM}\right) \tag{15}$$

depends on GM, ω and $\bar{W}$ only, it does not depend on the harmonics $n \neq 0$. We wish to specify

$$\bar{W} = W_0; \tag{16}$$

then

$$\bar{R} = 6\,370\,990.29 \text{ m}. \tag{17}$$

Let us note that the mean radius $\bar{\varrho}$ of surface $W = W_0$ can be expressed as:

$$\bar{\varrho} = \frac{1}{S}\int_S \varrho \,\mathrm{d}S, \tag{18}$$

$$\varrho = R_0\left[1 + A_0^{(0)} + \sum_{n=2}^{\bar{n}}\sum_{k=0}^{n}(A_n^{(k)}\cos k\Lambda + B_n^{(k)}\sin k\Lambda)P_n^{(k)}(\sin\phi)\right], \tag{19}$$

$$\begin{aligned}\mathrm{d}S &= (EG - F^2)^{1/2}\mathrm{d}\phi\,\mathrm{d}\Lambda\\ &= \varrho\left\{\left[\varrho^2 + \left(\frac{\partial\varrho}{\partial\phi}\right)^2\right]\cos^2\phi + \left(\frac{\partial\varrho}{\partial\Lambda}\right)^2\right\}^{1/2}\mathrm{d}\phi\,\mathrm{d}\Lambda;\end{aligned} \tag{20}$$

The expressions for the harmonic coefficients $A_n^{(k)}$ and $B_n^{(k)}$ as functions of $J_n^{(k)}$, $S_n^{(k)}$ and q can be found, e.g., in Burša (1970); E, G and F stand for the fundamental Gaussian quantities of the first order in the first quadratic form.

Retaining the terms of the order of 10^{-9} in magnitude, one gets ($\nu = a_0/R_0$)

$$\begin{aligned}\bar{\varrho} = R_0(1 &+ A_0 + B_0 + A_0B_0 + \frac{1}{5}A_2B_2 + \frac{1}{9}A_4B_4\\ &+ \frac{1}{13}A_6B_6)(1 + B_0)^{-1},\end{aligned} \tag{21}$$

$$A_0 = \frac{1}{3}\nu^{-3}q + \frac{2}{5}\nu^{-6}q^2 - \frac{1}{15}\nu^{-1}J_2^{(0)}q - \frac{2}{5}\nu^4(J_2^{(0)})^2 + \\ + \frac{24}{35}\nu^{-9}q^3 - \frac{8}{35}\nu^{-4}J_2^{(0)}q^2 + \frac{1}{21}\nu(J_2^{(0)})^2q + \\ + \frac{2}{5}\nu^6(J_2^{(0)})^3, \tag{22}$$

$$A_2 = \nu^2 J_2^{(0)} - \frac{1}{3}\nu^{-3}q - \frac{4}{7}\nu^{-6}q^2 + \frac{5}{21}\nu^{-1}J_2^{(0)}q - \frac{4}{7}\nu^4(J_2^{(0)})^2 - \\ - \frac{8}{7}\nu^{-9}q^3 + \frac{4}{7}\nu^{-4}J_2^{(0)}q^2 - \frac{1}{21}\nu(J_2^{(0)})^2q + \\ + 3\nu^6(J_2^{(0)})^3 - \frac{12}{7}\nu^6 J_2^{(0)}J_4^{(0)} + \frac{2}{21}\nu J_4^{(0)}q, \tag{23}$$

$$A_4 = \nu^4 J_4^{(0)} + \frac{6}{35}\nu^{-6}q^2 - \frac{6}{35}\nu^{-1}J_2^{(0)}q - \frac{36}{35}\nu^4(J_2^{(0)})^2 + \\ + \frac{216}{385}\nu^{-9}q^3 - \frac{192}{385}\nu^{-4}J_2^{(0)}q^2 + \frac{6}{77}\nu(J_2^{(0)})^2q + \\ + \frac{108}{55}\nu^6(J_2^{(0)})^3 - \frac{120}{77}\nu^6 J_2^{(0)}J_4^{(0)} - \frac{19}{77}\nu J_4^{(0)}q, \tag{24}$$

$$A_6 = \nu^6 J_6^{(0)} - \frac{8}{77}\nu^{-9}q^3 + \frac{12}{77}\nu^{-4}J_2^{(0)}q^2 - \frac{6}{77}\nu(J_2^{(0)})^2q + \\ + \frac{18}{11}\nu^6(J_2^{(0)})^3 - \frac{30}{11}\nu^6 J_2^{(0)}J_4^{(0)} + \frac{5}{33}\nu J_4^{(0)}q; \tag{25}$$

