ON THE DYNAMICS OF DUST GRAINS IN A HIERARCHICAL ENVIRONMENT

II. Triple Case

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Abstract. The star formation process usually leads to the formation of protoplanetary disks. Planets are thought to arise from the material of these disks. Amongst the stars in the solar neighbourhood, single systems like our own are a minority. Most stars are found in binaries or in systems of even higher multiplicity. In this paper, we extend the simulations presented in Paper I (de la Fuente Marcos and de la Fuente Marcos, 1998a) to hierarchical triple systems. As in Paper I, we study the stage of planetary formation during which the particulate material is still dispersed as centimetre-to-metre sized primordial aggregates. We investigate the response of the particles, in a protoplanetary disk with radius R_D = 100 AU around a solar-like star, to the gravitational field of bound perturbing companions in a moderately wide (300–1600 AU) orbit. As for this problem no analytic description of the orbital evolution of the particles exists, we perform numerical integrations using a Bulirsch–Stoer integrator. For this purpose, we have carried out a series of simulations of coplanar hierarchical configurations with three stars using a direct integration code that models gravitational and viscous forces. The massive protoplanetary disk is around one of the components of the triple system. As in Paper I, the evolution in time of the dust sub-disk depends mainly on the nature (prograde or retrograde) of the relative revolution of the stellar companions, and on the temperature and mass of the circumstellar disk. The perturbation of prograde companions induces a trailing spiral structure across the protoplanetary sub-metric dust sub-disk. Metre-sized particles are affected by strong precession. Our results show that the lifetime of particles in a disk in a hierarchical triple system is slightly shorter relative to its value in Paper I, although the actual value depends on the nature, prograde or retrograde, of the outer companion. The lifetime of particles in a hierarchical triple system including a prograde inner binary and a retrograde outer body is longer than in an equivalent triple with all the companions prograde. Dust disks in hierarchical triple systems with both companions in retrograde motion with respect to the particles in the disk show the shortest lifetimes. Our previous calculations suggest that disk luminosities in binary systems are several orders of magnitude higher than those for single stars. Circumstellar disks in triple systems may be 5–50% more luminous depending on the relative direction of rotation.

Keywords: Accretion, accretion disks, celestial mechanics, stellar dynamics, solar system: formation, circumstellar matter, planetary systems, triple systems, stars: pre-main sequence

1. Introduction

The star formation process in molecular clouds usually leads to the formation of multiple stellar systems, mostly binaries. In fact, amongst the youngest stars, the frequency of companions appears to be higher than in older systems (e.g., Ghez,

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1995). Loose T associations show a binary (in general multiple) frequency much higher than that observed in typical open clusters as the Pleiades. This difference does not result from the evolution of the binary systems during the pre-main sequence but it is intrinsic to the type of clustering. Low binary frequency is typical of stars formed in dense protostellar clusters, while the higher binary frequency is observed in T Tauri star clusters. If multiple systems are so common, they may have a main role in planet formation.

Nearly one-third of all binary star systems are thought to be members of larger multiple systems. Most of these are hierarchical triples, in which the (inner) binary is orbited by a third body in a much wider orbit (Tokovinin, 1997a, b). Hierarchical triple systems with G-dwarf components are not unusual in the solar neighbourhood. Duquennoy and Mayor (1991) give the following ratio of single:double:triple:quadruple among the 164 nearest G-dwarf stars: 1.5(91 systems):1(62):0.105(7):0.026(2); they also point out that the number of triple and quadruple systems may be larger. However, the number of triple systems with well-determined orbital elements is still small (Tokovinin, 1999).

Presence of circumstellar disks around T Tauri stars has been suspected for a longtime, but it was only since the late 1980's that these disks were directly detected, using high-angular millimetre imaging (Beckwith and Sargent, 1996; Koerner, 1997; Holland et al., 1998; Koerner et al., 1998). These images revealed extended gas structures which appeared to be in Keplerian rotation around the central object. T Tauri disks are composed of gas and dust. The mass in the disk, M_D, is in the range 0.001–0.01$M_{\odot}$, and matter is still accreting onto the star. On the other extreme, in disks with planetary systems, most of the mass is on the planets, a few 0.001$M_{\odot}$, and only a small amount of dust ($\ll 10^{-6} M_{\odot}$) remains. The coexistence of disks with stellar companions is attested to by a comparison of high-resolution multiple surveys with the results of imaging and long-wavelength flux measurements. Disks are found to be reduced in mass for binaries with separations in the 10–100 AU range, similar to the typical disk size. However, circumstellar disks in binaries wider than 100 AU and circumbinary disks around spectroscopic binaries are not obviously different from disks around single stars with respect to either their global properties or their frequency of occurrence. Circumstellar disks have been detected around several binaries: HK Tau, HR 4796A, or GG Tau3. Circumbinary disks have been found around GG Tau, UY Aur, BD+31°463, UZ Tau, DQ Tau, and GW Ori. Higher multiplicity systems can present both circumbinary and circumstellar disks as for example, GG Tau and the quadruple system UZ Tau. Recently, Koresko (2000) using the Keck 1 has found that the T Tauri infrared companion is a double system with a separation of about 7 AU. This would make T Tauri at least a hierarchical triple system. Monin and Bouvier (2000) obtained the first near-infrared images of the circumtertiary disk in the young triple system HV Tau. See e.g., *Protostars and Planets IV*, 2000, The University of Arizona Press, for a recent review on this subject. The GAIA mission will be able to identify the complete sample of stars (about 300,000) within 200 pc centred on the Sun

(Lattanzi et al., 2000) and this will allow accurate determination of the frequency of planetary systems as a function of parent star, as well as of planetary mass and separation. Such a data set will provide, at the same time, a precise determination of the number of planetary systems in binaries and hierarchical systems.

This paper extends the calculations presented in de la Fuente Marcos and de la Fuente Marcos (1998a, hereafter Paper I) to hierarchical triple systems. In Paper I we showed that for a binary system, the evolution in time of the dust sub-disk depends mainly on the nature (prograde or retrograde) of the relative revolution of the binary companion, and on the temperature and mass of the circumstellar disk. Calculations in Paper I show that for binary companions near the limit of tidal truncation of the disk, the perturbation leads to an enhanced accretion rate onto the primary, decreasing the lifetime of the particles in the protoplanetary disk with respect to the case of a single star. As a consequence of this enhanced accretion rate, the mass of the disk decreases faster, which leads to a longer resultant lifetime for particles in the disk. On the other hand, binary companions may induce tidal arms in the dust phase of protoplanetary disks. Spiral perturbations with $m = 1$ may increase in a factor 10 or more the dust surface density in the neighbourhood of the arm, facilitating the growth of the particles. In massive disks, the survival time of particles is significantly shorter than in less massive structures. As in Paper I, we study the stage of planetary formation during which the particulate material is still dispersed as centimetre-to-metre sized primordial aggregates. During this stage, particles are able to settle toward the midplane into a layer of mass density comparable to or much greater than that of the gas. Nonlinear, coupled interactions between the particles and the nebula gas become significant and ultimately determine the vertical profiles of the particle density and the mean velocities of the particles and the nearby gas. This is the environment in which the earliest planetesimals probably form. The primary goal of this paper is the study of the variation of the lifetime of particles in the disk and their dynamics for a reasonably wide range of hierarchical triple configurations.

Triple systems tend to be hierarchical, usually with a close binary and a more distant third star, as other configurations are generally unstable and are unlikely to persist and be detected. To first-order, a hierarchical triple system can be separated into an inner orbit (comprising the two close stars) and an outer orbit (comprising the third star and the centre-of-mass of the inner binary). This approximation is most valid when the distance to the third star far exceeds the separation between the inner two stars. Disks in triple systems may be divided into three main configurations: (1) If the disks surround each component of the multiple system individually the situation is similar to single stars and we refer to this configuration as circumstellar disks. (2) If one disk is surrounding a binary system we consider a circumbinary disk. (3) In the case of a circumtriple disk the complete triple system is surrounded by a disk. For higher multiplicity we can define analogous configurations. If a multiple (binary or higher) system is surrounded by a disk, it may have an inner clearing as a result of resonant interactions. Only case (1) is considered in this paper; cases (2) and (3) will be considered in a forthcoming paper. In our present calculations the analysis of the orbital evolution on the solid material includes a relatively wide range of planetesimal masses and nebula parameters (mass and temperature), as well as a range in pair separations and masses. We investigate the dynamical evolution of the dust particles using numerical simulations where the Bulirsch–Stoer method (Bulirsch and Stoer, 1966) of numerical solution of the equations of motion is utilized. We begin in Section 2 by reviewing the basic equations of the problem. In Section 3 we describe the calculations that we have performed. The results of the numerical integrations of the triple case are presented in Section 4. In Section 5 we discuss the effect of a captured, retrograde outer companion on the long-term evolution of the dust sub-disk. Finally, in Section 6 we make a summary of the main results of this research. Throughout this paper we assume that all the components of the model remain coplanar. We also assume that the stellar companions start on a circular orbit, and that the particles in the disk have no mutual interactions or size growth, as in Paper I. These assumptions and simplifications make it easier to compare our present results with those in Paper I. Some of these constraints will be relaxed in a future paper.

