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Erratum

Cascante, C., Ortega, J.M. and Verbitsky, I.E.: 'Wolff's inequality for radially nonincreasing kernels and applications to trace inequalities', *Potential Anal.* **16** (2002), 347–372.

The purpose of the paper was to prove a Th. Wolff-type inequality for radially nonincreasing kernels, and apply it to study some trace inequalities. We would like to point out that Lemma 2.6, and hence some of its consequences, are not valid as stated. However, the main results of the paper remain true for the version of Wolff's potential $\mathcal{W}_{K\,p}$ introduced in the paper, which was defined by (2.5), but not necessarily for the "larger" version $\overline{\mathcal{W}}_{K\,p}$ defined by (2.6). More precisely, the following changes are needed:

- In the statement of Lemma 2.6, λ_Q has to be replaced by $\lambda_Q\,\chi_Q$, where χ_Q is the characteristic function of the dyadic cube Q. Then the proof given in the paper is correct. It is in this form that Lemma 2.6 was mostly used in the paper.
- Proposition 4.2 and Proposition 4.4(a) can be improved so that the case $s > 2$ is covered as well. In fact, the following proposition holds, which replaces Lemma 2.7 and Proposition 4.4(b), which are not true as stated. The proof uses the revised Lemma 2.6 and induction in $k < s \leqslant k+1$ as in the proof of Proposition 4.4(a). In what follows, it is assumed that σ is a positive locally finite Borel measure on $\mathbf{R}^n$, and λ_Q are nonnegative reals such that $\lambda_Q = 0$ if $|Q|_\sigma = 0$.

PROPOSITION 0.1. *Let $1 < s < \infty$. Then there exist constants $C_i > 0$, $i = 1, 2, 3$, which depends only on s, such that, for any $\Lambda = (\lambda_Q)_{Q\in\mathcal{D}}$, $\lambda_Q \in \mathbf{R}^+$,*

$$A_1(\Lambda) \leqslant C_1\, A_2(\Lambda) \leqslant C_2\, A_3(\Lambda) \leqslant C_3\, A_1(\Lambda),$$

where A_1, A_2 and A_3 are defined in Section 4 of the paper.

We can now complete the proof of Theorem 4.3, the discrete Wolff theorem, for the potential $\mathcal{W}^d_{K\,p}$, just by applying Proposition 0.1 with $\lambda_Q = K(r_Q)r^n_Q\mu(Q)$, $\mathrm{d}\sigma = \mathrm{d}x$, and $s = p'$, from which it follows that

$$\mathcal{E}_{K_{\mathcal{D}}\,p}(\mu) \simeq \int_{R^n} \mathcal{W}^d_{K\,p}\mu\,\mathrm{d}\mu.$$