$$B_0 = 2A_0 + (A_0)^2 + \frac{4}{5}(A_2)^2, \tag{26}$$

$$B_2 = 2A_2 + 2A_0A_2 + \frac{5}{7}(A_2)^2 + \frac{4}{7}A_2A_4 - \frac{6}{7}A_0(A_2)^2, \tag{27}$$

$$B_4 = 2A_4 - \frac{18}{35}(A_2)^2 + 2A_0A_4 + \frac{72}{35}A_0(A_2)^2 + \frac{100}{77}A_2A_4, \tag{28}$$

$$B_6 = 2A_6 - \frac{30}{11}A_2A_4. \tag{29}$$

Numerically,

$$\bar{\varrho} = 6\ 371\ 006.38 \text{ m}, \tag{30}$$

which significantly differs from the value (17). Note that the surface area S in Equation (18) was expressed as

$$S = 4\pi R_0^2(1 + B_0). \tag{31}$$

In this context, we wish to clarify the physical meaning of the geopotential scale factor

$$R_0 = \frac{GM}{W_0} = (6\,363\,672.54 \pm 0.05)\ \text{m}. \tag{32}$$

After substituting (16) into (15), and introducing R_0 by Equation (32), one gets the relation

$$R_0 = \bar{R}\left(1 + \frac{1}{3}\,\frac{\omega^2 \bar{R}^3}{GM}\right)^{-1}. \tag{33}$$

It is just the length of the semi-minor axis of the rotating body, with its mass concentrated at its center and with the equipotential surface boundary. Such a fictitious body is called Roche model, see Kopal (1989). The geocentric radius ρ of the equipotential surface $W = W_0$ appropriate to the Roche Earth's model is specified as

$$W_0 = \frac{GM}{\rho}\left\{1 + \frac{1}{3}q\left(\frac{a_0}{\rho}\right)^{-3}[1 - P_2^{(0)}(\sin\phi)]\right\}. \tag{34}$$

No particle outside of its center gives rise to the gravity potential of the body. It may appear strange, however, this is the fundamental property of the Roche model (see Kopal, 1989). The body rotates with the uniform angular velocity, excepting particles situated only on the rotation axis which is the small axis of the Roche-body. The gravity potential at the points situated at the ends of the rotation axis is equal to the gravitational potential, then $W_0 = GM/R_0$. Consequently, any variation in (15) and/or in (18), or in the volume τ enclosed by the reference sphere, the radius (17), and/or by the equipotential surface $W = W_0$ gives rise to the variation in W_0. On the contrary, W_0 is invariant with respect to the harmonics $n \neq 0$. For example, the tidal potential, as well as, the free nutation disturbing potential (due to the Chandlerian polar motion) and temporal variations of the second zonal geopotential (Stokes) coefficient J_2 being harmonic ($n \geq 2$), do not disturb W_0.

Let us now pose the question as follows: to what extent is the geopotential on the surface $W = W_0$ of the area S, disturbed by the transfer of mass between the body O of the world oceans and the body A of the atmosphere?

A simplified spherical model will be used. The body of A is represented by the homogeneous spherical layer, the geocentric radii R_1, R_2 (see Figure 2) and the density σ_A. The body O is also approximated by the homogeneous spherical layer with radii R, R_1 and the density σ_O. Let P be an arbitrary point within the body of

A, the position of which be specified by the geocentric radius ρ. The gravitational potential V at P due to the bodies A and O, then is

$$V(P) = V(\rho) = V_A(\rho) + V_O(\rho), \tag{35}$$

$$V_A(\rho) = 2\pi G\sigma_A\, R_2^2 - \frac{2}{3}\pi G\sigma_A\, \rho^2 - \frac{4}{3}\pi G\sigma_A \frac{R_1^3}{\rho}, \tag{36}$$

$$V_O(\rho) = \frac{GM_O}{\rho}; \tag{37}$$

$$M_O = \frac{4}{3}\pi\sigma_O(R_1^3 - R^3)$$

where M_O is the mass of the body O and G is the Newtonian gravitational constant. The gravitational acceleration at P due to bodies A and O is evidently

$$\begin{aligned} g(P) \;=\; & -\frac{\partial V(P)}{\partial\rho} = \frac{4}{3}\pi G\sigma_A\rho - \frac{4}{3}\pi G\sigma_A\frac{R_1^3}{\rho^2} + \\ & + \frac{4}{3}\pi G\frac{\sigma_O}{\rho^2}(R_1^3 - R^3). \end{aligned} \tag{38}$$