2. Formulation

First, we describe mathematically the problem to solve in its most general configuration, primary + disk, secondary and third body. The gravitational perturbation of the disk on the secondary and the third body is neglected because the gravitational force provided by a disk of about $0.01 M_\odot$ on a star 300 AU apart is several orders of magnitude smaller than that of a $1 M_\odot$ star (see Figure 1, Paper I). Therefore, under this approximation the stellar three-body problem can be considered as the combination of perturbed two-body motion of the close pair plus perturbed two-body motion of the third body with respect to the centre of mass of the close pair. Following Harrington (1972), we consider three finite masses, M_0, M_1, M_2, moving such that M_0 and M_1 are the close pair (inner binary). By taking into account the conservation of linear momentum, the motion of the system can be described by considering the Jacobian system of variables, the vector $\mathbf{r}_1$ from M_0 to M_1, and the vector $\mathbf{r}_2$ from the barycentre of M_0 and M_1 to M_2. The equations of motion then become the following:

$$\ddot{\mathbf{r}}_1 = -A_1 \left[\frac{\mathbf{r}_1}{|\mathbf{r}_1|^3} + A_2 \left(\frac{\mathbf{r}_{02}}{|\mathbf{r}_{02}|^3} - \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^3} \right) \right] , \tag{1}$$

$$\ddot{\mathbf{r}}_2 = -A_3 \left[M_0 \frac{\mathbf{r}_{02}}{|\mathbf{r}_{02}|^3} + M_1 \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^3} \right] , \tag{2}$$

where $A_1 = G\ (M_0 + M_1)$, $A_2 = M_2/(M_0 + M_1)$, $A_3 = G(M_0 + M_1 + M_2)/(M_0 + M_1)$ (note miss-print in Paper I), and G is the gravitational constant. The vector

$\mathbf{r}_{02} = \mathbf{r}_2 + M_1/(M_0+M_1)\mathbf{r}_1$ is the vector from M_0 to M_2, and $\mathbf{r}_{12} = \mathbf{r}_2 - M_0/(M_0+M_1)\mathbf{r}_1$ is the vector from M_1 to M_2.

To model the dust disk, we use a thin disk model of the Nebula, in which the surface densities (both for gas and particles) and the temperature are given by decreasing power-laws. Hydrostatic equilibrium in the vertical direction is also adopted. Under the standard assumption of hydrostatic balance in the thickness H of the nebula ($H \approx C_s\Omega$, where C_s is the sound velocity and Ω is the Keplerian angular velocity), the dynamical problem remains a two-dimensional one. In such a pre-planetary nebula, for a given particle there is competition between gas drag and inertia. The main parameter to characterize this process is T_s, the stopping time, i.e., a characteristic friction time-scale which depends on the mass and velocity of the particles but also on the distance to the star. The drag regime depends on the size of the particles relative to the mean free path of the gas molecules ($\lambda \approx m_H\mu/\rho_g\sigma_{H_2}$, $\sigma_{H_2} \approx 2 \times 10^{-9}$ m^2):

$$T_s = \begin{cases} \frac{\rho_p s}{\rho_g C_s} & \text{if } s \leq 9/4\lambda \text{ (Epstein regime),} \\ \frac{8\rho_p s}{3\rho_g C_D u} & \text{otherwise (Stokes regime),} \end{cases} \tag{3}$$

where ρ_g is the density of the gas, C_s is the thermal velocity, ρ_p is the density of the solid material, s is the radius of the particle, and C_D is a non dimensional coefficient which depends on the Reynolds number, $\mathrm{Re} = 2su\rho_g/\eta_g$ (η_g is the viscosity of the gas, $\eta_g = 1/2\rho_g\lambda C_s$). The thermal velocity is given by $\sqrt{8\kappa T/\pi\mu m_H}$, where κ is the Boltzmann constant, μ is the mean molecular mass ($\mu \approx 2.34$), and m_H is the mass of an hydrogen atom. Following Weidenschilling (1977), the dimensionless drag coefficient is given by:

$$C_D = \begin{cases} \frac{24}{\mathrm{Re}} & \text{if Re} < 1, \\ \frac{24}{\mathrm{Re}^{0.6}} & \text{if } 1 < \text{Re} < 800, \\ 0.44 & \text{if Re} > 800. \end{cases} \tag{4}$$

The particles inside the disk are assumed to follow a Safronov initial mass function, $\xi(m) = n\exp(-m/\langle m\rangle)$ (Safronov, 1969), where $\xi(m)$ is the number of particles per unit mass interval. This is conveniently represented by the mass generating function

$$m(X) = m_l - \langle m\rangle \ln\left[1 - (1 - \mathrm{e}^{((m_l - m_u)/\langle m\rangle)})X\right] , \tag{5}$$

where m_l is the minimum mass, m_u is the maximum, $\langle m\rangle$ is the mean mass and X is a random variable uniformly distributed in the interval [0, 1], and $\mathrm{d}X/\mathrm{d}m \propto \xi(m)$ ($\langle m\rangle \geqq m_l$). The mass generating function has been computed in the way described, for example, in Kroupa et al. (1991).

On the other hand and before solving the motion equations, we need to specify the temperature and surface density profiles of the basic unperturbed state. For the purpose of this work, we have adopted a simple formulation for the physical characteristics of the protoplanetary nebula. We take the total mass density $\sigma(r)$ and temperature $T(r)$ of the disk to be the form

$$\sigma(r) = \sigma_o \left(\frac{r}{r_o}\right)^{-p} , \tag{6}$$

$$T(r) = T_o (\frac{L}{L_\odot})^{q/2} \left(\frac{r}{r_o}\right)^{-q} . \tag{7}$$

where r_o is a reference radius with Keplerian orbital frequency Ω_o ($\sqrt{GM_o/r_o^3}$), temperature T_o, mass density σ_o, and velocity v_K, and L is the luminosity of the star in solar units. We assume that the disk is vertically isothermal at temperature $T(r)$, and we take $L = 1\ L_\odot$ throughout the paper.

The gas vertical scale height H is

$$H = c/\Omega = \sqrt{\frac{2R}{\mu}} T^{1/2} \Omega^{-1}. \tag{8}$$

The average gas density ρ_g is

$$\rho_g = \frac{\sigma}{2H} = \rho_o \left(\frac{r}{r_o}\right)^{-p+q/2-3/2} , \tag{9}$$

where $\rho_o = \sqrt{\mu/8RT_o}\sigma_o\Omega_o$. For the purposes of this paper, the midplane gas density is taken to be the same as this average value. The gas pressure is given by:

$$P = \frac{\rho_g RT}{\mu} . \tag{10}$$

In this work we are interested in disks associated with T Tauri stars. As an example, for the circumstellar disk around T Tauri N (Akeson et al., 1998) the temperature $T_{10\mathrm{AU}} = 26^{+34}_{-13}$ K, p is in the range 0.5-2.0, and q in 0.4-0.75. The probability that p is ≥ 1.5 is 65%.

On the other hand, the small perturbation to the radial force due to pressure support is (e.g., Cuzzi et al., 1993)

$$\begin{aligned} \psi &= -\left(\frac{1}{2\rho_g r \Omega^2}\right) \frac{\partial P}{\partial r} \\ &= \frac{(p + q/2 + 3/2) RT_o}{2 r_o^2 \Omega_o^2 \mu} \left(\frac{r}{r_o}\right)^{(1-q)} . \end{aligned} \tag{11}$$

Therefore, the Cartesian components of the velocity of the gas, assuming initial unperturbed circular Keplerian flow are now:

$$V_X^g = -(1-\psi)\Omega Y \,,$$
$$V_Y^g = (1-\psi)\Omega X \,, \tag{12}$$

where X, Y are the Cartesian coordinates in a frame of reference at rest and centred on the disk-host star. The reference surface mass density in terms of the global disk parameters is given by

$$\sigma_o = \left(\frac{2-p}{2\pi r_o^2}\right)\left(\frac{R_D}{r_o}\right)^{p-2} M_D \,, \tag{13}$$

where R_D and M_D are the radius and the mass of the disk, respectively.

The motion equations for a solid particle submitted to the attraction of the star and to the friction drag of the Nebula gas are:

$$\frac{d^2\mathbf{r}}{dt^2} = -GM_0\frac{\mathbf{r}}{r^3} - \frac{\mathbf{u}}{T_s} + \mathbf{\Phi} + \mathbf{\Psi}, \tag{14}$$

where $\mathbf{u} = \mathbf{v} - \mathbf{V}_g$ and $\mathbf{V}_g$ is the gas velocity, $\mathbf{\Phi}$ is the contribution of the two other stellar companions:

$$\mathbf{\Psi} = -GM_1\frac{\mathbf{r}-\mathbf{r}_1}{|\mathbf{r}-\mathbf{r}_1|^3} - GM_2\frac{\mathbf{r}-\mathbf{r}_{02}}{|\mathbf{r}-\mathbf{r}_{02}|^3} \,, \tag{15}$$

and $\mathbf{\Psi}$ is the contribution of the self-gravity of the disk. T_s is the stopping time, i.e., a characteristic friction time-scale which depends on the mass and velocity of the particles but also on the distance to the star. The contribution of the self-gravity of the disk has been included as in Paper I (see Paper I for details).