- The proof of the discrete trace inequality, Theorem 4.5, with $\overline{\mathcal{W}}^d_{K\,p}\mu$ replaced by $\mathcal{W}^d_{K\,p}\mu$, follows the lines of the proof of the equivalence of (i) and (ii) of Theorem 3.4.
- The extension to the continuous version of Wolff's theorem, Theorem 2.9, for the continuous potential $\mathcal{W}_{K\,p}$ in place of $\overline{\mathcal{W}}_{K\,p}$ can be obtained from the discrete one as follows: By the Kerman–Sawyer inequality, in the definition of the energy $\mathcal{E}_{K\,p}(\mu)$ we can replace the kernel $K(x)$ by $K(cx)$ for any $c > 0$ since $\bar{K}$ in the maximal operator M_K satisfies the doubling condition. Next, a Fefferman–Stein-type argument as in Proposition 2.8 gives that $T_K f(x)$ is pointwise bounded by the average of the shifted dyadic potentials $T_{\tilde{K}_{\mathcal{D}+z}} f(x)$, where we put $\tilde{K}(x) = K(x/2)$ in place of $K(x)$, and use the fact that K is nonincreasing.
  Now we can apply the dyadic Wolff inequality to show that
  $$\begin{aligned}\mathcal{E}_{K\,p}(\mu) &\leqslant C \sup_z \int_{\mathbf{R}^n} \widetilde{\mathcal{W}}^{d,z}_{K\,p}\mu \,\mathrm{d}\mu \\ &= C \sup_z \sum_{Q\in\mathcal{D}+z} K\left(\frac{r_Q}{2}\right)\bar{K}(r_Q)^{p'-1}|Q|\,\mu(Q)^{p'}.\end{aligned}$$
  We can estimate $\mu(Q)^{p'}$ by the finite sum: $\sum \mu(Q')^{p'}$ where Q' are the preceding generation of the 2^n cubes of sidelength $r_Q/2$ contained in Q. Then $K(r_Q/2) = K(r_{Q'})$, and $\bar{K}(r_Q)$ as well as $|Q|$ has the doubling property. Notice that actually we can put any constant $0 < c < 1$ in place of $1/2$, i.e., in the expression
  $$\int_{\mathbf{R}^n} \mathcal{W}^d_{K\,p}\mu \,\mathrm{d}\mu = \sum_Q K(r_Q)\bar{K}(r_Q)^{p'-1}|Q|\,\mu(Q)^{p'}$$
  we can replace $K(x)$ by $K(cx)$ for any $c > 0$. Finally, the pointwise inequality (2.9) gives Wolff's inequality for the continuous kernel:
  $$\mathcal{E}_{K\,p}(\mu) \leqslant C \int_{\mathbf{R}^n} \mathcal{W}_{K\,p}\mu \,\mathrm{d}\mu.$$
- The continuous trace inequality for the potential $\mathcal{W}_{K\,p}$ stated in Theorem 3.4, is also derived from the discrete trace inequality given in Theorem 4.5. We just need to observe that in the expression
  $$\int_{\mathbf{R}^n} (\mathcal{W}^d_{K\,p}\mu)^r \,\mathrm{d}\mu$$
  we can replace $K(x)$ by $K(cx)$, for any $c > 0$, as well. Here $r = q(p-1)/(p-q) > 1$ since $q > 1$. Indeed, if we linearize, we get
  $$\begin{aligned}\int_{\mathbf{R}^n} (\widetilde{\mathcal{W}}^d_{K\,p}\mu)^r \,\mathrm{d}\mu &\leqslant \int_{\mathbf{R}^n} \widetilde{\mathcal{W}}^d_{K\,p}\mu\, g \,\mathrm{d}\mu \\ &= \sum_Q K(cr_Q)\bar{K}(r_Q)^{p'-1}|Q|\mu(Q)^{p'}\frac{1}{\mu(Q)}\int_Q g\,\mathrm{d}\mu,\end{aligned}$$

for some $g \geqslant 0$ with $||g||_{L^r(\mu)} \leqslant 1$. Next we estimate again $\mu(Q)^{p'}$ by a finite sum of $\sum_{Q'} \mu(Q')^{p'}$ where Q' are in a fixed number (depending on c) of the preceding generations of dyadic cubes contained in Q. Also, we use the inequality

$$\frac{1}{\mu(Q)} \int_Q g \, \mathrm{d}\mu \leqslant \frac{1}{\mu(Q')} \int_{Q'} M_\mu^d g \, \mathrm{d}\mu.$$

Thus

$$\int_{\mathbf{R}^n} (\widetilde{\mathcal{W}}_{K\,p}^d \mu)^r \, \mathrm{d}\mu \leqslant C \int_{\mathbf{R}^n} \mathcal{W}_{K\,p}^d \mu \, M_\mu^d g \, \mathrm{d}\mu,$$

and Hölder's inequality together with the fact that the dyadic maximal operator M_μ^d is bounded on $L^{r'}(d\mu)$ gives that

$$\int_{\mathbf{R}^n} (\widetilde{\mathcal{W}}_{K\,p}^d \mu)^r \, \mathrm{d}\mu \leqslant C \int_{\mathbf{R}^n} (\mathcal{W}_{K\,p}^d \mu)^r \, \mathrm{d}\mu.$$

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