Let point P be now shifted to P_1 situated at the ocean–atmosphere boundary sphere. Then, substituting $\rho = R_1$ we get from (36)

$$V(P_1) = 2\pi G\sigma_A(R_2^2 - R_1^2) + \frac{GM_O}{R_1} \tag{39}$$

and from (38)

$$g(P_1) = \frac{GM_O}{R_1^2}. \tag{40}$$

Let us suppose that the boundary sphere of the radius R_1 be fixed in the space, however the transfer of mass δM occur between bodies O and A, while satisfying the condition

$$M_A + M_O = \text{constant}; \tag{41}$$

M_A stands for the mass of the atmosphere by Verniani (1966)

$$M_A = (5.136 \pm 0.007) \times 10^{18}\,\text{kg}, \tag{42}$$

$$(GM_A = 3.43 \times 10^8\,\text{m}^3\,\text{s}^{-2}).$$

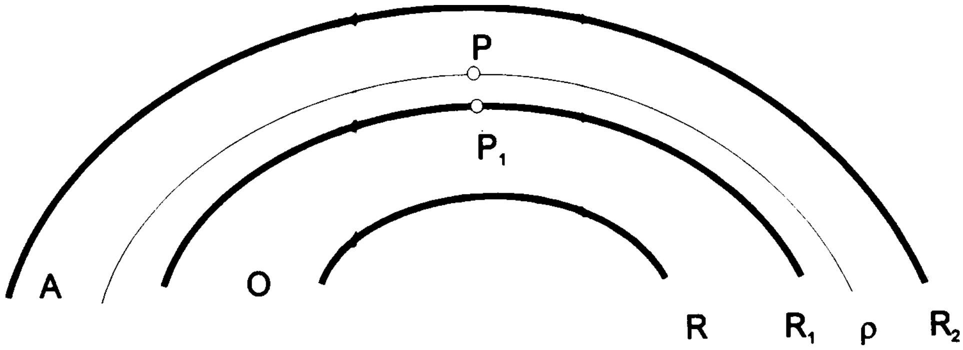

Figure 2. Spherical body models of the atmosphere (A) – ocean (O).

According to Trenberth (1981), the seasonal transfer of mass amounts

$$\delta M = 1.3 \times 10^{16}\,\mathrm{kg}. \tag{43}$$

Because of (43), potential $V(P_1)$ should vary about

$$|\delta V(P_1)| \approx \frac{G\delta M}{R_1}\frac{R_2 - R_1}{2R_1}\left(1 - \frac{2}{3}\frac{R_2 - R_1}{R_1}\right) + \\ +\text{terms containing}\left(\frac{R_2 - R_1}{R_1}\right)^3. \tag{44}$$

Numerically, assuming $R_2 - R_1 \approx 30$ km, then the seasonal variation is about

$$|\delta V(P_1)| = 3.2 \times 10^{-4}\,\mathrm{m^2\,s^{-2}} \tag{45}$$

which amounts only to about 0.03 mm in the radial distortion. However, the variation in the gravity acceleration within the model adopted is relatively large:

$$\delta g(P_1) = 2.1\mu\mathrm{gal}. \tag{46}$$

This is due to the variation of δM only. The corresponding shift of the perturbed equipotential surface is about 7 mm.

Note that value (7) includes M_A, i.e., $GM = GM_E + GM_A$; M_E here stands for the Earth's mass. This raises a question about the exactness of the W_0-determination at P_1 (Figure 2) when the value (7) is used. Analogously to Equation (39), the gravitational potential at P_1 due to the above model is equal to

$$V(P_1) = 2\pi G\sigma_A(R_2^2 - R_1^2) + \frac{GM_E}{R_1}. \tag{47}$$

Since

$$2\pi G\sigma_A(R_2^2 - R_1^2) = \frac{GM_A}{R_1}(1 + \frac{1}{2}\frac{R_2 - R_1}{R_1}), \tag{48}$$