3. Simulations

Even with the idealized forms adopted for the equilibrium disk properties, the parameter space accessible to multiple star/disk systems has too many dimensions to survey completely. As in Paper I, we concentrate on an interesting subspace by fixing the power-law indices p = 3/2 and q = 1/2. These two values are reasonable in the light of recent determinations for flat-spectrum T Tauri stars (Akeson et al., 1998). The mass, temperature (T_o), and radius of the disk, and the mass, nature (prograde or retrograde), and separation of the components of the multiple system now become the free parameters of our simulations. Moreover, we restrict ourselves to the study of a triple system, coplanar and hierarchical; i.e., a protoplanetary disk around a star with a widely separated binary companion, and with a third star at a distance several times that of the close binary separation. The circumbinary and circumtriple cases will be studied in a separate paper. The disk particles initially describe Keplerian orbits about the primary, M_0. Since the particles are non-interacting, the results are independent of the number of disk particles used, as long as the mass attributed to any one particle is much less than either of the stellar masses. Particles may be integrated one at a time because we are ignoring all inter-particle interactions in our models, as well as the effect of the disk material on the triple orbit during the numerical experiment. The motion of both the disk particles and the point masses is integrated using the Bulirsch–Stoer method (Press et al., 1992). We use an unsoftened gravitational force for each star. The accuracy of this method was checked and we found energy conserved to better than one part in 10^{12} and angular momentum to one part in 10^{14} (full details of the calculations appear in de la Fuente Marcos (2001) and in Paper I). We are studying the mean effects on a dust sub-disk with a certain range in sizes. In order to ensure that we have enough particles in our disk to obtain meaningful results we choose a representative ring of particles. The value of the number of particles that it contains, N, was chosen equal to 1000, with the initial mass function discussed in Section 1. Unfortunately, the computational time required to evaluate more than 1000 particles in this ring is prohibitively long, and thus we choose $N = 1000$ as a compromise between computational speed and statistical significance of the results. Even if we consider a disk radius of 100 AU we restrict our calculations to the inner part of the disk ($\sim$ 30 AU) and we follow the inwards evolution of the particles until they leave the region of the Giant Planets ($\sim$ 5 AU, in our own Solar System). We believe that this part of the disk may be the most interesting to study as regards giant planet formation, because exo-planets in very close orbits may be explained as a result of migration processes (Lin et al., 1996; Trilling et al., 1998; Murray et al., 1998).

We will present representative results which describe the behaviour of the particle phase near the midplane for a variety of particle radii and a variety of nebula locations. We will consider a circumstellar nebula with midplane temperature at 1 AU in the range of 280 K, but also a protoplanetary disk with midplane temperature at 1 AU in the range of 1000 K. This is the temperature believed to characterize FU Orionis outbursts, which are thought to manifest brief episodes of high temperature and viscosity (Hartmann et al., 1993). FU Ori objects, a class of eruptive variables whose luminosities are dominated by accretion, exhibit deep absorption features due to their low surface gravity disk atmospheres (Hartmann and Kenyon, 1996). The nebula temperature regulates relative velocity between solid particles and nebula gas through pressure gradient. The ψ factor is directly proportional to T_o, therefore an increase in temperature translates into a larger pressure gradient and shorter lifetimes for particles. On the other hand, the Ψ factor in the equations of motion is directly proportional to the mass of the nebula. Larger nebula masses imply higher velocity of the gas and shorter gas vertical scale H.

In this paper we study the case of a triple system and we restrict ourselves to the following circumstances:

1. The stellar orbits are coplanar;
2. the initial conditions are recursively circular.

We choose initial conditions such that the inner binary would have a circular orbit in the absence of the third star and the outer body has also a circular orbit if the inner binary were replaced by its combined mass at its instantaneous centre of gravity. We always start with the two orbits in phase. Parameters to be specified at the beginning of each simulation are the inner and outer radius and the character (prograde or retrograde) of the inner binary. Section 4 includes results for prograde outer companions. The effect of retrograde, captured companions is analysed in Section 5.

4. Triple Case

As pointed out in the Introduction Section, current observational estimates suggest that nearly 30% of all binary stars are in triple systems. Triple system configurations are too general to be taken into account here. In this section we consider a stable coplanar hierarchical triple system. The time-scale for planet formation is uncertain; it was thought to be roughly 10 Myr, as judged from both astronomical and solar system constraints (Strom et al., 1989, 1993; Podosek and Cassen, 1994). However, the discovery of the controversial planet candidate TMR-1C (Terebey et al., 1998, 2000) in Taurus suggests (if confirmed) that this limit may be lowered to about 3×10^5 y in the case of giant gaseous planets, assuming that TMR-1C has been formed in the standard way. It is hard to believe that planetary formation can take place in a dynamically chaotic environment; therefore, if we consider the old estimation, the hierarchical system must be stable at least for 10 Myr. Due to it, we have to determine whether the system is hierarchical or not at the beginning of the simulation. Triple stars are unstable if the ratio of the orbital period of the enclosed binary to the period of the third component exceeds a critical value. Dynamical stability requires that the ratio of outer period to inner period must be larger than a factor of $\backsimeq$ 3–6, if both orbits are nearly circular and all three bodies are of comparable mass (Kiseleva et al., 1994a, b). However, for our present purpose it is better to use the minimum Y_o necessary for stability, where Y_o is the ratio of outer pericenter to inner apocenter. In our case this is simply the ratio of semi-major axes because we restrict ourselves to hierarchical triple systems with initially doubly-circular orbits. In dynamically stable hierarchical triples the distant component always pumps an eccentricity into the binary on a time scale shorter than the orbital period of the binary. In its turn, the binary also pumps an eccentricity into the outer orbit, although the inner orbit seems to be more sensitive to perturbations. If we apply the stability criterion by Eggleton and Kiseleva (1995) as well as their definition of stability: that a hierarchical triple system is stable if it persists continuously in the same hierarchical configuration (which excludes exchange as well as disintegration), we obtain that for a system with $M_1/M_0 = 1$ and $A_2 = 0.025$ (Section 2) then $Y_o > 2.32$, if $M_1/M_0 = 1$ and $A_2 = 0.5$ then $Y_o > 3$, and if $M_1/M_0 = 1$ and $A_2 = 1.0$ then $Y_o > 3.37$. These are the three cases that we will study in this section. At this point we can compare the results from this criterion with the recent stability criterion of Mardling and Aarseth (1999). The critical ratio of the outer periastron distance of the mass M_2 to the inner apastron distance of $M_0 + M_1$ is given by

$$Y_0^{\min} = \mathcal{C} \left[\frac{1 + e_{\text{out}}}{(1 - e_{\text{out}})^{1/\alpha}} \frac{1 + A_2}{(1 + e_{\text{in}})^{3-1/\alpha}} \right]^{\alpha/(3\alpha-1)} , \tag{16}$$

where e_{out}, e_{in} are the outer and inner eccentricities, respectively, $A_2 = M_2/(M_0 + M_1)$, $\mathcal{C} = 2.8$, and $\alpha = 2$. This criterion has been verified (for mass ratios in range 0.01–100 of the outer body and wide range of values for e_{out}) by systematic calculations (Mardling and Aarseth, private communication).⋆ This stability criterion has proved very useful in N-body simulations of star clusters containing a significant proportion of primordial binaries. Applying the criterion (16) to the systems considered in this section we obtain 2.83, 3.29 and 3.69, for $A_2 = 0.025$, $A_2 = 0.5$ and $A_2 = 1.0$, respectively. This criterion seems to be as reliable as the one by Eggleton and Kiseleva for our present purpose. In our calculations $Y = 2.33$, 3.33 and 5.33, respectively.

In Figure 1 we show two examples of trajectories from our numerical experiments. The trajectory of a decimetre-sized particle appears in panel (a). Both the inner and the outer companion are prograde. As in Paper I, prograde means that the orbit is counterclockwise (as the orbits of the particles in the disk). The direction of rotation of the companions is very important because their gravitational perturbation induces a spiral structure across the protoplanetary dust sub-disk. Spiral arms can be classified by their orientation relative to the direction of rotation of the disk and the companions. A trailing arm is one whose outer tip points in the direction opposite to disk rotation, while the outer tip of a leading arm points in the direction of rotation. In our calculations, trailing arms appear when the inner companion goes around the center of mass of the binary in the direction of rotation of the disk. When the companion star goes in the opposite direction, a leading arm is found. Panel (b) shows the trajectory of a metre-sized particle. Now, the gravitational perturbation of the falling particle is stronger and the orbit precess rapidly. No clear arm is found. Formation of spiral arms may have important implications on the topic of giant planet formation. Particles spend most of their orbital time in the neighbourhood of the spiral arm, increasing the local surface density of solid material by a significant factor. Spiral perturbations may also have important implications on the processing of primordial material in spiral density waves in a protoplanetary nebula (Wood, 1996).

⋆ This criterion has been modified (Aarseth, private communication), canceling the dependence on the inner eccentricity. This change is not important in our case because we start from circular orbits.

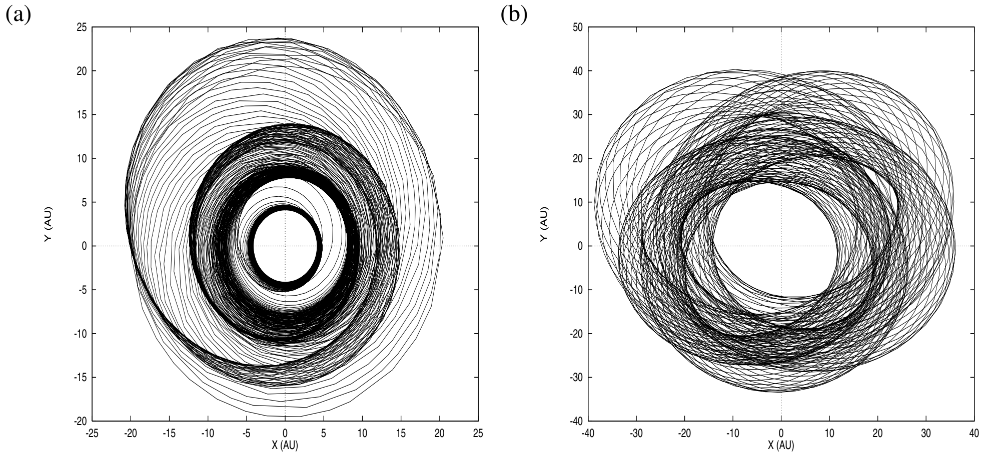

Figure 1. (a) Example of a decimetre-sized particle trajectory when the rotation of both the binary companion and the outer body is prograde, i.e., counterclockwise, with respect to the disk. The perturbation from the companions produces a leading arm as the outer tip of the arm points in the direction of rotation. The binary companion is solar-like and the outer companion is an A5 star ($2M_{\odot}$, Popper 1980). The binary separation is 300 AU and the distance from the center of mass of the close pair to the outer body is 1000 AU. The mass of the nebula is $0.001M_{\odot}$ and its temperature, 280 K. The trajectory is plotted in a frame of reference centred on the disk-host star. (b) Example of a particle trajectory for a metre-sized particle. In this case the outer companion of the double star is 1600 AU. The trajectory exhibits no clear spiral arm due to strong precession. As in (a) the trajectory is plotted in a frame of reference centred on the disk-host star.

4.1. 0.05 $M_{\odot}$ OUTER COMPANION

In this first hypothetical case we consider an initially circular binary with radius 30 AU and both components solar-like, with a distant brown dwarf companion 700 AU away in an initially circular orbit around the centre of mass of the binary. In Paper I we pointed out that the effect of a massive binary companion on the dynamics of centimetric particles is almost negligible, hence we only consider decimetric and metric particles here. The protoplanetary disk is around one of the components of the binary. As in Paper I we consider a low-mass nebula (0.001 $M_{\odot}$) and a minimum mass solar nebula (0.01 $M_{\odot}$), and two different temperatures (250 K and 1000 K at 1 AU). Figures 2 and 3 show the results for this configuration.

Figure 2 is for a low-mass nebula and we observe that lifetimes for prograde systems are longer, although this effect depends on the size of the particles; bigger particles have the longest spiral-in time. The perturbation of the outer body for a low-mass nebula does not alter significantly the lifetimes of decimetric particles relative to their values in the equal-mass binary discussed in Paper I and reduces slightly those for metric particles. The gravitational perturbation is stronger for metric particles. A minimum mass solar nebula shown in Figure 3 does not change this trend. Lifetimes for metric particles are fairly similar to those in Paper I. Increasing the temperature up to 1000 K halves the lifetime of the particles both for a low-mass nebula and a minimum mass nebula.

4.2. 1 $M_\odot$ OUTER COMPANION

Figures 4 and 5 illustrate the behaviour of a dust sub-disk for an initially circular binary with radius 300 AU and both components solar-like, with a solar-like outer companion 1000 AU away in an initially circular orbit around the centre of mass of the binary. In Figure 4 we plot the evolution of the semi-major axis for a low-mass nebula. As in other examples, we observe that lifetimes for prograde systems are longer, although this effect depends on the size of the particles; the bigger the particle, the longest the spiral-in time. The perturbation of the outer body for a low-mass nebula does not alter significantly the lifetimes of decimetric particles and reduces slightly those for metric particles as compared with Paper I. The perturbation is stronger for metric particles. A minimum mass solar nebula shown in Figure 5 does not change this trend. As in the previous case, increasing the temperature halves the lifetime of the particles for both values of the nebular mass.

4.3. 2 $M_\odot$ OUTER COMPANION

Here we consider an initially circular binary with radius 300 AU and both components solar-like, with a $2M_\odot$ outer companion 1600 AU away, in an initially circular orbit around the centre of mass of the binary. As in the other cases we do not study the behaviour of centimetric particles. Figures 6 and 7 show the results for this configuration. In Figure 6 we see the evolution of the radial position with time for a low-mass nebula. As in other cases, lifetimes for prograde systems are longer. The increase in percentage depends on the size of the particles as we pointed out in previous sections. The perturbation of the massive outer body tends to lengthen the lifetimes of the particles as compared with those from a less massive outer companion, although the increase is a few percent. The effect is more significant for metric particles. Increasing the mass of the nebula reduces the perturbative effects of the outer bodies (Figure 7). As for less massive companions, increasing the temperature to 1000 K at 1 AU halves the lifetime of the particles.

5. Discussion

Triple systems may not have a common mode of origin. Three main formation mechanisms have been considered: fission, multiple condensations (fragmentation), and many-body encounters (capture). Fission and fragmentation are able to produce primordial triple systems. The first process is associated with physical instabilities rather than dynamical interactions. It represents an attractive alternative for the origin of very close triples. On the other hand, stable hierarchical systems of

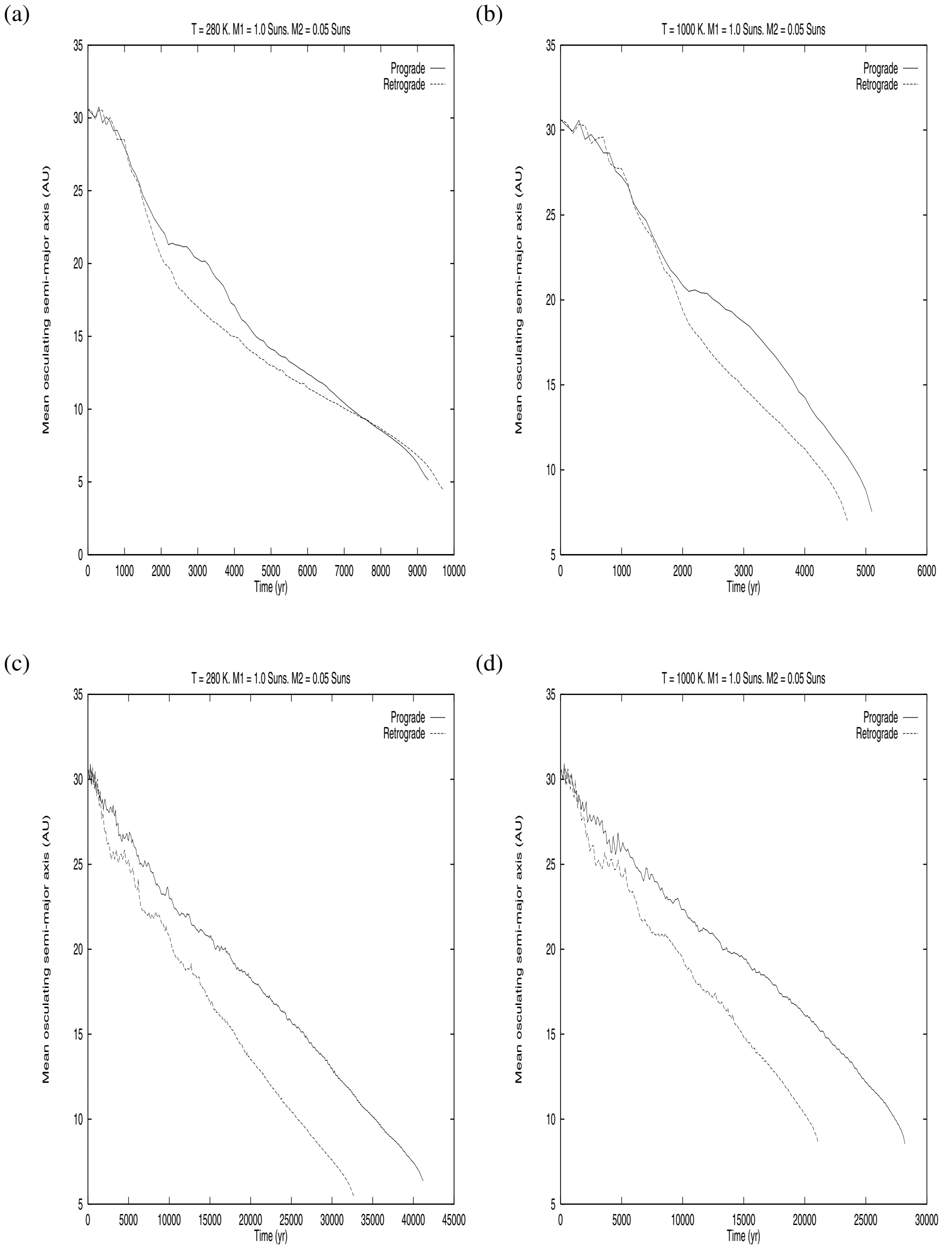

Figure 2. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.001 M_\odot$ submitted to the perturbation of a binary companion of $1 M_\odot$ at 300 AU and a sub-stellar outer companion of $0.05 M_\odot$ at 700 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