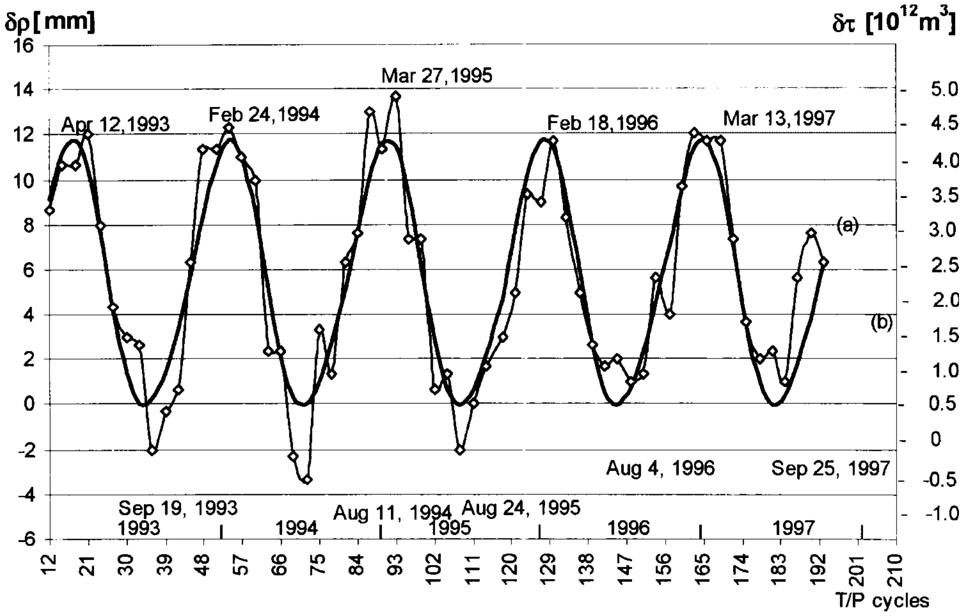

Figure 3. Variations (curve (a)) in the mean radius ρ of the global ocean level and in its volume τ, harmonics $\bar{n} = 8$ retained; $\bar{IB}$ correction applied. The curve (b) represents the analytical expression (6).

(47) can be rewritten as

$$V(P_1) = \frac{GM_E + GM_A}{R_1} + \frac{1}{2}\frac{GM_A}{R_1}\frac{R_2 - R_1}{R_1}. \tag{49}$$

That is why, in computing $V(P_1) = GM/R_1$ we neglect the second term on the right hand side of (49) which amounts to about 0.13 m^2 s^{-2} for $GM_A = 3.427 \times 10^8$ m^3 s^{-2}, i.e., about 1.3 cm in the radial direction. However, the total geopotential due to the atmosphere at P_1 amounts to about 53.79 m^2 s^{-2} and the corresponding stationary vertical shift of the equipotential surface is about 5.5 m.

The analysis of the T/P altimeter data 1993–1997 (cycles 11–194) with emphases on seasonal variations of MSS resulted in variations $\delta\varrho$ in the mean radius ϱ of the global ocean level and/or in the corresponding variations $\delta\tau$ in the volume τ of the ocean, see Figure 3, the curve (a). The best fitting analytical representation, the curve (b) is

$$\begin{aligned} \delta\varrho \;=\; & (5.6 \pm 0.2) + (5.9 \pm 0.3)\cos[\omega t - 61.1° \pm 2.9°)] + \\ & +(0.6 \pm 0.3)\cos 2[\omega t - (90.3° \pm 15.0°)]. \end{aligned} \tag{50}$$

Since we are interested only in the long-term variations of W_0, the numerical values of the above, purely periodical, model with seasonal periods, are not relevant here. We only wished to illustrate the point, namely that the variations in W_0 may be due to the transfer of mass between the atmosphere and the ocean.

However, besides of the mass transfer discussed above, there are other phenomena responsible for the variations in the volume τ, e.g., changes in the salinity of sea water and its temperature (steric sea level change).

5. Conclusions

The T/P altimeter data covering the seven year period of 1993–1999, cycles 11–261, used for the determination of W_0 and R_0-parameters, indicate that there are no long-term variations at the 1 mm level.

The W_0 and/or R_0 values were considered to be stable during the 1993–1999 period at least at the 0.01 m^2 s^{-2} and 1 mm level, respectively. In other words, within the 1 mm/y uncertainly level, no statistically significant mean sea level increase was observed in our analysis.

Values W_0 and/or R_0 are relatively quite stable, they depend only on GM, ω and τ and, that is why, W_0 or R_0 are considered to be suitable for adoption as primary astrogeodetic parameters.

Furthermore, the W_0-value provides a scale parameter for the Earth that is independent of the tidal reference system.

The W_0-value can also uniquely define the geoidal surface and it is required for a number of applications, including the General Relativity corrections in precise time keeping and time definitions. W_0 is essential for transformation of the observed Terrestrial Time (TT) to the Geocentric Coordinate Time (TGC) which is the currently adopted conventional (relativistic) reference time frame.

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