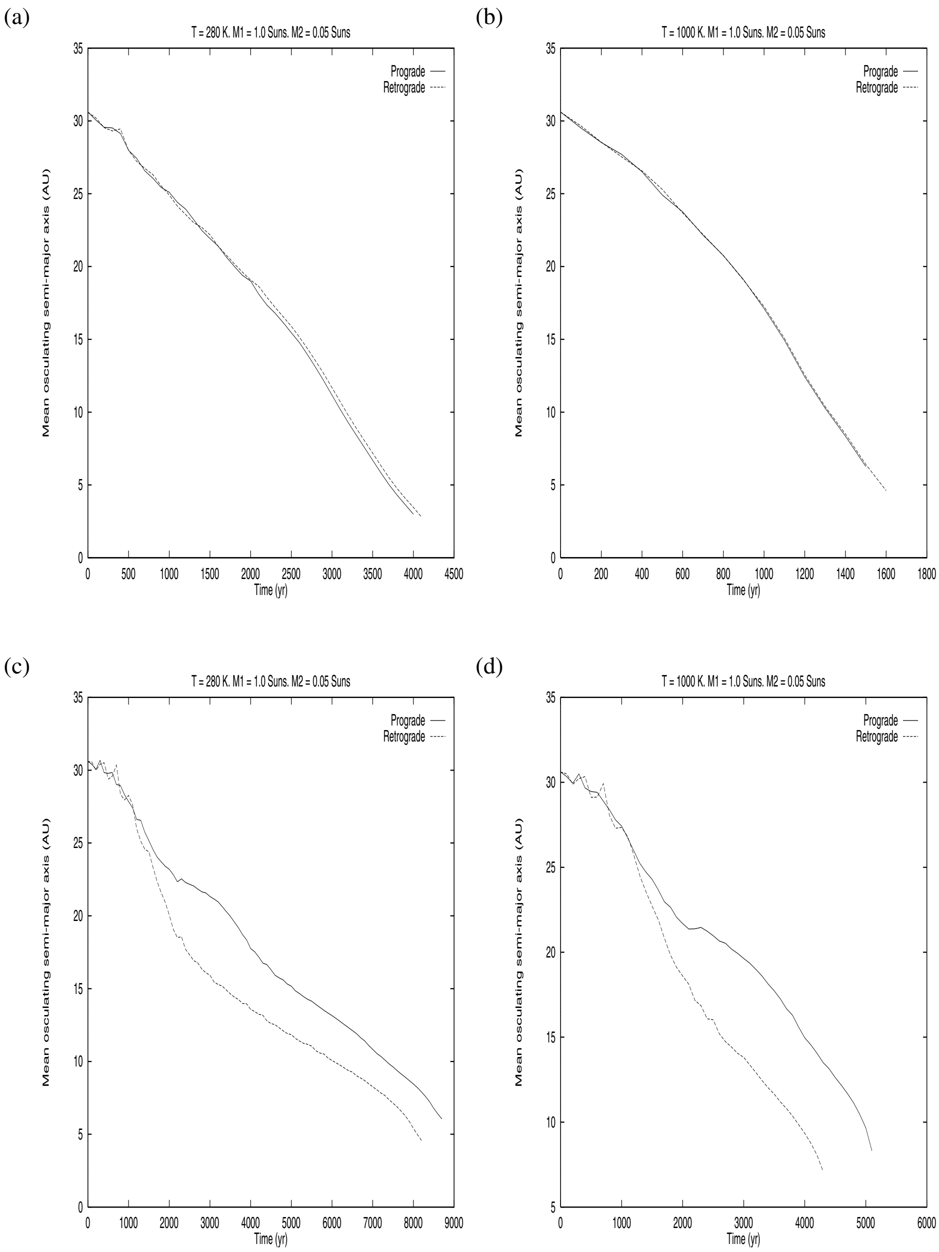

Figure 3. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.01 M_{\odot}$ submitted to the perturbation of a binary companion of $1 M_{\odot}$ at 300 AU and an outer companion of $0.05 M_{\odot}$ at 700 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

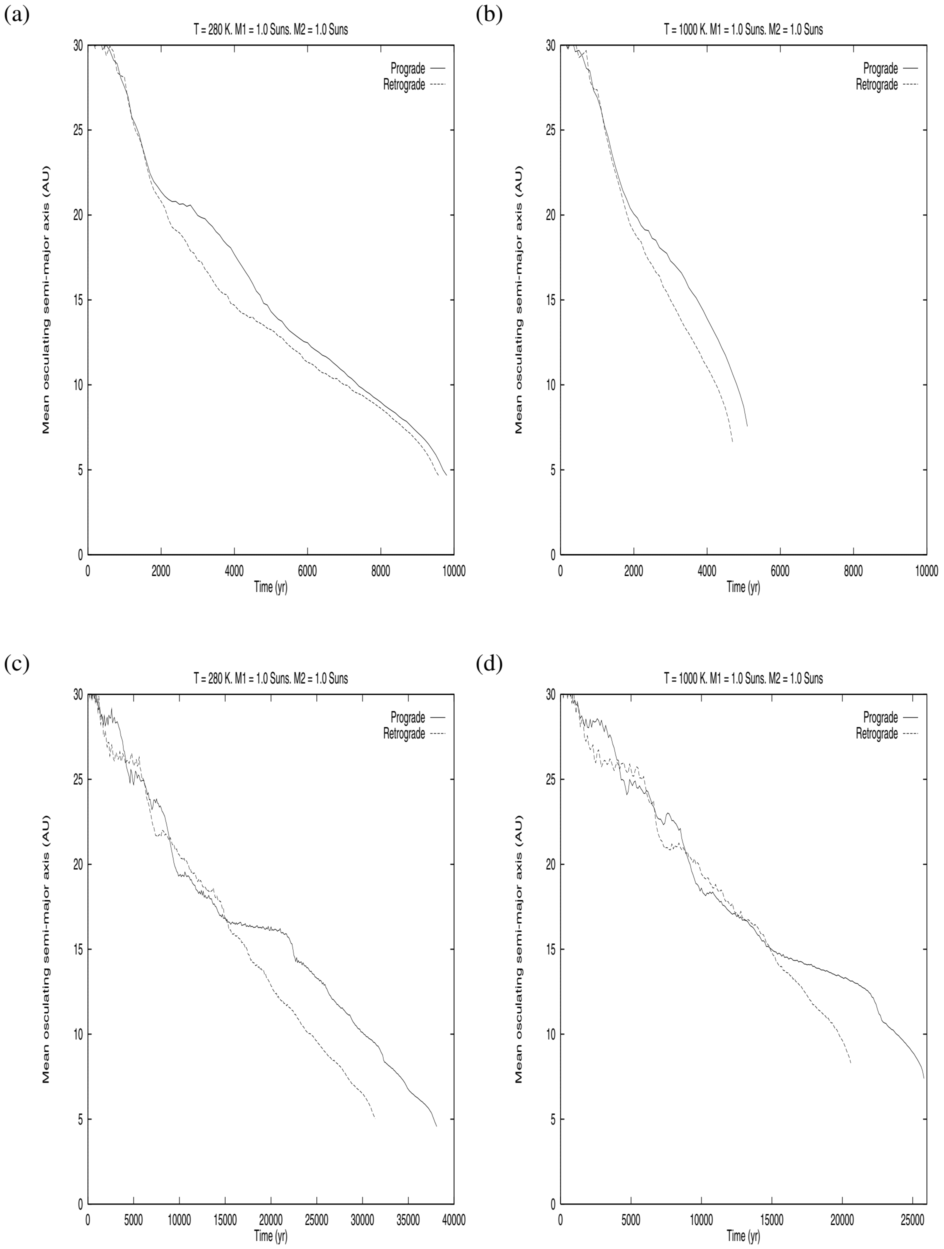

Figure 4. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.001 M_\odot$ submitted to the perturbation of a binary companion of $1 M_\odot$ at 300 AU and an outer companion of $1 M_\odot$ at 1000 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

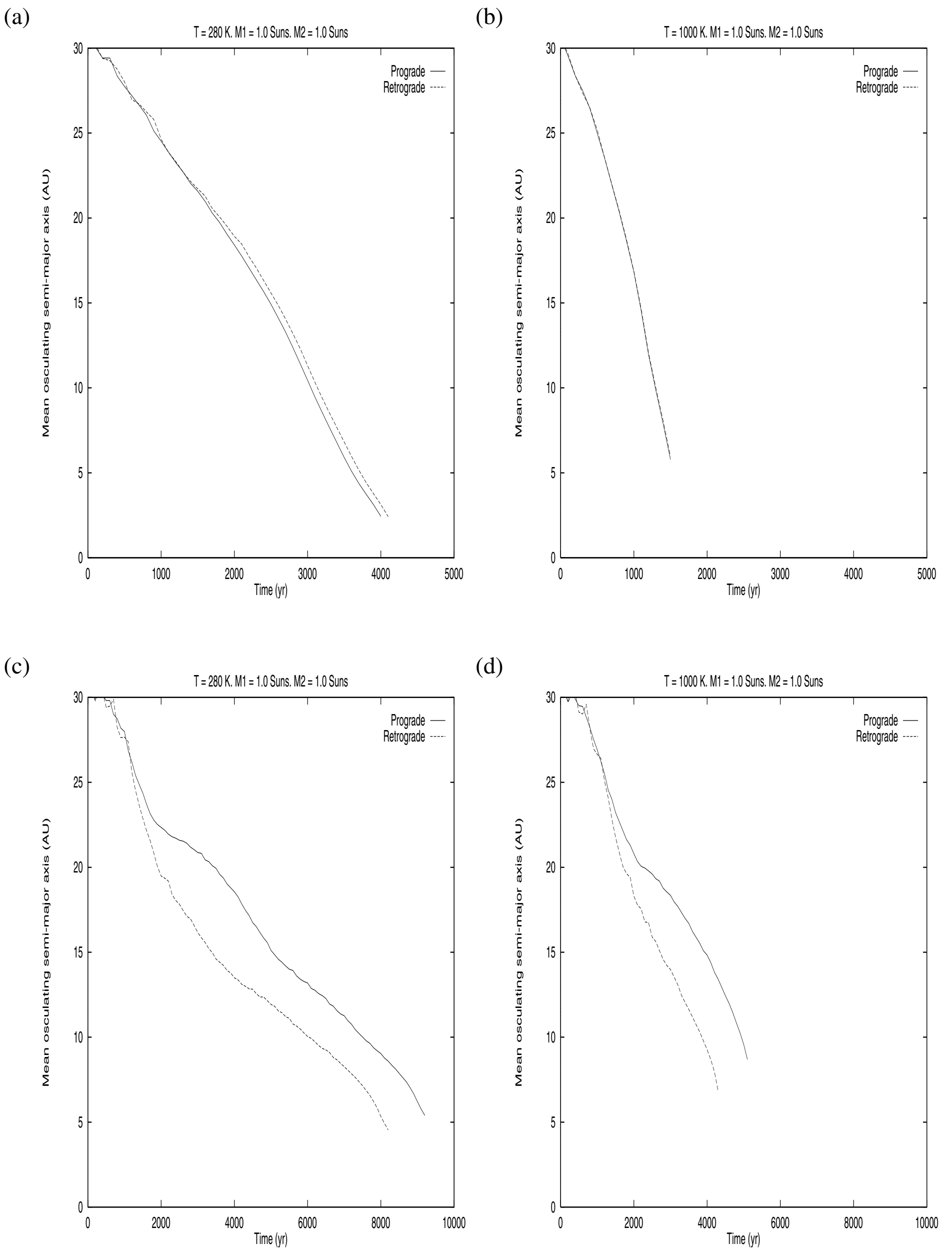

Figure 5. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.01 M_{\odot}$ submitted to the perturbation of a binary companion of $1 M_{\odot}$ at 300 AU and an outer companion of $1 M_{\odot}$ at 1000 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

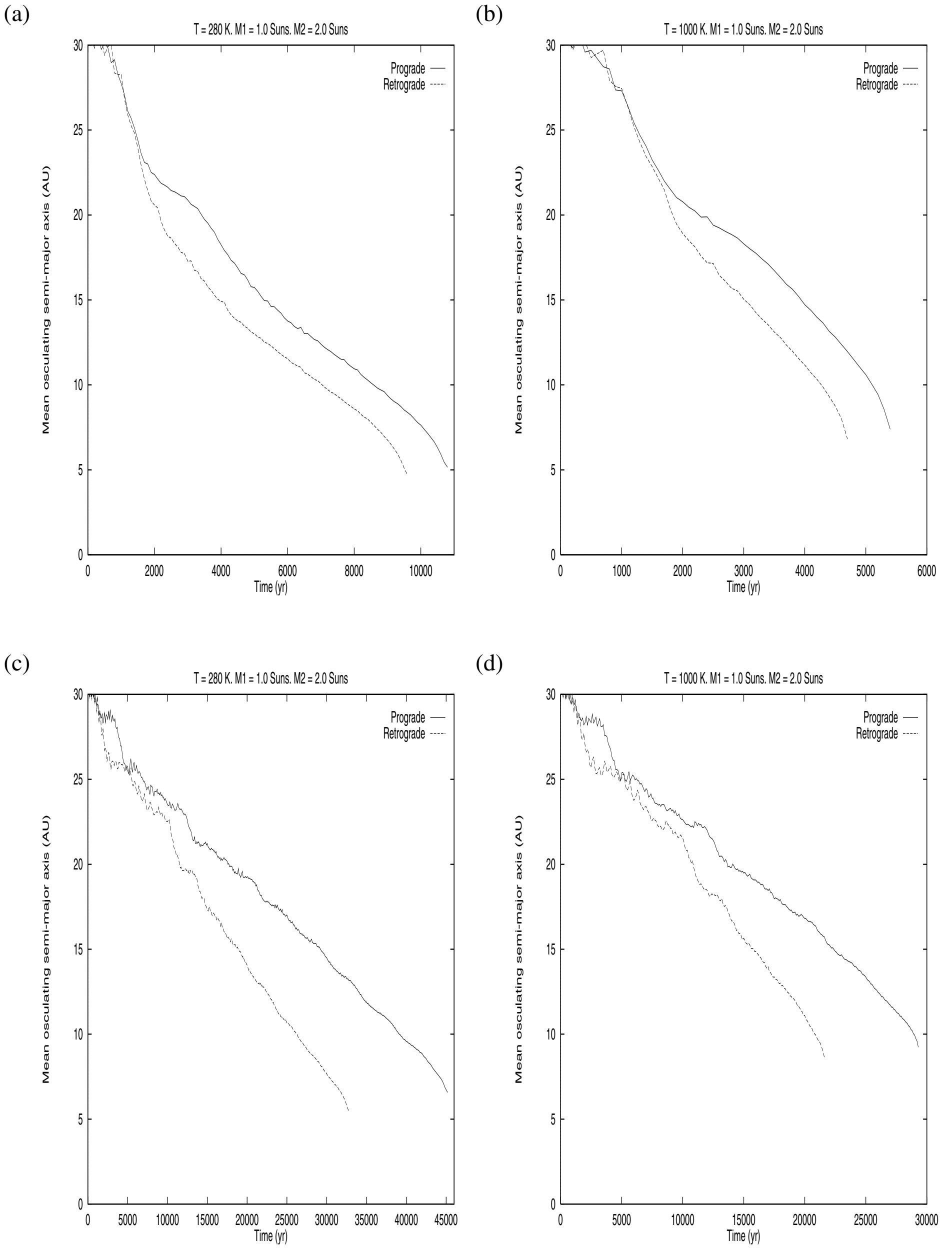

Figure 6. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.001 M_\odot$ submitted to the perturbation of a binary companion of $1 M_\odot$ at 300 AU and an outer companion of $2 M_\odot$ at 1600 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

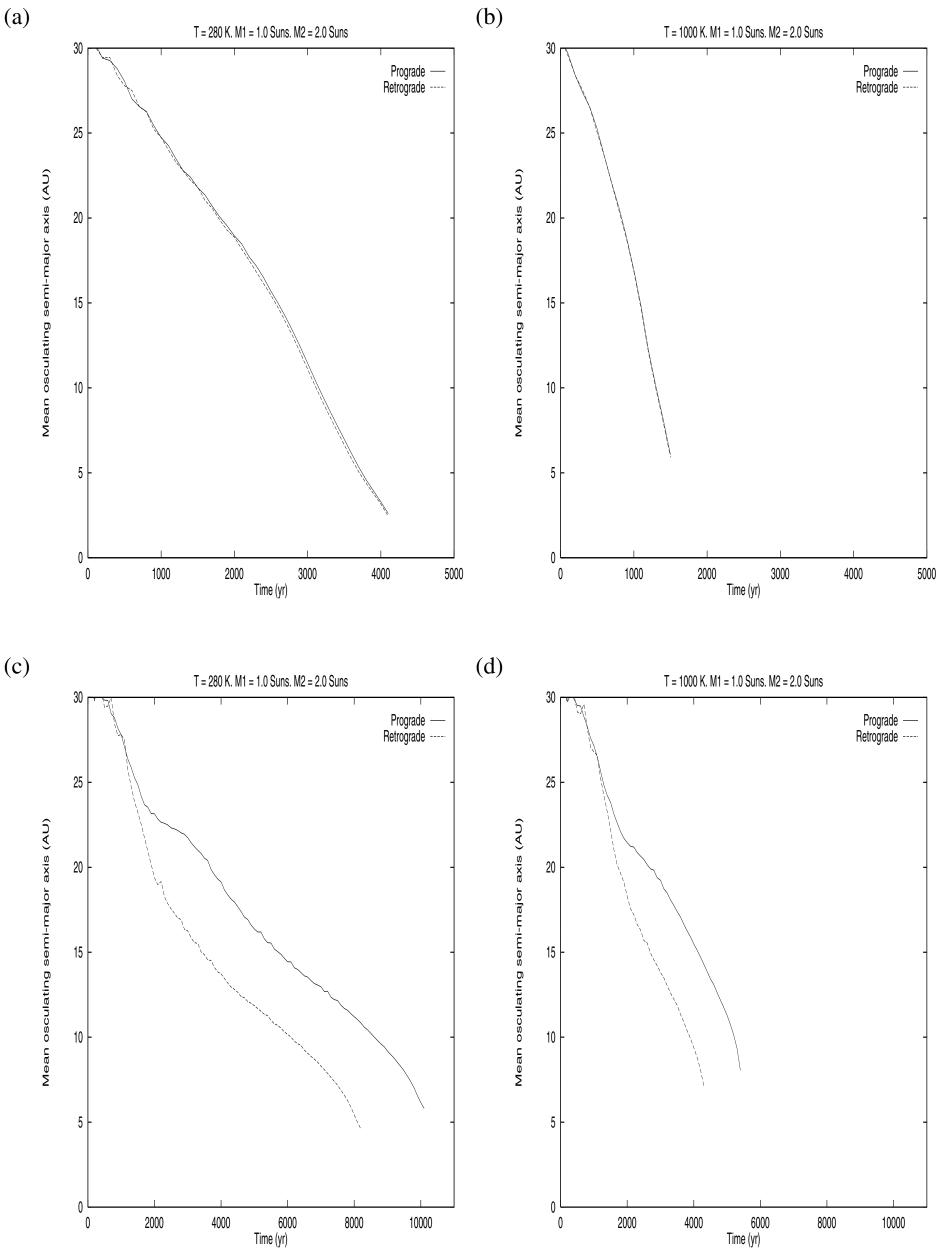

Figure 7. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.01M_{\odot}$ submitted to the perturbation of a binary companion of $1M_{\odot}$ at 300 AU and an outer companion of $2M_{\odot}$ at 1600 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

multiple (not only triple) protostellar cores can form through gravitationally driven fragmentation during the collapse of an isolated gas cloud (Boss, 1991). The third mechanism of triple formation is based on four or more strongly interacting stars producing a bound triple by capture. The probability of suitable stellar encounters is extremely small at typical interstellar densities. Conditions near star cluster centres are most favourable. Hierarchical triple star systems can play an important role in the dynamical evolution of dense star clusters. The cores of these objects are thought to contain a small but dynamically significant population of triple systems formed through dynamical interactions between primordial binaries. Both stable and unstable triples can form easily through exchange and resonant interactions between binaries. Production of hierarchical triple systems is also possible by the dynamical decay of small open clusters (Harrington, 1975).

It is likely that primordial hierarchical triple systems are prograde, and non-primordial triples have one or two retrograde components. In Section 4 we investigated the response of the solid material to both prograde and retrograde inner companions but the outer object was ever prograde. However, the probability of having a prograde binary and a retrograde, captured third body is higher. Figure 8 illustrates the evolution of the mean osculating semi-major axis of decimetre-sized particles submitted to various external gravitational perturbations. The triple system studied here is near the stability limit (Y = 3.33 vs. 3.37 for stability). We see that there is a slightly longer lifetime for particles with a retrograde outer companion. Systems with both companions in retrograde motion show the shortest lifetime. This effect also appears for low-mass outer companions.

Figures 9 and 10 are equivalent to Figures 2 and 3 but now the inner binary is prograde and the direction of rotation of the third body, the brown dwarf, changes. Lifetimes for systems with prograde outer companion are almost 20% longer than those in fully prograde systems.

We have found that, over a wide range of parameters, a three-body interaction results in an enhanced accretion rate onto the disk-host star. The exact value of this rise depends on several parameters but the most important as regards global effects is the nature (prograde or retrograde) of the companions. The lifetime of particles in a system including a prograde (primordial) inner binary and a captured (retrograde) outer body is longer than in an equivalent (primordial) triple with all the companions prograde. These predictions can be tested observationally by studying the luminosity of the disk. If we assume that the disk bolometric luminosity, L_D is derived mostly from the inward transport of material, i.e., we neglect any contribution to the luminosity from reprocessing of stellar photons, then an upper limit to the luminosity is:

$$L_D \approx \frac{G M_0 \dot{M}}{2 R_0}\,, \tag{17}$$

where M_0 and R_0 are the mass and radius of the disk-host star and $\dot{M}$ is the average mass accretion rate. On the other hand, $\dot{M} \approx 2\pi\sigma R_D v_r$, where σ is the surface

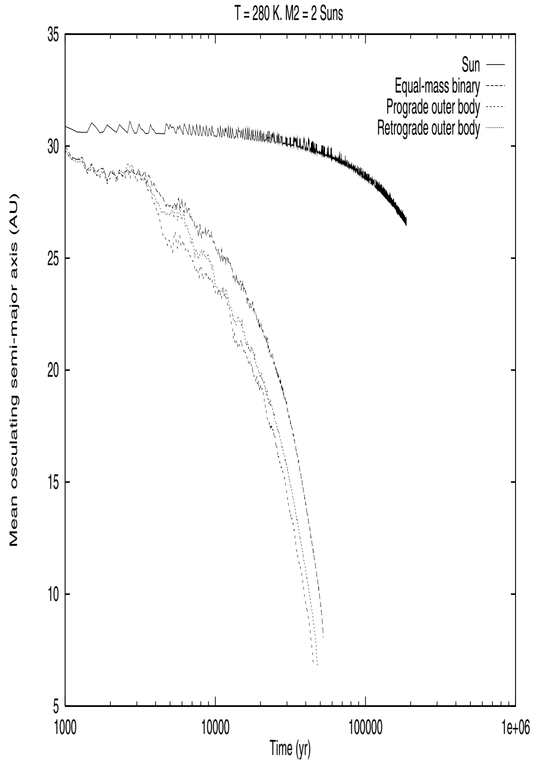

Figure 8. Comparison of the evolution of the osculating semi-major axis of a metric particle (T_o = 280 K, M_D = $0.001 M_{\odot}$) under the perturbation of the Sun, a solar-like companion (single binary), and a solar-like inner companion and a $2M_{\odot}$ outer companion in both prograde and retrograde orbits. A solar-like, prograde, binary companion at 300 AU boosts the accretion rate of metre-sized solid particles by about two orders of magnitude relative to its value for a single (without any companion) star. An additional third body ($2M_{\odot}$) at 1000 AU rises the mass input by a small amount relative to its value for an equal-mass, solar-like, prograde binary. The actual value of the accretion rate for hierarchical triple systems depends on the nature (prograde or retrograde) of the outer companion. Retrograde, massive, outer companions tend to stabilize the dust sub-disk relative to an equivalent prograde companion.

density and v_r is the average inwards, radial velocity in the disk. Our calculations suggest that disk luminosities in binary systems are several orders of magnitude higher relative to the value for single stars. Circumstellar disks in triple systems may be 5–50% more luminous depending on the relative direction of rotation. Surveys of volume-limited stellar samples show that at least 15% of A-K main sequence stars have some far-infrared dust excess with fractional dust luminosity ($f_d = L_d/L_{\star}$) greater than or equal to α Lyr's value of about 2×10^{-5} (Backman and Paresce, 1993; Plets, 1997; Dominik et al., 1998; Fajardo-Acosta et al., 1999). On the other hand, there is a large spread in the mass accretion rates and disk masses as a function of the age. Some of this range is probably due to uncertainties in age determinations, errors in accretion rates, time variability, and a range in

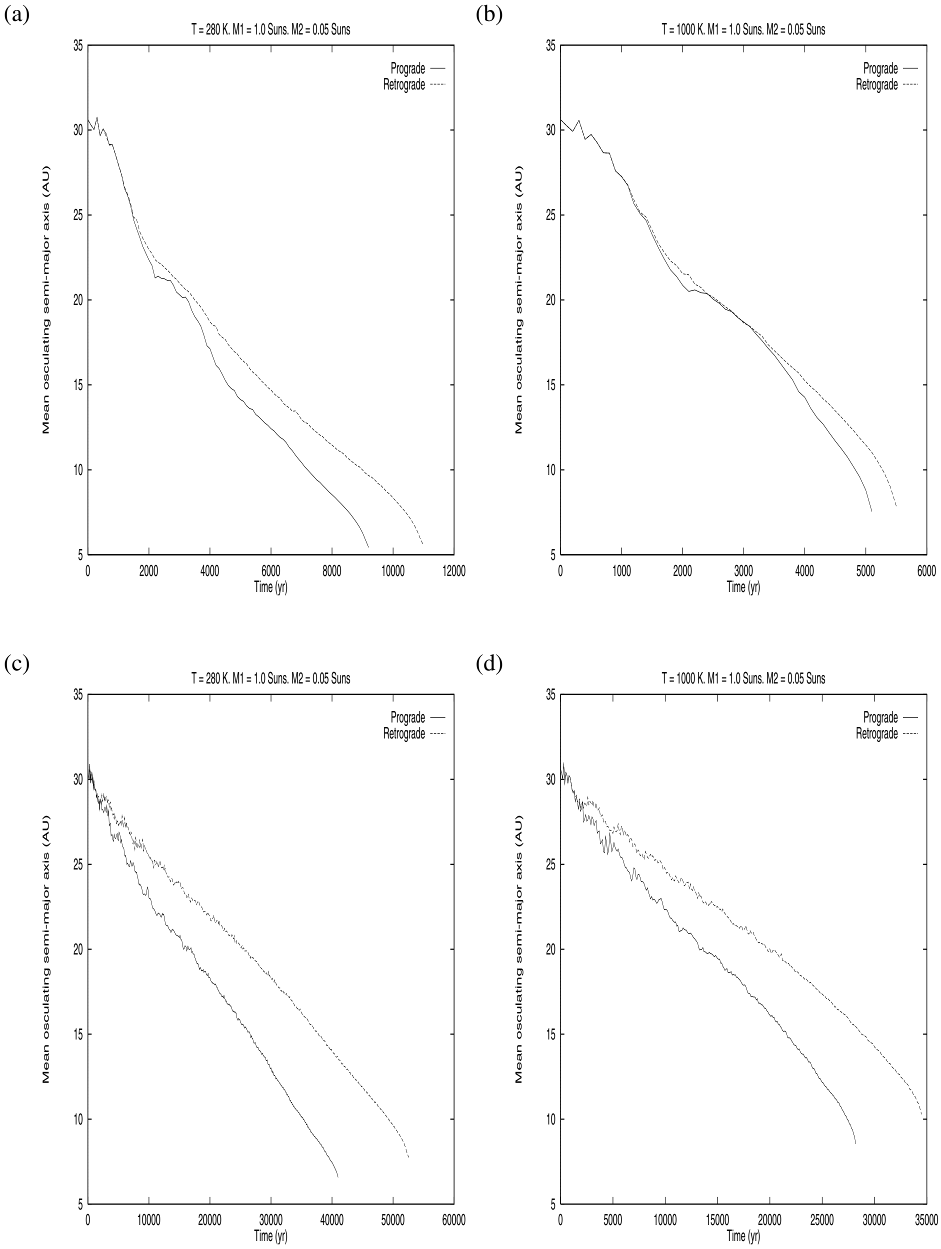

Figure 9. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.001 M_{\odot}$ submitted to the perturbation of a binary companion of $1 M_{\odot}$ at 300 AU and a sub-stellar outer companion of $0.05 M_{\odot}$ at 700 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

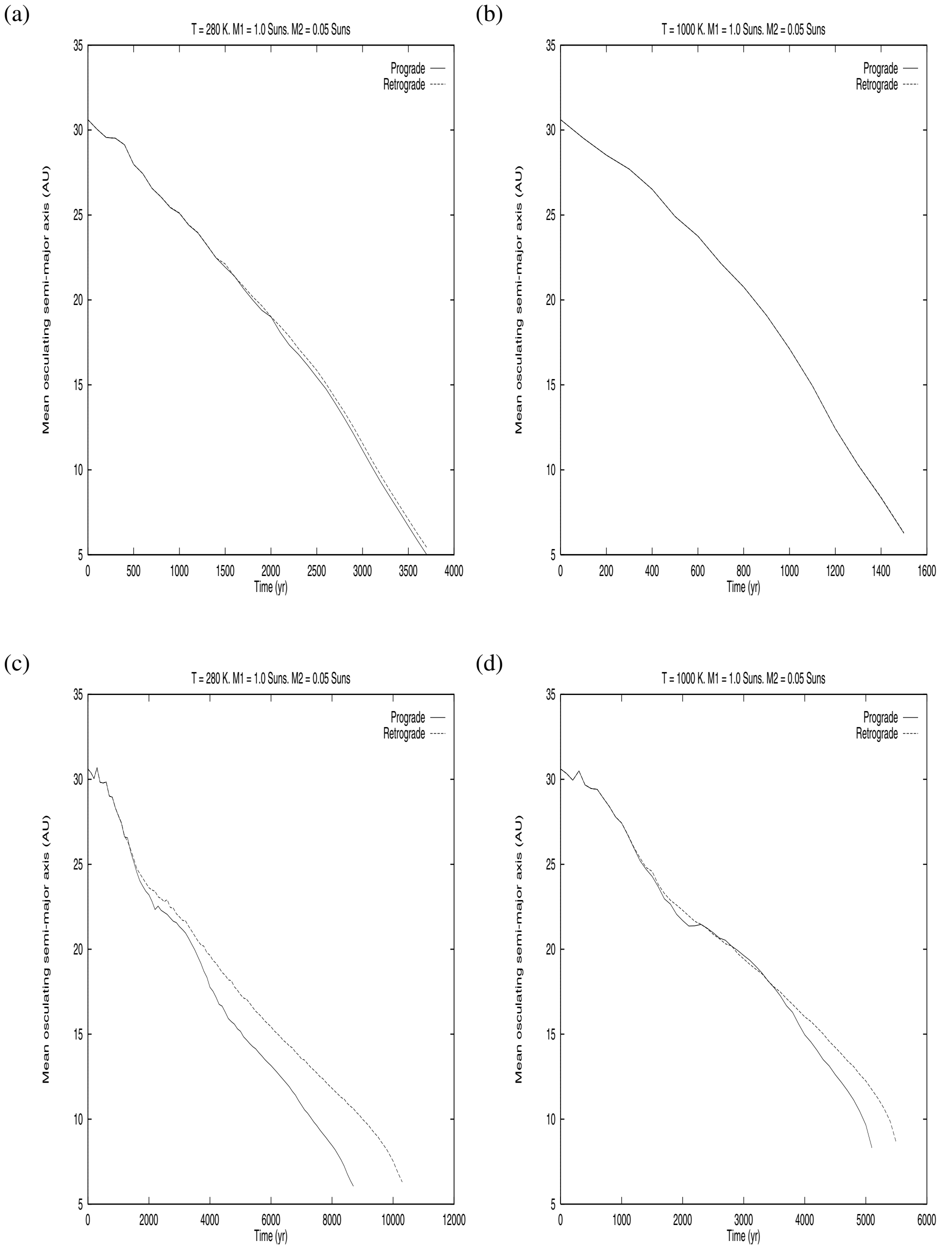

Figure 10. (a) Evolution of the mean osculating semi-major axis for a disk of decimetric particles with a mass of $0.01M_{\odot}$ submitted to the perturbation of a binary companion of $1M_{\odot}$ at 300 AU and an outer companion of $0.05M_{\odot}$ at 700 AU. The temperature of the disk is 280 K at 1 AU. (b) As (a) but with a temperature of 1000 K at 1 AU. (c) As (a) but for metric particles. (d) As (b) but for metric particles.

initial conditions, although our calculations suggest that multiplicity may also play a non-negligible role.

On the other hand, also exo-planets are found in binary or even higher multiplicity systems. As an example, the planet orbiting the star 16 Cygni B is the first planet ever detected in a hierarchical triple system. This star belongs to a particularly interesting system (Cochran et al., 1997; Hauser and Marcy, 1999) about 21.4 pc from Earth. This multiple star consists of a G-dwarf inner binary with a separation of $\sim$ 835 AU, and a distant M-dwarf companion. In spite of the large uncertainties concerning the parameters of the system it can be considered hierarchical. 16 Cygni B is thought about a close match to our Sun in brightness and temperature. No large planets have been detected yet around the other two stars. The companion to 16 Cygni B has a mass that could be as little as 1.6 times that of Jupiter, so it is probably a true planet rather than a brown dwarf. It circles the star every 2.2 years in a highly eccentric orbit (e = 0.67), which means that its distance from the star ranges between 0.56 and 2.83 AU, averaging about 1.67. Mechanisms for generating giant planets in highly eccentric orbits have been considered recently (Artymowicz, 1993, 1997; Rasio and Ford, 1996; Holman et al., 1997; Katz, 1997; Lin and Ida, 1997; Mazeh et al., 1997; de la Fuente Marcos and de la Fuente Marcos, 1997, 1998b, 2000; Laughlin and Adams, 1998; Ford et al., 2000). Our results suggest that the dynamics of the solid material in the hypothetical protoplanetary disk around 16 Cygni B was very similar to that of dust particles in our own Solar System.

6. Conclusions

We have studied the dynamics of the solid component of a fully viscous, coupled, two phases protoplanetary disk, in which the gas is pressure supported and the particles are coupled to the gas only by drag forces. In this work the circumstellar nebula is assumed to be in a triple system, in a hierarchical arrangement. In the light of the results from our calculations, hierarchical triple systems seem to be as hostile as binary systems to harbor planets. If triple companions can boost even more the global accretion rate onto the disk-host star relative to its value in binary systems, the mass of the protoplanetary disk will decrease faster. A less massive nebulae increases the lifetime of the solid particles. On the other hand, grain growth is also intensified due to one-armed spiral perturbations generated by the tidal interaction of the dust sub-disk with the companions.

Additional results include that lifetimes for decimetric and metric particles are longer if the orbits of the companion stars are prograde with respect to the disk. Centimetric and smaller particles seem to be mostly unaffected by the presence of a perturber under the approximations considered throughout this work. Therefore the dynamics of dust particles in a circumstellar disk in a hierarchical triple system is not very different from that described in Paper I for binaries. Totally different is the case of a circumbinary or circumtriple system. This case will be analysed in the following paper of this series.

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